

## Effects of a Magnetic Field on Heat Conduction in Some Ferrimagnetic Crystals\*

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The thermal conductivities of single-crystal bars of several ferrites have been measured over the temperature range 1.4° to 25°K in both zero external magnetic field and in a field of about 9.4 kilo-oersteds. For each specimen the zero-field conductivity is a smooth function of the temperature over the entire range considered. The magnitudes of the conductivities of the manganese and cobalt-zinc ferrite specimens are quite low, approximately 1.5 mw/cm<sup>2</sup>°K at 4°K. Upon application of a magnetic field parallel to the direction of heat current flow, the conductivities of the manganese ferrite and cobalt-zinc ferrite specimens are increased. The magnetoconductivities are rather large. For example,  $\Delta K/K_0=0.2$  for manganese ferrite at 2°K. Such a field produces no detectable change in the conductivity of the manganese-zinc ferrite. The experimental results are interpreted on the basis of the assumption that the heat current is primarily a phonon current limited by phonon-magnon scattering.

### INTRODUCTION

HEAT conduction in many solids is adequately described in terms of energy transport via electrons and/or quantized lattice vibrations (phonons), with due allowance for the interaction of these "excitations" with one another and with impurities, imperfections, or crystal boundaries. Other types of elementary excitations are thought to occur in certain solids, and it is natural to inquire what role, if any, these may play in the transport of heat. Theoretical speculation on the subject has been, until recently, quite limited.

An excitation which may well produce observable effects is the magnon, or quantized spin wave, occurring in spontaneously magnetized solids at temperatures below their Curie points. Rezanov and Cherepanov<sup>1</sup> have calculated the thermal conductivity of a ferromagnetic metal, treating the magnons present as Bose-Einstein particles. The conductivity is found to be essentially electronic, the magnons serving primarily to scatter electrons, thus reducing the electronic conductivity in much the same way as do phonons. The isolation and identification of the magnon effect in this situation is probably difficult and has not been attempted.

A more attractive experimental possibility is that pointed out by Sato.<sup>2</sup> He compares the thermal conductivity due to magnons with the phonon conductivity in a ferromagnetic insulator, assuming no phonon-magnon interaction or ferromagnetic anisotropy. It is

further assumed that at sufficiently low temperatures the magnon free path and the phonon free path become boundary limited, temperature independent, and of equal magnitude. For a typical substance, then, it is found that the magnon conductivity, varying as  $T^2$ , exceeds the phonon conductivity, varying as  $T^3$ , in the liquid helium temperature range. To the extent that real ferromagnetic insulators satisfy the simplified conditions of the calculation, it should be possible to detect magnon conductivity in the liquid helium region, identifying it perhaps merely by its temperature dependence.

It would appear to be possible to test this picture by measuring the low-temperature thermal conductivities of ferrimagnetic crystals. Of the substances in this category, ferrite crystals are the most easily prepared in a form suitable for measurement. These materials are semiconductors and many compositions behave essentially as insulators at liquid helium and hydrogen temperatures. While the interactions among magnetic ions in a ferrite may all be antiferromagnetic, analysis of the spin-wave spectrum shows that the important branch at low temperatures is one for which the dispersion law is of the same form as for a ferromagnet.<sup>3</sup> As a result, the magnetization and the magnon specific heat and thermal conductivity of a ferrite crystal should show the same temperature dependence as for an idealized ferromagnetic insulator.

The practicality of testing Sato's speculations by simply measuring the low-temperature thermal conductivities of ferrite crystals is doubtful. Even in the unlikely event that the simplifications of the model are reproduced in a given substance, the magnitude and temperature dependence of the conductivity over the available temperature range would not provide unambiguous identification of the mechanisms involved. The significance of the experiment is greatly increased, however, if we introduce an adjustable parameter, namely, an externally applied magnetic field. If such a field were found to produce any appreciable change in

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<sup>1</sup> A. J. Rezanov and V. J. Cherepanov, Proc. Acad. Sci. U.S.S.R. 93, 641 (1953).

<sup>2</sup> H. Sato, Progr. Theoret. Phys. (Kyoto) 13, 119 (1955).

