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Weak-Coupling Currents and Symmetries of Strong Interactions

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The general isotopic properties of bilinear currents which will lead to the $|\Delta S|\leq 1$ and $|\Delta I|=\frac{1}{2}$ rules for weak decay processes are examined. The latter rule is re-expressed in terms of an equivalent mathematical statement which permits one to obtain the usual predictions in a simple manner. In general, when the strangeness-conserving part of such a current is an isotopic vector, the strangeness-changing part can be a linear combination of $I=\frac{1}{2}$ and $I=\frac{3}{2}$ currents. The existence of an $I=\frac{3}{2}$ current could be established by experiments on the decays $K\rightarrow\pi+$leptons, or on high-energy neutrino capture, $\nu+N\rightarrow\mu+\Sigma$. Experiments on K_{e4} decays could test the bilinearity of the current.

The assumption that the vector part of such a current, both strangeness changing and nonchanging, is quasi-conserved (i.e., neglecting certain mass differences) in the presence of the strong interactions fixes the specific form of the current and further implies symmetries for the strong couplings. The various transformations which leave invariant a Yukawa-type strong interaction as well as their associated currents are found. A new possible symmetry group of the strong interactions is examined: a 14 parameter group usually denoted as $G2$. In the presence of both π and K couplings, it is found that $I=\frac{1}{2}$ and $\frac{3}{2}$ currents are quasi-conserved when the strong Lagrangian has a 7-dimensional rotational symmetry, while for the $I=\frac{1}{2}$ alone, the symmetry required is $G2$. In the presence of only π-baryon couplings, only $I=\frac{1}{2}$ currents can be quasi-conserved. Certain predictions for the K_{e3} and K_{e4} modes of decay and for $\Sigma^-\rightarrow n+e^-+\nu$ follow from the weak currents determined in this way.

INTRODUCTION

In recent years, it has become increasingly evident that there seems to exist some deep underlying connection between the strong interactions of elementary particles and their weak decay interactions. This has been evidenced in weak decays by the apparent lack of renormalization of the vector part of the β-decay interaction on the one hand, and on the other by the success of the $|\Delta I|=\frac{1}{2}$ rule. It is the purpose of this paper to investigate, within the framework of conventional theory, these possible connections between the strong and weak interactions.

To this end, the first section is an examination of the isotopic character of the currents which could give rise to weak decays. The $|\Delta I|=\frac{1}{2}$ rule is expressed in a mathematically equivalent way. In the framework of a current-current type Lagrangian which satisfies $|\Delta S|\leq 1$, and the $|\Delta I|=\frac{1}{2}$ rule, the most general bilinear current is established. The strangeness-changing part is found to be a linear combination of an $I=\frac{1}{2}$ and an $I=\frac{3}{2}$ current. By introducing the leptons in a phenomenological way, it is possible to make certain experimental predictions on the K_{e3} and $K_{\mu 3}$ decays. Further predictions, which depend essentially only upon charge independence of the strong interactions and a bilinear character of the current of strongly interacting particles, are made for the K_{e4} mode of decay. At this point, we discuss several features of an hypothetical vector meson responsible for weak decay processes.

In order to further specify these currents, the assumption is made that they should be quasi-conserved in the presence of strong interactions. In the second section, it is demonstrated how this requirement not only specifies the current but also a symmetry of the strong interactions.

In the third section all the bilinear currents and associated symmetries are found which are allowed by a Yukawa-type strong-interaction Lagrangian in which both K and π couplings are present and also in which only π couplings are present. In the former case, a new possible symmetry of the strong interactions is found which is a 14-parameter group, usually denoted by $G2$.

The fourth section is devoted to the experimental predictions which these symmetries imply for processes involving only strong or strong and electromagnetic interactions.

The experimental predictions for weak decay processes arising from these quasi-conserved currents is then discussed in the fifth section.

I. WEAK INTERACTIONS—ISOTOPIC SPIN CHARACTER

Up to the present, there have been two general points of view with regard to the fundamental Lagrangian responsible for the interactions between elementary particles. An extreme version of one of these considers as basic the four-fermion interaction. From this point of view, the weak interaction between four fermions is of the basic form while the strong and electromagnetic interactions are viewed as, in some sense, being phenomenological descriptions of more fundamental four-fermion interactions. In comparison,

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the other extreme point of view stresses the concept of an interaction between a current and a boson field. Here, the electromagnetic interaction is fundamental while the strong and weak interactions are, in some sense, phenomenological descriptions of the more basic current-boson type. Recently, with the experimental verification of the V-A coupling for the weak interaction, it has become theoretically appealing to consider the basic weak coupling to be of the current-boson type and the four-fermion decay processes to be phenomenological descriptions of this more fundamental coupling. This analogy with electrodynamics, in fact, has even been extended to the point of considering this weak current to be conserved, leading in a natural fashion to the experimentally observed absence of renormalization for the vector part in β decay.[1,2] In the present paper, we adopt the view that (i) *the Lagrangian responsible for weak decays is of the current-boson or current-current type.*

It has been suggested that the weak-interaction Lagrangian is of the form

$$\mathcal{L}=(G/\sqrt{2})\mathcal{J}_\mu{}^\dagger\mathcal{J}^\mu, \tag{1}$$

where G is the weak-coupling constant given in reference 1, and $\mathcal{J}_\mu$ is a single charged current which transforms in space-time as a linear combination of a vector and an axial vector.[1]

On the other hand, there appear to be two selection rules which are rather well satisfied experimentally, namely, to order G, (ii) $|\Delta S|\leq 1$ *in all decays,* and (iii) *in decays involving only strongly interacting particles and in which strangeness changes by one unit, the isotopic spin obeys* $|\Delta I|=\frac{1}{2}$. As has been previously noted, in a current-current type interaction a single charged current cannot lead to a $|\Delta I|=\frac{1}{2}$ rule.[3,4] It would seem necessary, therefore, that this current be generalized in order that rules (ii) and (iii) might be satisfied.

A rather natural generalization is to regard $\mathcal{J}_\mu$ as having several charge components, $\mathcal{J}_\mu{}^{(i)}$ (where i represents the various charge states). Thus, instead of Eq. (1), we have

$$\mathcal{L}=(G/\sqrt{2})\textstyle\sum_i \mathcal{J}_\mu{}^{(i)\dagger}\mathcal{J}^{\mu(i)}. \tag{2a}$$

Of course, if the process occurs via an intermediate vector meson, $W_\mu{}^{(i)}$, the basic interaction would be[5]

$$\mathcal{L}=g\textstyle\sum_i \mathcal{J}^{\mu(i)}W_\mu{}^{(i)}+\text{H.c.} \tag{2b}$$

By taking into account requirements (ii) and noting the existence of leptonic modes of decay, $\mathcal{J}_\mu{}^{(i)}$ may be written as

$$\mathcal{J}_\mu{}^{(i)}=J_\mu{}^{(i)}+l_\mu{}^{(i)}=j_\mu{}^{(i)}+s_\mu{}^{(i)}+l_\mu{}^{(i)}, \tag{3}$$

where the currents of the strongly interacting particles, $j_\mu{}^{(i)}$ and $s_\mu{}^{(i)}$, carry zero and one unit of strangeness, respectively, and $l_\mu{}^{(i)}$ is the leptonic current. It is clear that, in order to satisfy (ii) for the terms $\sum_i s_\mu{}^{(i)\dagger}s^{\mu(i)}$, the $s_\mu{}^{(i)}$ can contain only terms in which the change in strangeness is solely of one sign.[6] This statement replaces the rule $\Delta Q=\Delta S=+1$ for charged currents. By convention, we choose $s_\mu{}^{(i)}$ to have $\Delta S=1$ (then $s_\mu{}^{(i)\dagger}$ corresponds to $\Delta S=-1$). If we restrict ourselves to the usual bilinear expressions, then the $j_\mu{}^{(i)}$ transform, in general, as linear combinations of the components of isotopic scalars, $j_\mu{}^{(0,0)}$, vectors, $j_\mu{}^{(1,m)}$, and a symmetric second rank tensor, $j_\mu{}^{(2,m)}$. Similarly, the $s_\mu{}^{(i)}$ transform as linear combinations of the components of isotopic spin $\frac{1}{2}$, $s_\mu{}^{(\frac{1}{2},m)}$, and $\frac{3}{2}$, $s_\mu{}^{(\frac{3}{2},m)}$ tensors. The existence of $j_\mu{}^{(1,m)}$ and $s_\mu{}^{(\frac{1}{2},m)}$ is necessary in order to describe the process $n\rightarrow p+e^-+\bar{\nu}_1$ and $K^+\rightarrow e^++\nu_1$, respectively. It follows that the components $j_\mu{}^{(2,m)}$ cannot exist in the present scheme because they, in conjunction with $s_\mu{}^{(\frac{1}{2},m)}$, would give rise to terms with $|\Delta S|=1$ and $|\Delta I|=\frac{3}{2}$, in contradiction with (iii). The possible isoscalar contribution to $j_\mu{}^{(i)}$, i.e., $j_\mu{}^{(0,0)}$, by the same token, can only exist if these $s_\mu{}^{(\frac{3}{2},m)}$ currents are not present.

The components of the total current, $J_\mu{}^{(i)}$, may now easily be written down.

$$\begin{aligned}
J_\mu{}^{(+2)}&=\beta_0 s_\mu{}^{(\frac{3}{2},\frac{3}{2})},\\
J_\mu{}^{(+1)}&=\rho_1 j_\mu{}^{(1,1)}+\alpha_1 s_\mu{}^{(\frac{1}{2},\frac{1}{2})}+\beta_1 s_\mu{}^{(\frac{3}{2},\frac{1}{2})},\\
J_\mu{}^{(0)}&=\rho_2 j_\mu{}^{(1,0)}+\alpha_2 s_\mu{}^{(\frac{1}{2},-\frac{1}{2})}+\beta_2 s_\mu{}^{(\frac{3}{2},-\frac{1}{2})}+\eta j_\mu{}^{(0,0)},\\
J_\mu{}^{(-1)}&=\rho_3 j_\mu{}^{(1,-1)}+\beta_3 s_\mu{}^{(\frac{3}{2},-\frac{3}{2})},
\end{aligned} \tag{4}$$

where the α_i, β_i, and ρ_i, and η are arbitrary real constants with the restriction $\eta\beta_i=0$ and where the upper index on the $J_\mu{}^{(i)}$ is the value, ΔQ, of the charge carried by the current. By our convention, $j_\mu{}^{(1,1)\dagger}=-j_\mu{}^{(1,-1)}$, $j_\mu{}^{(1,0)\dagger}=j_\mu{}^{(1,0)}$, $j_\mu{}^{(0,0)\dagger}=j_\mu{}^{(0,0)}$. In order to further specify the relationship amongst the various α's, β's, and ρ's, we must invoke condition (iii), that is, the $|\Delta I|=\frac{1}{2}$ rule.

Let us digress for a moment in order to express the $|\Delta I|=\frac{1}{2}$ rule in an equivalent mathematical statement which will be useful for later discussions. The following theorem depends only upon the conservation of charge and baryon number, and upon the relation $Q=I_3+Y_3=I_3+\frac{1}{2}(N+S)$. It does not depend upon the hypothesis that the weak interaction involves a current.

Theorem: The selection rule $|\Delta I|=\frac{1}{2}$ is equivalent to the statements

$$(I_-,\mathcal{L}_w{}^S)=0, \tag{5a}$$

$$(I_+,\mathcal{L}_w{}^{S\dagger})=0, \tag{5b}$$

[1] R. P. Feynman and M. Gell-Mann, Phys. Rev. **109**, 193 (1958).

[2] S. S. Gerstein and J. B. Zeldovich, Zhur. Eksp. i Teoret. Fiz. **29**, 698 (1955) [translation: Soviet Phys.—JETP **2**, 576 (1956)].

[3] S. B. Treiman, Nuovo cimento **15**, 916 (1960).

[4] T. D. Lee and C. N. Yang, Phys. Rev. Letters **4**, 307 (1960) and Phys. Rev. (to be published).

[5] The symbol for a particle denotes the operator which annihilates it.

