Analysis of Long-Range Alpha-Emission Data

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Alpha barrier penetrabilities for the long-range alpha particles of Po²¹² and Po²¹⁴ are calculated by using the diffuse exponential nuclear potential derived from optical-model analysis of alpha-particle elasticscattering data. The calculations are made on the same basis as reported by Rasmussen in two previous publications. Partial half-lives for alpha and gamma emission are calculated on the assumption that the long-range alpha decay is unhindered with respect to the ground-state alpha decay.

INTRODUCTION

 \mathbf{R} ASMUSSEN has reported alpha-decay barrierpenetration calculations using an exponential nuclear potential.^{1,2} Details of the calculation were discussed,¹ and numerical results for all measured alphaparticle transitions of even-even¹ and odd-mass² nuclei were given. The alpha-nuclear potential used was the real part of a potential given by Igo to fit alpha elastic-



FIG. 1. Level scheme and decay scheme of Po^{214} as given by Bishop (reference 4). E_{α} in Mev includes electron screening correction. Alpha intensities in square brackets relative to N_{α} (ground state) $\equiv 10^{6}$. E_{γ} in kev. Gamma intensities in parenthesis relative to N_{γ} (609 kev) $\equiv 1$.

¹ J. O. Rasmussen, Phys. Rev. **113**, 1593 (1959). ² J. O. Rasmussen, Phys. Rev. **115**, 1675 (1959). scattering data³:

$$V(r) = -1100 \exp\left\{\frac{1.17A^{\frac{1}{2}} - r}{0.574}\right\} \text{ Mev.}$$

This potential is used here to calculate with the IBM-650 computer the alpha-decay barrier-penetration factors for the long-range alpha particles from the excited states of Po²¹² and Po²¹⁴. The centrifugal potential for the angular momentum of the alpha particle, as determined from the decay schemes, is included in these calculations. The penetration factors are used to calculate the partial half-lives of the levels for alpha decay, assuming that the decay is unhindered with respect to

the ground-state decay. The relative alpha-particle and gamma-ray abundances are then used to calculate partial half-lives for the gamma transitions from these same states.

Po^{214}

In the case of Po²¹⁴, two different proposed level schemes and decay schemes are used. The first of these, due to Bishop⁴ (see Fig. 1), assumes that all of the long-range alpha groups arise from transitions to the ground state of Pb²¹⁰. The second, due to Hauser⁵ (see Fig. 2), proposes that three of the long-range alpha groups arise from transitions to excited states of Pb²¹⁰. This proposal was made to account for the fact that

FIG. 2. Level scheme and decay scheme of Po^{214} as given by Hauser (reference 5). E_{α} in Mev includes electron screening correction. Alpha intensities in square brackets relative to N_{α} (ground state) = 10⁶. Pb²¹⁰ level populated in alpha decay: 0=ground state; 1=first excited state; 2=second excited state.

⁸ G. Igo, Phys. Rev. Letters 1, 72 (1958).
 ⁴ G. R. Bishop, Nuclear Phys. 5, 358 (1958).
 ⁵ U. Hauser, Z. Physik 150, 599 (1958).

