

low-geometry counter,⁹ and its geometry had been determined to be 0.00366%. Repetition of the astatine count established that no appreciable amount of astatine was evaporating in the counter.

Immediately after it had been counted, the plate was covered with 1 mil cellophane to prevent the loss of Bi²⁰⁷ recoils. After twenty-five astatine half-lives the gamma spectrum of the sample was measured. The cover was then removed, and the sample was counted in an alpha proportional counter of 50.9% geometry which had also been calibrated against a high precision counter. The count obtained was that expected from the decay of pure At²¹¹ (apparent half-life: 7.248 hr). Alpha pulse analysis at this time detected no foreign activity, and less than one disintegration per minute of long-lived activity remained after the astatine had entirely decayed.

The gamma spectrum was taken with the sample mounted 1.0 cm below the face of a 3-in. diameter by 3 in. deep cylindrical thallium-activated sodium iodide crystal. A "Penco" Model IV 100-channel pulse-height analyzer was used. Coincidence corrections were made using the direct-reading dead-time meter of the instrument.

The spectrum obtained was characteristic of Bi²⁰⁷.¹⁰ The area under the 569-keV photopeak was measured after subtracting off the continuum resulting from scattering of more energetic gamma rays, the correction amounting to about 20%. The counting efficiency of this photopeak was taken to be 35.6% of the physical geometry of the counter.¹⁰

The physical geometry of the gamma counter was determined from the observed ratio of alpha to 60-keV gamma activity of a sample of chemically and radiochemically pure Am²⁴¹, using Magnusson's¹¹ intensity data. The alpha activity of the americium was determined in the calibrated proportional counter described previously. The gamma spectrum was measured first without added absorber and then through 130 mg/cm² of lead to eliminate x rays and the 26-keV gamma. In this way the fractional contribution of the escape peak of the 60-keV gamma could be evaluated. Using Allen's¹² absorption data, the absorber present in the crystal housing was determined from the attenuation of the x rays and the 26-keV gamma to be equivalent to 550 mg/cm² of aluminum. Absorption corrections were applied to both the americium and bismuth spectra. Allowance for this thickness of aluminum was also included in the sample-to-crystal distance. From the americium data the physical geometry of the counter

⁹ H. P. Robinson, National Research Council Publication No. 573, 1958 (unpublished).

¹⁰ R. L. Heath, Atomic Energy Commission Research and Development Report IDO-16408, TID-4500, Ed. 13, 1957 (unpublished).

¹¹ L. B. Magnusson, Phys. Rev. **107**, 161 (1957).

¹² S. J. M. Allen, *Handbook of Chemistry and Physics* (Chemical Rubber Publishing Company, Cleveland, Ohio, 1952), p. 2235.

TABLE I. Half-life of At²¹¹, in hours.

7.239±0.011
7.259±0.020
7.212±0.0068
7.191±0.0095
7.196±0.0086
7.215±0.0029
7.216±0.0050
7.196±0.018
7.213±0.013
Weighted mean: 7.214±0.0035
95% confidence level: ±0.007

was calculated to be 37.3%. The theoretical value for a point source at exactly 1 cm distance is 37.1%.¹⁰

The 1.06- and 1.77-MeV gamma rays of Bi²⁰⁷ have respective abundances of 79% and 9%,⁶ and their respective total absolute counting efficiencies in our counter were 18.2% and 15.6%.¹⁰ Since these rays are coincident with the 569-keV gamma ray,⁶ we may calculate that 15.8% of the intensity of the latter has been lost due to "coincidence summing."¹⁰

RESULTS

The results of nine determinations of the half-life of At²¹¹ appear in Table I. Uncertainties are standard deviations unless otherwise specified. The uncertainty of each determination arises from the scatter of the points in the least-squares analysis. It reflects the length of time the sample was followed and the length and frequency of the individual counts in the determination. The uncertainty of the weighted mean was computed from the deviations of the individual values from the mean.¹³ It is 1.7±0.4 times that which would be calculated solely from the individual uncertainties.¹³ Hence the individual uncertainties are probably reasonably indicative of the reliability of the half-life values to which they pertain.