[6] We adopt the convention that for the covariant $(\bar{a},b)$, $\Delta S=S_a-S_b$, $\Delta Q=Q_a-Q_b$, $\Delta T_3=T_{3a}-T_{3b}$, etc.

where $\mathfrak{L}_w{}^S$ is that part of the weak Lagrangian, $\mathfrak{L}_w$, which involves only the strongly interacting particles and for which $\Delta S=+1$. The operators $I_\pm$ are the usual isotopic spin raising and lowering operators.

Proof: Because $\mathfrak{L}_w{}^S$ must conserve charge and baryon number, the third component of isotopic spin of $\mathfrak{L}_w{}^S$ is $I_3=-S/2=-\frac{1}{2}$. If the selection rule $|\Delta I|=\frac{1}{2}$ holds, $\mathfrak{L}_w{}^S$ must transform as an isospinor with $I_3=-\frac{1}{2}$, i.e., as $(\frac{1}{2},-\frac{1}{2})$. Equation (5a) follows immediately because the operation $(I_-,\mathfrak{L}_w{}^S)$ decreases I_3 by one unit. The reciprocal is also true: if Eq. (5a) is satisfied, then the $|\Delta I|=\frac{1}{2}$ rule follows. In fact, if $\mathfrak{L}_w{}^S$ contained terms such as $(\frac{3}{2},-\frac{1}{2})$, $(\frac{5}{2},-\frac{1}{2})$, etc., Eq. (7a) would not be true. Equation (5b) follows, of course, from Eq. (5a). (Since the strong interactions are assumed to be charge independent, the effective matrix elements for a physical process must also transform as the component of an isotopic spinor, even after including the corrections induced by the strong interactions.)

As a matter of practical interest, one can very simply obtain the usual predictions of the $|\Delta I|=\frac{1}{2}$ rule by judicious use of Eqs. (5a) and (5b) without explicit reference to the spurion method (for examples of the method, see Appendix).

Now, let us note that $\mathfrak{L}_w{}^S$, constructed from the currents of Eq. (4) has the form (when $\eta=0$)

$$\mathfrak{L}_w{}^S=(G/\sqrt{2})\textstyle\sum_i j_\mu{}^{(i)\dagger}[J^{\mu(i)}-j^{\mu(i)}]. \tag{6}$$

We now assume that $\mathfrak{L}_w{}^S$ satisfies Eq. (5a), i.e., the $|\Delta I|=\frac{1}{2}$ rule. By remembering the well-known commutation relations for the lowering operators [we use $I_\pm\equiv(I_1\pm iI_2)/\sqrt{2}$]

$$(I_-,Y^{(j,m)})=(1/\sqrt{2})[(j+m)(j-m+1)]^{\frac{1}{2}}Y^{(j,m-1)}, \tag{7}$$

where the $Y^{(j,m)}$ symbolize the various currents $j_\mu{}^{(1,m)}$, $s_\mu{}^{(\frac{1}{2},m)}$, and $s_\mu{}^{(\frac{3}{2},m)}$, it follows that

$$\begin{aligned}(I_-,\mathfrak{L}_w{}^S)=0=\,&j_\mu{}^{(1,0)}[\rho_2(I_-,J^{\mu(0)})-\rho_3J^{\mu(-1)}+dj_\mu{}^{(1,-1)}]\\&-j_\mu{}^{(1,-1)}[\rho_1(I_-,J^{\mu(+1)})-\rho_2J^{\mu(0)}-dj_\mu{}^{(1,0)}],\end{aligned} \tag{8}$$

where $d=\frac{1}{2}(\rho_1{}^2+\rho_3{}^2-2\rho_2{}^2)$. Thus, the commutation rules for the $J_\mu{}^{(i)}$ are found to be

$$\begin{aligned}(I_-,J_\mu{}^{(0)})&=\frac{\rho_3}{\rho_2}J_\mu{}^{(-1)}+\frac{c-d}{\rho_2}j_\mu{}^{(1,-1)};\\(I_-,J_\mu{}^{(+1)})&=\frac{\rho_2}{\rho_1}J_\mu{}^{(0)}+\frac{c+d}{\rho_1}j_\mu{}^{(1,0)};\\(I_-,J_\mu{}^{(-1)})&=0,\end{aligned} \tag{9}$$

where c is an arbitrary constant. By explicit use of Eq. (9), we are led to the following components for $J_\mu{}^{(i)}$:

$$\begin{aligned}J_\mu{}^{(+2)}&=\beta_0s_\mu{}^{(\frac{3}{2},\frac{3}{2})},\\J_\mu{}^{(+1)}&=j_\mu{}^{(1,1)}+\alpha s_\mu{}^{(\frac{1}{2},\frac{1}{2})}+\beta s_\mu{}^{(\frac{3}{2},\frac{1}{2})},\\J_\mu{}^{(0)}&=\rho_2j_\mu{}^{(1,0)}+\frac{\alpha}{\rho_2\sqrt{2}}s_\mu{}^{(\frac{1}{2},-\frac{1}{2})}+\frac{\sqrt{2}\beta}{\rho_2}s_\mu{}^{(\frac{3}{2},-\frac{1}{2})}+\eta j_\mu{}^{(0,0)},\\J_\mu{}^{(-1)}&=\rho_3j_\mu{}^{(1,-1)}+\frac{\sqrt{3}\beta}{\rho_3}s_\mu{}^{(\frac{3}{2},-\frac{3}{2})},\end{aligned} \tag{10}$$

where we have set $\rho_1=1$ and the condition $\eta\beta=0$ is understood.[7]

If $J_\mu{}^{(i)}$ is coupled to an intermediate vector meson, then since $J_\mu{}^{(i)}$ is not, in general, Hermitian, it will be necessary that this meson field have the following modes:

$$\begin{aligned}&\text{if}\quad \alpha=\beta=0{:}\ W_\mu{}^{(i)},\quad W_\mu{}^{(i)\dagger}(i=1,0)\quad\text{where}\quad W_\mu{}^{(0)\dagger}=W_\mu{}^{(0)},\\&\text{if}\quad \beta=0,\alpha\neq0{:}\ W_\mu{}^{(i)},\quad W_\mu{}^{(i)\dagger}(i=1,0,-1),\\&\text{if}\quad \alpha\neq0,\beta\neq0{:}\ W_\mu{}^{(i)},\quad W_\mu{}^{(i)\dagger}(i=2,1,0,-1).\end{aligned} \tag{11}$$

For arbitrary values of the constants in Eq. (10), it is clear that the strangeness-conserving contributions to $\sum_i J_\mu{}^{(i)\dagger}J^{\mu(i)}$ are not isotopic scalars. However, for certain choices of these parameters, it is possible to construct a Lagrangian in which $|\Delta I|=0$ or $\frac{1}{2}$. There are two such sets of parameters.

Case a: $\eta=\beta=\beta_0=0$, $\rho_2=1/\sqrt{2}$, $\rho_3=0$,

$$\begin{aligned}J_\mu{}^{(+1)}&=j_\mu{}^{(1,1)}+\alpha s_\mu{}^{(\frac{1}{2},\frac{1}{2})},\\J_\mu{}^{(0)}&=(1/\sqrt{2})j_\mu{}^{(1,0)}+\alpha s_\mu{}^{(\frac{1}{2},-\frac{1}{2})}.\end{aligned} \tag{12}$$

Case b: $\eta=\alpha=0$, $\beta_0=\beta$, $\rho_2=\sqrt{2}$, $\rho_3=\sqrt{3}$,

$$\begin{aligned}J_\mu{}^{(+2)}&=\beta s_\mu{}^{(\frac{3}{2},\frac{3}{2})},\\J_\mu{}^{(+1)}&=j_\mu{}^{(1,1)}+\beta s_\mu{}^{(\frac{3}{2},\frac{1}{2})},\\J_\mu{}^{(0)}&=\sqrt{2}j_\mu{}^{(1,0)}+\beta s_\mu{}^{(\frac{3}{2},-\frac{1}{2})},\\J_\mu{}^{(-1)}&=\sqrt{3}j_\mu{}^{(1,-1)}+\beta s_\mu{}^{(\frac{3}{2},-\frac{3}{2})}.\end{aligned} \tag{13}$$

As for the associated bosons, case (a) requires $W_\mu{}^{(+1)\dagger}=-W_\mu{}^{(-1)}$, while case (b) imposes no further requirements. Case (a) corresponds to the choice of currents and bosons made by Lee and Yang.[4]

It should be noted that the differences in the experimental predictions between Eqs. (10) and (12), when $\beta=0$, for a current-current type theory arise only in weak scattering processes of strongly interacting particles, which, of course, are completely masked by the strong interactions. In a current-boson version, on the other hand, the decays of the intermediate bosons will differ in the two cases.

It is now necessary to consider the way in which the leptons are to be introduced into the theory. Since the *usual* charge space formalism seems inapplicable to leptons, there is no straightforward application of the requirements which prescribed $J_\mu{}^{(i)}$ which will lead to

[7] These currents, of course, have the proper Clebsch-Gordan coefficients for combining a spin 1 with a spin $\frac{1}{2}$ to give a spin $\frac{1}{2}$ and for a spin 1 with a $\frac{3}{2}$ to also give a spin $\frac{1}{2}$. We use the usual phase convention of E. U. Condon and G. H. Shortley, *Theory of Atomic Spectra* (Cambridge University Press, New York, 1953).

a leptonic current. That is not to say that such a method does not exist, but rather that the possibilities have not yet been fully investigated. We would, therefore, prefer to consider the complete answer to this question to be beyond the scope of the present work. Rather, we shall introduce the leptonic currents in a phenomenological manner. Experimentally, it is rather well established that no neutral or doubly charged leptonic currents exist. We shall, therefore, introduce only the following[8]:

$$l_\mu^{(+1)} = \bar{\nu}_1\gamma_\mu(1+\gamma_5)e^- \pm \bar{\mu}^+\gamma_\mu(1-\gamma_5)\nu_2. \quad (14)$$

We have assumed the lepton current to have a symmetry between the μ and e, and we have introduced twin neutrinos in order to avoid the difficulty of $\mu \rightarrow e+\gamma$.[9] This leptonic current will now be added to $J_\mu^{(+1)}$ and its Hermitian conjugate to $J_\mu^{(-1)}$, with arbitrary constants, in order to form the complete weak interaction current $\mathcal{J}_\mu^{(i)}$.

$$\begin{aligned} &\mathcal{J}_\mu^{(+2)} = J_\mu^{(+2)}; \quad \mathcal{J}_\mu^{(+1)} = J_\mu^{(+1)} + \epsilon_1 l_\mu^{(+1)}; \\ &\mathcal{J}_\mu^{(0)} = J_\mu^{(0)}; \quad \mathcal{J}_\mu^{(-1)} = J_\mu^{(-1)} - \epsilon_2 l_\mu^{(+1)\dagger}. \end{aligned} \quad (15)$$

The fact that $\epsilon_1 \neq 0$, is necessary in order to describe leptonic decays with $\Delta Q = \Delta S$, such as $K^+ \rightarrow e^+ + \nu_1$, $\Lambda^0 \rightarrow p + e^- + \bar{\nu}_1$, etc.[10–12] However, there is very little information about the existence or absence of leptonic decays with $\Delta Q = -\Delta S$, such as $\Sigma^+ \rightarrow n + e^+ + \nu_1$, $K^0 \rightarrow \pi^+ + e^- + \bar{\nu}_1$, etc. These decays can occur only if $\beta\epsilon_2 \neq 0$. Thus, if these processes are found in nature, the $I=\frac{3}{2}$ currents must exist. On the other hand, their absence would not necessarily imply that the $I=\frac{3}{2}$ currents do not exist.

Let us now consider further experimental results which might be used to fix the remaining constants. Okubo *et al.*[13] have considered the relations among the various $K_{\mu 3}$ modes of decay which occur when the strongly interacting current transforms as a spinor. We shall proceed in a similar way for the more general case.

In order to make the predictions more definite, we will restrict ourselves to the case $\epsilon_2 = 0$. This assumption can be tested independently by searching for the processes with $\Delta Q = -\Delta S$, or by comparing the total rates for $K_1^0 \rightarrow \pi^\pm +$leptons and $K_2^0 \rightarrow \pi^\pm +$leptons (if $\epsilon_2 = 0$, these two rates should be equal, regardless of the existence or absence of $I=\frac{1}{2}$ and $I=\frac{3}{2}$ currents).

