approximation A has led to an $F_5(\xi)$ in *apparently* good agreement with the exact F_5. On the other hand, the lowest order perturbation theory approximation reproduces F_5 exactly. Yet one would rather not say that therefore perturbation theory is more accurate than dispersion theory plus approximation A. It is at least satisfying that approximation A does not lead to an answer in gross disagreement with the exact result. However, note that although the disagreement between Eqs. (30) and (55) is slight, it is precisely such a slight difference, which, if it occurred in the case of F_4, could have led to a finite result for both δm and Z_2^{-1}. The main conclusion to be drawn from this may be that with the use of approximation A, the dispersion relation technique is unreliable in the calculation of high-energy effects.

Within the framework of our approximations

$$F_5(\xi)=-\frac{e}{\xi+m^2}\left(\frac{m^2}{\xi+m^2}\right)^{\alpha/8\pi} \quad \text{for} \quad |\xi|\gg m^2.$$

Yet it is an exact consequence of the gauge invariance of electrodynamics that $F_5=-e/(\xi+m^2)$. Therefore, our method of choosing relevant intermediate states in Eq. (33) has destroyed this symmetry of the full theory. Unfortunately, we do not see what finite set of intermediate states would be sufficient to maintain the gauge invariance of the theory in the approximation.

If we were optimistic about these difficulties we should note that the δm is infinite because we have used approximation B. It is easy to convince oneself that a slight modification of the Compton scattering amplitude at high energies would be sufficient to produce an integral equation for F_4 and thus a possible damping of the pole term in Eq. (52) as $-\xi\rightarrow\infty$.

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$K_2{}^0$ and the Equivalence Principle*†

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It is shown that the existence of the long-lived neutral K meson, and the absence of its decay into two pions, establishes that the gravitational masses of the K^0 and $\bar{K}^0$ are equal to a few parts in 10^{-10} of the K inertial mass. This is of interest since the $\bar{K}^0$ is the antiparticle of the K^0, and is not identical with the K^0. The gravitational mass of such a nonidentical antiparticle has never been directly measured.

Also, the $\bar{K}^0$ has opposite strangeness to the K^0. Thus the argument rules out any linear dependence of the gravitational mass on the strangeness quantum number, a point on which all previous experiments say nothing.

These observations are in accord with, and serve as a confirmation of, the equivalence principle of Einstein.

SINCE the discovery of antinucleons, the interesting possibility that antimatter may have gravitational mass opposite in sign to its inertial mass has been widely discussed.[1–3] Although such a possibility would necessarily involve major modifications in present theoretical ideas,[2] it is generally regarded as something to be settled by experiment.

Schiff[4] has recently put forth considerable evidence against the antigravity idea, by showing that negative gravitational mass of the positrons in the virtual pairs of the Coulomb field of the nucleus would very likely (i.e., barring fortuitous cancellations) produce an observable effect in the Eötvös experiment.[5] The argument is necessarily somewhat indirect, since the antiparticles are virtual rather than real. In any case, it is useful to extend the proof to other types of particles.

We consider here the effect of gravity on the $K_2{}^0$, and show that it affords a direct measurement of the difference between the gravitational mass of a particular particle, the K^0, and the gravitational mass of its antiparticle, the $\bar{K}^0$. We conclude that this difference is zero, to an accuracy of a few $\times10^{-10}M_K$. This is in disagreement with the antigravity hypothesis, and instead affords an extremely precise check, in a new context, of the equivalence principle of Einstein.

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[1] P. Morrison and T. Gold, *Essays on Gravity* (Gravity Research Foundation, New Boston, New Hampshire, 1957), p. 45.

[2] P. Morrison, Am. J. Phys. **26**, 358 (1958).

[3] D. Matz and F. A. Kaempffer, Bull. Am. Phys. Soc. **3**, 317 (1959).

[4] L. I. Schiff, Phys. Rev. Letters **1**, 254 (1958); for a more complete account, see Proc. Nat. Acad. Sci. **45**, 69 (1959).

[5] R. V. Eötvös, D. Pekar, and E. Fekete, Ann. Physik **68**, 11 (1922).

The K_2^0 is a coherent linear combination of particle and antiparticle states.[6] It therefore forms a sort of natural interferometer, for investigating the gravitational masses of the K^0 and $\bar{K}^0$.

We begin by noting that the K_2^0, whether because of CP invariance or for other reasons, experimentally does not decay into the $\pi^+ + \pi^-$ mode characteristic of the K_1^0. For any other linear combination of K^0 and $\bar{K}^0$ there will be a K_1^0 component and a finite rate for decay into two pions: the K_2^0 is just that linear combination which cannot decay into two pions.