<sup>3</sup> H. Kaplan, Phys. Rev. 86, 121 (1952); see also T. A. Kaplan, Phys. Rev. 109, 782 (1958).

the thermal conductivity of an insulating ferrimagnet, the presumption would be strong that the magnetic spin system was participating in some way in the process of heat transport. A substance describable by the independent-excitation model, for example, would exhibit a magnetoconductivity of characteristic sign and magnitude. It is instructive to make a rough calculation of this particular limiting value of the magnetoconductivity in order to appreciate the feasibility of experiments of this type.

Calculations of the specific heats of ferrites by Kouvel and Brooks<sup>4</sup> show that at a given temperature the magnon specific heat may be significantly reduced upon application of a magnetic field. An analogous effect occurs in the magnon thermal conductivity. This becomes evident if one repeats Sato's calculation including in the expression for the energy of a magnon of wave number  $\mathbf{k}$  the Zeeman term, i.e.,

$$E = Ck^2 + g\mu H, \quad (1)$$

where  $C$  is a constant proportional to the exchange energy,  $g$  is the spectroscopic splitting factor,  $\mu$  the Bohr magneton. For appropriate crystalline directions,  $H$ , as used in this expression, may be regarded as the resultant of an external field and an anisotropy field.

The magnon contribution to the conductivity, assumed independent of phonon contribution, can be calculated from the general expression deduced by Klemens,<sup>5</sup> viz.,

$$K = \frac{1}{3} \sum_j \int \frac{\partial U_j(\mathbf{k}, T)}{\partial T} v_{vj}(\mathbf{k}) l_j(\mathbf{k}) d\mathbf{k}, \quad (2)$$

where the summation is over all possible polarizations.  $\partial U_j(\mathbf{k}, T)/\partial T$  is the contribution to the specific heat,  $v_{vj}(\mathbf{k})$  is the group velocity, and  $l_j(\mathbf{k})$  is the mean free path of particles of wave number  $\mathbf{k}$ . If one assumes an effective magnon mean free path,  $l_s$ , independent of  $\mathbf{k}$  (wave number), then this expression yields for the magnon thermal conductivity at low temperatures,

$$K_{SH} = \frac{l_s k^3 T^2}{3\pi h C} \sum_{n=1}^{\infty} \left[ \left( \frac{g\mu H}{\kappa T} \right) \frac{21}{n} + \left( \frac{4g\mu H}{\kappa T} \right) \frac{1}{n^2} + \frac{6}{n^3} \right] \times \exp\left(-\frac{n\mu g H}{\kappa T}\right), \quad (3)$$

which, for  $H=0$ , reduces to the result implicit in Sato's work,

$$K_{S0} = 0.765 l_s k^3 T^2 / h C. \quad (4)$$

Using Eq. (2) and the Debye model, we calculate the phonon conductivity at low temperatures assuming again a constant mean free path  $l_p$ . The result is

$$K_p = 326 \kappa^4 l_p T^3 / h^3 v^2. \quad (5)$$

<sup>4</sup> J. S. Kouvel and H. Brooks, Technical Report No. 198, Cruft Laboratory, Harvard University, May 20, 1954 (unpublished).

<sup>5</sup> P. G. Klemens, in *Handbuch der Physik*, edited by S. Flügge (Springer-Verlag, Berlin, 1956), Vol. 14, p. 198.

TABLE I. Summary of specimen properties.

	MnFe <sub>2</sub> O <sub>4</sub>	Co(Zn)Fe <sub>2</sub> O <sub>4</sub>	Mn(Zn)Fe <sub>2</sub> O <sub>4</sub>
Major metallic components (wt. %)	38.4 Fe 33.7 Mn	55.8 Fe 11.6 Co 6.46 Zn	49.2 Fe 20.95 Mn 6.17 Zn
Average resistivity (ohm-cm)			
Room temp.	2×10 <sup>4</sup>	0.2	0.2
77°K	~10 <sup>12</sup>	53	172
4.2°K	>>10 <sup>12</sup>	≥3×10 <sup>10</sup>	≥7×10 <sup>9</sup>
Length (cm)	4.92	4.37	4.29
Cross section (cm <sup>2</sup> )	0.434	0.329	0.218
Room-temperature saturation magnetization (cgs units/g)	52.7	99.2	86
Orientation of the long axis	[410]	[534]	[252]

Here  $v$  is the average sound velocity  $\kappa$  is Boltzmann's constant.

