Stark Broadening of Spectral Lines by High-Velocity Charged Particles*

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The broadening of the Lyman α line by high-velocity charged particles is calculated in the classical path approximation without the completed-collision assumption. For noninteracting perturbers, the divergence at large impact parameters associated with usual impact theories does not arise. Interactions between the perturbers are introduced by the pair correlation function. The resulting line shape is valid for frequencies larger than those permitted by the impact theory.

INTRODUCTION

THE usual impact theories of the broadening of spectral lines by independent perturbers (in the classical path approximation) lead to a result which diverges logarithmically for large impact parameters when the radiating atom undergoes a linear Stark effect.[1,2] It is customary to avoid this divergence by cutting off the range of the force at the Debye length[2] λ or to use a shielded Coulomb potential[1]; the motivation being that the interactions between the perturbers produce a shielding effect. However, a shielded Coulomb potential is a time-average effect whereas the broadening is due to the fluctuations of the field on the radiating atom. Therefore the problem arises as to whether the use of a shielded Coulomb potential is adequate for computing the broadening of spectral lines in the impact approximation.[3]

The source of the divergence rests in the assumption that any collision that has its time of closest approach in a certain time interval is completed in that time interval.[4] This falsifies the distant collisions since the duration of a collision increases with increasing distance of closest approach. Although the distant collisions have a small effect on the radiating atom, there are very many of them, and if one assumes that they are completed in too short a time, one overestimates their effect and thereby produces the divergence.

The purpose of this paper is to show that, by avoiding the completed-collision assumption, the broadening of the line can be formulated in such a way that the divergence associated with large impact parameters does not occur for independent perturbers. It will also be shown that the effect of the interactions between perturbers can be treated from first principles, thus avoiding the questionable use of a shielded Coulomb potential.

The divergence occurs in the *weak-collision* contribution to the broadening and we therefore restrict ourselves to it. Since our concern is with making this part of the impact theory rigorous our computed line shapes are just for the weak collisions. Methods already exist[1] to convert this into realistic line shapes that include contributions from strong collisions and ion effects.

To carry out this program we take, as a simple example exhibiting the linear Stark effect, the Lyman α line. We assume that the temperature and density are such that the *impact approximation*[1,5] is valid, i.e., that there is only one strong collision at a time.

Our results justify the use of the joint assumption of completed collisions and cutoff for frequencies (measured from the line center) small compared with (v/λ), and give a new result for frequencies of the order (v/λ) and larger.

I

The broadening of spectral lines rests on the evaluation of the average of the time development operator $\langle U(t)\rangle$ in the interaction representation. In the notation of Baranger,[5] $U(t)$ is given by

$$U(t)=1+\sum_{n=1}^{\infty}(-i)^n\int_0^t dt_1\int_0^{t_1}dt_2\cdots\int_0^{t_{n-1}}dt_n\,V'(t_1)\cdots V'(t_n). \quad (1)$$

In Eq. (1) we have set $\hbar=1$ and the matrix $V'(t)$ is related to the interaction matrix $V(t)$ between the radiator and *all* the perturbers by

$$V'(t)=e^{iH_0t}V(t)e^{-iH_0t}. \quad (2)$$

We assume that the interaction $V(t)$ causes transition only between the four degenerate components of the state $n=2$. When the base functions,

$$\varphi_1=\psi_{200},\quad \varphi_2=\psi_{211},\quad \varphi_3=\psi_{21-1},\quad \varphi_4=\psi_{210},$$

are used, the matrix V' (i.e., that part connecting these four states) can be written as

$$V'=\tfrac{3}{2}ea_0\Gamma, \quad (3)$$

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[1] H. R. Griem, A. C. Kolb, and K. Y. Shen, Phys. Rev. **116**, 4 (1959).
[2] H. Margenau and M. Lewis, Revs. Modern Phys. **31**, 569 (1959).
[3] The problem of including interactions between perturbers in the static (statistical) theory has recently been considered by M. Baranger and B. Mozer, Phys. Rev. **115**, 521 (1959).
[4] This assumption is also responsible for the divergence in the Boltzmann equation.
[5] M. Baranger, Phys. Rev. **111**, 494 (1958).