[8] We use the notation $\gamma_0^\dagger = \gamma_0$, $\gamma_i^\dagger = -\gamma_i$ $(i=1, 2, 3)$.

[9] G. Feinberg, Phys. Rev. **110**, 1482 (1958); J. Schwinger, Ann. Phys. **2**, 407 (1957).

[10] F. Eisler, R. Plano, A. Prodell, N. Samios, M. Schwartz, J. Steinberger, M. Conversi, P. Franzini, I. Mannelli, R. Stangelo, and V. Silvestrini, Nevis Cyclotron Report No. 67 (unpublished).

[11] F. S. Crawford, M. Cresti, M. L. Good, G. R. Kalbfleisch, M. L. Stevenson, and H. K. Ticho, Phys. Rev. Letters **1**, 377 (1958).

[12] P. Nordin, J. Orear, L. Reed, A. H. Rosenfeld, F. T. Solmitz, H. D. Taft, and R. D. Tripp, Phys. Rev. Letters **1**, 380 (1958).

[13] S. Okubo, R. E. Marshak, E. C. G. Sudarshan, W. B. Teutsch, and S. Weinberg, Phys. Rev. **112**, 665 (1958).

In the case $\epsilon_2 = 0$ the matrix element for $\bar{K}^0 \rightarrow \pi^- + \mu^+ + \bar{\nu}_2$, is, by virtue of our current,

$$M(\bar{K}^0 \rightarrow \pi^- + \mu^+ + \bar{\nu}_2) = 0,$$

while

$$\begin{aligned} &M(K^+ \rightarrow \pi^0 + \mu^+ + \bar{\nu}_2) \\ &\quad = (G/\sqrt{2})\langle \pi^0 | \alpha s_\lambda^{(\frac{1}{2},\frac{1}{2})\dagger} + \beta s_\lambda^{(\frac{3}{2},\frac{1}{2})\dagger} | K^+ \rangle \bar{u}_\mu \gamma_\lambda (1-\gamma_5) u_{\nu 2}, \end{aligned}$$

and

$$\begin{aligned} &M(K^0 \rightarrow \pi^- + \mu^+ + \bar{\nu}_2) \\ &\quad = (G/\sqrt{2})\langle \pi^- | \alpha s_\lambda^{(\frac{1}{2},\frac{1}{2})\dagger} + \beta s_\lambda^{(\frac{3}{2},\frac{1}{2})\dagger} | K^0 \rangle \bar{u}_\mu \gamma_\lambda (1-\gamma_5) u_{\nu 2}. \end{aligned}$$

But the isotopic character implies the following relations:

$$\begin{aligned} \langle \pi^0 | s_\lambda^{(\frac{1}{2},\frac{1}{2})\dagger} | K^+ \rangle &= (1/\sqrt{2}) \langle \pi^- | s_\lambda^{(\frac{1}{2},\frac{1}{2})\dagger} | K^0 \rangle; \\ \langle \pi^0 | s_\lambda^{(\frac{3}{2},\frac{1}{2})\dagger} | K^+ \rangle &= -\sqrt{2} \langle \pi^- | s_\lambda^{(\frac{3}{2},\frac{1}{2})\dagger} | K^0 \rangle. \end{aligned}$$

It, therefore, follows that the transition rates are related as

$$\begin{aligned} R(K_1^0 \rightarrow \pi^\mp + \mu^\pm + \nu) &= R(K_2^0 \rightarrow \pi^\mp + \mu^\pm + \nu) \\ &= R(K^0 \rightarrow \pi^- + \mu^+ + \bar{\nu}_2) \\ &= \gamma R(K^+ \rightarrow \pi^0 + \mu^+ + \bar{\nu}_2), \end{aligned} \quad (16)$$

where γ is a parameter which takes on the value 2 for a pure $I=\frac{1}{2}$ current and the value $\frac{1}{2}$ for a pure $I=\frac{3}{2}$ current. For arbitrary mixtures of $I=\frac{1}{2}$ and $\frac{3}{2}$, the value of γ cannot be predicted without further specifying the structure of the currents. The same relations will hold for K_{e3}. It should be noted that any deviation of γ from 2 is an indication that there exists an $I=\frac{3}{2}$ current as well as an $I=\frac{1}{2}$ current. As stated before, the vector and/or the axial vector part of the $I=\frac{1}{2}$ current must exist to explain $\Lambda \rightarrow p + e^- + \bar{\nu}_1$, as well as $K^+ \rightarrow \mu^+ + \bar{\nu}_2$, etc. On the other hand, this is one of the few crucial experiments for determining whether, in addition, there is also an $I=\frac{3}{2}$ current. By using the experimental lifetimes and branching ratios for the K^+, it is found that the total rate for K_2^0 into one π and leptons is

$$\begin{aligned} &R(K_2^0 \rightarrow \pi^\pm + \text{leptons}) \\ &\quad = (13.4 \pm 1.4) \times 10^6 \text{ sec}^{-1} \text{ for pure } I=\tfrac{1}{2} \quad (17a) \\ &\quad = (3.4 \pm 0.4) \times 10^6 \text{ sec}^{-1} \text{ for pure } I=\tfrac{3}{2}. \quad (17b) \end{aligned}$$

It should be noted that Eq. (17b) is valid only in the particular case of $\epsilon_2 = 0$, while Eq. (17a) is completely unambiguous. For a mixture of $I=\frac{1}{2}$ and $\frac{3}{2}$, as before, no definite prediction can be made. Thus, any deviation, in any direction, from the pure $I=\frac{1}{2}$ prediction implies the presence of an $I=\frac{3}{2}$ current. The experimental result of Crawford *et al.*[14] is $(20.4_{-5.6}^{+7.2}) \times 10^6$ sec^{-1}. It would be very desirable to improve the accuracy in these various experiments in order to determine whether the $I=\frac{3}{2}$ current actually exists.

Another possible experiment for determining the existence of an $I=\frac{3}{2}$ current is to examine the high

[14] F. S. Crawford, M. Cresti, R. L. Douglass, M. L. Kalbfleisch, and M. L. Stevenson, Phys. Rev. Letters **2**, 361 (1959).

energy neutrino capture experiment suggested in reference 4, namely,

$$\nu_2+n\rightarrow\mu^++\Sigma^-,\qquad \nu_2+p\rightarrow\mu^++\Sigma^0. \tag{18}$$

For pure $I=\frac{1}{2}$ and $I=\frac{3}{2}$ currents, these cross sections are in the ratio 2 to 1 and 1 to 2, respectively.

It is interesting to notice that the general structure of the weak-interaction Lagrangian discussed above, Eqs. (2a) and (10), leads to some very general experimental predictions which are independent of the existence or absence of the $I=\frac{3}{2}$ currents. In fact, if we assume that the currents of the strongly interacting particles involve only expressions bilinear in the fields and that the strong interactions are charge independent, it is clear that

$$(I_+,(I_+,J_\mu^{(+1)}))=0, \tag{19a}$$

$$(I_-,(I_-,J_\mu^{(+1)\dagger}))=0. \tag{19b}$$

These follow simply from the fact that a bilinear combination of particles whose charge $|Q|\leq 1$ cannot be formed into a current with $I>2$.

Let us now consider processes of the type $K^+\rightarrow\pi^{+,0}+\pi^{-,0}+\mu^++\bar{\nu}_2$ and $K^0\rightarrow\pi^0+\pi^-+\mu^++\bar{\nu}_2$. Assuming that the leptons interact locally, the matrix element for the processes are of the form

$$\langle\pi\pi|J_\lambda^{(+1)\dagger}|K^+\rangle\bar{u}_\mu\gamma_\lambda(1-\gamma_5)u_{\nu 2}.$$

Now, from Eq. (19b) we readily obtain the relation

$$2\langle(1/\sqrt{2})(\pi^0\pi^-+\pi^-\pi^0)|J_\lambda^{(+1)\dagger}|K^0\rangle +\sqrt{2}\langle(1/\sqrt{2})(\pi^+\pi^-+\pi^-\pi^+)|J_\lambda^{(+1)\dagger}|K^+\rangle -2\langle\pi^0\pi^0|J_\lambda^{(+1)\dagger}|K^+\rangle=0.$$

If we further assume $\epsilon_2=0$ and the validity of time reversal invariance, we can relate the $(K^+)_{e4}$ and $(K_2{}^0)_{e4}$ processes. When the relative motion of the two pions is predominantly in states of even angular momentum, we obtain

$$2M_1{}^e+M_2{}^e-\sqrt{2}M_3{}^e=0, \tag{20}$$

where $M_1{}^e$, $M_2{}^e$, and $M_3{}^e$ stand for the matrix elements of $K_2{}^0\rightarrow\pi^0+\pi^-+\mu^++\bar{\nu}_2$ (or $K_1{}^0\rightarrow\pi^0+\pi^-+\mu^++\bar{\nu}_2$), $K^+\rightarrow\pi^++\pi^-+\mu^++\bar{\nu}_2$, and $K^+\rightarrow\pi^0+\pi^0+\mu^++\bar{\nu}_2$, respectively, when the π's are in even relative angular momentum states. This in turn implies the triangular inequalities

$$2R_1{}^{e\frac{1}{2}}\leq\sqrt{2}R_3{}^{e\frac{1}{2}}+R_2{}^{e\frac{1}{2}}, \tag{21}$$

where $R_1{}^e$, $R_2{}^e$, and $R_3{}^e$ are the three decay rates corresponding to $M_1{}^e$, $M_2{}^e$, and $M_3{}^e$. If the $I=\frac{3}{2}$ current were not present, we could use

$$(I_-,J^{(+1)\dagger})=0 \tag{22}$$

instead of Eq. (19). In this particular case, we would obtain $M_1{}^e=0$, $M_2{}^e=\sqrt{2}M_3{}^e$, which implies

$$R_1{}^e=0;\quad R_2{}^e=2R_3{}^e. \tag{23}$$

The predictions (21) and (23) hold also, of course, for decays involving e^+ instead of μ^+, provided the pions are predominantly in states of even relative angular momentum.

It is also possible to verify Eqs. (19) by means of neutrino capture experiments of the type $\bar{\nu}_1+N\rightarrow e^++\Sigma+\pi$. One obtains an equality between the matrix elements of five different processes involving the various π, Σ, and nucleon charge states.

The above results are independent of the possible existence of any intermediate bosons. Let us now examine those results which follow from the existence of such particles. Lee and Yang have recently discussed the implications of the "schizon" which is associated with an isotopic spin $\frac{1}{2}$ strangeness-changing current [see Eqs. (11) and (12)]. In their scheme, the "schizon" character arises since such a boson is simultaneously a member of a quartet of bosons (two doublets, similar to the K-particles) and a member of a triplet. In the general case $\alpha,\beta\neq 0$, it is not possible to make both the interactions $\sum_i j_\mu{}^{(i)}W^{\mu(i)}$ and $\sum_i s_\mu{}^{(i)}W^{\mu(i)}$ isotopic scalars by assigning a dual or "schizon" character to the $W^{\mu(i)}$. For this reason, it is not possible, in general, to make simple predictions for the decays of the W particles. On the other hand, if $\alpha=0$, one can adopt the scheme of Eqs. (11) and (13), which gives rise, in a manner similar to that of Eqs. (11) and (12), to a "schizon" character for the bosons. In this case, the meson is simultaneously a member of an octet (two quartets) and a triplet.

Several other points concerning the intermediate mesons should be mentioned. First, if $\epsilon_2=0$, high-energy neutrino capture will produce $W^{(+1)}$'s but not $W^{(-1)\dagger}$'s. The reason for this is that, in this case, the leptons are only coupled to one of the charged currents. However, if the W's are produced by the scattering of strongly interacting particles, e.g.,

$$\pi^++p\rightarrow W^++p \tag{24a}$$

then both $W^{(+1)}$ and $W^{(-1)\dagger}$ are produced. Finally, if there exists a doubly charged meson, then the following process is possible

$$\pi^++p\rightarrow W^{++}+\Sigma^0\quad(\text{or }\Lambda). \tag{24b}$$

Since the leptons are not coupled to $W^{(+2)}$, this would seem to be the best method of producing such a doubly charged component.