TABLE I. Po ²¹⁴ alpha-barrier penetration factors and parti	half-lives of excited states based	on the decay scheme due to Bishop. ^a
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Energy level (kev)	E_{α} with screening correc- tion (Mev)	N_{lpha} rela- tive to 10 ⁶ ground- state decays ^b	α angu- lar mo- men- tum	Barrier- penetration factor, P	Pa	rtial α ulf-life (sec)	N_{γ} relative to 10 ² ground- state decays	Eγ (kev)	Partial γ half-life, $\tau_{\frac{1}{2}}$ (sec)	Single-particle half-life,° $ au_{sp}$ (sec)	$ au_{ m sp}/ au_{ m i}$
0	7.714	106	0	1.58×10^{-16}	1.63	6×10⁻₄	• • •		•••	• • •	
609.3	8.312	0.43	2	4.65×10^{-15}	5.5	$\times 10^{-6}$	42	609.3	5.6×10^{-12}	$4.0 \times 10^{-11}(E2)$	7.1
1281.3	8.971	0.45	0	3.43×10^{-13}	7.5	$\times 10^{-8}$	(1.3)(?)	1281.3	2.6×10^{-12}	•••	
1378.0	9.066	$(2.7)^{d}$	2	3.46×10^{-13}	7.5	$\times 10^{-8}$	5.9	1378	$(3.4 \times 10^{-12})^{d}$	$6.8 \times 10^{-13} (E2)$	$(0.2)^{d}$
		. ,					4.6	769	$(4.4 \times 10^{-12})^{d}$	$1.3 \times 10^{-11} (E2)$	(3.0) ^d
									. ,	$5.5 \times 10^{-14} (M1)$	(0.012)d
1415.9	9.103	22.0	0	6.93×10^{-13}	3.7	$\times 10^{-8}$	$(0.3)_{K}$	e	•••	•••	` • • • <i>`</i>
							1.3	806.3	6.3×10-11	$9.9 \times 10^{-12} (E2)$	0.16
1668.0	9.351	0.38	2	1.52×10^{-12}	1.7	$\times 10^{-8}$	0.8	1668.4	8.1×90^{-13}	$2.6 \times 10^{-13} (E2)$	0.32
1847.6	9.527	1.35	2	3.64×10^{-12}	7.1	×10−9	1.7	1848.5	5.6×10^{-13}	$1.6 \times 10^{-13} (E2)$	0.29
							6.3	1238.3	1.5×10^{-13}	$1.2 \times 10^{-12} (E2)$	8.0
										$1.3 \times 10^{-14} (M1)$	0.087
2016.7	9.693	0.35	2	8.11×10^{-12}	3.2	$\times 10^{-9}$	0.6	2016.7	1.9×10^{-13}	$1.0 \times 10^{-13} (E2)$	0.53
							7.6	1408.0	1.5×10^{-14}	$6.1 \times 10^{-13} (E2)$	41
										$8.9 \times 10^{-15} (M1)$	0.59
2144	9.818	1.06	4	4.71×10^{-12}	5.5	$\times 10^{-9}$	(≤ 1)	1534	$\geq 5.8 \times 10^{-13}$	$4.0 \times 10^{-13} (E2)$	≤ 0.69
2270.2	9.941	0.36	4	8.34×10^{-12}	3.1	×10-9	(≤ 0.8)	1661	$\geq 1.4 \times 10^{-13}$	$2.7 \times 10^{-13}(E2)$	≤ 1.9
2440.8	10.109	1.67	2	5.48×10^{-11}	4.7	$\times 10^{-10}$	2.1	2441	3.7×10^{-14}	$3.9 \times 10^{-14} (E2)$	1.1
2508.5	10.176	0.38	2	7.36×10^{-11}	3.5	$\times 10^{-10}$	0.21	2508.5	6.3×10^{-14}	$3.4 \times 10^{-14} (E2)$	0.54

• See reference 4. • G. H. Briggs, Revs. Modern Phys. 26, 1 (1954). • Calculated from the formulas given by Moszkowski (reference 6). • Calculated assuming that the partial γ half-life of the 1378-kev gamma is five times the single-particle gamma half-life. • Conversion electrons. Strength parameter $|\rho|\kappa = 0.035$ [see E. L. Church and J. Weneser, Phys. Rev. 103, 1035 (1956).]