where the nonzero elements of Γ are

$$\Gamma_{12}=\Gamma_{31}=\Gamma_{13}{}^{*}=\Gamma_{21}{}^{*}=\sqrt{2}(E_x+iE_y),$$
$$\Gamma_{14}=\Gamma_{41}=2E_z,$$

and a_0 is the radius of the first Bohr orbit. E_x, E_y, E_z are the components of the electric field (assumed uniform over the atom) at the radiating atom due to *all* the perturbers. Performing an average over all possible time sequences, we have

$$\langle U(t)\rangle=1-\frac{9a_0{}^2e^2}{4}\int_0^t dt_1\int_0^{t_1} dt_2\langle\Gamma(t_1)\Gamma(t_2)\rangle+\cdots, \quad (4)$$

since $\langle E_x\rangle=\langle E_y\rangle=\langle E_z\rangle=0$. *If the perturbers are independent of each other*, it can be shown that

$$\langle\Gamma(t_1)\Gamma(t_2)\rangle=4NP\langle\mathcal{E}_z(t_1)\mathcal{E}_z(t_2)\rangle, \quad (5)$$

where the nonzero matrix elements of P are $P_{11}=3$, $P_{22}=P_{33}=P_{44}=1$; $\mathcal{E}_z$ is the z component of the electric field from *one* perturber and N is the total number of perturbers. The higher (nonzero) terms in Eq. (5) do not reduce to anything simple and we shall restrict the development to the terms exhibited in Eq. (4). To this approximation we have

$$\langle U(t)\rangle\approx 1-9e^2a_0{}^2NP\int_0^t dt_1\int_0^{t_1} dt_2\langle\mathcal{E}_z(t_1)\mathcal{E}_z(t_2)\rangle. \quad (6)$$

The neglected terms are necessary for a correct description of the strong collisions and the long-time behavior of the weak collisions. Strong and weak collisions can be given an approximate meaning in the following way. Consider the average effect of a complete collision of one perturber with speed v and distance of closest approach ρ, the average being taken over directions of perturber motion. As we shall see in the next section, there exists a distance $\rho_c=3e^2a_0v^{-1}$ such that the average effect of a collision with $\rho\gtrsim\rho_c$ produces only a small change in the state of the radiating atom. Strong and weak collisions are then those that have $\rho<\rho_c$ and $\rho\gtrsim\rho_c$, respectively. We assume here that the velocity of the perturbers is high enough so that the impact approximation is valid, i.e., that there is only one strong collision at a time. Since our interest is in the weak collisions we shall neglect the effects of the strong collisions. Methods already exist for incorporating the strong collisions, and the conditions are known for when they are negligible.[1]

The neglect of higher terms restricts $\langle U(t)\rangle$ to short times, but this is adequate in a discussion of the wings of the line and, under certain conditions, the core can be obtained by repeated application of Eq. (6) to successive time intervals.[5]

In Sec. II we shall show how, with the completed-collision assumption, the usual weak-collision result evolves from Eq. (6) and shall obtain ρ_c. In Sec. III we shall obtain the broadening without the assumption of completed collisions and cutoff.

II

The integral in Eq. (6) can be written as

$$\frac{1}{2}\left\langle\left(\int_0^t dt'\ \mathcal{E}_z(t')\right)^2\right\rangle, \quad (7)$$

and since the perturber moves on a straight line

$$\mathcal{E}_z(t)=-e[\rho_z+v_z(t-t_i)]r_i{}^{-3},$$
$$r_i{}^2=\rho^2+v^2(t-t_i)^2. \quad (8)$$

Here ρ is the distance of closest approach, t_i is the time of closest approach, and subscript z means z component. The integral in Eq. (7) becomes

$$\int_0^t dt'\ \mathcal{E}_z(t')=-e\rho_z\alpha_i(t)-ev_z\beta_i(t),$$

$$\alpha_i\equiv\int_0^t dt'\ r_i{}^{-3}(t');\quad \beta_i(t)\equiv\int_0^t dt'(t'-t_i)r_i{}^{-3}(t').$$

In the completed-collision assumption one considers the impact to be completed either inside or outside the time interval $(0,t)$ depending on whether t_i is inside or outside the interval. This means that we take

$$\alpha_i=2/\rho^2v,\quad \beta_i=0 \quad \text{if} \quad 0\leqslant t_i\leqslant t,$$
$$\alpha_i=\beta_i=0 \quad \text{if} \quad t_i<0 \quad \text{or} \quad t_i>t.$$