The existence of the intermediate boson also has interesting effects on the various parameters which characterize muon decay, such as the mean life τ_μ and the parameters ρ and ξ.

When the electromagnetic corrections are calculated on the basis of the universal four-fermion interaction, the theoretical value quoted for τ_μ has been[15]

$$\tau_\mu=2.31\pm0.05\ \mu\text{sec}. \tag{25a}$$

On the basis of a new determination of the end-point

[15] T. Kinoshita and A. Sirlin, Phys. Rev. 113, 1652 (1959).

energy of O^{14}, this value is [16]

$$\tau_\mu = 2.30 \pm 0.03\ \mu\text{sec}. \quad (25b)$$

On the other hand, if the weak interactions are mediated by a boson of mass m_W, the direct decay mean life (i.e., uncorrected by the electromagnetic interactions) becomes[17]

$$\tau_\mu{}^W = \tau_\mu [1 + \tfrac{3}{5} m_\mu{}^2 / m_W{}^2]^{-1}. \quad (25c)$$

As is well known, the mass of such a boson must be larger than m_K in order to avoid a rapid decay $K \to W + \gamma$. Taking into account Eqs. (25b) and (25c) and for a value of $m_W \sim 1000 m_e$, one obtains approximately

$$\tau_\mu{}^W \approx 2.24 \pm 0.03\ \mu\text{sec}, \quad (25d)$$

which is to be compared with the experimental result,[16]

$$\tau_\mu{}^{\text{exp}} = 2.21 \pm 0.005\ \mu\text{sec}. \quad (25e)$$

For such a value of m_W, the ρ and ξ parameters have the values

$$\rho^W = 0.763; \quad \xi^W = 1.02_6, \quad (25f)$$

which are consistent with the present experimental results.[18]

II. CONSERVED CURRENTS—IMPLICATIONS

The previous considerations on the weak-interaction current has depended only upon general transformation properties in isotopic spin space. But aside from the striking experimental rules (ii) and (iii), there is another experimental fact which seems to indicate some deep underlying connection between the strong couplings of elementary particles and their weak decay interactions. This is the apparent lack of renormalization of the vector part of the β-decay interaction.[1,2]

It is possible, entirely by analogy with electrodynamics, to achieve this absence of renormalization by requiring that the current involved be conserved in the presence of the strong interactions. Although this may not be the only way of achieving such a result, it is currently the most appealing way, from a theoretical point of view, and hence will be used as the basis for the following analysis. This conservation of current implies a definite connection between the strong and weak interactions—namely, the conservation implies a symmetry of the strong interactions, charge independence, while at the same time it prescribes the exact current of strongly interacting particles for the non-strangeness-changing weak Lagrangian. It should, of course, be noted that, unlike electrodynamics, this current is not absolutely conserved; its conservation is broken by those interactions which violate charge independence, such as the electromagnetic field, and which, therefore, supposedly give rise to the various multiplet mass splittings.[19]

Now, in order to introduce a weak interaction which is responsible for strangeness-changing decays, it seems entirely reasonable just to add to the current described above another current which carries a strangeness of one unit. Then, by analogy, if it is required that this new current also be conserved, there should exist some further symmetry of the strong interactions as well as a definite prescription for writing down this new current. Again, it should be noted that this current will not be absolutely conserved[20]; its conservation will be broken by those interactions which violate the symmetry between particles which differ in strangeness by one unit, such as by those which are responsible for the mass differences between, for example, the Σ and the nucleon or Ξ, or the K and π. Such currents, which are conserved in the limit of neglecting certain mass differences, will be denoted as "quasi-conserved."

Let us briefly examine the form of the current which would be responsible for β decay and for possibly all other strangeness-conserving decays. In the case of neutron decay, one of the terms of this current must be $\bar{p}\gamma_\mu n$. It might easily be expected that other terms such as $\bar{\Sigma}^+\gamma_\mu\Sigma_0$, $\bar{\Xi}^0\gamma_\mu\Xi^-$, $\pi^-\partial_\mu\pi^0$, $\pi^0\partial_\mu\pi^-$, etc., might also be present, but as yet experimentally undetected, so that the proper current should be of the form

$$j_\mu = \bar{p}\gamma_\mu n + a\bar{\Sigma}^+\gamma_\mu\Sigma^0 + b\bar{\Sigma}^0\gamma_\mu\Sigma^- + c\bar{\Xi}^0\gamma_\mu\Xi^- + \cdots,$$

where $a, b, c, \cdots$ are arbitrary constants which are yet to be specified. If this current were, in addition, assumed to have some specific transformation property in isotopic spin space, then certain relations among some of these constants would be implied, e.g., that of a vector would imply $a = -b$. Many unrelated and thus arbitrary constants, however, still would remain. One way of fixing these remaining constants, for example, might be to assume that the *isotopic form* of j_μ is the same as the isotopic vector which is coupled to the π-meson in the strong interactions of the Yukawa type (e.g., see references 3 and 13). This would imply that $c = g_{\Xi\pi}/g_{N\pi}$, etc., where the g's are the coupling constants for the strong interactions. This current, of course, would not be conserved in general and hence would give rise to renormalization effects.

The assumption of a conserved current, on the other hand, fixes these constants in another way, namely $a = -b = -\sqrt{2}$, $c = 1$, etc., which makes j_μ just that conserved current that invariance under isotopic spin transformations implies exists. In fact, this invariance actually implies the existence of three conserved cur-

[16] Reported by R. P. Feynman at the Tenth Annual Rochester Conference on High-Energy Nuclear Physics, August, 1960 (to be published); R. K. Bardin, C. A. Barnes, W. A. Fowler, and P. A. Seeger (to be published). Without radiative corrections $\tau_\mu = 2.251 \pm 0.012$. In Eq. (25b), we have added the corrections of reference 15 evaluated with the usual definition of universality, i.e., equality of bare coupling constants, and have added 1% to the estimated error to take into account uncertainties of the calculation.

[17] T. D. Lee and C. N. Yang, Phys. Rev. **108**, 1611 (1957); A. Sirlin, Phys. Rev. **111**, 337 (1958).

[18] R. J. Plano and A. Lecourtois, Bull. Am. Phys. Soc. 4, 82 (1959).

[19] R. E. Behrends and A. Sirlin, Phys. Rev. Letters **4**, 186 (1960).

[20] S. Okubo, Nuovo cimento **13**, 292 (1959).

rents, i_μ^1, i_μ^2, i_μ^3, or more simply the isotopic vector current i_μ^i. Thus, the requirement of the conservation of a current, in addition to preventing renormalization of the charge, fixes, in a specific manner, the relative magnitudes of the various terms which appear in an arbitrary current.

The strangeness-changing current can be analyzed in an analogous manner. We can easily convince ourselves that the form of such a current (which changes strangeness by one unit) is

$$s_\mu = a\bar{n}\gamma_\mu\Sigma^- + b\bar{p}\gamma_\mu\Lambda + c\bar{\Sigma}^+\gamma_\mu n + d\bar{\Sigma}^+\gamma_\mu\Xi^0 + \cdots.$$

Now, in order to fix the various arbitrary constants, we will assume, by analogy with the strangeness-conserving current i_μ, that s_μ is also a quasi-conserved current. This then implies a symmetry of the strong interactions higher than charge independence, i.e., the invariance of the Lagrangian under a transformation which changes strangeness by one unit. Since we do not know of the existence of such a symmetry, we must solve both problems at once. To this end, we will make the general analyses of the next section.

III. SYMMETRIES AND CONSERVED CURRENTS

The invariance of a Lagrangian under a transformation implies the existence of a conserved vector current, and conversely, the existence of a conserved vector current implies the existence of a transformation which leaves the Lagrangian invariant. It is this latter statement which we will use as the basis for the following analysis of the strong interactions. Namely, we will construct the most general vector current, impose the condition of its conservation, and thereby determine the symmetry of the Lagrangian as well as the specific form of the current. The Lagrangian which we will consider is the most general, charge-independent, Yukawa-type Lagrangian,

$$\begin{aligned}\mathcal{L}_{\text{int}} = {} & g_{\pi N}\bar{N}\tau_i i\gamma_5 N\pi_i + g_{\pi\Xi}\bar{\Xi}\tau_i i\gamma_5\Xi\pi_i \\ & - ig_{\pi\Sigma}\epsilon_{ijk}\bar{\Sigma}_i i\gamma_5\Sigma_j\pi_k + g_{\pi\Lambda}(\bar{\Lambda}\mathcal{O}_{\pi\Lambda}\Sigma_i\pi_i + \bar{\Sigma}_i\mathcal{O}_{\pi\Lambda}{}^\dagger\Lambda\pi_i) \\ & + g_{\Sigma N}\bar{N}\tau_i\mathcal{O}_{\Sigma N}\Sigma_i K + g_{\Sigma\Xi}\bar{\Xi}\tau_i\mathcal{O}_{\Sigma\Xi}\Sigma_i K_G \\ & + g_{\Lambda N}\bar{N}\mathcal{O}_{\Lambda N}\Lambda K + g_{\Lambda\Xi}\bar{\Xi}\mathcal{O}_{\Lambda\Xi}\Lambda K_G, \qquad (26)\end{aligned}$$

where

$$N = \begin{pmatrix} p \\ n \end{pmatrix},\quad \Xi = \begin{pmatrix} \Xi^0 \\ \Xi^- \end{pmatrix},\quad K = \begin{pmatrix} K^+ \\ K^0 \end{pmatrix},\quad K_G = \begin{pmatrix} -\bar{K}^0 \\ \bar{K}^+ \end{pmatrix},$$

and the $\mathcal{O}_{ab}$ are the space-time operators of the interactions (e.g., 1, $i\gamma_5$).

In the first part of this section, we shall base our analysis on the full Lagrangian, i.e., we shall assume that all the $g_i \neq 0$. The basic assumption of this approach is that the currents which appear in the weak interaction are to be quasi-conserved in the presence of all strong interactions, both K and π. On the other hand, in the second part, we shall assume that the K couplings are weaker than the π couplings and that the weak currents are quasi-conserved only in the presence of the stronger π-baryon interactions. This latter approach allows for the possibility that the K interactions have less symmetry than those of the pions and hence could give rise to the various mass splittings between the baryons.

The general method is as follows. We construct the most general current of one charge by forming an arbitrary linear combination of all possible vector covariants which are bilinear in either baryons or mesons. We then demand that the divergence of such a current should vanish. With the help of the equations of motion derived from the Lagrangian above, we obtain a set of simultaneous algebraic equations involving the coefficients of the vector covariants of the arbitrary current, the various strong-coupling constants, g_i, the masses, m_i of the baryons and μ_i of the bosons, and the various space-time interactions, $\mathcal{O}_i$. As mentioned above, we assume first that all the $g_i \neq 0$ in order to solve this set of equations. The solutions give various relations amongst the coefficients of the currents, the g_i, m_i, μ_i, and $\mathcal{O}_i$. With this knowledge, it is an easy matter to find the symmetries of the Lagrangian which gives rise to the determined current. In the second part, we proceed in exactly the same manner except that we simplify the Lagrangian by setting all the K-coupling constants equal to zero.

Since the details of the calculation are too cumbersome to present, we shall list only the results obtained in the manner described above. In order to do this in a compact notation, it is convenient to introduce the seven-dimensional charge space which has been previously discussed in the literature.[21] Briefly, in this charge space, an eight-component spinor represents the baryons and a seven-dimensional vector, the bosons. Thus

$$\psi = \begin{bmatrix} p \\ n \\ \Xi^0 \\ \Xi^- \\ \Sigma^+ \\ Y^0 \\ Z^0 \\ \Sigma^- \end{bmatrix},\quad \phi = \begin{bmatrix} K_1 \\ K_2 \\ K_3 \\ K_4 \\ \pi_1 \\ \pi_2 \\ \pi_3 \end{bmatrix}\quad \begin{aligned} Y^0 &= (\Lambda^0 - \Sigma^0)/\sqrt{2}, & Z^0 &= (\Lambda^0 + \Sigma^0)/\sqrt{2}, \\ K_1 &= (\bar{K}^+ + K^+)/\sqrt{2}, & K_2 &= (\bar{K}^+ - K^+)/\sqrt{2}i, \\ K_3 &= (\bar{K}^0 + K^0)/\sqrt{2}, & K_4 &= (\bar{K}^0 - K^0)/\sqrt{2}i, \\ \pi_i &= (\pi^- + \pi^+)/\sqrt{2}, & \pi_2 &= (\pi^- - \pi^+)/\sqrt{2}i. \end{aligned} \qquad (27)$$

[21] J. Tiomno, Nuovo cimento **6**, 69 (1957); R. E. Behrends, Nuovo cimento **11**, 424 (1959); D. C. Peaslee, Phys. Rev. **117**, 873 (1960); J. M. Sourian, Compt. rend. **250**, 2807 (1960).

