Photoproduction and Detection of the Two-Meson Bound State*†

JACK L. URETSKY AND T. R. PALFREY, JR.
Purdue University, Lafayette, Indiana

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We investigate the possibility of the detection of a photoproduced two-pion bound system. The general detection problem is discussed briefly; the branching ratio between $2\pi^0$ and $2\gamma^0$ decay modes is calculated; and the total photoproduction cross section is estimated in terms of the binding energy of the two-pion state both by field-theoretic and by phase-space arguments. We conclude that if the binding energy is of the order of 10 Mev the state should be detectable in photoproduction experiments, and the binding energy should be measurable.

I. INTRODUCTION

FLEETINGLY, from time to time, there must exist in nature a particle that is a bound state of two π mesons. The necessity for the existence of such a particle is a trivial consequence of our faith in the efficacy of Coulomb forces at not-too-small distances. It is pertinent to the point of this paper that such a particle has never been observed, and we shall find, in the course of our inquiry, that the lack of observability is directly traceable to the weak binding that is expected to characterize such a mesonic atom.

If one accepts this conclusion, then an interesting question presents itself. Suppose that, as a consequence of the mediation of short-range forces, the binding energy of the bi-mesonic particle is somewhat greater than one would predict from consideration of purely Coulomb effects. How large does the binding energy have to be in order for the particle to be detectable with present day apparatus? We attempt to answer this question in the context of a rather specific model.

If the binding energy of our composite particle is small it does not seem unreasonable to suppose that it has the same ground state quantum numbers that one would expect from purely electrostatic interactions. In the absence of any specific knowledge of the mesonic forces involved we shall, then, make the provisional assumption that we are dealing with a spin zero particle. Further, if we suppose that the binding energy is less than the charged-neutral pion mass difference it would be possible for the $\pi^+\pi^-$ combination to have a bound state while the other pion pair combinations are devoid of bound states. If, for example, the $\pi^+\pi^-$ is bound by a very few Mev then there would be a decay mode into a neutral pion pair because of the neutral π-charged π mass difference. Two-photon decay is possible of course, regardless of the magnitude of the binding energy.

It is clear that we are considering at this point a neutral spin-zero particle, to be denoted by the symbol, b^0. We suspect that this particle is extremely short lived. For example, a quantum electrodynamic perturbation-theoretic calculation of the lifetime of the Coulombic ground state before it decays into two photons gives 10^{-12} second as the result, and the b^0 could be expected to have a more transitory existence by several orders of magnitude. In short, it would appear that the b^0 is exceedingly difficult to detect. The possibility that experiments already conducted would have established its existence is discussed in the concluding section.

The property of the b^0 that makes its detection possible with present day experimental techniques is its decay mode into two photons whose energy, in the reference system where the decaying particle is at rest, is unique. To utilize this property we have chosen to study an experiment in which the b^0 is photoproduced from a proton, and a decay photon is then detected in coincidence with the recoil proton. Because there are two possible fast decay modes ($2\pi^0$ and 2γ) for the b^0 the interesting quantity to the experimenter will be an *effective* cross section σ_{eff}, which we define to be the total photoproduction cross section σ_b, multiplied by the probability of decaying into two photons.

In Sec. II we discuss the experimental details and estimate the minimum value of σ_{eff} that we can hope to measure. Section III is devoted to estimating the branching ratio for the two decay modes. In Sec. IV we obtain a relationship between the b^0 cross section and the total cross section for producing a π^+-π^- pair at the same incident photon energy, thus permitting us to make a numerical estimate of σ_b. This section makes use of a field-theoretic formalism and a statistical model for pion-pair photoproduction. Section V is devoted to showing that the essential results of the previous section may be obtained from simple intuitive arguments and a phase space calculation. The last summarizes our results.

