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Difference in the Multiple Scattering of Electrons and Positrons

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The difference in the multiple scattering of electrons and positrons has been calculated on the basis of the work of Nigam, Sundaresan, and Wu. The results are compared with the experimental work of Henderson and Scott and are found to be in good agreement.

I. INTRODUCTION

IN a recent paper Nigam, Sundaresan, and Wu[1] have pointed out that the formula given by Molière[2] for the scattering cross section of a charged particle by an atom in his theory of multiple scattering is incorrect. This is because Molière's calculation of the scattering amplitude[1] includes an inconsistent expansion of the phase shift in powers of $\alpha_1=zZe^2/\hbar v$. In the paper of Nigam *et al.*, hereafter to be referred as paper A, Dalitz's[3] expression for the scattering cross section of a charged particle by the screened Coulomb field of an atom is used, and the distribution function for multiple scattering is calculated in powers of α_1 in a consistent manner. One interesting result of paper A is that, in contrast to Molière's theory, it predicts different screening angles for electron and positron scattering and, consequently, different distribution functions for multiple scattering. Mohr[4] has calculated the difference of electron and positron multiple scattering by using Dalitz's scattering cross section but essentially retaining Molière's result for the screening angle. In this note we have employed the results of paper A and checked them against the experiment of Henderson and Scott[5] on electron-positron multiple scattering.

[1] B. P. Nigam, M. K. Sundaresan, and T. Y. Wu, Phys. Rev. **115**, 491 (1959).
[2] G. Molière, Z. Naturforsch. **2a**, 133 (1947), and **3a**, 78 (1948).
[3] R. H. Dalitz, Proc. Roy. Soc. (London) **A206**, 509 (1951).
[4] C. B. O. Mohr, Proc. Phys. Soc. (London) **A67**, 730 (1954).
[5] C. Henderson and A. Scott, Proc. Phys. Soc. (London) **A70**, 188 (1957).

II. SUMMARY AND RESULTS OF PAPER A

In Molière's work[2] all the effect of the deviation from the first Born approximation on the distribution function for multiple scattering is contained in the quantity B which depends only on the screening angle χ_α. The screening angle χ_α, for the scattering of a charged particle by the screened Coulomb field of an atom is given by[6,7]

$$\ln\frac{2}{\chi_\alpha}-\frac{1}{2}=\int_0^1\frac{q(y)}{y}dy, \quad y=\sin(\chi/2), \qquad (1)$$
$$q(y)=\sigma(y)/\sigma_R(y),$$

where χ is the angle of scattering and $q(y)$ is the ratio of the scattering cross section (with screening) to the Rutherford cross section (no screening) for single scattering. The screening angle χ_α is defined such that in the first Born approximation, when $q(y)=\sigma_B(y)/\sigma_R(y)$,

$$\chi_\alpha\rightarrow\chi_0=\hbar\lambda/p, \qquad (2)$$

for the scattering of an electron of momentum p by an exponentially screened potential $V(r)=-(Ze^2/r)e^{-\lambda r}$. In Molière's paper the potential used is $V(r)=-(Ze^2/r)\omega(r\lambda_0)$, where the Thomas-Fermi function $\omega(r\lambda_0)$ is represented by a sum of three exponentials and λ_0

[6] S. A. Goudsmit and J. L. Saunderson, Phys. Rev. **57**, 24 (1940), and **58**, 36 (1940).
[7] H. A. Bethe, Phys. Rev. **89**, 1256 (1953).

TABLE I. Numerical values of $f^{(1)\prime}/\alpha\beta\chi_c\sqrt{B}$.

