O^{14} Decay Energy and the Fermi Interaction Constant

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The threshold energy of the $C^{12}(He^3,n)O^{14}$ reaction has been measured precisely with the use of a Van de Graaff accelerator and an electrostatic analyzer, the value being 1436.2±0.9 kev. From this value, the O^{14} beta-decay end-point energy (for decay leading to the 2.312-Mev state in N^{14}) is computed to be 1.8000 ±0.0065 Mev, based on the 1956 table of masses, and 1.8097 ±0.0015 Mev, based on the 1960 table of masses. A revised value of the Fermi interaction constant in beta decay is calculated and applied in the conserved vector current theory of Feynman and Gell-Mann. When (1) the radiative corrections and other corrections are applied to the decay of O^{14}, (2) the corrected ft value is used to compute the vector coupling constant in beta decay, (3) the value of this vector coupling constant is assumed to be the same as that for the muon decay and is used to calculate the lifetime of the muon, and (4) this lifetime is corrected for radiation effects, the predicted mean life of the muon becomes 2.289±0.013 μsec (based on the 1960 table of masses and the radiative corrections of Kinoshita and Sirlin) or 2.245±0.013 μsec (based on the 1960 table of masses and the corrections, radiative and otherwise, of Durand and collaborators). The former value is 3.6% greater than a weighted average of several recent measurements of the muon mean life, 2.210±0.003 μsec, while the latter value is only 1.6% greater, and is within the combined experimental and theoretical uncertainties. However, the definitions of coupling constants used by Durand and collaborators differ somewhat from those used by others.

INTRODUCTION

In the Fermi theory of beta decay, the decay rate and the decay energy determine the "coupling strength" or "interaction constant." For several years, it has been known that the values of the interaction constants for the different classes of weak interactions are approximately equal. Feynman and Gell-Mann[1] have advanced a conserved vector current hypothesis to explain why the vector coupling constant should be the same for beta decay as for muon decay. The decay of O^{14} offers one of the best opportunities for a precise determination of the vector coupling constant G_V for beta decay since (1) the decay channel populating the lowest $T=1$ state in N^{14} involves a pure Fermi transition, (2) the branching ratio[2] for this pure Fermi transition is almost 100% (and accurately known) because of the chance cancellation in the matrix element for the decay channel populating the ground state of N^{14}, and (3) the overlap of wave functions for the initial and final states (both members of the same isotopic spin multiplet) is almost perfect.[3] The precise determination of the O^{14} ft value is therefore very important.

Gerhart[4] and Sherr *et al.*[2] have measured the partial half-life for the pure Fermi transition in the decay of O^{14}, but the end-point energy of Gerhart[4] is in disagreement with that which is calculated from the $C^{12}(He^3,n)O^{14}$ reaction threshold energy measurement of Bromley *et al.*[5] And the threshold energy measurement of Bromley *et al.* is in rather poor agreement with the value obtained by Butler[6] for the same reaction. Since at present, the $C^{12}(He^3,n)O^{14}$ reaction threshold energy offers the most precise method for determination of the O^{14} decay energy (and hence the f value), an independent precision measurement of this threshold energy is highly desirable. Such is the purpose of the present experiment.

The equality of the vector coupling constants, as calculated from the experimental lifetimes and decay energies for beta decay and muon decay, may be considered a test of the validity of the conserved vector current hypothesis of Feynman and Gell-Mann, and also a test of the universality of the Fermi interaction constant.

A second reason for the importance of the $C^{12}(He^3,n)O^{14}$ threshold energy measurement is that this reaction threshold offers a convenient calibration point for particle accelerators using He^3 beams. A small amount of carbon contamination usually appears on a target inside an accelerator vacuum system, so the above reaction threshold can usually be observed without a change of target. Only one absolute measurement has been reported[7] for He^3 bombarding energies.