II. DETECTABILITY OF PHOTOPRODUCTION CROSS SECTION

The decay of the b^0 into two gamma rays provides a method of distinguishing it from other particles or combinations of particles. As an example we examine in this section the problem of the detection of photoproduced b^0 in the reaction:

$$\gamma+p\rightarrow b^0+p, \tag{1}$$

with subsequent decay of the b^0 by its two-gamma mode,

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First we compute the approximate counting rates for detection of the photoproduction event, and then discuss confusable backgrounds. The proposed method of detection is to count a proton in coincidence with a single high-energy gamma ray whose energy is approximately measured (e.g., in a total absorption Čerenkov detector). This detection method is chosen as that of highest efficiency which still gives a chance of distinguishing the background events described in Eqs. (2)–(4).

Because the production of two pions by photons from hydrogen peaks at about 600 Mev, it is probably advantageous to do the measurements in that energy region. The kinematics of the b^0 production by 600-Mev photons at 90° c.m. angle are about as follows: Recoil proton energy and angle, 140 Mev, 37° lab; b^0 energy and angle, 180 Mev, 61° lab. A decay gamma ray moving in the direction of the b^0 will have a laboratory energy of about 420 Mev.

If, for example, one detects the protons produced by those gamma-rays lying within 20 Mev of the tip of the bremsstrahlung spectrum, uses a proton solid angle in the laboratory of about 0.05 steradian and a gamma-ray counter whose geometrical efficiency is about 10%, then the number of detected b^0 per equivalent quantum will be about $2Nt\sigma_{\text{eff}}\times 10^{-5}$. Nt is the target thickness in atoms per cm^2, σ_{eff} is the effective total cross section for photoproduction of the b^0, as defined in the first section (here assumed to be isotropic in the c.m. system). If one sets one count per hour as a reasonable lower limit for acceptable counting rates, and assumes a beam of 10^{11} equivalent quanta per minute, then the lowest detectable σ_{eff} is of the order of 10^{-32} cm^2 with a 5-in. liquid hydrogen target.

Confusable reactions which must be distinguished from b^0 photoproduction are the following:

$$\gamma+p\rightarrow\gamma'+p', \tag{2}$$

$$\gamma+p\rightarrow p+\pi^0+\pi^0, \tag{3}$$

$$\gamma+p\rightarrow p+\pi^0. \tag{4}$$

The first of these, proton Compton effect, can be eliminated on the basis of kinematics. A photon of 600-Mev incident energy scattering into the photon detector will give insufficient recoil energy to the proton for it to be detected in the proton telescope.

Reaction 3, the simultaneous production of two neutral pions, can be eliminated by requiring a high-energy loss in the photon counter. In the unlikely event that both neutral pions come off at the angle at which the b^0 is to be detected, *and* that two of the four decay gamma rays from the neutral pions are detected in the counter, the process becomes indistinguishable from b^0 photoproduction. It is to be noted that the efficiency for detection in this case goes as the square of the gamma-ray counter efficiency, and is hence highly reduced.

Reaction 4, single neutral pion production, although kinematically distinguishable from b^0 production, gives rise to high counting rates in both the proton counter and the photon counter, and it might be supposed that it would lead to a prohibitively high chance coincidence rate. This effect was calculated for the target and detector configuration described above, with the further assumption that the accelerator used had about a 3% beam duty cycle. Under these conditions the chance coincidence rate, typically, would be equal to the b^0 rate if σ_{eff} were on the order of 10^{-33} cm^2. This, therefore, would only be a serious limitation if more intense beams were available, in which case it would be necessary to use a two-photon detection system in coincidence with the proton counter.

III. BRANCHING RATIO

We have already remarked that the b^0 has two alternative modes of decay, and it is apparent from the previous section that our ability to detect the particle is critically dependent upon the two photon decay probability. We shall estimate this quantity by calculating the branching ratio $(2\pi^0/2\gamma)$ for the bi-mesonic atom. We then argue that a weakly bound b^0 should have the same branching ratio.

We may support this contention by arguing that the decay rate for either mode is made up of some matrix element multiplied by the square of the bound-state wave function for zero separation of the constituent particles. Thus, the expressions for the decay rates of a bi-mesonic atom and a b^0 (in a particular mode) are identical except for the bound-state wave function. In a branching ratio calculation the dependence upon the wave function cancels.