Let us estimate the magnitude of the conductivity expected for an ideal ferrimagnetic insulator at helium temperatures assuming independent phonon and magnon contributions and mean free paths limited by boundary scattering. For a typical ferrite  $C \sim 10^{-28}$  erg-cm<sup>2</sup>, so that Eq. (4) gives  $K_{S0} \sim 0.30 \times 10^9 l_s T^2$  mw/cm<sup>2</sup>-°K. The average sound velocity will be roughly  $v \sim 3 \times 10^5$  cm/sec, so that from Eq. (5) we find  $K_p \sim 0.45 \times 10^9 l_p T^3$  mw/cm<sup>2</sup>-°K. Equating  $l_s$  and  $l_p$  to the mean diameter of the specimen rod, say 0.5 cm, we find at 2°K the total conductivity in the absence of any external field to be  $K_0 = K_{S0} + K_p \sim 2.4 \times 10^8$  mw/cm<sup>2</sup>-°K. Using now Eq. (3), we may estimate the fractional change in the conductivity produced by application of a field of 10<sup>4</sup> oe to the same idealized crystal. This magnetoconductivity,  $\Delta K/K_0 (= K_{SH} - K_{S0}/K_p + K_{S0})$ , is equal to -0.07 at 2°K. At 3°K,  $\Delta K/K_0$  would become -0.03 while  $K_0$  would increase to  $\sim 7.4 \times 10^8$  mw/cm<sup>2</sup>-°K. Note that  $\Delta K/K_0$  would not be changed by making  $l_s$  and  $l_p$  shorter so long as they remained equal to one another.

The thermal magnetoconductivity,  $\Delta K/K_0$ , in the independent excitation limit is thus expected to be rather large. This has suggested the possibility that even in real ferrimagnetic crystals a thermal magnetoconductivity might be detected in the helium region with available fields. We have, therefore, measured the thermal conductivities of several ferrite crystals at low temperatures and the effect produced by an externally applied magnetic field. An effect has been found but of a kind which apparently requires for its interpretation modification of the simplified picture outlined above.

#### EXPERIMENTAL PROCEDURE

The thermal conductivity measurements have been carried out by a standard method<sup>5,6</sup> in which a sta-

<sup>6</sup> J. L. Olsen and H. M. Rosenberg, in *Advances in Physics*, edited by N. F. Mott (Taylor and Francis, Ltd., London, 1953), Vol. 2, p. 28.

tionary temperature gradient is maintained in a small rod of specimen material by means of an electrical heater attached to one end. For this purpose it was necessary to fabricate from the rough single crystal boules, kindly supplied by Linde Air Products Corporation, rodlike specimens of uniform cross section. A careful surface grinding procedure yielded rectangular bars approximately 2 in.  $\times$   $\frac{1}{4}$  in.  $\times$   $\frac{1}{4}$  in. No additional treatment of the specimens was attempted. Table I contains a summary of the properties of the specimens employed in the present work. It should be noted that these crystals were prepared by the flame fusion technique, and, as a result, were of nonuniform composition. The analysis quoted for a given specimen was obtained for scrap material from the same boule. It merely suggests the general composition. An attempt to deduce from it a chemical formula characteristic of the specimen as a whole would hardly be meaningful. We shall identify the three specimens as (1) manganese ferrite,  $\text{MnFe}_2\text{O}_4$ , (2) cobalt-zinc ferrite,  $\text{Co}(\text{Zn})\text{Fe}_2\text{O}_4$ , and (3) manganese-zinc ferrite,  $\text{Mn}(\text{Zn})\text{Fe}_2\text{O}_4$ , bearing in mind that these designations are not proper chemical ones.