A preliminary account of the present experiment has been written.[8]

EXPERIMENTAL PROCEDURE

The singly charged He^3 beam, supplied by the NRL 5-Mv Van de Graaff Accelerator, was passed first through a magnetic analyzer for beam-component separation and preliminary analysis and then through a two-meter radius electrostatic analyzer[9] whose slit widths were adjusted to give 0.05% beam energy resolution. The C^{12} target, prepared by the molecular cracking of carbon onto a thin strip of molybdenum in an atmosphere of methyl iodide, was about 15 kev thick

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[7] K. L. Dunning, J. W. Butler, and R. O. Bondelid, Phys. Rev. **110**, 1076 (1958).
[8] R. O. Bondelid, J. W. Butler, A. del Callar, and C. A. Kennedy, Naval Research Laboratory Quarterly on Nuclear Science and Technology, January, 1960 (unpublished), p. 7.
[9] R. O. Bondelid and C. A. Kennedy, Phys. Rev. **115**, 1601 (1959).

to 1-Mev He^3 particles. The target was protected from vacuum system contaminants by an enclosing surface kept at liquid-nitrogen temperature. The neutron detector consisted of three BF_3 proportional counters embedded in a small amount of paraffin. Both the neutron detector and the target cold trap have been previously described.[7]

EXPERIMENTAL RESULTS

The neutron yield, as a function of electrostatic-analyzer potentiometer setting, is shown by the solid circles of Fig. 1 (ordinate scale on the right). For reasons discussed in a previous communication[10] we choose to determine the threshold intercept by extrapolating the plot of the two-thirds power of the neutron yield. This plot and the straight-line extrapolation are shown by the crosses in Fig. 1 (ordinate scale on the left). The intercept and its probable error, determined arithmetically by application of the method of least squares, are found to be 0.95668±0.00036 volt.

In order to relate this potentiometer setting to the energy of the He^3 particles, it is necessary to make corrections for the relativistic effect, the internal and external magnetic fields of the analyzer, and the energy of the electron that is traveling with the singly charged He^3 particle. In addition to these corrections it is necessary to know the calibration factors of the electrostatic analyzer. These include a determination of the analyzer geometry and of the resistor-stack ratio. At the time of the present experiment the geometry of the electrostatic analyzer was very well known, but a subsequent series of experiments indicated that the resistor-stack ratio had changed from the calibrated value because a short circuit had developed across several turns of one of the resistors. Because the subsequent experiments were performed with protons as bombarding particles, it was not considered feasible to repeat the threshold-energy determination of the $C^{12}(He^3,n)O^{14}$ reaction. However, since the $T^3(p,n)He^3$ reaction threshold was measured both with the damaged resistor stack, as used in the present experiment, and with a later repaired and recalibrated resistor stack, it is possible to compute a precise value of the $C^{12}(He^3,n)O^{14}$ threshold energy relative to the $T^3(p,n)He^3$ reaction threshold, measured to be 1019.7±0.5 kev on an absolute scale.[10] The result thus obtained for the $C^{12}(He^3,n)O^{14}$ reaction threshold energy is 1436.2±0.9 kev, where the uncertainty is the probable error referred to the absolute scale; that is, it includes both the probable error in the relative measurement and the probable error in the absolute measurement of the $T^3(p,n)He^3$ reaction threshold.

The change in analyzer calibration due to the damaged resistor was 0.12%. Because of the remote possibility that the resistor was damaged after completion of the present experiment (instead of prior to the present experiment as indicated by the relative $T^3(p,n)He^3$ reaction threshold measurements), it is possible (but very unlikely) that the final value quoted above contains a systematic error of 0.12%, the quoted value being low.

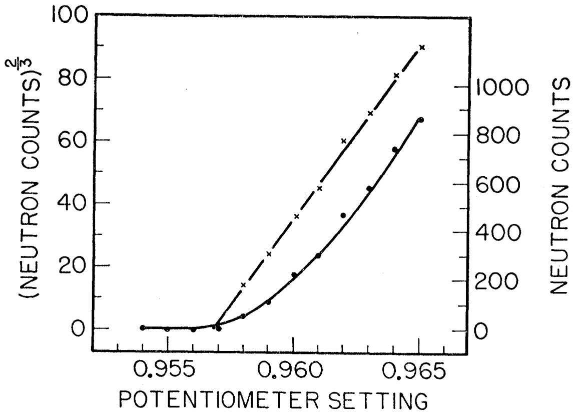

FIG. 1. Neutron yield near threshold for the $C^{12}(He^3,n)O^{14}$ reaction. The solid circles are the neutron counts (right-hand ordinate) as a function of potentiometer setting on the electrostatic analyzer. The crosses give the neutron counts to the $\frac{2}{3}$ power (left-hand ordinate).