Energy level (kev)	E_{α} with screen- ing cor- rection (Mev)	N_{lpha} relative to 10 ⁶ ground- state decays ^b	α angu- lar mo- men- tum	Barrier- penetration factor, P	Pa ha (rtial α llf-life (sec)	N_{γ} relative to 10^2 ground- state decays	Eγ (kev)	Partial γ half-life, $\tau_{\frac{1}{2}}$ (sec)	Single-particle half-life,° $ au_{ m sp}$ (sec)	$ au_{ m sp}/ au_{rac{1}{2}}$
0	7 714	106	0	1.58×10^{-16}	1.63	6×10^{-4}					$(\delta^2 = 0.111 \text{ Mev})$
0	6.951	$\sim 10^2 d$	2	2.99×10^{-19}	8.6	×10-2					$(\delta^2 = 0.0096 \text{ Mev})$
600.3	8 312	0 43	$\overline{2}$	4.65×10^{-15}	5.5	$\times 10^{-6}$	42	609.3	5.6×10^{-12}	$4.0 \times 10^{-11} (E2)$	7 1
1281 3	8 971	0.45	$\overline{2}$	2.08×10^{-13}	1.2	$\times 10^{-7}$	1.3	1281.3	4.3×10^{-12}	$9.8 \times 10^{-13} (E2)$	0.23
1415.0	9 103	22.0	ō	6.93×10^{-13}	3.7	$\times 10^{-8}$	$(0.3)_{K}$	e			
1110.7	2.100		÷	01/07/1-0		/ . = -	1.3	806.3	6.3×10^{-11}	$9.9 \times 10^{-12} (E2)$	0.16
1847 6	9 527	1.35	2	3.64×10^{-12}	7.1	$\times 10^{-9}$	1.7	1848.5	5.6×10^{-13}	$1.6 \times 10^{-13} (E2)$	0.10
1011.0	1.041	1.00		0.01/(10		/(10	6.3	1238.3	1.5×10^{-13}	$1.0 \times 10^{-12} (E2)$	8.0
							010		110/(10	$1.3 \times 10^{-14} (M1)$	0.087
2016 7	0.603	0.35	2	8.11×10^{-12}	32	$\times 10^{-9}$	0.6	2016.7	1.9×10^{-13}	$1.0 \times 10^{-13} (E2)$	0.53
2010.7	2.070	0.00	-	0.11/(10	0.2	//10	7.6	1408.0	1.5×10^{-14}	$61 \times 10^{-13} (E2)$	41
									2107(20	$8.9 \times 10^{-15} (M1)$	0.50
2440.8	10 100	1.67	2	5.48×10^{-11}	47	$\times 10^{-10}$	2.1	2441	37×10^{-14}	$3.9 \times 10^{-14} (E2)$	11
2110.0	0 346	0.38	õ	2.43×10^{-12}	1.06	× 10 ⁸			1.9×10^{-13}		0.21
2508.5	10 176	0.38	4	2.40×10^{-11}	1 07	×10-9	<1.1	1900	$>3.7 \times 10^{-14}$	$1.4 \times 10^{-13}(E2)$	< 3.8
2000.0	10.170	0.00	3	4.55×10^{-11}	57	$\times 10^{-10}$	· · · ·		$>2.0 \times 10^{-14}$	$1.4 \times 10^{-13} (E2)$	<7.0
			0	1.00/10	0.1	X10			2.07(10	$3.6 \times 10^{-15} (M1)$	<0.18
2700	10 363	1 1 2	2	1 65 × 10 ¹⁰	1.56	$\times 10^{-10}$	5×10^{-2}	2700	3.5×10^{-13}	$2.4 \times 10^{-14} (E2)$	0.060
2100	10.000	1.14	ĩ	2.27×10^{-10}	1 14	×10-10	0/(10	1,00	2.6×10^{-13}	$2.4 \times 10^{-14} (F2)$	0.002
				2.27 / 10	****	/(10			=107(10	$1.3 \times 10^{-15} (M1)$	0.072
7886	10 546	0.23	2	3.54×10^{-10}	73	$\times 10^{-11}$	4.3×10^{-2}	2886	3.9×10^{-14}	$1.0 \times 10^{-14} (F2)$	0.005
2000	10.540	0.20	ĩ	4.87×10^{-10}	5.3	×10-11	1.07(10	-000	2.8×10^{-14}	$1.7 \times 10^{-14} (E2)$	0.44
			Ŧ	1.07 \ 10	0.0	X10			2.0/(10	$1.0 \times 10^{-15} (M1)$	0.01
3046	10 703	~ 0.1	2	671×10^{-10}	38	$\times 10^{-11}$	3.7×10^{-2}	3046	1.0×10^{-14}	$1.3 \times 10^{-14} (E2)$	1 3
0010	0 030	0.36	õ	4.15×10^{-11}	62	$\times 10^{-10}$			6.0×10^{-13}		0.022
	1.907	0.00	5	1.10/(10	0.2	//~*			0.07(10		0.022

TABLE II. Po²¹⁴ alpha-barrier penetration factors and partial half-lives of excited states based on the decay scheme due to Hauser.^a

* See reference 5.
^b G. H. Briggs, Revs. Modern Phys. 26, 1 (1954).
• Calculated from the formulas given by Moszkowski (reference 6).
• Calculated from the formulas given by Moszkowski (reference 6).
• C. J. Walen and G. Bastin, Proceedings of the International Congress on Nuclear Physics, 1958 (Dunod Publishing Company, Paris, 1959), pp. 910-916.
• Conversion electrons. Strength parameter is 0 k = 0.035 [see E. L. Church and J. Weneser, Phys. Rev. 103, 1035 (1956)].

gamma rays from the levels corresponding to some of the alpha groups have not been detected.