The saturation magnetizations given in Table I were kindly obtained for us by Simon Foner of Lincoln Laboratory. The value for the manganese ferrite specimen is lower than that for the pure material (80 gauss-cm<sup>2</sup>/g). This is consistent with the fact that the crystal contains excess manganese. Small regions in the specimen consisting essentially of oxides of manganese would probably not contribute to the spontaneous magnetization. The saturation magnetization of the manganese-zinc ferrite specimen is essentially that<sup>7</sup> of a material having the composition  $\text{Mn}_{0.7}\text{Zn}_{0.3}\text{Fe}_2\text{O}_4$ . Similarly, the value obtained for the cobalt-zinc ferrite specimen is roughly that<sup>7</sup> characteristic of a pure material of the composition  $\text{Co}_{0.5}\text{Zn}_{0.5}\text{Fe}_2\text{O}_4$ . Unfortunately, the need for rodlike geometry made it impossible in the present experiments to select a simple crystalline direction for the specimen axis. Clearly this would be desirable in subsequent studies.

The measured electrical resistivities of the specimens at three temperatures are shown in Table I. It will be seen that in the liquid helium range the resistivities are so large that it is unlikely that significant heat transport via free carriers occurs in these materials.

The specimen was mounted in a copper vacuum chamber which could either be filled with He gas at low pressure or evacuated to a pressure of less than  $10^{-7}$  mm of Hg. The vacuum jacket could be immersed either in liquid He or in liquid (or solid) H<sub>2</sub>. One end of the specimen was held by a copper clamp connected directly to the wall of the vacuum jacket and thus was in good thermal contact with the refrigerant. A heater of approximately  $10^8$  ohms was wound on the free end of the specimen. At two intermediate positions along the rod carbon resistance thermometers (one-half watt

Allan-Bradley resistors nominally 270 ohms) were attached by means of small copper clamps.

With this arrangement the thermal conductivity is given by

$$K = \dot{Q}L/A\Delta T,$$

where  $L$  is the distance between thermometers,  $A$  the cross-sectional area,  $\dot{Q}$  the power dissipated in the heater, and  $\Delta T$  the temperature difference indicated by the two thermometers.

During each experimental run, both thermometers were calibrated against the vapor pressure of the refrigerant. In the helium range the temperatures were those of Clement's T55E scale.<sup>8</sup> Above the lambda point appropriate correction for hydrostatic pressure head was applied to the measured pressures. In the hydrogen range the vapor pressure-temperature values obtained at the National Bureau of Standards<sup>9</sup> were employed. The calibration data were fitted approximately with the semiempirical expression suggested by Clement<sup>10</sup>:

$$[(\log_{10}R)/T]^3 = a + b \log_{10}R.$$

Having chosen a reasonable value for the constant  $b$ , precise values of  $a$  were computed for each calibration point and plotted against  $\log_{10}R$ . A smooth curve drawn through these points provided not only a continuous calibration in the helium and hydrogen ranges but also one for the region from 4.2 to 14°K. The accuracy of a temperature measurement by this method was approximately one millidegree in the helium region and ten millidegrees over the hydrogen range. In a typical experiment in the helium range,  $\Delta T \sim 0.05 - 0.1^\circ\text{K}$ , while at hydrogen temperatures,  $\Delta T \sim 1.0^\circ\text{K}$ . The bath temperature could be held constant for periods up to an hour by means of a pressure regulator designed by Burk.<sup>11</sup>

In order to minimize the effects of demagnetization, measurements in an external field were performed with the field applied parallel to the long axis of the specimen. This configuration necessitated the use of an external Dewar assembly of rather large diameter and limited the maximum field attainable with the Weiss-type electromagnet to 9400 oe. It should be emphasized that the thermometers were calibrated during each run for each value of magnetic field. In this way the small magnetoresistance of the thermometers introduced no error.

## RESULTS

Several different sets of measurements were taken on each specimen over the temperature range 1.4° to 25°K.

<sup>8</sup> J. R. Clement, J. K. Logan, and J. Gaffney, *Phys. Rev.* **150**, 743 (1955).

<sup>9</sup> H. W. Wooley, R. B. Scott, and F. G. Brickwedde, *J. Research Natl. Bur. Standards* **41**, 379 (1948).

<sup>10</sup> J. R. Clement, *Temperature* (Reinhold Publishing Company, New York, 1955), Vol. II, p. 382.

<sup>11</sup> D. L. Burk, thesis, Carnegie Institute of Technology, 1957 (unpublished); see also Technical Report No. 2, Carnegie Institute of Technology, August, 1958 (unpublished).