$\theta/\chi_c\sqrt{B}$	0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0	3.0	4.0
$f^{(1)\prime}/\alpha\beta\chi_c\sqrt{B}$	−11.123	−9.085	−6.848	−4.952	−3.254	−1.740	−0.371	+0.908	+2.179	+3.569	+0.716	+0.170	+0.063

$=Z^{\frac{1}{3}}/0.855a_0$, a_0 being the Bohr radius. In the first Born approximation, this Thomas-Fermi field gives, upon numerical integration,

$$\chi_\alpha \rightarrow \chi_0 \simeq 1.12(\hbar\lambda_0/p). \tag{2a}$$

In the paper of Nigam *et al.*,[1] the calculations are carried out by using Dalitz's[3] relativistic formula, derived in the second Born approximation, for the scattering of a spin $\frac{1}{2}$ particle of charge z ($z=-1$ for an electron) by an exponentially screened Coulomb field:

$$V(r)=(zZe^2/r)e^{-\lambda r}, \tag{3}$$

where the screening parameter $\lambda=\mu\lambda_0$, μ being an adjustable parameter of the order of unity. The parameter μ is introduced to compensate for the use of a single exponential as the screening factor of the Coulomb field of an atom instead of a sum of three exponentials as done by Molière. The expression for the screening angle χ_α is then obtained by calculating the angular distribution function[6,7]

$$f(\theta,t)=\sum_{l=0}^{\infty}(l+\tfrac{1}{2})P_l(\cos\theta) \times\exp\left\{-Nt\int_0^\pi d\chi\,\sin\chi\sigma_D(\chi)[1-P_l(\cos\chi)]\right\}, \tag{4}$$

where $f(\theta,t)\sin\theta d\theta$ is the actual number of scattered particles between θ and $\theta+d\theta$; t is the thickness of the foil. The expression for the screening angle χ_α is obtained by calculating the integral in the exponential of Eq. (4) and combining all the contribution from the small angles into a single term. This gives

$$\chi_\alpha=\chi_0\left\{1+2\alpha\chi_0\left[\frac{1-\beta^2}{\beta}\ln\chi_0+\frac{0.2310}{\beta}+1.448\beta\right]\right\}, \tag{5}$$

where

$$\chi_0=\mu\frac{\hbar}{p}\left(\frac{Z^{\frac{1}{3}}}{0.885a_0}\right), \quad \alpha=-zZ/137, \quad \beta=v/c.$$

The corresponding expression in Molière's theory is

$$\chi_\alpha=\chi_0\{1.13+3.76\alpha^2/\beta^2\}^{\frac{1}{2}}. \tag{6}$$

Notice that since in Eq. (5), χ_α depends on α, we will get different values of the screening angle for electron ($z=-1$) and positron ($z=+1$) scattering. In Molière's theory χ_α depends on α^2, Eq. (6), and there is no difference in electron and positron scatterings. The multiple scattering can then be described in terms of the parameters χ_α', ξ, b, and B defined as follows.

$$\ln(2/\chi_\alpha)=\ln(2/\chi_\alpha')-\tfrac{1}{2}+C-(2\alpha\chi_0/\beta)(1-\beta^2)(1-C), \tag{7a}$$

$$\xi=1+(2\alpha\chi_0/\beta)(1-\beta^2), \tag{7b}$$

$$b=\xi\ln(\chi_c^2/4)-\ln(\chi_\alpha'^2/4), \tag{7c}$$

$$b=B-\xi\ln B, \tag{7d}$$

where C is Euler's constant $=0.577$. Finally, the distribution function can be approximated by

$$f(\theta,t)\simeq K(\chi_c^2B)^{-1}\left\{f^{(0)}+\frac{1}{B}(f^{(1)\prime}+f^{(1)}) +\frac{1}{2!B^2}(f^{(2)\prime}+f^{(2)})+\cdots\right\}, \tag{8}$$

where

$$\int_0^\infty du\,u^{1+\frac{1}{2}\chi_c^2[\beta^2+\pi\alpha\beta-\frac{1}{4}\xi]}J_0\left(\frac{\theta}{\chi_c\sqrt{B}}u\right)\exp(-u^2/4) \times\begin{cases}1 & =f^{(0)},\\ -\pi\alpha\beta\chi_c(B)^{\frac{1}{2}}u & =f^{(1)\prime},\\ \xi(u^2/4)\ln(u^2/4) & =f^{(1)},\\ -2\pi\alpha\beta\chi_c(B)^{\frac{1}{2}}\xi(u^2/4)\ln(u^2/4) & =f^{(2)\prime},\\ [\xi(u^2/4)\ln(u^2/4)]^2 & =f^{(2)},\end{cases} \tag{9}$$

and

$$K=\exp\left\{\frac{B\chi_c^2}{16}\left[1+\frac{8\pi\alpha\beta}{B}+2\xi\frac{\ln 2}{B} +\frac{8(\beta^2+\pi\alpha\beta)}{B}(C-\ln\chi_c\sqrt{B})\right]\right\},$$

$\chi_c^2=4\pi Nte^4z^2Z(Z+1)/(pc\beta)^2$.