The same argument may be made in field theoretic language. The photon decay diagram (for example) is made up of a b^0-two pion vertex and a two pion-two photon vertex joined by two intermediate pion lines. The neutral pion decay diagram is identical except that the final vertex is a π-π scattering vertex. Thus, the *ratio* of decay rates is independent of the first vertex which, we shall see later, is just the bound-state wave function (in momentum space) of the b^0. The requirement that the b^0 be weakly bound limits the intermediate states that must be considered to the two-pion state.

The electromagnetic decay of the bi-mesonic atom is calculated in lowest order perturbation theory which is to say that the strong interactions of the pions are neglected. If there is an enhancement of the decay resulting from the strong interactions, then we are underestimating the decay rate. This would be satisfactory for our purposes. In any event, the analogous calculation for neutral pion decay suggests that we may not be badly in error.

If we use the Lorentz gauge for the calculation, then the only contribution to the matrix element is from the

ϕ^2A^2 term in the Lagrangian (the contact term). The result, except for a factor of 2 resulting from the even parity of the atomic ground state is formally identical to the formula for the ^1S decay of positronium[1]

$$\omega_\gamma = (\pi\alpha^2/8\mu^2)\,|f(0)|^2, \tag{5}$$

where ω is the decay rate, $f(0)$ the wave function of atom at the origin, α the fine-structure constant, and μ the mass of the π meson.

The decay into neutral pions may be analyzed in terms of the scattering process $(\pi^+\pi^- \to 2\pi^0)$ at zero energy for the incident particles. The scattering is then completely determined by the π-π scattering lengths and the π^+-π^0 mass difference. The analysis follows very closely upon the lines laid out by Jackson, Ravenhall, and Wyld[2] in analyzing k-meson nucleon scattering except that we have fewer channels of the scattering matrix to deal with. Again we quote only the result which is

$$\omega_\pi = \frac{8\pi}{9}\frac{\Delta\mu}{\mu}|f(0)|^2 \frac{|a_2-a_0|^2}{1+\frac{1}{9}(\Delta\mu/\mu)^2|2a_2+a_0|^2} \approx (8\pi/9)(\Delta\mu/\mu)\,|f(0)|^2|a_2-a_0|^2. \tag{6}$$

Here we have set a_0 and a_2 to be the isotopic spin zero and two scattering lengths expressed in the units of the pion Compton wavelength, and $\Delta\mu$ is the π^+-π^0 mass difference. The final equality is a good approximation unless one of the scattering lengths is extremely large: i.e., unless there is an s-wave resonance in π-π scattering at zero energy. We shall ignore this possibility.

We have then the branching ratio

$$B(\pi/\gamma) \sim 10^3|a_0-a_2|^2. \tag{7}$$

It seems reasonable to suppose that $|a_0-a_2|^2$ lies somewhere in the range zero to four pion Compton wavelengths squared. The calculations of Chew and Mandelstam[3,4] though not completely applicable to this case, suggest $\frac{1}{10}$ as a reasonable value, and we shall use this estimate in the remainder of the paper. We conclude that about 1% of our b^0 particles will undergo electromagnetic decay.

IV. PHOTOPRODUCTION CROSS SECTION

The S matrix (or, what is equivalent, the T matrix) for the photoproduction of a b^0 from a proton is

$$S_{fi} = \langle p'Q|kp\rangle, \tag{8}$$

with the momentum variables p, k, p', Q of, respectively, an incoming proton and photon and an outgoing proton and bi-meson.