<sup>7</sup> C. Guillaud and H. Creveaux, *Compt. rend.* **230**, 1458 (1950).

The results obtained in zero external field with the specimens in a state of remanent magnetization reproduced those taken in zero field with the specimens thoroughly demagnetized.

The thermal conductivities of the three specimens in zero external magnetic field are displayed as functions of temperature over the range 1.4° to 25°K in Fig. 1. The magnitudes of the conductivities of the  $\text{MnFe}_2\text{O}_4$  and  $\text{Co(Zn)Fe}_2\text{O}_4$  specimens are seen to be roughly comparable and extremely small, amounting to about 0.4  $\text{mw/cm}^\circ\text{K}$  at 2°K and rising to approximately 15  $\text{mw/cm}^\circ\text{K}$  at 20°K. These values are lower than those reported for single crystals of any other insulating substances at these temperatures. They are comparable with the conductivities of disordered dielectrics and of alloys such as stainless steel.<sup>12</sup>

At the lowest temperatures, the thermal conductivity of the  $\text{Mn(Zn)Fe}_2\text{O}_4$  specimen, while also small, is an order of magnitude greater than the conductivities of the other two crystals.<sup>13</sup> The temperature variation of

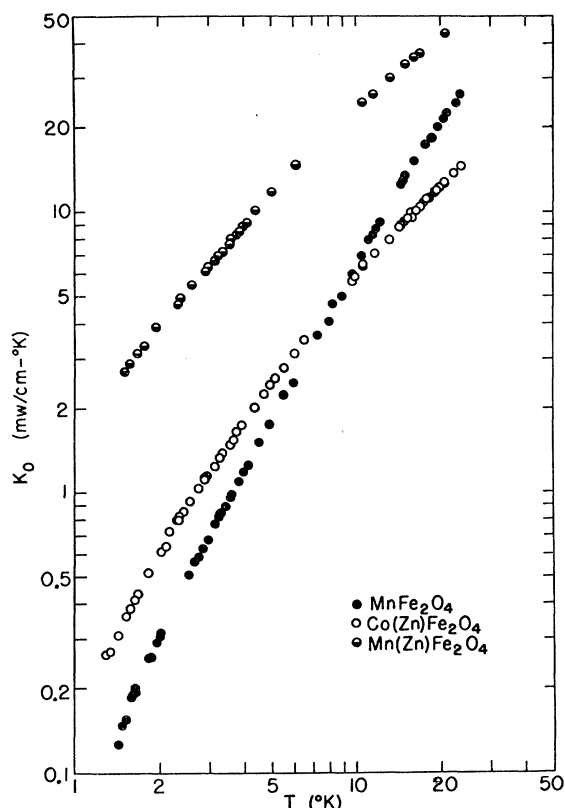


FIG. 1. A log-log plot of the temperature variation of the thermal conductivities of three ferrite specimens between 1.4° and 25°K.

<sup>12</sup> For a summary of thermal conductivity values, the reader is referred to the National Bureau of Standards Circular No. 556 (U. S. Government Printing Office, Washington, D. C., 1954).

<sup>13</sup> Dr. Guy K. White has kindly informed us of measurements in this range on a nickel ferrite crystal by G. K. White and S. B. Woods (unpublished). The temperature dependence of  $K$  is similar to that seen in our  $\text{MnFe}_2\text{O}_4$  specimen. The magnitude of  $K$  is more nearly that of our  $\text{Mn(Zn)Fe}_2\text{O}_4$  specimen.