The absolute ratio of initial At²¹¹+Po²¹¹ alpha activity to residual Bi²⁰⁷ 569-keV gamma activity was found to be 1.16×10⁵. Using our 7.214-hr half-life for At²¹¹, a 40.9% abundance for its alpha branch,² and a 2.2% total conversion coefficient for the 569-keV Bi²⁰⁷ gamma ray,⁶ we shall set the half-life of Bi²⁰⁷ at 38±3 yr. The uncertainty has been estimated at the 95% confidence level by assigning a 5% uncertainty to each of the three principal sources of error: integration of the Bi²⁰⁷ gamma peak, determination of the physical geometry of the gamma counter, and evaluation of the counting efficiency of the Bi²⁰⁷ gamma ray.

DISCUSSION

Our half-life for At²¹¹ is in perfect agreement with Gray's¹⁴ value of 7.20±0.05 hr, and in reasonable

¹³ Raymond T. Birge, Phys. Rev. **40**, 207 (1932).

¹⁴ P. R. Gray, Phys. Rev. **101**, 1306 (1956).

agreement with the 7.5-hr value originally reported by the discoverers of the element.¹⁵ Our half-life for Bi²⁰⁷ agrees well with the lower limit set by Neumann and Perlman,² but is significantly greater than the values reported by Harbottle⁵ and by Sosniak and Bell.⁴ The reason for this discrepancy is not immediately apparent. The 8-year half-life obtained by Cheng *et al.*³ is completely out of line, and it is difficult to believe that it pertains to the same isotope.

¹⁵ D. Corson, K. MacKenzie, and E. Segrè, Phys. Rev. 57, 459 (1940).

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Properties of Finite Nuclei

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The properties of O¹⁶, Ca⁴⁰, and Zr⁹⁰ have been determined, using the Brueckner-Gammel-Weitzner theory of finite nuclei. Self-consistent solutions of the Hartree-Fock equations as modified by Brueckner and Goldman have been obtained. The properties computed include binding energy, mean proton and neutron radii, separation energies, spin-orbit splittings, nonlocal and state-dependent single-particle potentials, surface depth of density and potentials, potential-density relation. The predictions of the theory are in semiquantitative agreement with experiment.

I. INTRODUCTION

IN a series of previous papers,¹ methods have been developed for the study of many-fermion systems and applied in detail to the determination of the properties of nuclear matter. Approximate extensions of these methods to the study of finite nuclei have also been proposed.² It is the purpose of this paper to review briefly the formulation and present status of the methods as applied to nuclear matter, to describe the theory of finite nuclei, and to give the results of a numerical study of the properties of finite nuclei.

II. NUCLEAR MATTER

In this section we shall not attempt to review the basis of the procedures used in the study of nuclear

matter, but give only a brief summary of the present status of the methods.

The results of this paper are based on the *K*-matrix approximation¹ for the ground-state energy of nuclear matter, which is

$$E = \sum_i \frac{p_i^2}{2m} + \frac{1}{2} \sum_{ij} (K_{ij,ij} - K_{ij,ji}). \quad (2.1)$$

The *K* matrix determines the interaction of pairs of nucleons moving in the nuclear medium, taking account of the exclusion principle and the binding effects of the average nuclear field. The determining equation for *K* is

$$K = v + vGK, \quad (2.2)$$

in which the propagator *G* describes motion in the nuclear field. Taking matrix elements of Eq. (2.2) with respect to the eigenstates of the unperturbed medium, we rewrite Eq. (2.2) as

$$K_{kl,ij} = v_{kl,ij} + \sum_{mn} v_{kl,mn} G_{mn}(ij) K_{mn,ij}, \quad (2.3)$$

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¹ See K. A. Brueckner and J. L. Gammel, Phys. Rev. 109, 1023 (1958), for a list of references.

² K. A. Brueckner, J. L. Gammel, and H. Weitzner, Phys. Rev. 110, 431 (1958); R. J. Eden and V. J. Emery, Proc. Roy. Soc. (London) A248, 266 (1958); and R. J. Eden, V. J. Emery, and S. Sampanthar, Proc. Roy. Soc. (London) A253, 177 (1959).