In this Euclidean space there are then a set of seven Hermitian anticommuting 8×8 matrices, Γ_i. The representation of these Γ_i associated with the spinors defined above are

$$\Gamma_{1,2,3}=1\times\sigma_{1,2,3}\times\sigma_1,\quad \Gamma_4=1\times 1\times\sigma_2,\quad \Gamma_{5,6,7}=\sigma_{1,2,3}\times 1\times\sigma_3, \tag{28}$$

where the σ_i are the usual three Pauli 2×2 spin matrices. There are 21 independent rotations in this space under which the spinors transform as $\psi'=e^{\frac{1}{2}\Gamma_{ij}\omega_{ij}}\psi$ [where $\Gamma_{ij}\equiv\frac{1}{2}(\Gamma_i\Gamma_j-\Gamma_j\Gamma_i)$] and the vectors as $\phi_i'=a_{ij}\phi_j$. For example, the three usual isotopic spin rotations are given as $e^{\frac{1}{2}i\tau_i\alpha_i}$ where

$$\tau_1=i[\Gamma_{67}+\tfrac{1}{2}(\Gamma_{14}-\Gamma_{23})],\quad \tau_2=i[\Gamma_{57}-\tfrac{1}{2}(\Gamma_{13}+\Gamma_{24})],\quad \tau_3=i[\Gamma_{56}+\tfrac{1}{2}(\Gamma_{12}-\Gamma_{34})].$$

In the following, we shall group the results by the relations amongst the strong coupling constants which are required in order that the current be conserved. We shall then explicitly list the conserved currents and also the specific rotations in the 7-dimensional space under which this Lagrangian is invariant. Since each of these sets of rotations form a group, the identification and the number of parameters will also be given.

Results when all $g_i\neq 0$

(I) No further conditions on g_i.

$$j_\mu^{(0,0)}=\bar{N}\gamma_\mu N+\bar{\Xi}\gamma_\mu\Xi+\bar{\Sigma}_i\gamma_\mu\Sigma_i+\bar{\Lambda}\gamma_\mu\Lambda,$$

$$i_\mu^{(1,1)}=-i_\mu^{(1,-1)\dagger}=\bar{p}\gamma_\mu n+\bar{\Xi}^0\gamma_\mu\Xi^- +\sqrt{2}(\bar{\Sigma}^0\gamma_\mu\Sigma^- -\bar{\Sigma}^+\gamma_\mu\Sigma^0)-i\sqrt{2}(\pi^-\partial_\mu\pi^0-\pi^0\partial_\mu\pi^-) -i(K^0\partial_\mu\bar{K}^+-\bar{K}^+\partial_\mu K^0),$$

$$i_\mu^{(1,0)}=-\frac{1}{\sqrt{2}}[\bar{p}\gamma_\mu p-\bar{n}\gamma_\mu n+\bar{\Xi}^0\gamma_\mu\Xi^0-\bar{\Xi}^-\gamma_\mu\Xi^- +2(\bar{\Sigma}^+\gamma_\mu\Sigma^+-\bar{\Sigma}^-\gamma_\mu\Sigma^-)-i2(\pi^+\partial_\mu\pi^- -\pi^-\partial_\mu\pi^+) -i(K^+\partial_\mu\bar{K}^+-\bar{K}^+\partial_\mu K^+)+i(K^0\partial_\mu\bar{K}^0-\bar{K}^0\partial_\mu K^0)],$$

$$y_\mu^3=\frac{1}{\sqrt{2}}[\bar{p}\gamma_\mu p+\bar{n}\gamma_\mu n-\bar{\Xi}^0\gamma_\mu\Xi^0-\bar{\Xi}^-\gamma_\mu\Xi^- +i(\bar{K}^+\partial_\mu K^+-K^+\partial_\mu\bar{K}^+)+i(\bar{K}^0\partial_\mu K^0-K^0\partial_\mu\bar{K}^0)].$$

Lagrangian invariant under $e^{i\lambda}$; $\exp[\Gamma_{67}+\frac{1}{2}(\Gamma_{14}-\Gamma_{23})]\alpha_1$, $\exp[\Gamma_{57}-\frac{1}{2}(\Gamma_{13}+\Gamma_{24})]\alpha_2$, $\exp[\Gamma_{56}+\frac{1}{2}(\Gamma_{12}-\Gamma_{34})]\alpha_3$; $\exp[\frac{1}{2}(\Gamma_{12}+\Gamma_{34})\alpha]$, respectively (1-parameter baryon gauge group; 3-parameter rotation group, R_3, which is the isotopic spin group; 1 parameter rotation group, R_2, which corresponds to hypercharge conservation).

(II) $g_{N\pi}=g_{\Xi\pi}$; $\eta_1 g_{N\Lambda}=\eta_2 g_{\Xi\Lambda}$; $\eta_1 g_{N\Sigma}=\eta_2 g_{\Xi\Sigma}$; $\eta_1=\pm 1$, $\eta_2=\pm 1$, $m_N=m_\Xi$; $\mathcal{O}_{N\Lambda}=\mathcal{O}_{\Xi\Lambda}$, $\mathcal{O}_{N\Sigma}=\mathcal{O}_{\Xi\Sigma}$.
Currents are $j_\mu^{(0,0)}$, $i_\mu^{(1,m)}$, y_μ^3, and

$$y_\mu^1=y_\mu^{2\dagger}=\eta_1\eta_2(\bar{p}\gamma_\mu\Xi^0+\bar{n}\gamma_\mu\Xi^-)-i(\bar{K}^+\partial_\mu\bar{K}^0-\bar{K}^0\partial_\mu\bar{K}^+).$$

Lagrangian invariant[22] under (I) and $\exp[\frac{1}{2}(\Gamma_{14}+\Gamma_{23})\alpha_1]$, $\exp[\frac{1}{2}(\Gamma_{24}-\Gamma_{13})\alpha_2]$ (baryon gauge group plus 6-parameter rotation group, R_4, which is decomposable into two invariant subgroups of 3-parameter rotations, R_3, which are the isotopic spin group and the hypercharge rotation group).

(III) $g_{\Lambda\pi}=\eta_2 g_{\Sigma\pi}$; $g_{N\Lambda}=\eta_3 g_{N\Sigma}$; $g_{\Xi\Lambda}=\eta_3 g_{\Xi\Sigma}$; $\eta_2=\pm 1$, $\eta_3=\pm 1$, $m_\Lambda=m_\Sigma$; $\mathcal{O}_{\Lambda\pi}=i\gamma_5$, $\mathcal{O}_{N\Lambda}=\mathcal{O}_{N\Sigma}$, $\mathcal{O}_{\Xi\Lambda}=\mathcal{O}_{\Xi\Sigma}$.
Currents are $j_\mu^{(0,0)}$, $i_\mu^{(1,m)}$, and

$$r_\mu^{(1,1)}=-r_\mu^{(1,-1)\dagger}=(1/\sqrt{2})[\eta_3(\bar{\Lambda}\gamma_\mu\Sigma^-+\bar{\Sigma}^+\gamma_\mu\Lambda)-\bar{\Sigma}^0\gamma_\mu\Sigma^-+\bar{\Sigma}^+\gamma_\mu\Sigma^0-i\sqrt{2}(\bar{K}^+\partial_\mu K^0-K^0\partial_\mu\bar{K}^+)],$$

$$r_\mu^{(1,0)}=-(1/\sqrt{2})[\eta_3(\bar{\Lambda}\gamma_\mu\Sigma^0+\bar{\Sigma}^0\gamma_\mu\Lambda)+\bar{\Sigma}^-\gamma_\mu\Sigma^- -\bar{\Sigma}^+\gamma_\mu\Sigma^+-i(\bar{K}^+\partial_\mu K^+-K^+\partial_\mu\bar{K}^+)+i(\bar{K}^0\partial_\mu K^0-K^0\partial_\mu\bar{K}^0)].$$

Lagrangian invariant under (I) and $\exp[\frac{1}{2}(\Gamma_{23}-\Gamma_{14})\alpha_1]$, $\exp[\frac{1}{2}(\Gamma_{13}+\Gamma_{24})\alpha_2]$, $\exp[\frac{1}{2}(\Gamma_{34}-\Gamma_{12})\alpha_3]$ (baryon gauge group, 1-parameter hypercharge rotation group, R_2, plus 6-parameter rotation group, R_4, which is decomposable into two invariant subgroups of 3-parameter rotations, R_3).

(IV) $g_{N\pi}=g_{\Xi\pi}=-g_{\Sigma\pi}=-\eta_1 g_{N\Sigma}=-\eta_2 g_{\Xi\Sigma}$; $g_{\Lambda\pi}=\eta_1 g_{N\Lambda}=\eta_2 g_{\Xi\Lambda}$; $\eta_1=\pm 1$, $\eta_2=\pm 1$; $m_N=m_\Xi=m_\Sigma$; $\mu_K=\mu_\pi$; $\mathcal{O}_{N\Sigma}=\mathcal{O}_{\Xi\Sigma}=i\gamma_5$; $\mathcal{O}_{\Lambda\pi}=\mathcal{O}_{\Lambda N}=\mathcal{O}_{\Lambda\Xi}$.
Currents are $j_\mu^{(0,0)}$, $i_\mu^{(1,m)}$, y_μ^i, and

$$s_\mu^{(\frac{3}{2},\frac{3}{2})}=\sqrt{3}[-\eta_1\bar{p}\gamma_\mu\Sigma^-+\eta_2\bar{\Sigma}^+\gamma_\mu\Xi^- -i(\bar{K}^+\partial_\mu\pi^- -\pi^-\partial_\mu\bar{K}^+)],$$

$$s_\mu^{(\frac{3}{2},\frac{1}{2})}=-\eta_1(\bar{n}\gamma_\mu\Sigma^- -\sqrt{2}\bar{p}\gamma_\mu\Sigma^0)-\eta_2(\bar{\Sigma}^+\gamma_\mu\Xi^0+\sqrt{2}\bar{\Sigma}^0\gamma_\mu\Xi^-)+i\sqrt{2}(\bar{K}^+\partial_\mu\pi^0-\pi^0\partial_\mu\bar{K}^+)-i(\bar{K}^0\partial_\mu\pi^- -\pi^-\partial_\mu\bar{K}^0),$$

$$s_\mu^{(\frac{3}{2},-\frac{1}{2})}=\eta_1(\bar{p}\gamma_\mu\Sigma^+ +\sqrt{2}\bar{n}\gamma_\mu\Sigma^0)-\eta_2(\bar{\Sigma}^-\gamma_\mu\Xi^- -\sqrt{2}\bar{\Sigma}^0\gamma_\mu\Xi^0)+i\sqrt{2}(\bar{K}^0\partial_\mu\pi^0-\pi^0\partial_\mu\bar{K}^0)+i(\bar{K}^+\partial_\mu\pi^+ -\pi^+\partial_\mu\bar{K}^+),$$

$$s_\mu^{(\frac{3}{2},-\frac{3}{2})}=\sqrt{3}[\eta_1\bar{n}\gamma_\mu\Sigma^+ +\eta_2\bar{\Sigma}^-\gamma_\mu\Xi^0+i(\bar{K}^0\partial_\mu\pi^+ -\pi^+\partial_\mu\bar{K}^0)],$$

plus Hermitian conjugate currents $s_\mu^{(\frac{3}{2},m)\dagger}$. Lagrangian invariant under (I), (II), and $\exp[\Gamma_{47}+\frac{1}{2}(\Gamma_{25}-\Gamma_{16})]\alpha_1$, $\exp[\Gamma_{37}+\frac{1}{2}(\Gamma_{15}+\Gamma_{26})]\alpha_2$, $\exp[\Gamma_{27}-\frac{1}{2}(\Gamma_{36}+\Gamma_{45})]\alpha_3$, $\exp[\Gamma_{17}+\frac{1}{2}(\Gamma_{46}-\Gamma_{35})]\alpha_4$, $\exp[\frac{1}{2}(\Gamma_{36}-\Gamma_{45})\alpha_5]$, $\exp[\frac{1}{2}(\Gamma_{46}+\Gamma_{35})\alpha_6]$, $\exp[\frac{1}{2}(\Gamma_{25}+\Gamma_{16})\alpha_7]$, $\exp[\frac{1}{2}(\Gamma_{15}-\Gamma_{26})\alpha_8]$ (baryon gauge group plus 14-parameter exceptional group $G2$, which is not decomposable into invariant subgroups).