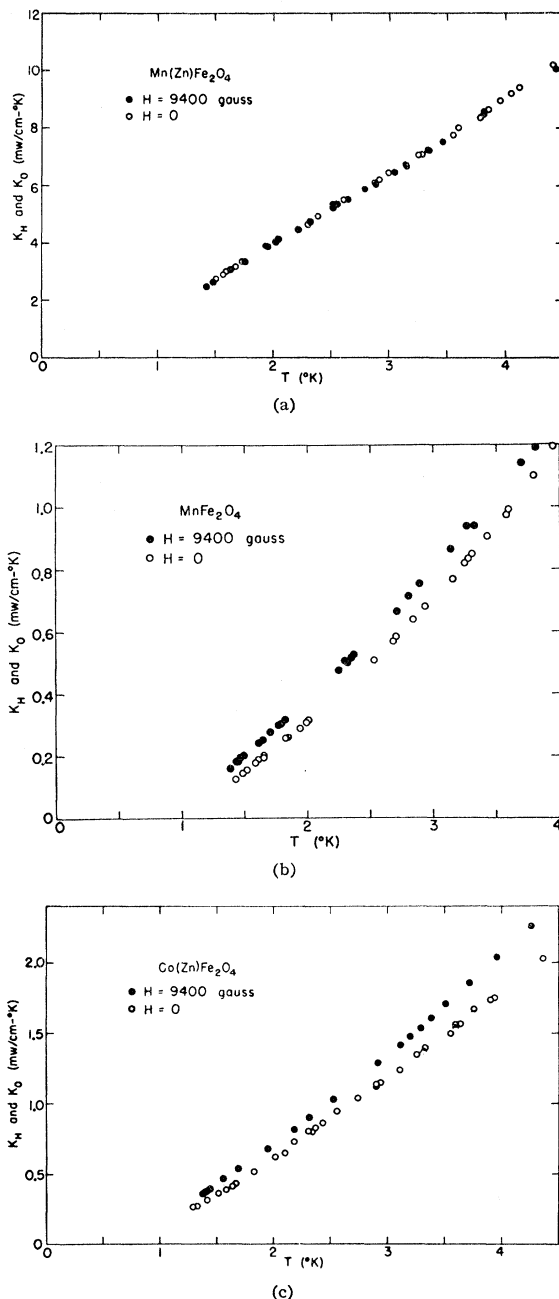


FIG. 2. The temperature variation of the thermal conductivity in both zero and 9400 gauss external longitudinal field of (a) manganese zinc ferrite; (b) manganese ferrite; (c) cobalt zinc ferrite.

$K_0$  is rather different in each of the specimens, and it would be difficult to reach any definite conclusion as to the dominant conductivity mechanism merely from this temperature dependence alone.

The results of the measurements for the three specimens in zero field and in a field of 9.4 kilo-oersteds applied parallel to the direction of heat current flow are summarized in Figs. 2(a), (b), and (c). For the  $\text{MnFe}_2\text{O}_4$

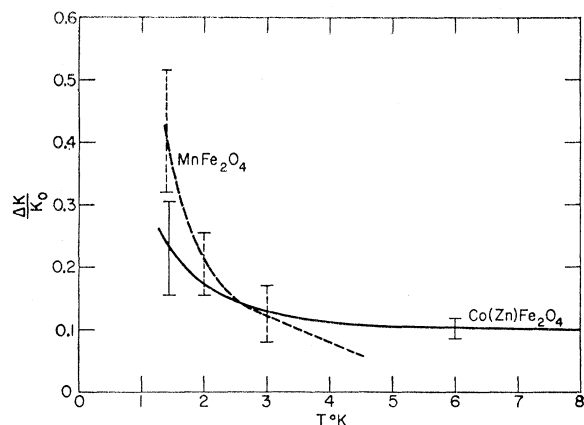


FIG. 3. The temperature variation of  $\Delta K/K_0$  for  $\text{MnFe}_2\text{O}_4$  (dashed curve) and  $\text{Co(Zn)Fe}_2\text{O}_4$  (solid curve). Bars indicate the estimated experimental uncertainties.

and  $\text{Co(Zn)Fe}_2\text{O}_4$  specimens at liquid helium temperatures, the field produces an increase in the thermal conductivity, i.e.,  $\Delta K/K_0 > 0$ . In the liquid hydrogen region  $\Delta K/K_0$  goes to zero for  $\text{MnFe}_2\text{O}_4$ , but is still measurable for the  $\text{Co(Zn)Fe}_2\text{O}_4$  sample. The results for the  $\text{Mn(Zn)Fe}_2\text{O}_4$  specimen, however, show no measurable magnetoconductivity throughout the range 1.4° to 25°K. The thermal conductivity was also measured for all three specimens in applied fields of 2.0 and 8.0 kilo-oersteds at several temperatures in this range. The need for preparing a complete calibration curve for both thermometers for any given external field, however, made it impractical to carry out the systematic measurement of the conductivity as a function of field at a fixed temperature. The observations made on the  $\text{MnFe}_2\text{O}_4$  and  $\text{Co(Zn)Fe}_2\text{O}_4$  specimens lie within the extremes of the zero-field and maximum-field values. In the case of  $\text{Mn(Zn)Fe}_2\text{O}_4$ , for which the intermediate-field data are the most complete, no significant change from the zero-field conductivities was found.