Let us now consider the effect if the K_2^0 were placed in a gravitational potential, ϕ. The K^0 component of the K_2^0 would have an increment $+M\phi/\hbar$ added to its De Broglie frequency (where M is the inertial mass of the K); but the $\bar{K}^0$ frequency would have $-M\phi/\hbar$ added to it, under the antigravity assumption. The K_2^0 would therefore no longer be an eigenstate of the system, but would periodically turn into a K_1^0, the frequency of the mixing being $\omega_m = 2M\phi/\hbar$. The system would still have two eigenstates, but both would now be capable of decaying into two pions; in fact, if the mixing frequency were large compared with $1/\tau_1$ (where τ_1 is the K_1^0 mean lifetime), the long-lived neutral K meson would cease to exist as a particle; both eigenstates would be shortlived, because of the intrinsic strength of the two-pion decay interaction. (In this limit the largest terms in the decay matrix are the terms $\pm M\phi/\hbar$, which are associated with K^0, $\bar{K}^0$, respectively; thus the decay matrix in the K^0, $\bar{K}^0$ representation is already "almost diagonal." The eigenstates in this limit are, therefore, to good approximation, K^0 and $\bar{K}^0$. Both would decay into two pions at the same rate, by the CPT theorem; experimentally, this rate is very large.)

The size of the effect is determined by the ratio of $2M\phi$ to $\hbar/\tau_1$. The latter is about 7×10^{-6} ev. The former, if we take for ϕ the gravitational potential of the earth, is about 0.7 ev, five orders of magnitude larger.[7]

Therefore, under the antigravity assumption, the K_2^0 would not exist as a particle, in disagreement with experiment.

Since the hypothetical effect is so large, we had best inquire further whether the inclusion of the gravitational term is indeed necessary. For this purpose consider the following Gedanken experiment: imagine a K^0 produced at rest, at the surface of the earth; let us then wait several K_1^0 mean lives, so that we have a K_2^0. Let us suppose further that the K_2^0 is stable against decay into two pions. Now imagine the particle to be raised, by some external agency, a distance h above its original position, and then being brought to rest. Our device has then done work Mgh on the K^0; and under the antigravity hypothesis, has had work Mgh done on it by the $\bar{K}^0$. The energies must now differ by $2Mgh$; and if the K^0 and $\bar{K}^0$ were at first degenerate, they would not be so after being raised.

The inclusion of the gravitational term is seen to be quite inescapable. Further, the De Broglie oscillations, unobservable for most particles, *are* observable in the $K^0 - \bar{K}^0$ system; for instance, the Fry-Sachs scheme for measuring the $K_1^0 - K_2^0$ mass difference involves just such an observation.

From where, then, shall ϕ be measured? If the earth were the only body in the universe, K^0 and $\bar{K}^0$ would be degenerate at an infinite distance from the earth, since there the influence of the earth would vanish. Therefore the only sensible choice would be $\phi = 0$ at infinity.

This makes it clear that it is an absolute potential we are dealing with, in the sense that we cannot add an arbitrary constant to ϕ. This is a concept foreign to physics. However, we cannot rule out antigravity on this ground alone. That absolute potentials never occur in electromagnetism, for example, is a part of gauge invariance. Now charge conservation follows from gauge invariance; In the $K^0 - \bar{K}^0$ system with antigravity, "gravitational charge" (i.e., gravitational mass) would *not* be conserved. The transition $K^0 \rightleftarrows \bar{K}^0$, brought about by the weak interactions, would violate it. The physical situation therefore could not be gauge-invariant, and an absolute potential could result.

We must also consider the following objection: if we think of the gravitational energy $Mg\phi$ as being stored in gravitational fields, then $Mg\phi_{\text{earth}}$ is stored over a region several earth radii in size. Since even the K_2^0 lives less than 10^{-7} sec, there is insufficient time for this large region to communicate with the particle during its brief existence. Therefore a newly born $\bar{K}^0$ (made by $K^0 \to 2\pi \to \bar{K}^0$) would not yet know that it was supposed to oscillate at a different frequency from that of the K^0, because the energy stored in the field at large distances would not yet have had time to change.

It seems to us that the reply must be that if energy is to be conserved, then, when $K^0 \to 2\pi \to \bar{K}^0$ occurs (in a theory with antigravity), a gravitational disturbance must originate at the particle, and spread out from it at the velocity of light. This disturbance must carry with it an amount of energy $(M_K - M_{\bar{K}})\phi$, which energy is then redistributed throughout the field as the solution approaches the static one we have discussed. Only in this way can the total energy remain independent of time, as it must be if energy is to be conserved.

A moments reflection will show that similar things occur in simply moving a massive object on the surface of the earth; one can lift a weight from one height to another in say a few milliseconds (without radiating any appreciable fraction of the energy in gravitational waves) but this is a time short compared to the time required for a signal to propagate several earth radii. Therefore, in this familiar case, a similar energy-conserving, nonradiative, disturbance must propagate

[6] M. Gell-Mann and A. Pais, Phys. Rev. **97**, 1387 (1955).
[7] We are accustomed to thinking of gravitational effects as weak. However, this is a coherent effect of all the particles in the Earth.

outward from the moved object and die out as the field energy is redistributed by it.