[22] The vector ψ and ϕ of Eq. (27) correspond to the following choice of phases: $\eta_1=\eta_2=\eta_3=1$. To consider the more general case, it is simply sufficient to replace ψ in Eq. (27) by a ψ' where $N'=\eta_1 N$, $\Xi'=\eta_2\Xi$, $\Lambda'=\eta_3\Lambda$.

(V) $g_{N\pi}=g_{\Xi\pi}=-g_{\Sigma\pi}=-\eta_3 g_{\Lambda\pi}=-\eta_1 g_{N\Sigma}=-\eta_2 g_{\Xi\Sigma}=-\eta_1\eta_3 g_{N\Lambda}=-\eta_2\eta_3 g_{\Xi\Lambda}$; $\eta_1=\pm 1$, $\eta_2=\pm 1$, $\eta_3=\pm 1$. (7-dimensional symmetry.) All baryon masses equal; all $\mathcal{O}$'s equal to $i\gamma_5$; $\mu_K=\mu_\pi$. Currents are $j_\mu^{(0,0)}$, $i_\mu^{(1,m)}$, $r_\mu^{(1,m)}$, $y_\mu{}^i$, $s_\mu{}^{(\frac{3}{2},m)}$, $s_\mu{}^{(\frac{3}{2},m)\dagger}$, and

$$s_\mu{}^{(\frac{1}{2},\frac{1}{2})}=(1/\sqrt{3})[-3\eta_1\eta_3\bar{p}\gamma_\mu\Lambda-3\eta_2\eta_3\bar{\Lambda}\gamma_\mu\Xi^-+\eta_1(\bar{p}\gamma_\mu\Sigma^0+\sqrt{2}\bar{n}\gamma_\mu\Sigma^-)-\eta_2(\bar{\Sigma}^0\gamma_\mu\Xi^--\sqrt{2}\bar{\Sigma}^+\gamma_\mu\Xi^0)$$
$$-i2(\bar{K}^+\partial_\mu\pi^0-\pi^0\partial_\mu\bar{K}^+)-i2\sqrt{2}(\bar{K}^0\partial_\mu\pi^--\pi^-\partial_\mu\bar{K}^0)],$$

$$s_\mu{}^{(\frac{1}{2},-\frac{1}{2})}=(1/\sqrt{3})[-3\eta_1\eta_3\bar{n}\gamma_\mu\Lambda+3\eta_2\eta_3\bar{\Lambda}\gamma_\mu\Xi^0-\eta_1(\bar{n}\gamma_\mu\Sigma^0-\sqrt{2}\bar{p}\gamma_\mu\Sigma^+)-\eta_2(\bar{\Sigma}^0\gamma_\mu\Xi^0+\sqrt{2}\bar{\Sigma}^-\gamma_\mu\Xi^-)$$
$$+i2(\bar{K}^0\partial_\mu\pi^0-\pi^0\partial_\mu\bar{K}^0)-i2\sqrt{2}(\bar{K}^+\partial_\mu\pi^+-\pi^+\partial_\mu\bar{K}^+)],$$

plus Hermitian conjugate currents $s_\mu{}^{(\frac{1}{2},m)\dagger}$. Lagrangian is invariant under baryon gauge group and 21-parameter rotation group, R_7.

It is important to observe that the quasi-conservation of the $I=\frac{1}{2}$ currents requires the seven-dimensional rotation symmetry,[23] while the quasi-conservation of the $I=\frac{3}{2}$ currents necessitates a lower symmetry, namely, $G2$.

Results When K-Couplings Absent[24]

(A) No further conditions on g_i. Symmetry is identical with symmetry (I) except that currents do not have $K\partial_\mu K$ type terms and there is the following additional symmetry:

$$\hat{\imath}_\mu{}^{(0,0)}=\bar{\Sigma}^+\gamma_\mu\Sigma^++\bar{\Sigma}^-\gamma_\mu\Sigma^-+\bar{\Sigma}^0\gamma_\mu\Sigma^0+\bar{\Lambda}\gamma_\mu\Lambda.$$

Lagrangian invariant under $\Sigma_i'=e^{i\lambda}\Sigma_i$, $\Lambda'=e^{i\lambda}\Lambda$ (one-parameter gauge group).

(B) $g_{N\pi}=g_{\Xi\pi}$, $m_N=m_\Xi$. Symmetry is identical with symmetry (II) except that currents do not have $K\partial_\mu K$ type terms.

(C) $g_{\Sigma\pi}=\eta g_{\Lambda\pi}$, $m_\Sigma=m_\Lambda$; $\mathcal{O}_{\Lambda\pi}=i\gamma_5$. Symmetry is identical with symmetry (III) except that currents do not have $K\partial_\mu K$ type terms.

(D) $g_{N\pi}=g_{\Sigma\pi}=\eta g_{\Lambda\pi}$, $m_\Sigma=m_N=m_\Lambda$, $\mathcal{O}_{\Lambda\pi}=i\gamma_5$. Currents of (A), (C), and

$$\hat{s}_\mu{}^{(\frac{1}{2},\frac{1}{2})}=\eta\bar{p}\gamma_\mu\Lambda+\bar{p}\gamma_\mu\Sigma^0+\sqrt{2}\bar{n}\gamma_\mu\Sigma^-,$$
$$\hat{s}_\mu{}^{(\frac{1}{2},-\frac{1}{2})}=\eta\bar{n}\gamma_\mu\Lambda-\bar{n}\gamma_\mu\Sigma^0+\sqrt{2}\bar{p}\gamma_\mu\Sigma^+$$

plus Hermitian currents $\hat{s}_\mu{}^{(\frac{1}{2},m)\dagger}$.

(E) $g_{\Xi\pi}=g_{\Sigma\pi}=\eta g_{\Lambda\pi}$, $m_\Sigma=m_\Xi=m_\Lambda$, $\mathcal{O}_{\Lambda\pi}=i\gamma_5$. Currents of (A), (C), and

$$\hat{s}_\mu{}^{(\frac{1}{2},\frac{1}{2})'}=\eta\bar{\Lambda}\gamma_\mu\Xi^--\bar{\Sigma}^0\gamma_\mu\Xi^-+\sqrt{2}\bar{\Sigma}^+\gamma_\mu\Xi^0,$$
$$\hat{s}_\mu{}^{(\frac{1}{2},-\frac{1}{2})'}=-\eta\bar{\Lambda}\gamma_\mu\Xi^0-\bar{\Sigma}^0\gamma_\mu\Xi^0-\sqrt{2}\bar{\Sigma}^-\gamma_\mu\Xi^-$$

plus Hermitian currents $\hat{s}_\mu{}^{(\frac{1}{2},m)'\dagger}$. It is important to observe that in the present case (K-couplings switched off) the $I=\frac{3}{2}$ currents cannot be quasi-conserved.

IV. STRONG INTERACTIONS—PREDICTIONS

In the previous section we determined the various possible symmetries of the strong interactions. These symmetries, of course, have certain experimental consequences for those physical processes which proceed through just the strong couplings. In this section, we shall discuss these predictions.

Of the symmetries when the K couplings are absent, the most interesting is when all the constants are equal, i.e., global symmetry. This has been discussed extensively in the literature.[25]

In the symmetry (I), the invariance under the baryon gauge group, the isotopic spin group and the one-parameter hypercharge group guarantees the conservation of baryons, isotopic spin, and hypercharge, respectively. The experimental implications of these symmetries are well known and will not be discussed here.

Symmetry (II), the hypercharge rotation group, has been discussed in some detail by Feinberg and Behrends.[26] There seems to be little experimental evidence either favoring or disagreeing with the predictions of this symmetry.

Symmetry (III), the "doublet approximation," has been discussed by Pais and shown to be in disagreement with experiment.[27]

Symmetry (IV), the exceptional group $G2$, which has not been discussed previously will form the basis of the present considerations.

According to the theory of continuous groups, there are four general classes of simple groups with varying numbers of parameters (orthogonal, unitary, etc.). Aside from these there are five exceptional groups, each with a fixed number of parameters. The exceptional group with the smallest number of parameters, 14, has been designated by Cartan as $G2$.[28] It is a nondecomposable subgroup of the 7-dimensional rotation group, R_7, and is in fact the subgroup which we have designated as symmetry (IV).

However, rather than give a complete analysis of this group and its properties which bear on the strong interactions, we shall follow a more pedestrian approach in order to determine some of the predictions for physical processes. If it should turn out that this group is, in some way, a symmetry of the strong interactions, or a part thereof, then, of course, a complete analysis will have to be made.

The group $G2$, as constituted in our symmetry (IV),

[23] This fact has been emphasized by V. M. Shekhter, J. Exptl. Theoret. Phys. 36, 581 (1959) [translation: Soviet Phys.—JETP 9, 403 (1959)].

[24] Currents for this truncated case are denoted by carets.

[25] For example, see M. Gell-Mann, Phys. Rev. **106**, 1296 (1957); D. Amati and B. Vitale, Fortschr. Physik **7**, 375 (1959).

[26] G. Feinberg and R. E. Behrends, Phys. Rev. **115**, 745 (1959).

[27] A. Pais, Phys. Rev. **110**, 574 (1958).

[28] E. Cartan, "Sur la structure des groups de transformations finis et continus", These, Paris, 1894, II edition 1933.

TABLE I. Finite-angle rotations of $G2$. The exponentials describe the rotations which take the first column into the corresponding column. The second column is the usual charge symmetry operation. The Λ is not transformed under $G2$.