Using now the familiar reduction formula techniques leads us to[5,6]

$$S_{fi} = 2\pi\delta^4(p_f-p_i)(M/2Q_0p_0')^{\frac{1}{2}}\bar{\mu}(p')\int d^4x\, e^{-ip'\cdot x} \times\langle 0|(f(x),J(0))_+|\gamma p\rangle = \delta^4(p_f-p_i)T_{fi}. \tag{9}$$

We have introduced $p_{\mu f}$ and $p_{\mu i}$, the components of the total initial and final four-momenta, the nucleon mass, M, and the nucleon and b^0 currents, f and J. The latter two quantities are defined by

$$(\gamma^\mu\partial/\partial x_\mu + M)\psi(x) = -f(x), \tag{10a}$$

$$[(\partial/\partial x)^2 - M_B{}^2]\phi_b(x) = -J(x). \tag{10b}$$

The $(\ )_+$ symbol defines the usual time-ordered product.

At this point we introduce what we shall call the "weak-binding approximation." The formal statement is that to a good approximation $J(x)$ can only couple the vacuum to the two pion state. All other matrix elements of the J operator will be ignored. The matrix element in Eq. (5) then becomes

$$\langle 0|(f(x),J(0))_+|\gamma p\rangle = \int d^3k_r\int d^3K \times\langle 0|J(0)|\pi\pi\rangle\langle\pi\pi|f(x)|\gamma p\rangle\theta(-x), \tag{11}$$

where K and k_r are, respectively, the total and relative momenta of the intermediate pion pair and $\theta(-x)$ requires x_0 to range only over negative values.

We should really include in Eq. (11) one more term containing a two-neutral-pion intermediate state. The missing term is then related to the product of the $(\pi^0)^2$ photoproduction amplitude times the amplitude for the decay of the b^0 in the pion mode. Because we are seeking a relationship involving experimentally known quantities we are forced to ignore this process. It seems *a priori* unlikely, however, that this omission will change the order of magnitude of our result.[7]

We may now apply the usual translation invariance arguments to Eq. (7) and substitute the result into Eq. (9), carrying out the indicated integrations. We are then led to

$$T_{fi} = 2\pi^4 i(M/2Q_0p_0')^{\frac{1}{2}}\bar{\mu}(p') \times\int d^3k_r \frac{\langle 0|J(0)|\pi\pi\rangle\langle\pi\pi|f|\gamma p\rangle}{Q_0-K_0+i\epsilon}. \tag{12}$$

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[1] J. M. Jauch and F. Rohrlich, *The Theory of Photons and Electrons* (Addison-Wesley Publishing Company, Inc., Reading, Massachusetts, 1955), p. 282.

[2] J. D. Jackson, D. G. Ravenhall, H. W. Wyld, Nuovo cimento **9**, 834 (1958).

[3] G. F. Chew and S. Mandelstam Phys. Rev. **119**, 467 (1960).

[4] G. F. Chew and S. Mandelstam and H. P. Noyes, Phys. Rev. **119**, 478 (1960).

[5] H. Lehmann, K. Symanzik, and W. Zimmerman, Nuovo cimento **1**, 205 (1955).

[6] N. N. Bogolyubov and D. V. Shirkov, *Introduction to the Theory of Quantized Fields* (Interscience Publishers, Inc., New York, 1959), Chap. IX.

[7] Recent Cornell experiments suggest that the $2\pi^0$ photoproduction cross section is small compared with that for charged pions. S. Richert and A. Silverman, Bull. Am. Phys. Soc. **5**, 237 (1960).

The matrix element $\langle 0|J(0)|\pi\pi\rangle$ may be identified as the "off the energy shell" T-matrix element for the capture of two charged pions into a bi-meson. If we suppose that a pair of π mesons interact through a potential, V, we may write (in the center-of-momentum system of the two pions)[8]

$$(2Q_0)^{-\frac{1}{2}}\langle 0|J(0)|\pi\pi\rangle = -(2\pi)^{-\frac{3}{2}}\langle\Psi_b|V|\phi(k_r)\rangle = (2\pi)^{-\frac{3}{2}}(B+k_r^2/\mu)\langle\Psi_b|\phi(k_r)\rangle, \quad (13)$$