Therefore, if t_i is within $(0,t)$ we have

$$\frac{1}{2}\left\langle\left(\int_0^t dt'\ \mathcal{E}_z(t')\right)^2\right\rangle_{\theta,\rho}=\frac{2e^2}{3v^2}\int_{\rho c}^{\rho m}\rho^{-2}W(\rho)d\rho, \quad (9)$$
$$W(\rho)\equiv 2\pi\rho/\pi\rho_m{}^2.$$

the subscripts θ,ρ indicating an average over angles and impact parameter ρ, and ρ_m being an upper limit. Equation (9) must be averaged over times of closest approach. If ν denotes the collision frequency, then $\nu t/N$ is the probability of a given perturber having a collision in the time interval $(0,t)$. We then have, using $\nu=\pi\rho_m{}^2nv$,

$$\langle U(t)\rangle\approx 1-(12\pi ne^4a_0{}^2Ptv^{-1})\ln(\rho_m/\rho_c), \quad (10)$$

where n is the number density. The dependence on ρ_m leads to a divergence for Coulomb forces since the limit $\rho_m\to\infty$ should be taken for independent perturbers. The appearance of ρ_c is associated with the neglect of strong collisions.

If ρ_m is kept finite and the second term of Eq. (10) is small compared to unity at a time equal to the correlation time of the field, then $\langle U(t)\rangle$ can be extended to longer times by using Eq. (10) for successive time intervals. The result is

$$\langle U(t)\rangle\approx\exp[-(12\pi ne^4a_0{}^2Ptv^{-1})\ln(\rho_m/\rho_c)]. \quad (11)$$

The first two terms of Eq. (11) are just those of Eq. (10).

We now select a suitable value for ρ_c. The deviation from unity of the time development operator for a single complete collision with impact parameter ρ is

$$6e^4a_0^2P/v^2\rho^2.$$

The critical impact radius ρ_c will be defined as the value of ρ that makes this term of the order of unity,[6] that is

$$\rho_c=3e^2a_0^2/v.$$

III

We return to the evaluation of Eq. (6) without the completed collision and cutoff assumptions. This is accomplished as follows: We first evaluate $\langle\mathcal{E}(t_1)\cdot\mathcal{E}(t_2)\rangle$ and note that this is just $3\langle\mathcal{E}_z(t_1)\mathcal{E}_z(t_2)\rangle$. The evaluation of $\langle\mathcal{E}(t_1)\cdot\mathcal{E}(t_2)\rangle$ involves two steps: One must calculate (a) the probability that the particle is at $\mathbf{r}$ at time t_2 and (b) the probability that at t_1 $(t_1\geqslant t_2)$ the particle is at some point on the sphere of radius $v(t_1-t_2)$. We must also eliminate the contribution from the strong collisions. It is convenient to modify, in an unessential way, the meaning of strong and weak collisions. In Sec. II the weak-collision contribution was obtained by averaging the effects of a completed collision over impact parameters ranging from ρ_c to ρ_m. However, in a completed collision with $\rho<\rho_c$, the contribution from the segments of the path that lie *outside* the sphere of radius ρ_c, is smaller than the total contribution of a completed collision with impact parameter ρ_c. It is therefore consistent with the spirit of weak effects to include these segments. The consequence of this modification on the results of Sec. II is to replace, in Eqs. (10) and (11), the factor $\ln(\rho_m/\rho_c)$ by $\{\ln(\rho_m/\rho_c)+\frac{3}{2}-2\ln 2\}$.

With this modification, $\langle\mathcal{E}(t_1)\cdot\mathcal{E}(t_2)\rangle$ for weak effects, is given by

$$\begin{aligned}&\frac{3e^2}{R^3}\left[\frac{1}{\rho_c}-\frac{v(t_1-t_2)}{4\rho_c^2}\right]\quad\text{for}\quad(t_1-t_2)\leqslant 2\rho_c/v,\\&\frac{3e^2}{R^3}\left[\frac{1}{v(t_1-t_2)}\right]\quad\text{for}\quad(t_1-t_2)\geqslant 2\rho_c/v.\end{aligned}\tag{12}$$

The entire system has been taken to be a sphere of radius R with the radiator at the center, and we have assumed $v(t_1-t_2)\ll R$; we shall eventually let $R\to\infty$. Equation (6) then becomes, upon defining $n=N/(\frac{4}{3}\pi R^3)$,

$$\begin{aligned}&\langle U(t)\rangle\approx 1-12e^4\pi na_0^2PI(t),\\&I(t)=\left[\frac{t}{2v}+\frac{t}{v}\ln\left(\frac{tv}{2\rho_c}\right)+\frac{2\rho_c}{3v^2}\right]\quad\text{for}\quad t\geqslant 2\rho_c/v\\&\qquad=\left[\frac{t^2}{2\rho_c}-\frac{vt^3}{24\rho_c^2}\right]\quad\text{for}\quad t\leqslant 2\rho_c/v.\end{aligned}\tag{13}$$

[6] The definitions of ρ_c and results, Eq. (11), are essentially the same as in reference 1 except that, for simplicity, we assume the perturbers to have the same speed.