The numerical results for Po^{214} are summarized in Tables I and II along with the decay-scheme data utilized. The alpha barrier-penetration factor, P, is given for each of the states, assuming spins and parities as indicated in Figs. 1 and 2. This penetration factor is then used to calculate the partial alpha half-life for each level:

$$t = t (P_g/P_e),$$

x, excited state α , ground state

where P_g and P_e are penetration factors for the groundstate and excited-state alpha decay.

This partial alpha half-life and the relative alpha and gamma abundances lead to partial gamma half-lives, which are compared with single-particle gamma halflives calculated from the formulas given by Moszkowski.⁶ The value obtained by Nierhaus and Daniel⁷ of 19% of the beta transitions in Bi²¹⁴ going directly to the ground state of Po²¹⁴ was used. The gamma-ray intensities in both cases are taken from the work of Bishop⁴ and of Dzhelepov *et al.*⁸

TABLE III. Ratio of reduced width to first excited state to reduced width to ground state for Po^{214} alpha-emitting levels.

Level (kev)	δ^2 (excited state)/ δ^2 (ground state)	
 0	0.086	
2441	5.1	
3046	58	

No alpha group has been detected from the 1378-kev level. By comparison with the E2 transitions from the 1281- and 1416-kev levels the partial gamma half-life of the 1378-kev gamma ray is assumed to be five times the single-particle value. This assumption gives a value of 2.7 alphas from this level per 10⁶ ground-state transitions. This number could easily be unresolved from the abundant alpha group from the 0+ 1416-kev level; thus, the data on this level are consistent with the alpha-decay data.

The calculations also show that it is reasonable that some of the gamma rays from certain alpha-emitting states have not been detected. On the other hand, the second decay scheme allows one to calculate relative reduced transition probabilities, δ^2 (see reference 1), for the decay of a few given levels to the first excited state and to the ground state of Pb²¹⁰. This ratio is shown in Table III, and is seen to increase greatly as the energy increases. This would indicate that the transition to the ground state becomes quite hindered as the energy of the initial state increases.

 ⁷ R. Nierhaus and H. Daniel, Z. Naturforsch. 12A, 1 (1957).
 ⁸ B. Dzhelepov, S. Shestopalova, and I. Uchevatkin, Nuclear

FIG. 3. Level scheme and decay scheme of Po²¹² as given by Emery and Kane (reference 9). E_{α} in Mev includes electron screening correction. E_{γ} in kev. All intensities (in parenthesis) relative to 100 Po²¹² ground-state transitions.

The results of these calculations do not differ appreciably from those of Hauser,⁵ who used a simpler potential and experimental level spacings. The calculations on the groups leading to excited states are very interesting. The large variation in reduced transition probabilities shown in Table III indicates that the assumption of alpha decay to the excited states of Pb²¹⁰ may not be valid.

\mathbf{Po}^{212}

The level scheme and decay scheme of Po^{212} due to Emery and Kane⁹ and also to Sergeev *et al.*¹⁰ is shown

FIG. 4. Half-lives of first excited (2+) levels of even-even nucleiratio of single-particle to experimental half-lives.

⁹ G. T. Emery and W. R. Kane, Phys. Rev. **118**, 755 (1960). ¹⁰ A. G. Sergeev, E. M. Krisyuk, G. D. Latyshev, Yu. N. Trofimov, and A. S. Remennyi, Soviet Phys.-JETP **6**, 878 (1958).

⁶S. A. Moszkowski, in *Beta- and Gamma-Ray Spectroscopy*, edited by K. Siegbahn (Interscience Publishers, New York, 1955), Chap. XIII.