where Ψ_b is the wave function of a bi-meson with binding energy. B, and $\phi(k_r)$ is the wave function of a pair of noninteracting pions with relative kinetic energy k_r. The last equality is obtained from

$$(H-k_r^2/\mu)\phi(k_r)=V\phi(k_r), \quad (14a)$$

$$H\Psi_b=-B\Psi_b, \quad (14b)$$

if H is the total (nonrelativistic) Hamiltonian for the two-pion system. The last matrix element of Eq. (13) will be recognized as the bi-meson wave function in momentum space. It will turn out that we shall only need this wave function for small relative momenta (relative kinetic energies of the order of or less than the binding energy); we shall content ourselves with using the asymptotic form

$$\Psi_b=(\mu^{\frac{3}{2}}B^{\frac{3}{2}}\pi^{-1})^{\frac{1}{2}}\exp[-(\mu B)^{\frac{1}{2}}r], \quad (15)$$

in terms of the relative radial coordinate, r.[9]

If we now make a nonrelativistic approximation for the denominators in Eq. (12) we may rewrite the equation by making use of Eqs. (13) and (14) as

$$T_{fi}=(\mu^{\frac{3}{2}}B^{\frac{3}{2}}\pi^{-2})^{\frac{1}{2}}\int d^3k_r(k_r^2+\mu B)^{-2}T_{fi}', \quad (16)$$

where T_{fi}' is the matrix element for the photoproduction of two charged pions in the system where the center of momentum of the two pions is at rest.

The integral over the pion relative momentum may be done by observing that contributions for k^2 larger than μB are strongly damped by the denominator. This suggests that for small binding energies (and a suitable well-behaved T_{fi}') we may evaluate T_{fi}' for k_r equal to zero and take it outside the integral. We are left with the result

$$T_{fi}=(\pi^2\mu^{\frac{3}{2}}B^{\frac{3}{2}})^{\frac{1}{2}}T_{fi}'(k_r^2=0), \quad (17)$$

where the last equality is appropriate to an arbitrary Lorentz frame. The total cross section for b^0 photoproduction is then equal to

$$\sigma_b(W)=[M^2/(2\pi)^2][(W^2-M^2)/16kW] \times(\mu^2/W^2)(B/\mu)^{\frac{3}{2}}|\mathfrak{M}|^2,$$

in terms of W, the total energy in the overall center-of-momentum system.

[8] B. Lippman and J. Schwinger, Phys. Rev. **79**, 481 (1951).

[9] Similar results have been obtained for the photoproduction of positronium by A. I. Alekseev, Zhur. Eksp. i Theoret. Fiz. **31**, 909 (1956) [translation: Soviet Phys.-JETP **4**, 771 (1951)], and preceding papers.

We may relate this result to the total cross section for photoproducing a charged pion pair by using a statistical model.[10] We write

$$T_{fi}'=\left(\frac{1}{2\pi}\right)^{-7/2}\left(\frac{M^2}{8q_0q_0'p_0p_0'k_0}\right)^{\frac{1}{2}}\mathfrak{M}, \quad (18)$$

where $\mathfrak{M}$ is a matrix element assumed to be constant; and is then given by Eq. (20a) in the following section. Carrying out the integrations [see Eq. (23) below], and ignoring all μ/M terms, we achieve the final result

$$\sigma_b=2\pi[\mu^2/(W^2+M^2)](B/\mu)^{\frac{3}{2}}f(\beta)^{-1}\sigma_{\pi\pi}, \quad (19a)$$

with the abbreviations

$$f(\beta)=\{1-\beta^{-1}(\beta^2-1)^{-\frac{1}{2}}\ln[\beta+(\beta^2-1)^{\frac{1}{2}}]\}, \quad (19b)$$

and

$$\beta=(W^2+M^2)/2MW. \quad (19c)$$

V. PHASE-SPACE ESTIMATE

The essential features of the preceding result may be obtained from elementary physical arguments starting with the remark that if two mesons are produced with zero relative kinetic energy they have unit probability of coagulating into a bound state. For electromagnetic capture the free-bound transition probability is roughly constant for relative kinetic energies less than the binding energy.[11] We shall approximate this situation by supposing that capture takes place with unit probability for relative kinetic energies less than the binding energy and zero probability for larger kinetic energies.