It is interesting to compare Eq. (13) with the usual result, Eq. (10). Equation (13) does not have an upper cutoff [ρ_m of Eq. (10)], i.e., the radiating atom is interacting with *all* perturbers in an infinite volume (except those within a sphere of radius ρ_c).

The results, Eq. (13), are for independent perturbers, *If the perturbers are not assumed to be independent of each other*, Eq. (4) can be written as

$$\langle U(t)\rangle\approx 1-3a_0^2e^2P\int_0^t dt_1\int_0^{t_1}dt_2\langle\mathbf{E}(t_1)\cdot\mathbf{E}(t_2)\rangle.\tag{14}$$

In the Appendix it is shown that, for times s smaller than the mean free time τ,

$$\langle\mathbf{E}(t_1)\cdot\mathbf{E}(t_2)\rangle=4\pi ne^2\frac{e^{-vs/\lambda}}{vs};\quad\frac{2\rho_c}{v}\ll s\ll\tau,\tag{15}$$

where $s=t_1-t_2$ and λ is the Debye length,

$$\lambda=(kT/4\pi ne^2)^{\frac{1}{2}}.$$

For short and long times, Eqs. (14) and (15) give

$$\langle U(t)\rangle-1\approx-\frac{KPt}{v}\left[\tfrac{1}{2}+\ln(tv/2\rho_c)\right],\quad\frac{2\rho_c}{v}<t\ll\frac{\lambda}{v}\tag{16a}$$

$$\approx-\frac{KPt}{v}\left[\tfrac{3}{2}-\gamma+\ln(\lambda/2\rho_c)\right],\quad\frac{\lambda}{v}\ll t\ll\tau;\tag{16b}$$

where $K\equiv 12e^4\pi na_0^2$, and $\gamma=$ Euler's constant. Equation (16a) agrees with Eq. (13), and Eq. (16b) essentially with Eq. (10). That is, the effect of the interactions reduces the time interval over which the noninteracting result is valid and justifies the usual approach for times large compared with (λ/v), the correlation time of the field.

The line shape $F(\omega')$ can be computed from[5]

$$F(\omega')=\frac{1}{\pi}\operatorname{Re}\int_0^\infty e^{i\omega' s}\operatorname{Tr}[e^{-iH_0s}\langle U(s)\rangle\rho D]ds,\tag{17}$$

where ρ is the density matrix and $D=\mathbf{d}|\psi_f\rangle\langle\psi_f|\mathbf{d}$; $\mathbf{d}$ is the electric dipole moment vector and ψ_f is the ground-state wave function. The ground-state energy has been set equal to zero. Assuming ρ to be diagonal, and using Eqs. (14), (15), and (17), $F(\omega')$ is

$$\begin{aligned}F(\omega')\approx&\frac{1}{\pi}\sum_i\rho_{ii}D_{ii}P_{ii}K\\&\times\left\{\frac{1}{\omega^2v}\left[\tfrac{3}{2}-\gamma-\ln\left(\frac{2\rho_cq^{\frac{1}{2}}}{v}\right)\right]+\frac{2}{vq}\right\},\\&\omega_L\ll\omega\ll v/2\rho_c,\end{aligned}\tag{18}$$

where

$$q\equiv\omega^2+(v/\lambda)^2;\quad\omega_L\equiv(K/v)\ln(\lambda/2\rho_c);\quad\omega\equiv|\omega'-E_0|.$$

The lower frequency limit ω_L arises from the truncated form of the time development operator upon which the development has been made.

For frequencies $\omega_L \ll \omega \ll (v/\lambda)$, the term within curly brackets in Eq. (18) reduces to

$$\frac{1}{\omega^2 v}\left[\tfrac{3}{2}-\gamma+\ln(\lambda/2\rho_c)\right], \tag{19}$$

which is essentially the result (in the wings) of the usual theory. For frequencies $(v/\lambda) \ll \omega \ll (v/2\rho_c)$, this term becomes

$$\frac{1}{\omega^2 v}\left[\tfrac{7}{2}-\gamma+\ln(v/2\rho_c\omega)\right], \tag{20}$$

which differs essentially from Eq. (19) by the presence of the frequency in the logarithmic term.