(Mev)	state decays ^a	men- tum	Barrier penetration factor, P	Partial α half-life (sec)	ground- state decays	E_{γ} (kev)	Partial γ half-life, $t_{\frac{1}{2}}$, (sec)	$\begin{array}{c} \text{Single-particle} \\ \gamma \text{ half-life} \\ t_{\rm sp}{}^{\rm b} \text{ (sec)} \end{array}$	$t_{ m sp}/t_{ m 1}$	
8.810	106	0	1.32×10^{-13}	3.04×10^{-7}			•••	•••		
9.522	35	2	3.30×10^{-12}	1.2×10^{-8}	10.2	727	4.1×10^{-12}	$1.7 \times 10^{-11}(E2)$	4.1	
10.452	20	2	2.24×10^{-10}	1.8×10^{-10}	0.6	953	6.0×10^{-13}	$4.4 \times 10^{-12} (E2)$	7.3	
								$2.9 \times 10^{-14} (M1)$	0.048	
10.573	180	0	5.95×10-10	6.7 ×10 ⁻¹¹	$\sim^{1}_{0.008}$	1074 с	1.2×10^{-12}	$2.4 \times 10^{-12} (E2)^{\prime}$	2.0	
r	ection (Mev) 8.810 9.522 10.452 10.573	$\begin{array}{c} \text{Ig con} & \text{ground} \\ \text{(Mev)} & \text{state} \\ \hline & \text{decays}^a \\ \hline & \\ 8.810 & 10^6 \\ 9.522 & 35 \\ 10.452 & 20 \\ \hline 10.573 & 180 \\ \end{array}$	$\begin{array}{c cccc} & \text{igtormation} & \text{interment} \\ \hline \text{(Mev)} & \text{state men-} \\ \hline \text{decays}^{\text{a}} & \text{tum} \\ \hline \hline 8.810 & 10^6 & 0 \\ 9.522 & 35 & 2 \\ 10.452 & 20 & 2 \\ \hline 10.573 & 180 & 0 \\ \hline \end{array}$	lag controlgroundmen- decays ^a penetration factor, P8.810 10^6 0 1.32×10^{-13} 9.522 35 2 3.30×10^{-12} 10.452 20 2 2.24×10^{-10} 10.573 180 0 5.95×10^{-10}	lag of method log of	lag of method log of method <th log="" of<="" td=""><td>$\begin{array}{c ccccccccccccccccccccccccccccccccccc$</td><td>lag of a ground modelDentration pointerDentration factor, PDentration (sec)Dentration factor, PDentration factor, P</td><td>$\begin{array}{c ccccccccccccccccccccccccccccccccccc$</td></th>	<td>$\begin{array}{c ccccccccccccccccccccccccccccccccccc$</td> <td>lag of a ground modelDentration pointerDentration factor, PDentration (sec)Dentration factor, PDentration factor, P</td> <td>$\begin{array}{c ccccccccccccccccccccccccccccccccccc$</td>	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	lag of a ground modelDentration pointerDentration factor, PDentration (sec)Dentration factor, PDentration factor, P	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$

TABLE IV. Po²¹² alpha-barrier penetration factors and partial half-lives of excited states.

^a G. H. Briggs, Revs. Modern Phys. 26, 1 (1954).
 ^b Calculated from the formulas given by Moszkowski (reference 6).
 ^c Conversion electrons. Strength parameter is |ρ|κ =0.041 [see E. L. Church and J. Weneser, Phys. Rev. 103, 1035 (1956)].

in Fig. 3. The results for Po²¹² are summarized in Table IV. The gamma intensities and multipolarities are taken from Emery and Kane9 and the level energies from Sergeev et al.10

Partial alpha and gamma half-lives are calculated as in the case of Po²¹⁴ above. The results here differ by about a factor of two from those of Emery and Kane,⁹ who also based their calculations on the ground-state decay while using a simpler potential.

Ratios of single-particle half-lives to experimental

half-lives^{11–13} for first excited (2+) states in even-even nuclei are shown in Fig. 4 as a function of mass number. The ratios calculated here for Po²¹² and Po²¹⁴ agree very well with the general trend, indicating that the assumption of unhindered alpha decay is probably good for these first excited states.

¹¹ D. Strominger, J. M. Hollander, and G. T. Seaborg, Revs. Modern Phys. 30, 585 (1958).
 ¹² R. H. Davis, A. S. Divatia, D. A. Lind, and R. D. Moffat, Phys. Rev. 103, 1801 (1956).
 ¹³ F. K. McGowan and P. H. Stelson, Phys. Rev. 99, 112 (1956).