Just as in the previous section we assume the statistical model for pion pair production. The total cross section is given by

$$\sigma_{\pi\pi}=\frac{M^2}{16\pi^3kW}|\mathfrak{M}|^2\int_M^{E_m}dE(E^2-M^2)^{\frac{1}{2}} \times\left(1-\frac{4\mu^2}{W^2+M^2-2EW}\right)^{\frac{1}{2}}, \quad (20a)$$

with

$$E_m=(2W)^{-1}(W^2+M^2-4\mu^2),$$

where k is the c.m. photon energy, $\mathfrak{M}$ a (constant) matrix element, and the integration is over the range of energy of the recoil nucleon. For photon energies of the order of the nucleon rest mass the integral in Eq. (20) is reasonably well approximated by setting the meson rest-mass equal to zero. We have, then,

$$\sigma_{\pi\pi}=\frac{M^2}{(2\pi)^3}\frac{W^4-M^4}{16kW^3}|\mathfrak{M}|^2f(\beta), \quad (21)$$

where $f(\beta)$ is defined in Eq. (20).

[10] J. V. Lepore and R. N. Stuart, Phys. Rev. **94**, 1724 (1954).

[11] See the results for the radiative capture of neutrons by protons. J. M. Blatt and V. Weisskopf, *Theoretical Nuclear Physics* (John Wiley & Sons, Inc., 1952), p. 606.

To obtain the cross section for b^0 production we carry out the integration in Eq. (21) only over the region corresponding to the relative kinetic energy of the two mesons (in the Lorentz frame where their total momentum vanishes) being less than the binding energy of the b^0. This corresponds to setting the lower limit in Eq. (21) to be

$$(2W)^{-1}(W^2+M^2-4\mu^2-4\mu B).$$

For small binding energies we may apply the mean-value theorem to obtain (neglecting all μ/M terms)

$$\sigma_b=\frac{M^2}{(2\pi)^3}\frac{W^2-M^2}{8kW}\frac{\mu^2}{W^2}\left(\frac{2B}{\mu}\right)^{\frac{3}{2}}. \qquad (22)$$

The result of this section, which is to be compared with Eq. (19a), is

$$\sigma_b=2[\mu^2/(W^2+M^2)](2B/\mu)^{\frac{3}{2}}f(\beta)^{-1}\sigma_{\pi\pi}. \qquad (23)$$

VI. CONCLUSIONS

We shall now proceed with a numerical estimate of the order of magnitude of binding energies that would make the b^0 detectable in a photoproduction experiment. It is implicit in all of our approximations that we are a reasonable distance above the two-meson threshold (otherwise we could not have dropped the μ/M terms). Consequently it seems reasonable to apply our formulas at the energy where the two meson production cross section is a maximum, namely at a laboratory photon energy of about 600 Mev. Here $\sigma_{\pi\pi}$ is about 50 μb, and Eq. (15a) and Eq. (19) both give

$$\sigma_b=10(B/\mu)^{\frac{3}{2}}\,\mu\text{b}. \qquad (24)$$

From the analysis of Sec. II we have already concluded that the smallest observable effective cross section (as defined in the introduction) is of the order of 10^{-2} μb. We have also, in Sec. III, estimated that the branching ratio to the electromagnetic decay mode is about 1% if double π^0 decay is also allowed; that is, if the binding energy is less than twice the π^+-π^0 mass difference. From Eq. (24) we conclude, then, that a b^0 of binding energy less than twice the π^+-π^0 mass difference ($\sim$9 Mev) would not be detected in the proposed experiment.