It is not our problem to see how a theory of antigravitation might contrive to satisfy the requirements of energy conservation and causality. We only say, if it does, then the K_2^0 must behave in the way we have described.

We conclude, then, that the existence of the K_2^0 destroys the antigravity hypothesis, at least for $\bar{K}^0$ mesons.

This being the case, we ask, instead: to within what accuracy are the K^0 and $\bar{K}^0$ gravitational masses equal, as shown by the experiments?

We are thus concerned now with the case $\omega_m \ll 1/\tau_1$. In this limit, a straightforward calculation shows that the ratio of two pion decays, induced by ω_m in the long-lived component, to the normal three-body decays, is

$$\eta = \frac{\epsilon^2\phi^2\tau_1\tau_2}{\hbar^2|1+i2\Delta\tau_1|^2}, \tag{1}$$

where ϵ is the difference in the gravitational masses of the K^0, $\bar{K}^0$, and Δ is the $K_1^0-K_2^0$ mass difference frequency. Solving for ϵ, we have

$$\epsilon = \frac{\eta^{\frac{1}{2}}\hbar|1+2i\Delta\tau_1|}{\phi(\tau_1\tau_2)^{\frac{1}{2}}}. \tag{2}$$

Experimentally, $\eta \leqslant 10^{-2}$.[8]

For ϕ, we may write

$$\phi = \phi_e + \phi_s + \phi_g + \phi_u + C, \tag{3}$$

where the first four terms are the contributions of the earth, the sun, our galaxy, and the rest of the universe, respectively. C is a nonarbitrary constant, to be discussed shortly.

The terms ϕ_e, ϕ_s, ϕ_g are defined to be zero at infinity, as discussed earlier for ϕ_e. The term ϕ_u we would like to define in the same way, but we are faced with the conceptual difficulty that we cannot "step outside the universe" to do so. Another way of saying this is that a single constant provided by a cosmological theory, might have to be added to ϕ; this is why we have written the last term, C, into Eq. (3).

Now we would not expect C to cancel out all the other terms, including ϕ_e; this would be a return to a geocentric universe. Likewise we would not expect it to cancel ϕ_s, or even ϕ_g; the sun and the galaxy are tiny local specks in the universe. But we cannot rule out that, in a future cosmology, C might cancel ϕ_u. This is not at all an academic point, as may be seen from Table I, which displays the K gravitational potential energies, and the corresponding limits on ϵ, as calculated from Eqs. (2) and (3). It is seen that the successively larger bodies produce successively larger effects. The writer feels that the limit set by the galaxy is the proper one to use, because of the cosmological uncertainty just referred to. We conclude, then, that the K^0 and $\bar{K}^0$ have the same gravitational mass to within a few parts in $10^{-10}M_K$ (for $\Delta\sim 1/\tau_1$). This is the result expected from the equivalence principle of Einstein, which asserts that gravitational mass and inertial mass are equal.

TABLE I. K^0 gravitational potentials and the corresponding limits on the difference in gravitational mass between the K^0 and $\bar{K}^0$.

Body	K^0 potential energy	$\epsilon/(M_k|1+2i\Delta\tau_1|)$
Earth	0.4 ev	$\leqslant 7\times10^{-8}$
Sun	6 ev	$\leqslant 5\times10^{-9}$
Galaxy	300 ev	$\leqslant 10^{-10}$
Universe	5 to 500 Mev	$\leqslant 10^{-14}$ to 10^{-16}

Under this assumption, no absolute potential is needed.

This result also rules out, within the stated accuracy, the possibility that the gravitational mass of all particles might have a term linear in the strangeness, i.e.,

$$M_g = M_i + \tfrac{1}{2}\epsilon S, \tag{4}$$

where M_g = gravitational mass, M_i = inertial mass.[9]

Such a term would have escaped detection in the Eötvös experiment, since $S=0$ for the stable matter used in the experiment. Thus we can say that strange particles have a gravitational mass that is independent of the strangeness, to a few parts in 10^{-10} of the inertial mass.

The arguments presented here have nothing to do with the interesting question of whether the weak interactions obey the equivalence principle; rather the weak interactions of the K^0 are here used only as a probe to observe whether the strong interactions, responsible for the greater part of the mass, obey the equivalence principle.

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[8] L. M. Lederman, *1958 Annual International Conference on High-Energy Physics at CERN*, edited by B. Ferretti (CERN Scientific Information Service, Geneva, 1958), p. 275.

[9] We are indebted to H. Lewis for this observation.