The Q value for the $C^{12}(He^3,n)O^{14}$ reaction is computed in a direct manner to be -1.1477 ± 0.0007 Mev. With this Q value and the 1960 table of masses[11] the mass excess of O^{14} (C^{12} scale) is computed to be 8.0073 ±0.0008 Mev, from which the mass of O^{14} is found to be 14.0085967±0.0000009 amu. With the value of the first excited state in N^{14}, to which 99.4% of the decays of O^{14} lead,[2] adjusted by Ajzenberg-Selove and Lauritsen[12] to be 2.312±0.0012 Mev, the positron end point of the decay of O^{14} to this state is computed to be 1.8097±0.0015 Mev. The f value, calculated with the aid of the tables of Moszkowski and Jantzen,[13] is 42.81 ±0.26. These values are listed in Table I.

If we use the 1956 table of masses,[14] the results given above become appreciably different. The philosophy adopted here is to use primarily the 1960 table, but to calculate the f value also with the 1956 table. The mass excess of O^{14} then is 12.1347 ±0.0058 Mev (1956 table, $O^{16}=0$) from which the mass of O^{14} is found to be 14.0130318±0.0000062 amu (1956 table, $O^{16}=16$). The O^{14} positron end point and f value determined as before are 1.8000±0.0065 Mev and 41.84±0.66, respectively.

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TABLE I. Numerical results from the measurement of the $C^{12}(He^3,n)O^{14}$ reaction threshold energy. The uncertainties in the mass, mass excess, and positron end point include the uncertainties given in the 1960 mass table as well as the one from the present experiment. The uncertainties in the f value include the uncertainty in the table of f values (±0.5%) in addition to the end-point uncertainty. The uncertainty in the ft value includes in addition the uncertainty in the partial half-life, whose value we have adopted as 71.4±0.2 sec.

Reaction threshold energy	1.4362	± 0.0009	Mev
Reaction Q value	−1.1477	± 0.0007	Mev
Mass excess of O^{14} ($C^{12}=0$)	8.0073	± 0.0008	Mev
Mass of O^{14} ($C^{12}=12$)	14.0085967	± 0.0000009	amu
Positron end point for the decay of O^{14} to the 2.312-Mev state of N^{14}	1.8097	± 0.0015	Mev
f	42.81	± 0.26	
ft	3057	±20	sec

DISCUSSION

There is a discrepancy among the three previous values obtained for the threshold energy of the $C^{12}(He^3,n)O^{14}$ reaction. The first value, reported by Butler,[6] is 1435±5 kev and was measured with the NRL 2-Mv Van de Graaff Accelerator, equipped with a 90° magnetic analyzer calibrated with the $Li^7(p,n)Be^7$ reaction threshold at 1881.1 kev. The second value, reported by Bromley *et al.*,[5] is 1449.6±2.8 kev and was measured with the Chalk River Van de Graaff Accelerator, equipped with a 90° magnetic analyzer calibrated with the $Li^7(p,n)Be^7$ reaction threshold at 1881.6 kev and the $Li^7(\alpha,\gamma)B^{11}$ reaction resonance at 958 kev. The third measurement, made by Dunning *et al.*[15] with the NRL 5-Mv Van de Graaff Accelerator, utilizing the 50° port of a magnetic analyzer, is 1435±5 kev based on the $Li^7(p,n)Be^7$ reaction threshold at 1881.1 kev. All of these magnetic analyzer measurements were made with nuclear magnetic resonance equipment used to determine the magnetic field strength.

The agreement of the present measurement with the two previous NRL measurements is very good, but there is a real discrepancy with the Chalk River value. One of their calibration points, the resonance at 958±1 kev in the $Li^7(\alpha,\gamma)B^{11}$ reaction, was measured[16] in 1950 with a generating voltmeter and is based in turn on a resonance in the $F^{19}(p,\alpha\gamma)O^{16}$ reaction at 873.5 kev. On the NRL energy scale, the $F^{19}(p,\alpha\gamma)O^{16}$ resonance occurs at 872.4±0.5 kev. This difference can perhaps account for a small part, about 2 kev, of the discrepancy. But there still remains a discrepancy of about 11 kev, and this situation casts some doubt on the validity of the calibration point at 958±1 kev. A remeasurement of this resonance energy is therefore highly desirable. A value of 950.3 kev would remove the discrepancy between the NRL and Chalk River values.