The temperature dependence of the magnetoconductivities of the  $\text{MnFe}_2\text{O}_4$  and  $\text{Co(Zn)Fe}_2\text{O}_4$  specimens in maximum field is summarized in Fig. 3. The horizontal bars show the estimated uncertainty to be assigned at the indicated temperatures. The rather large values of the magnetoconductivities and especially their positive signs are most striking. It is interesting to note that the magnitudes of the magnetoconductivities of these two substances are comparable, as are their conductivities. If the change in conductivity upon application of a magnetic field were roughly the same, regardless of specimen composition, then it would not be surprising that no magnetoconductivity is observed in  $\text{Mn(Zn)Fe}_2\text{O}_4$ . At 2.5°K, for example,  $\Delta K$  for  $\text{Co(Zn)Fe}_2\text{O}_4$  is 0.1 mw/cm-°K, so that  $\Delta K/K_0 \sim 0.1 \pm 0.025$ . Since the precision of the  $\text{Mn(Zn)Fe}_2\text{O}_4$  results is essentially the same, an observable  $\Delta K$  would have to be at least an order of magnitude greater than that of the

$\text{Co(Zn)Fe}_2\text{O}_4$ . Thus, we cannot conclude from the present observations that  $\text{Mn(Zn)Fe}_2\text{O}_4$  does not exhibit thermal magnetoconductivity.

#### DISCUSSION

The noteworthy features of the observations are best emphasized by comparison with expectations based on the independent excitation model. First of all, the temperature variation of the zero-field conductivities at the lowest temperatures is different for each substance. Only in the case of  $\text{Co(Zn)Fe}_2\text{O}_4$  below 2.2°K is  $K$  roughly proportional to  $T^2$  as the simple picture would predict. In this region the conductivity of  $\text{MnFe}_2\text{O}_4$  actually varies more rapidly with temperature whereas  $\text{Mn(Zn)Fe}_2\text{O}_4$  exhibits a slower temperature variation. Secondly, the magnitudes of the zero-field conductivities are much smaller than would be anticipated if the phonon and magnon free paths were limited by the specimen boundary. Whereas our idealized specimen was expected to show a conductivity of  $\sim 2 \times 10^3$  mw/cm-°K at 2°K, one actually observes conductivities smaller by a factor of  $\sim 10^3$  or  $10^4$ . The mean free path of the dominant excitation is apparently  $\sim 10^{-3}$  or  $10^{-4}$  cm, while the minimum specimen dimension is 0.5 cm. Finally, a magnetoconductivity has only been observed in those specimens having the lowest zero-field conductivity, and, what is probably most significant, the sign of this magnetoconductivity is positive, contrary to expectation.

We are not in a position to present a comprehensive quantitative interpretation of these diverse facts. However, the facts themselves suggest certain plausible modifications of the original simple picture. As was noted, the mean free path of excitations responsible for heat transport is much smaller than the specimen dimensions. This could result from inhomogeneities in the specimen material, and also from mutual interaction of excitations of different types, viz., phonons and magnons. Furthermore, it is not obvious that all inhomogeneities would restrict the mean free paths of phonons and magnons to the same extent. We are dealing with ferrites having the spinel lattice in which disordering of the magnetic ions as well as their inhomogeneous distribution is very easily achieved, particularly in crystals grown by the flame fusion method. It is possible to conceive of boundaries in such a medium which would be significant for magnons and relatively unimportant for phonons.