Our results show that for frequencies smaller than (v/λ) the results of the usual theory are valid, i.e., the line is a Lorentz shape for frequencies smaller than (v/λ). For frequencies on the order of (v/λ) and greater, the line shape is no longer of a Lorentz shape; the shape being given by Eq. (18) which for frequencies larger than (v/λ) simplifies to Eq. (20). The method presented here is of course applicable to all Stark broadening problems when the perturbers are sufficiently fast. We have chosen the case of the Lyman α line, perturbed by one kind of particle of a *single speed*, since this example is general enough to exhibit the particular problems we have discussed.

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APPENDIX

The ensemble average of the product of $\mathbf{E}(t_1)$, the total electric field at a point at the time t_1, and $\mathbf{E}(t_2)$, the total electric field at the same point at the time t_2, is given by

$$\langle \mathbf{E}(t_1)\cdot\mathbf{E}(t_2)\rangle = \sum_{i,j}\int W(\mathbf{R}_i' t_1;\mathbf{R}_j t_2)\,\mathcal{E}(\mathbf{R}_i') \cdot \mathcal{E}(\mathbf{R}_j)d\mathbf{R}_i' d\mathbf{R}_j, \tag{1a}$$

where $\mathcal{E}(\mathbf{R}_j)$ is the field at the point produced by the jth particle that is located at $\mathbf{R}_j$ and $W(\mathbf{R}_i' t_1;\mathbf{R}_j t_2)$ is the probability of finding the ith particle at $\mathbf{R}_i'$ at t_1, and the jth particle at $\mathbf{R}_j$ at t_2.

In general

$$W(\mathbf{R}_i' t_1;\mathbf{R}_i t_2) = W(\mathbf{R}_i' t_1)K(\mathbf{R}_i' t_1|\mathbf{R}_i t_2), \tag{2a}$$

$$W(\mathbf{R}_i' t_1;\mathbf{R}_j t_2) = \int W(\mathbf{R}_i' t_1;\mathbf{R}_j'' t_1) \times K(\mathbf{R}_j'' t_1|\mathbf{R}_j t_2)d\mathbf{R}_j'', \tag{3a}$$

where K is a conditional probability. In thermal equilibrium we have

$$\text{(a)}\quad W(\mathbf{R}_i', t_1) = V^{-1},$$
$$\text{(b)}\quad W(\mathbf{R}_i' t_1;\mathbf{R}_j'' t_1) = V^{-2}C(r);\quad r=|\mathbf{R}_i'-\mathbf{R}_j''|,$$
$$\text{(c)}\quad K(\mathbf{R}_i' t_1|\mathbf{R}_i t_2) = K(\mathbf{R}_i'|\mathbf{R}_i s);\quad s=t_2-t_1.$$

Using these relations along with Eq. (2a) and Eq. (3a), Eq. (1a) can be written as

$$\begin{aligned}\langle \mathbf{E}(t_1)\cdot\mathbf{E}(t_1+s)\rangle &= V^{-1}\sum_i \int K(\mathbf{R}_i'|\mathbf{R}_i s)\mathcal{E}(\mathbf{R}_i')\cdot\mathcal{E}(\mathbf{R}_i)d\mathbf{R}_i' d\mathbf{R}_i \\ &\quad + V^{-1}\sum_{i,j,i\neq j}\int K(\mathbf{R}_j''|\mathbf{R}_j s)\mathcal{E}(\mathbf{R}_j) \cdot \mathbf{I}(\mathbf{R}_j'')d\mathbf{R}_j d\mathbf{R}_j'', \\ \mathbf{I} &\equiv V^{-1}\int C(r)\mathcal{E}(\mathbf{R}_i')d\mathbf{R}_i'.\end{aligned} \tag{4a}$$

The effect of the interaction between the perturbers is contained in the terms K and C. For the noninteracting case, only the first term contributes since $C=1$ and $\mathbf{I}$ is then zero. For the weakly interacting case, i.e., high temperatures and low densities, we may approximate Eq. (4a) as follows:

(a) We assume that the weak interaction does not materially alter K from its noninteracting value for times s that are not too long. The first term of Eq. (4a) then gives the result obtained in Sec. III [Eq. (12)]. For $s \gtrsim 2\rho_c/v$, it is

$$4\pi n e^2(1/vs). \tag{5a}$$

(b) For $C(r)$ we use the only known approximation,[7]

$$C(r)=\exp[-e^2\alpha/kT];\quad \alpha=e^{-r/\lambda}/r, \tag{6a}$$

where λ is the Debye length.