Gerhart[4] made a direct measurement of the positron end point of the decay of O^{14} populating the 2.312-Mev state in N^{14} and found it to be 1.835±0.008 Mev. This value is also in rather poor agreement with the present results. Again the reason is not known, and an independent measurement of the positron end point is also highly desirable.

VECTOR COUPLING CONSTANT

Feynman and Gell-Mann[1] used the ft value[17] of Bromley *et al.*[5] for the decay of O^{14} to recalculate the vector coupling constant G_V, obtaining the value $(1.41\pm0.01)\times10^{-49}$ erg cm^3.[18] When they equated this revised value of G_V to G_μ according to their conserved vector current hypothesis, they computed a predicted mean life of the muon, 2.26±0.04 μsec, which they compared with the experimental measurements of Bell and Hincks,[19] 2.22±0.02 μsec, and Dudziak *et al.*,[20] 2.21±0.02 μsec. Feynman and Gell-Mann ignored the radiative correction factors, but Berman[21] calculated the radiative correction factors for the decay of O^{14} and the muon, obtaining a correction to the observed O^{14} half-life of +2.6% and a correction of −0.44% to be applied to the observed muon mean life. His value of the predicted mean life of the muon is 2.33±0.05 μsec, making the agreement between predicted and measured values poorer. Kinoshita and Sirlin[22] also calculated the radiative correction factors in the decay of O^{14}, obtaining a correction of +1.7% to be applied to the observed O^{14} half-life, and obtained a predicted muon lifetime of 2.31±0.05 μsec. If these calculations were corrected for the partial half-life[17] of the Fermi transition in the decay of O^{14}, giving an ft value of 3105, the predicted lifetimes would be as follows: Feynman and Gell-Mann, 2.27±0.04 μsec; Berman, 2.34±0.05 μsec; Kinoshita and Sirlin, 2.32±0.05 μsec.

Between the preliminary report[8] of the present experiment and this final report, a number of relevant experiments and calculations have been performed. Durand *et al.*,[23] using the techniques of dispersion theory, have calculated the corrections to the decay of O^{14} (including competition from K capture, electron screening, electromagnetic form factors, and "second forbidden" nuclear matrix elements as well as radiative). Their over-all correction (including the Coulomb cor-

[15] K. L. Dunning, R. O. Bondelid, and J. W. Butler (unpublished results, 1957).

[16] W. E. Bennett, P. A. Roys, and B. J. Toppel, Phys. Rev. **82**, 20 (1951).

[17] This ft value of Bromley *et al.*, reference 5, was calculated apparently with the over-all experimentally observed half-life of O^{14} (Gerhart, reference 4), instead of the partial half-life (Sherr *et al.*, reference 2). The corrected value is 3105.

[18] The dimensions of the universal Fermi coupling constant have been incorrectly given as erg/cm^3 by several authors; for example, in references 1 and 22 and also by J. Bernstein and R. R. Lewis, Phys. Rev. **112**, 232 (1958).

[19] W. E. Bell and E. P. Hincks, Phys. Rev. **84**, 1243L (1951).

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[22] T. Kinoshita and A. Sirlin, Phys. Rev. **113**, 1652 (1959).

[23] L. Durand, III, L. F. Landovitz, and R. B. Marr, Phys. Rev. Letters **4**, 620 (1960).

TABLE II. Comparison of the values of the vector coupling constants G_V and G_μ and a comparison of the predicted muon mean life τ_μ with the experimental mean life, 2.210±0.003 μsec. The uncertainties given are those due to the uncertainty in the *ft* value, and do not include the uncertainties due to the calculations of corrections.

	No corrections	Berman[a]	Kinoshita and Sirlin[b]	Durand *et al.*[c]
G_V (10^{-49} erg cm^3)	1.418±0.004	1.400±0.004	1.406±0.004	1.426±0.004
G_μ (10^{-49} erg cm^3)	1.428±0.002	1.432±0.002	1.432±0.002	1.438±0.002
ΔG[d] (%)	0.7	2.2	1.8	0.8
τ_μ (μsec)	2.241±0.013	2.309±0.013	2.289±0.013	2.245±0.013
$\Delta\tau$[e] (%)	1.4	4.5	3.6	1.6