	$\exp[\pm\frac{1}{4}\pi(2\Gamma_{57}-\Gamma_{13}-\Gamma_{24})]$	$\exp[\pm\frac{1}{4}\pi(2\Gamma_{37}+\Gamma_{15}+\Gamma_{26})]$	$\exp[\pm\frac{1}{4}\pi(2\Gamma_{17}+\Gamma_{46}-\Gamma_{35})]$	$\exp[\pm\frac{1}{4}\pi(\Gamma_{46}+\Gamma_{35})]$	$\exp[\pm\frac{1}{4}\pi(\Gamma_{13}-\Gamma_{24})]$	$\exp[\pm\frac{1}{4}\pi(\Gamma_{15}-\Gamma_{26})]$
p	$\pm n$	$\mp\Sigma^+$	$-\Xi^-$	p	$\mp\Xi^0$	$\mp\Sigma^-$
n	$\mp p$	Ξ^0	$\pm\Sigma^-$	$\mp\Sigma^+$	$\mp\Xi^-$	n
Ξ^0	$\pm\Xi^-$	n	$\mp\Sigma^+$	$\pm\Sigma^-$	$\pm p$	Ξ^0
Ξ^-	$\mp\Xi^0$	$\mp\Sigma^-$	$-p$	Ξ^-	$\pm n$	$\mp\Sigma^+$
Σ^+	$-\Sigma^-$	$\pm p$	$\pm\Xi^0$	$\pm n$	Σ^+	$\pm\Xi^-$
Σ^0	$-\Sigma^0$	$-\Sigma^0$	$-\Sigma^0$	Σ^0	Σ^0	Σ^0
Σ^-	$-\Sigma^+$	$\pm\Xi^-$	$\mp n$	$\mp\Xi^0$	Σ^-	$\pm p$
K^+	$\pm K^0$	$\mp\pi^+$	$-\bar{K}^+$	K^+	$\pm\bar{K}^0$	$\mp\pi^-$
K^0	$\mp K^+$	$-\bar{K}^0$	$\pm\pi^-$	$\mp\pi^+$	$\mp\bar{K}^+$	K^0
$\bar{K}^0$	$\mp\bar{K}^+$	$-K^0$	$\pm\pi^+$	$\mp\pi^-$	$\mp K^+$	$\bar{K}^0$
$\bar{K}^+$	$\pm\bar{K}^0$	$\mp\pi^-$	$-K^+$	$\bar{K}^+$	$\pm K^0$	$\mp\pi^+$
π^+	$-\pi^-$	$\pm K^+$	$\mp\bar{K}^0$	$\pm K^0$	π^+	$\pm\bar{K}^+$
π^0	$-\pi^0$	$-\pi^0$	$-\pi^0$	π^0	π^0	π^0
π^-	$-\pi^+$	$\pm\bar{K}^+$	$\mp K^0$	$\pm\bar{K}^0$	π^-	$\pm K^+$

contains two subgroups (not invariant subgroups) which are the 3-parameter isotopic spin rotation group and the 3-parameter hypercharge rotation group. It should be noted that it does not contain the symmetry (III) which disagrees so definitely with experiment. It, therefore, cannot be dismissed, out of hand, as being incorrect. Physically, $G2$ is the group of all rotations in the seven-dimensional space which does not transform the Λ particle [the transformation of Λ into Σ's is physically the symmetry (III)]. [It is easy to see physically that this set of rotations, symmetry (IV), forms a group: namely, each of these rotations mixes the remaining seven baryons among themselves, but does not mix any of the seven with a Λ. Therefore, any combination of rotations will always lead to a rotation which does not transform the Λ.]

In Table I we have listed the finite-angle rotations of $G2$ which will be useful in our analysis. The angle was chosen so that the rotation would take one particle into another rather than a linear combination of others.

We now consider the four-point function for a scattering, say, $\pi^+ + p \to \pi^+ + p$,

$$\langle T[\bar{\psi}_p(x_1)\psi_p(x_2)\phi_{\pi^+}(x_3)\phi_{\pi^+}{}^*(x_4)]\rangle_0, \tag{29}$$

where T is the time-ordering operator and $\langle\ \ \rangle_0$ means the vacuum expectation value in the physical vacuum. This quantity is related by a simple linear integral transform to the matrix element operator. We will symbolize this quantity by $\langle p\pi^+; p\pi^+\rangle$.

If U is the unitary operator which induces one of the rotations, then it will commute with the Hamiltonian when the various mass and coupling-constant relations necessary for invariance are satisfied [in this case the restrictions for the symmetry (IV)], and hence it will leave the vacuum invariant. It thus follows, by using Table I, that the rotation $\exp[\pm\frac{1}{4}\pi(2\Gamma_{57}-\Gamma_{13}-\Gamma_{24})]$ changes the four-point function for $\pi^+ + p \to \pi^+ + p$ in the following manner:

$$\begin{aligned}\langle p\pi^+; p\pi^+\rangle &= \langle Up\pi^+; p\pi^+U^\dagger\rangle \\ &= \langle n\pi^-; n\pi^-\rangle.\end{aligned} \tag{30}$$

This is the usual charge symmetry relation. Similarly, the rotation $\exp[\pm\frac{1}{4}\pi(\Gamma_{15}-\Gamma_{26})]$ leads to the following relations:

$$\begin{aligned}\langle p\pi^+; p\pi^+\rangle = \langle \Sigma^-\bar{K}^+; \Sigma^-\bar{K}^+\rangle &= \langle n\pi^-; n\pi^-\rangle \\ &= \langle nK^+; nK^+\rangle.\end{aligned} \tag{31}$$

In this manner, it is possible to list the following relations among the various experimentally feasible processes:

$$\begin{aligned}
\Omega_1 &= \langle p\pi^+; p\pi^+\rangle = \langle n\pi^-; n\pi^-\rangle = \langle pK^0; pK^0\rangle \\
&= \langle nK^+; nK^+\rangle, \\
\Omega_2 &= \langle \Sigma^+K^+; p\pi^+\rangle = \langle \Sigma^-K^0; n\pi^-\rangle = \langle nK^+; pK^0\rangle \\
&= \langle pK^0; nK^+\rangle, \\
\Omega_3 &= \langle p\pi^-; p\pi^-\rangle = \langle n\pi^+; n\pi^+\rangle = \langle p\bar{K}^0; p\bar{K}^0\rangle \\
&= \langle n\bar{K}^+; n\bar{K}^+\rangle, \\
\Omega_4 &= \langle n\pi^0; p\pi^-\rangle = -\langle p\pi^0; n\pi^+\rangle = \langle \Sigma^+\pi^0; p\bar{K}^0\rangle \\
&= -\langle \Sigma^-\pi^0; n\bar{K}^+\rangle, \\
\Omega_5 &= \langle \Sigma^0K^0; p\pi^-\rangle = -\langle \Sigma^0K^+; n\pi^+\rangle = -\langle \Sigma^0\pi^+; p\bar{K}^0\rangle \\
&= \langle \Sigma^0\pi^-; n\bar{K}^+\rangle, \\
\Omega_6 &= \langle \Sigma^0K^+, p\pi^-\rangle = \langle \Sigma^+K^0; n\pi^+\rangle = \langle \Xi^0K^+; p\bar{K}^0\rangle \\
&= -\langle \Xi^-K^0; n\bar{K}^+\rangle, \\
\Omega_7 &= \langle pK^+; pK^+\rangle = \langle nK^0; nK^0\rangle, \\
\Omega_8 &= \langle p\bar{K}^+; p\bar{K}^+\rangle = \langle n\bar{K}^0; n\bar{K}^0\rangle, \\
\Omega_9 &= \langle \Sigma^+\pi^-; p\bar{K}^+\rangle = \langle \Sigma^-\pi^+; n\bar{K}^0\rangle = -\langle n\bar{K}^0; p\bar{K}^+\rangle \\
&= -\langle p\bar{K}^+; n\bar{K}^0\rangle, \\
\Omega_{10} &= \langle \Sigma^-\pi^+; p\bar{K}^+\rangle = \langle \Sigma^+\pi^-; n\bar{K}^0\rangle = \langle \Xi^0K^0; p\bar{K}^+\rangle \\
&= -\langle \Xi^-K^+; n\bar{K}^0\rangle, \\
\Omega_{11} &= \langle \Sigma^0\pi^0; p\bar{K}^+\rangle = \langle \Sigma^0\pi^0; n\bar{K}^0\rangle, \\
\Omega_{12} &= \langle \Lambda K^0; p\pi^-\rangle = \langle \Lambda K^+; n\pi^+\rangle = -\langle \Lambda\pi^+; p\bar{K}^0\rangle \\
&= -\langle \Lambda\pi^-; n\bar{K}^+\rangle, \\
\Omega_{13} &= \langle \Lambda\pi^0; p\bar{K}^+\rangle = \langle \Lambda\pi^0; n\bar{K}^0\rangle.
\end{aligned} \tag{32}$$

Due to isotopic spin conservation, there exist certain relations among these four-point functions. These are found, for example, by writing the $\Sigma^0\pi^+$ and $\Sigma^+\pi^0$ in terms of isotopic spin 2 and spin 1 functions and then noting that $p\bar{K}^0$ is a pure spin 1 function. In this manner the following relations may be found:

$$\Omega_4=\Omega_5;\quad \Omega_{12}=-\sqrt{2}\Omega_{13};\quad 2\Omega_{11}-\Omega_{10}-\Omega_9=0;$$
$$\sqrt{2}\Omega_4+\Omega_3-\Omega_1=0;\quad \sqrt{2}\Omega_5+\Omega_6-\Omega_2=0. \quad (33)$$

In order to compare with experiment, it is necessary to first form the matrix elements from these operators and then their cross sections. The formation of the matrix elements involves taking these operators between physical particle states. This implies that the matrix elements will depend explicitly on the masses of the incoming and outgoing particles, the functional form of which is unknown from the present considerations. Therefore, it would seem that a comparison of a set of reactions that are related by the above symmetries but whose incoming or outgoing products do not have corresponding masses is possible only at energies which are sufficiently large so that the mass differences are negligible.

At sufficiently high energies, the three-term relations in Eq. (33) can be used to form the usual triangular inequalities among the magnitudes of the various amplitudes. If we denote the amplitude for the process represented by the operator Ω_i by A_i, then the triangular inequalities are

$$\Delta(2A_{11},A_{10},A_9)\geq 0;\quad \Delta(\sqrt{2}A_4,A_3,A_1)\geq 0;$$
$$\Delta(\sqrt{2}A_5,A_6,A_2)\geq 0, \quad (34)$$

where $\Delta(a,b,c)\geq 0$ means $|a|+|b|-|c|\geq 0$, $|a|-|b|+|c|\geq 0$, and $-|a|+|b|+|c|\geq 0$.

With the present experimental evidence at moderate energies, it seems difficult to establish whether any of these relations are valid or not. However, if an additional assumption is made, it is possible to make an experimental test of the symmetry. Namely, if we assume that the electromagnetic field is introduced into the Lagrangian according to the principle of minimal electromagnetic coupling, we then note that the total Lagrangian, strong plus electromagnetic, is invariant under

$$\text{the rotation}\quad \exp[\pm\tfrac{1}{4}\pi(\Gamma_{15}-\Gamma_{26})] \quad\text{and}\quad A_\mu\to -A_\mu,$$
$$\text{the rotation}\quad [\pm\tfrac{1}{4}\pi(2\Gamma_{37}+\Gamma_{15}+\Gamma_{26})] \quad\text{and}\quad A_\mu\to A_\mu, \quad (35a)$$

since the first rotation takes all charged particles into particles with the opposite charge and neutrals into neutrals, while the second rotation takes particles into other particles of the same charge. In the manner of reference 26, we now consider the three-point functions, say for the Λ, i.e.,

$$\langle T[\bar{\psi}_\Lambda(x_1)\psi_\Lambda(x_2)A_\mu(x_3)]\rangle_0, \quad (36)$$

which is related by a simple linear integral transform[29] to the *e-m* vertex operator for the Λ, symbolized by $\Gamma_\Lambda{}^\mu$. By applying the first transformation, we find that $\Gamma_\Lambda{}^\mu=-\Gamma_\Lambda{}^\mu=0$. In a similar manner we obtain the results 1–5 of reference 26. However, in addition, we obtain the following results

$$\Gamma_n{}^\mu=-\Gamma_{\Xi^0}{}^\mu=-\Gamma_n{}^\mu=0,$$
$$\Gamma_p{}^\mu=\Gamma_{\Sigma^+}{}^\mu=-\Gamma_{\Xi^-}{}^\mu=-\Gamma_{\Sigma^-}{}^\mu. \quad (37)$$

Since these are the total electromagnetic vertex operators, it follows that both the magnetic moment and the charge distributions of the neutron are zero. Experimentally, the charge distribution of the neutron does indeed seem to be zero, but that of the magnetic moment is definitely not. However, two observations are important at this point: (a) This result depends essentially on the use of the principle of minimal electromagnetic coupling. It is conceivable that this principle might not be valid. For example, if the baryons had intrinsic anomalous magnetic moments (Pauli terms), then this principle would be violated, and by the same token, the results we just found would not be valid. (b) The effect of the $K-\pi$ mass difference on the predictions of the $G2$ symmetry is particularly important when evaluating quantities proportional to powers of the momentum transfer q_μ at small values of q^2. In fact, a second order calculation using the $G2$ symmetry and the principle of minimal electromagnetic coupling gives corrections proportional to $(\mu_K-\mu_\mu)/m$, where m is the baryon mass (the $\Sigma-N$ mass difference has been neglected in this calculation). Numerically, one obtains a result $\mu_n\sim -1$ nuclear magnetons. Such large corrections to the predictions as occur in this case would tend to make this a poor process for testing the $G2$ symmetry.

These two observations suggest that no definite conclusions with respect to the validity of $G2$ can be drawn from electromagnetic processes (at least at small momentum transfers) and that it might be preferable to test this symmetry by nonelectromagnetic phenomena.