The situation is quite different if the b^0 binding energy is greater than 9 Mev, for in this case the neutral pion decay channel is closed off, and essentially all of the decays will be into the two-photon channel. In this case, taking B, the binding energy as 10 Mev, Eq. (24) leads to an estimated cross section of about $\frac{1}{5}$ μb. It follows that the experiment we have proposed is adequate to detect a b^0 of this binding energy.

In principle it should also be possible to establish the existence of a b^0 by observing τ^+ decay.[12] For example, one might expect to find two-body decays of the kind:

$$\tau^+\rightarrow\pi^++b^0,$$

followed by neutral decay of the b^0. This type of event would look like τ' decay, except for the fact that the π^+ could have an energy greater than the 53 Mev end point of the π^+ spectrum in τ' decay. This situation would arise if the pion binding energy of the b^0 were greater than twice the $\pi^\pm$-π^0 mass difference.

In order to estimate whether or not one should already have seen such an event, we have calculated the probability that two of the π's in a τ^+ decay come out in a bound state as a b^0. The model and method of calculation are quite analogous to those for photoproduction given in Sec. V of this article. The result is that the rates are connected by the approximate ratio:

$$(\tau^+\rightarrow\pi^++b^0)/\tau_{\text{total}}{}^+\approx0.04(B/10)^{\frac{3}{2}},$$

where B is now the binding energy in Mev of the two pions in the b^0.

We wish to compare this number with the number of τ' events known and studied. To do this, we note that the decay probability of τ is about four times that of τ', so that

$$(\tau^+\rightarrow\pi^++b^0)\approx0.16(B/10)^{\frac{3}{2}}\tau'. \qquad (25)$$

We examine this relationship under the three cases: $B<10$, $B\approx10$, $B>10$.

Case I ($B<10$ Mev). The total number of τ' observed and studied in experiments seeking to establish the pion energy spectrum of τ' is about 90.[13–15] If B is much less than 10 Mev, the b^0 decay mode is improbable, and the decay is not kinematically distinguishable from τ' in a single event. We conclude that the present π^+ energy spectrum from τ' decays is not inconsistent with this possibility.

Case II ($B\approx10$ Mev). In this case one would expect to see a group of events of the type τ' with pion energies in the neighborhood of the end-point energy for τ' decay. Range straggling, dip correction errors, etc., would limit the resolution of this group to a few Mev in nuclear track plate experiments. Of 90 τ' events plotted in energy spectra, roughly 10 are observed to have energies greater than 40 Mev, and the observed distributions are not inconsistent with a 0^- Dalitz distribution. The principal questions therefore are: (1) How many of the π^+ in the upper energy "bins" of the data could be from the b^0 decay mode without disturbing the data unduly, and (2) how reliable is the estimate of

[12] We should also mention the experiment of C. Bernardini, R. Querzoli, G. Salvini, A. Silverman, and G. Stoppini, Nuovo cimento 14, 268 (1959). These experimenters searched for a particle such as the b^0 by observing the momentum spectrum of the recoil proton in the process $\gamma+p\rightarrow p+?$. We conclude, after examining their published results, that a b^0 of less than about 30-Mev binding energy would not have been detected in this experiment.

[13] G. Alexander, R. H. W. Johnston, and C. O'Cealleigh, Nuovo cimento 6, 478 (1957).

[14] S. Taylor, G. Harris, J. Orear, J. Lee, and P. Baumel, Phys. Rev. 114, 359 (1959).

[15] D. M. Ritson, A. Pevsner, S. C. Fung, M. Widgoff, G. T. Zorn, S. Goldhaber, and G. Goldhaber, Phys. Rev. 101, 1085 (1956).

Eq. (25)? Neither of these questions can be answered definitely. However, at $B=10$ Mev the number of b^0 decays observed should have been 15. This estimate could easily be in error by a factor of two, and the frequency of observation of pions in τ' decay in the upper bins also could be in error for statistical or even systematic reasons. We conclude that we cannot exclude the existence of a b^0 of 10-Mev binding energy from the present τ' decay data.