We will assume, therefore, that: (1) the mean free path of magnons is significantly smaller than that of phonons; (2) that phonons and magnons interact. These hypotheses permit, then, a consistent interpretation of what is probably the most striking observation, viz., the negative thermal magnetoconductivity. We see this as follows: one can imagine a magnon mean free path sufficiently small ( $< 10^{-4}$  cm) that phonons dominate the total conductivity. The free path of phonons ( $\sim 10^{-3}$  or  $10^{-4}$  cm) is then limited in part by scattering

on magnons. Application of an external field effectively reduces the concentration of magnons and thus the magnon scattering of phonons. The phonon mean free path is thereby increased, resulting in an observed rise in the total conductivity, even though the magnon conductivity has been reduced.

A phonon free path of  $10^{-4}$  cm corresponds to a phonon relaxation time of  $\tau \sim 10^{-8}$  sec, assuming the sound velocity to be  $3 \times 10^5$  cm/sec. According to our assumptions this  $\tau$  is to be identified as essentially a phonon-magnon relaxation time. One must ask whether, on the basis of current theories of the phonon-magnon interaction, such a short relaxation time is at all plausible. The theory of phonon-magnon interaction has been discussed by Kittel and Abrahams.<sup>14</sup> They calculate phonon-magnon relaxation processes using essentially the magnetoelastic energy as an interaction Hamiltonian. One concludes from their work that the indirect processes, in which, for example, a phonon decays into two magnons, are quite improbable at low temperatures. The direct process, however, in which a phonon decays into a single magnon is more probable by many orders of magnitude,<sup>15</sup> and might well account for a relaxation time of  $\sim 10^{-8}$  sec.

It is expected that direct processes are resonant, i.e., will occur when both phonons and magnons have equal energies and wave numbers. It is instructive to estimate the phonon wave number associated with the resonance condition. We equate the phonon energy,  $E_p = \hbar v k_p$ , and the magnon energy in the absence of a field,  $E_s = C k_s^2$ , and let the phonon and magnon wave numbers be equal. Then  $k = \hbar v / C \sim 3 \times 10^6$  cm<sup>-1</sup>. An estimate of the dominant phonon wave number is obtained by equating the phonon energy to  $\kappa T$ . Thus,  $k_p \sim \kappa T / \hbar v$ . At 7°K, for example, this would yield  $k_p \sim 3 \times 10^6$  cm<sup>-1</sup>. Thus in the temperature range covered by our experiments the resonance condition for direct processes is satisfied, and it is quite likely that these processes will be significant in determining the phonon-magnon relaxation time. It may be argued that resonant processes would affect only phonons of a very limited range of wave numbers, and, therefore, at low temperatures do little to assist the relaxation of the phonon

system as a whole. As was pointed out, the ferrite crystals used in these measurements are probably disordered magnetically. The work of Clogston *et al.*<sup>16</sup> suggests that, under these circumstances, direct transitions can occur in which crystal momentum need not be conserved; thus many phonons might be involved.

These qualitative considerations suggest the explanation of the observed behavior of the thermal conductivity of certain ferrite crystals in external magnetic fields. It seems certain that the independent-excitation model, whose predictions stimulated the present investigation, is not realistic. The phonon-magnon interaction would seem to play an essential role in heat transport in real crystals. A full treatment of this problem must, of course, deal also with the effects of demagnetization, anisotropy, and domain structure which have been ignored in this discussion. Their importance will be known only after the performance of additional experiments outlined below.

The present investigation constitutes a preliminary study of thermal conductivity in ferrimagnetic crystals at low temperatures. Several obvious extensions of this work are in progress or are contemplated. First of all, measurements of conductivity as a function of field up to very high fields would help to establish clearly the nature of the scattering processes. Secondly, the interpretation of the observations would be greatly simplified by the use of specimens of simple chemical composition oriented in such a way that a magnetic field might be applied along an easy direction of magnetization. Finally, measurements of considerable interest could be performed on ferrimagnets of better defined magnetic structure, e.g., magnetite and in particular the garnets.

#### ACKNOWLEDGMENTS

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<sup>14</sup> C. Kittel and E. Abrahams, *Revs. Modern Phys.* **25**, 233 (1953).

<sup>15</sup> We are grateful to Professor C. Kittel for illuminating correspondence on this and related matters.

<sup>16</sup> A. M. Clogston, H. Suhl, L. R. Walker, and P. W. Anderson, *J. Phys. Chem. Solids* **1**, 129 (1956).