Other predictions follow from the transformation (35a), namely, $\langle n\pi^0; nA_\mu\rangle=-\langle n\pi_0; nA_\mu\rangle=0$, i.e., photoproduction of π^0's from neutrons is zero. Also, $\langle\Lambda A_\mu;\Sigma^0\rangle=-\langle\Lambda A_\mu;\Sigma^0\rangle=0$, i.e., the decay $\Sigma^0\to\Lambda^0+\gamma$ is forbidden. Again, these disagree with experiment but are subject to the comments given above.

V. WEAK INTERACTIONS—PREDICTIONS

In a previous section, we derived the vector currents which are quasi-conserved in the presence of the strong interactions when both the K and π couplings are present and when only the π couplings are present. We now will make the following assumption: *(iva) The vector part of the currents which appear in the weak*

[29] See, e.g., Y. Takahashi, Nuovo cimento **6**, 371 (1957).

interaction are quasi-conserved in the presence of all strong interactions, both K and π, *as expressed in the Lagrangian of Eq. (26)*; or (*ivb*) *the vector part of the currents which appear in the weak interaction are quasi-conserved in the presence of only the strong* π*-baryon interactions of Eq. (26)*. These alternative assumptions will, of course, give rise to different experimental predictions.

The most general vector part of the current $J_\mu^{(i)}$ under (*iva*) may then be written

$$
\begin{aligned}
J_\mu^{(+2)V} &= \beta_{0V} s_\mu^{(\frac{3}{2},\frac{3}{2})}, \\
J_\mu^{(+1)V} &= i_\mu^{(1,1)} + \rho_{1V}' r_\mu^{(1,1)} + \alpha_V s_\mu^{(\frac{1}{2},\frac{1}{2})} \\
&\qquad + \beta_V s_\mu^{(\frac{3}{2},\frac{1}{2})} + \epsilon_1 l_\mu^{(+1)}, \\
J_\mu^{(0)V} &= \rho_{2V} i_\mu^{(1,0)} + \rho_{2V}' r_\mu^{(1,0)} + \frac{\alpha_V}{\rho_{2V}\sqrt{2}} s_\mu^{(\frac{1}{2},-\frac{1}{2})} \\
&\qquad + \frac{\sqrt{2}}{\rho_{2V}} \beta_V s_\mu^{(\frac{3}{2},-\frac{1}{2})} + \eta_V j_\mu^{(0,0)} + \eta_V' y_\mu^3, \\
J_\mu^{(-1)V} &= \rho_{3V} i_\mu^{(1,-1)} + \rho_{3V}' r_\mu^{(1,-1)} + \frac{\sqrt{3}}{\rho_{3V}} \beta_V s_\mu^{(\frac{3}{2},-\frac{3}{2})} \\
&\qquad - \epsilon_2 l_\mu^{(+1)\dagger},
\end{aligned}
\tag{38}
$$

where these various currents are explicitly listed in the discussion of the symmetries[30] (I) to (V), and the conditions $\eta\beta = \eta'\beta = 0$ are understood. As was seen in the last section, symmetry (III) is definitely not a symmetry of the strong interactions.[27] However, this symmetry must be satisfied in order that $r_\mu^{(1,m)}$ and $s_\mu^{(\frac{1}{2},m)}$ be quasi-conserved. Thus, if we require condition (*iva*) to be satisfied, we are forced to exclude the $r_\mu^{(1,m)}$ and $s_\mu^{(\frac{1}{2},m)}$ currents, i.e., $\alpha_V = 0 = \rho_{iV}$.[31]

The most general vector part of the current $J_\mu^{(i)}$ under assumption (*ivb*) is

$$
\begin{aligned}
J_\mu^{(+1)V} &= \hat{i}_\mu^{(1,1)} + \rho_{1V}' \hat{r}_\mu^{(1,1)} + \alpha_V \hat{s}_\mu^{(\frac{1}{2},\frac{1}{2})} \\
&\qquad + \alpha_V' \hat{s}_\mu^{(\frac{1}{2},\frac{1}{2})\prime} + \epsilon_1 l_\mu^{(+1)}, \\
J_\mu^{(0)V} &= \rho_{2V} \hat{i}_\mu^{(1,0)} + \rho_{2V}' \hat{r}_\mu^{(1,0)} + \frac{1}{\sqrt{2}} \frac{\alpha_V}{\rho_{2V}} \hat{s}_\mu^{(\frac{1}{2},-\frac{1}{2})} \\
&\qquad + \frac{1}{\sqrt{2}} \frac{\alpha_V'}{\rho_{2V}} \hat{s}_\mu^{(\frac{1}{2},-\frac{1}{2})\prime} + \eta_V \hat{j}_\mu^{(0,0)} + \eta_V' \hat{y}_\mu^3 \\
&\qquad + \eta_V'' \hat{t}_\mu^{(0,0)}, \\
J_\mu^{(-1)V} &= \rho_{3V} \hat{i}_\mu^{(1,-1)} + \rho_{3V}' \hat{r}_\mu^{(1,-1)},
\end{aligned}
\tag{39}
$$

where these various currents are given in the discussion of the symmetries (A) to (E). In this case, there are no definite results from the strong interactions which necessitates the exclusion of any of the above currents.

Because $i_\mu^{(1,1)}$ and $\hat{i}_\mu^{(1,1)}$ are the usual quasi-conserved currents which arise from isotopic spin conservation, we obtain the well-known lack of renormalization of the vector part of neutron β decay. For the same reason, both currents will give rise to the decay $\pi^+ \to \pi^0 + e^+ + \nu_1$ at the predicted rate.[1] The other predictions which follow from such a current are valid here also.

For decays not involving leptons, no predictions can be made without further specifying the form of the axial vector part of the current $J_\mu^{(i)A}$. Thus, for example, the absence of an isotopic spin $\frac{1}{2}$ part of the vector current is *not* sufficient to guarantee a lack of asymmetry in the $\Lambda \to N + \pi$ decays.

Let us now turn our attention to strangeness-changing decays which involve leptons. If we neglect terms of relative order $(\mu_K^2 - \mu_\pi^2)/m_N^2$ in the matrix elements, then assumption (*iva*) permits the prediction of the ratio of the K_{e3} and $K_{\mu 3}$ modes of decay, valid to all orders in the strong coupling. Sugawara[32] has shown that this ratio is

$$R(K^+ \to \pi^0 + e^+ + \nu_1)/R(K^+ \to \pi^0 + \mu^+ + \bar{\nu}_2) = 1.55. \tag{40}$$

This result is independent of the relative strengths of the strangeness-changing currents, α and β. By using the Gell-Mann and Rosenfeld[33] averages, the experimental value is 1.04 ± 0.22.

It also follows, when this current is quasi-conserved under assumption (*va*), that we can obtain the absolute rate, $R(K^+ \to \pi^0 + e^+ + \nu_1)$, as a function of $\beta\epsilon_1$, independently of any closed loop diagrams involving the strong couplings. The analogous case of $\pi^+ \to \pi^0 + e^+ + \nu_1$ was discussed by Feynman and Gell-Mann.[1] The rate is, up to terms of relative order $(\mu_K^2 - \mu_\pi^2)/m_N^2$ and further neglecting the electron mass,[34]

$$
\begin{aligned}
&R(K^+ \to \pi^0 + e^+ + \nu_1) \\
&\qquad = [(4\beta^2 \epsilon_1^2 G^2/(2\pi)^3] \mu_K (\mu_{\pi^0})^4 (0.54).
\end{aligned}
\tag{41}
$$

Substituting the observed rate, we obtain

$$(\epsilon_1\beta)^2 \approx 1.2 \times 10^{-2}. \tag{42}$$

By using this value of $(\epsilon_1\beta)^2$, it is possible to obtain a lower limit on the rate for the decay $\Sigma^- \to n + e^- + \bar{\nu}_1$. If in this decay we neglect the mass of the electron, it is well known that there is no interference between the vector and the axial vector contributions to the total rate, so that the pure vector part provides a lower

[30] F. Gürsey has independently discussed the possible existence of a quasi-conserved $I = \frac{3}{2}$ current [private communication and Ann. Phys. (to be published)].

[31] It is perhaps interesting to note that if the $I = \frac{1}{2}$ currents were accepted and the seven dimensional rotation symmetry were, in some sense, a symmetry of the strong interactions, one could determine α and β from the ratio of the $K^+ \to \pi^0 + e^+ + \nu_1$ and $K^0 \to \pi^- + e^+ + \nu_1$ decays and from their absolute rates. If it were further assumed that in $(\bar{p}\Lambda)$ the vector and axial vector parts enter in the combination $1 \pm \gamma_5$ in the effective matrix element, one would obtain a prediction for the β decay of the Λ of about 1.2 parts in a thousand, which is consistent with experiment.

[32] M. Sugawara, Phys. Rev. **112**, 2128 (1958).

[33] M. Gell-Mann and A. H. Rosenfeld, Annual Review of Nuclear Science (Annual Reviews, Inc., Palo Alto, California, 1957), Vol. 7, p. 407.

[34] J. C. Pati, S. Oneda, and B. Sakita (to be published).

limit. We obtain[35]

$$R(\Sigma^- \to n+e^-+\bar{\nu}_1)/R(\Sigma^- \to n+\pi^-) > 1.7\times10^{-4} \quad (43)$$

for $\tau_{\Sigma^-}=1.6\times10^{-10}$ sec. Similar results can be calculated for the decays $\Xi^- \to \Sigma^0+e^-+\bar{\nu}_1$ and $\Xi^0 \to \Sigma^+ +e^-+\bar{\nu}_1$.

We again point out that the pure $I=\frac{3}{2}$ current, which arises from assumption (*iva*), will lead to the predictions of Eq. (17b), which appear, at present, to be in disagreement with experiment. However, this result implies the additional assumption $\epsilon_2=0$ which can be investigated independently [see the discussion after Eq. (15)].

For the condition (*ivb*), we have the prediction of Eq. (17a), and the absence of decays with $\Delta Q=\Delta S$. Without making an additional assumption concerning the form of the axial vector part of the current, no further predictions for assumption (*ivb*) are feasible.

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APPENDIX

In this Appendix, we illustrate the fact that the relationships implied by the $|\Delta I|=\frac{1}{2}$ rule can be readily obtained by means of Eqs. (5a) and (5b). Let us, for example, consider the decays $\Sigma \to N+\pi$. We write the matrix element

$$M_1=\langle n\pi^-|I_-\mathcal{L}_w{}^sI_+|\Sigma^-\rangle = \langle (1/\sqrt{2})p\pi^- + n\pi^0|\mathcal{L}_w{}^s|\Sigma^0\rangle. \quad (A.1)$$

As I_- commutes with $\mathcal{L}_w{}^s$, we also have

$$M_1=\langle n\pi^-|\mathcal{L}_w{}^s\tfrac{1}{2}(I^2-I_3{}^2-I_3)|\Sigma^-\rangle=\langle n\pi^-|\mathcal{L}_w{}^s|\Sigma^-\rangle. \quad (A.2)$$

We consider now

$$M_2=\langle (1/\sqrt{2})p\pi^- + n\pi^0|I_-\mathcal{L}_w{}^sI_+|\Sigma^0\rangle = -\langle\sqrt{2}p\pi^0 - n\pi^+|\mathcal{L}_w{}^s|\Sigma^+\rangle. \quad (A.3)$$

On the other hand, by using Eq. (7a) again,

$$M_2=\langle (1/\sqrt{2})p\pi^- + n\pi^0|\mathcal{L}_w{}^s|\Sigma^0\rangle = M_1. \quad (A.4)$$

Thus, we obtain

$$\langle n\pi^-|\mathcal{L}_w{}^s|\Sigma^-\rangle = -\sqrt{2}\langle p\pi^0|\mathcal{L}_w{}^s|\Sigma^+\rangle + \langle n\pi^+|\mathcal{L}_w{}^s|\Sigma^+\rangle, \quad (A.5)$$

which is the well-known relation of the $|\Delta I|=\frac{1}{2}$ rule.

[35] R. E. Behrends and C. Fronsdal, Phys. Rev. **106**, 345 (1957).