The second term of Eq. (4a) is then

$$-\frac{4\pi n e^2}{vs}[1-e^{-vs/\lambda}], \tag{7a}$$

and finally

$$\langle \mathbf{E}(t_1)\cdot\mathbf{E}(t_1+s)\rangle = 4\pi n e^2 e^{-vs/\lambda}/vs;\quad s\gtrsim 2\rho_c/v. \tag{8a}$$

Equation (8a) is valid for times s that are short enough so that the perturber motion is essentially that of a free particle. If we define the mean free time τ as the time for which the perturber changes its velocity by

[7] L. D. Landau and E. M. Lifshitz, *Statistical Physics* (Addison Wesley Publishing Company, Inc., Reading, Massachusetts, 1958), Sec. 74.

an amount equal to its velocity, i.e., by the condition

$$\langle(\Delta\mathbf{v})^2\rangle=\langle(\mathbf{v})^2\rangle,$$

where

$$\Delta\mathbf{v}\equiv e/m\int_0^\tau \mathbf{E}(t_1)dt_1, \tag{9a}$$

then the restriction on s is $s\ll\tau$. We can estimate the order of magnitude of τ by using Eq. (8a) in Eq. (9a) even though $\mathbf{E}$, the field produced by the perturbers in Eq. (8a), is at a fixed point in space whereas $\mathbf{E}$ in Eq. (9a) is the field at the moving perturber. The result is

$$\tau=\left(\frac{\lambda}{v}\right)\left(\frac{\lambda}{d}\right)^3\frac{36\pi}{\ln(\lambda/a)}, \tag{10a}$$

where $d=n^{-\frac{1}{3}}$ and a is a lower cutoff distance [replacing ρ_c in Eq. (8a)] that can be taken as the mean distance of closest approach. Since τ is much greater than (λ/v), it is possible to have s large enough to guarantee Eq. (16b).

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Recombination of Ions and Electrons

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A process of electron-ion recombination is considered, involving three bodies (one ion and two electrons), in which an electron, as a result of a collision with another electron, loses enough energy to be captured in one of the excited electronic orbits of the ion and then ends in the ground state by emission of one or more light quanta. It is shown that such a process might account for the large values of the recombination coefficient found experimentally.

I. INTRODUCTION

RADIATIVE recombination is the process in which an electron comes within a small distance of a positive ion and is captured in one of the low-lying electronic orbits, with the emission of a light quantum. Quantum mechanical calculations [1,2] on such a process predict recombination coefficients of the order of 10^{-12} cm^3/sec. Experimentally, recombination in many gases has been studied and in all those cases in which it is almost certain to occur between positive ions and electrons, recombination coefficients of the order of 10^{-10} cm^3/sec have been found.[3–5] So far none of the many processes considered to eliminate such a discrepancy seems to account for the large recombination coefficients found experimentally. "Neither dissociative recombination nor effects due to negative ions are likely to be important in the gases investigated."[6]

The purpose of this paper is to show that there is a process of electron-ion recombination which might account for the values of the recombination coefficients found experimentally. Consider a fully ionized gas, consisting entirely of singly charged ions and electrons. A process is possible, involving three bodies (one ion and two electrons), in which an electron, as a result of a collision with another electron, loses enough energy to be captured in one of the excited electronic states of the ion and then ends in the ground state by emission of one or more light quanta. This process is by no means new and has been considered implicitly, for instance, in the study of stellar atmospheres.[7] It appears, however, to have been somehow overlooked in the explanation of any one of the recombination experiments mentioned above.

The calculations presented here are for the case of a fully ionized hydrogen gas. The conclusions arrived at might be expected to be at least qualitatively valid also for other atomic gases.

II. THEORY

Consider a fully ionized hydrogen gas. As a result of a collision between two electrons, one of them may lose enough of its kinetic energy to be captured in a close orbit around an ion (say in a state of total quantum number n and orbital angular momentum l). Once the electron is bound, either of two processes can occur: (a) The electron is re-ejected into the continuum by collision with another electron, or (b) the electron makes a radiative transition to a lower level, from which it can still be re-ejected into the continuum or make another radiative transition. Recombination will be

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