Case III ($B>10$ Mev). The energies of the decay π^+ in $\tau^+ \rightarrow b^0+\pi^+$ decay become greater as the b^0 mass decreases, until finally at about $B=15-20$ Mev the decays would be kinematically distinguished in nuclear track plate experiments. The estimate of Eq. (25) becomes increasingly less reliable as B increases, also, but it is still expected that as B increases, so will the fraction of τ decays going by the b^0 mode. We conclude that the binding in the b^0 is less than $20-30$ Mev on the basis of the fact that too few (~ 2) π^+ from decays[14,16] apparently like τ' decay have been found with energies greater than the end point energy of 53 Mev.

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[16] These events are discussed by K. Hiida, Nuovo cimento **13**, 1117 (1959).

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Cloud-Chamber Study of Hard Collisions of Cosmic-Ray Muons with Electrons*

R. F. DEERY† AND S. H. NEDDERMEYER
Department of Physics, University of Washington, Seattle, Washington
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The energy distribution of the secondary electrons produced in targets of carbon or paraffin by the μ-e scattering process for muon momenta in the range 5–50 Bev has been measured for electron energies up to 10 Bev, or c.m. momentum transfer up to 100 Mev. A vertical array of three cloud chambers immersed in a magnetic field of 11 000 gauss was used with a fourfold coincidence system. Two flat rectangular proportional counters, suitably biased, together with two Geiger-Müller trays, provided fair rejection of uneventful penetrating particles and a high efficiency for selection of the narrow electronic showers characteristic of the high-energy electromagnetic events. On 5900 counter triggered photographs there were 291 accepted events, having one or more ($\pm$) electrons with energy ≥ 0.10 Bev, believed to originate in μ-e collisions in the (carbon or paraffin) target above the top chamber, from incident muons in the momentum interval 5–50 Bev. The data are compared with a calculation based on the Bhabha formula for spin $\frac{1}{2}$ muons, taking into account the momentum distribution of the incident muons, the energy loss and shower development in the target and the chamber walls, and a theoretical efficiency factor. Arguments are given to show that direct pair production and bremsstrahlung of the muons in the target and in the Pb shield above the apparatus produce negligible effects. The experiment permits a reliable measurement of only the relative distribution. When arbitrarily normalized, the calculated distribution is in fairly good agreement with the data, except for a small systematic difference suggesting an excess of observed events for the harder collisions. Although the discrepancy is interpretable as a statistical fluctuation, the data are fitted much better over the entire range when the basic cross section is modified by a "form factor," F^2 (*greater* than unity), with $F=1+|q^2|\lambda_\mu{}^2$ where q is the invariant of the 4-momentum transfer in units of $\hbar$, and λ_μ is the Compton wavelength of the muon. This may be the first indication of a deviation from standard quantum electrodynamics for hard μ-e collisions. More strongly it shows that, if there is a deviation, it is not representable by a form factor less than unity.

I. INTRODUCTION

THE essential content of this paper may be appreciated from the Abstract, the plots of the results in Sec. IV, and the conclusions, Sec. VI. The background discussion in the introduction is long because the experiment is really an old one which may have new and interesting aspects. Section II includes the design, the experimental conditions and "boundary" data, with a general discussion of the problems of analysis of the results; Sec. III, apparatus details, calibration and measurement procedures; Sec. IV, the results; and Sec. V, details of the analysis.

Background

The production of high-energy secondary electrons by close collisions of cosmic-ray particles with atomic electrons was one of the first processes to be studied extensively in cosmic rays,[1] and together with other studies, ultimately led to the discovery of the muon.[2] Although the early mistaken but generally believed

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† Now at the California Institute of Technology, Pasadena, California.

[1] C. D. Anderson and S. H. Neddermeyer, *Proceedings of the International Conference on Physics, London, 1934* (Cambridge University Press, Cambridge, 1934), p. 171.
[2] S. H. Neddermeyer and C. D. Anderson, Phys. Rev. **51**, 884 (1937); Phys. Rev. **54**, 88 (1938); Revs. Modern Phys. **11**, 191 (1939).