[a] Calculated with the use of the radiative corrections of Berman, reference 21. His corrections to be applied to the observed O^{14} half-life amounted to +2.6%, and his corrections to be applied to the observed muon mean life amounted to −0.44%.
[b] Calculated with the use of the radiative corrections of Kinoshita and Sirlin, reference 22. Their corrections to the observed O^{14} half-life amounted to +1.7%, and we use Berman's correction of −0.44% to the observed muon mean life.
[c] Calculated with the use of the corrections, radiative and otherwise, of Durand *et al.*, reference 23. Their total corrections to the observed half-life of O^{14} amounted to −1.12%, and to the observed muon mean life, −1.32%. Their definitions of G_V and G_μ are somewhat different from those of others.
[d] ΔG represents the difference between the value of G_V and the value of G_μ expressed as a percentage.
[e] $\Delta\tau$ represents the difference between the predicted muon mean life and a weighted average of the measured values, 2.210±0.003 μsec, expressed as a percentage.

rection of MacDonald[3] to the nuclear matrix element) to the observed O^{14} half-life is −1.12%. They combined this correction with the O^{14} beta end-point energy from the preliminary report[8] of the present experiment, 1.8000±0.0065 Mev (based on the 1956 table of masses), and obtained a predicted muon mean life of 2.23±0.05 μsec, in good agreement with several very recent experimental measurements: Fischer *et al.*,[24] 2.200±0.015 μsec; Reiter *et al.*,[25] 2.211±0.003 μsec; and Telegdi *et al.*,[26] 2.208±0.005 μsec.

If we use the corrections of Durand *et al.* with the O^{14} end-point energy from the present experiment based on the 1960 table of masses, the agreement between predicted and experimental values for the muon lifetime is not quite so good. However, shortly before the manuscript of the present experiment was submitted, we received preprint copies of two other related experiments indicating that the O^{14} half-life previously used was too high. The *ft* value in Table I includes these latest half-life measurements, and new values of G_V, G_μ, and τ_μ have been calculated. These new O^{14} half-life measurements were made by Bardin *et al.*,[27] who obtained the value 71.1±0.2 sec and Hendrie and Gerhart,[28] who obtained the value 70.91±0.04 sec. The partial half-life for the Fermi transition based on the branching ratio of Sherr *et al.*,[2] is approximately 0.6% larger than the observed half-life. We have adopted the value 71.4±0.2 sec for the average of the partial half-lives, and it is this value which has been used to obtain the *ft* value of Table I.

Values of G_V have been computed using the formula

$$G_V=2^{\frac{1}{2}}gC_V=\left[\frac{2\pi^3\ln 2}{2}\frac{\hbar/mc^2}{(ft)_{O^{14}}}\right]^{\frac{1}{2}} mc^2(\hbar/mc)^3, \qquad (1)$$

which is a variation of the one discussed by Konopinski.[29] These values, computed from the uncorrected *ft* value of Table I and from the variously corrected *ft* values, are given in Table II. These values for G_V may be compared with the coupling constant calculated from muon decay using the $V-A$ theory. Using the most recent measurement[26] of muon mass, 206.76±0.03 electron masses, a weighted average of the muon mean life,[24–26] 2.210±0.003 μsec, and the formula[29]

$$G_\mu^2=6\pi^3\hbar^7c^6/\tau_\mu W_0^5, \qquad (2)$$

where W_0 is the maximum energy the electrons may have and is almost exactly half the muon rest mass, we calculate G_μ to be $(1.428\pm0.002)\times10^{-49}$ erg cm^3. If we use the muon radiative correction of Berman,[21] the corrected G_μ is 1.432×10^{-49} erg cm^3. The muon radiative corrections of Durand *et al.* give a value of $(1.438\pm0.002)\times10^{-49}$ erg cm^3 for G_μ which is in fair agreement with their value of G_V, $(1.426\pm0.004)\times10^{-49}$ erg cm^3, but they used different definitions for G_V and G_μ from the usual ones in Lagrangian theory, and the theoretical significance of this equality is therefore not clear. The deviations of G_V and G_μ are listed in Table II in the row beneath the values of G_μ.

Table II also lists the predicted values of τ_μ obtained when one applies the respective corrections to the O^{14} and muon decay rates. These values may be directly compared with a weighted mean of the observed[24–26] values of τ_μ, 2.210±0.003 μsec.

The last row of Table II lists the differences between the predicted values of τ_μ and the experimental value, expressed as a percentage. During the calculation of the numbers in Table II, all interim numbers were carried with two extra digits, the final entries then being rounded off.

ACKNOWLEDGMENTS

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