The exponent of u in the integrand can be taken as unity. The numerical values of $f^{(0)}$ and $f^{(1)}$ are given by Bethe[7] and the values of $f^{(1)\prime}$ for specific cases are given in paper A; the contributions of $f^{(2)}$ and $f^{(2)\prime}$ being ignored. In Table I, we give the numerical values of $f^{(1)\prime}/\alpha\beta\chi_c\sqrt{B}$.

III. MULTIPLE SCATTERING OF POSITRONS AND ELECTRONS

The experiment on the multiple scattering of positrons and electrons has been performed by Henderson and Scott.[5] We reproduce in part their Table I (our

TABLE II. Data and observed[a] $1/e$ widths for multiple scattering.

Energy Mev	Material	t (mg cm^{-2})	B	$\chi_c^2\times10^2$	θ_ω(obs)	θ_ω(calc)
0.40	Au	1.98	4.06	2.62	19.8	18.7
	Ag	2.64	4.96	2.29	18.3	19.3
	Al	6.04	6.03	1.64	18.8	18.0
0.59	Ag	2.64	4.85	1.20	13.2	13.8
1.2	Ag	2.64	4.71	0.367	7.25	7.54

[a] See reference 5.

Table II). θ_ω is the half-width (or $1/e$ width) and θ_ω is defined as the angle at which the intensity falls to a factor $1/e$ of its maximum value. In order to compare their experimental results for the difference of electron (θ_ω^-) and positron (θ_ω^+) $1/e$ widths with theory, they assumed, for the purpose of calculations, that the Molière's distributions for electrons and positrons differed only in the value of the parameter χ_c^2B. They noted that for a given scattering foil, the plot of the electron-positron count ratios calculated from Molière distributions with a given difference of χ_c^2B against the cube of the counter to foil distance is a straight line; the slope of the straight line varying linearly with the difference of χ_c^2B. Thus, the value which fitted the experimental straight line was taken to be the estimate of the difference of χ_c^2B.

Mohr[4] has calculated the difference in the multiple scattering of electrons and positrons, using Dalitz's[3] formula. He calculated the difference in the root mean square angle of scattering for electrons and positrons as given by Molière's theory,

$$\theta_{\rm rms}^2=\tfrac{1}{2}\theta_{\rm max}^2B = \int_0^{\theta_{\rm max}}\theta^2P(\theta)d\theta, \qquad (10)$$

where $P(\theta)$ is the probability of single scattering through an angle between θ and $\theta+d\theta$, which for small angles was taken as

$$P(\theta)\simeq 2\theta_{\rm max}^2\sigma_D{}^c d\theta/\theta^3, \qquad (11)$$

$\sigma_D{}^c$ being a corrected Dalitz cross section, evaluated by assuming the equality

$$\frac{\sigma_D{}^{c-}-\sigma_D{}^{c+}}{\sigma_D{}^{-}-\sigma_D{}^{+}}=\left[\frac{\sigma_D{}^{c-}-\sigma_D{}^{c+}}{\sigma_D{}^{-}-\sigma_D{}^{+}}\right]_{\lambda=0}, \qquad (12)$$

and B is the parameter as defined by Molière,[2] which is same for electron and positron [Eqs. (6) and (7), with $\xi=1$]. However, it is clear from the results of paper A, Eqs. (5) and (7), that B is different for electron and positron scattering since the screening angle as derived from Dalitz's[3] formula is different for the two cases. A consistent calculation for the difference in the multiple scattering of electrons and positrons should therefore use the formulation of paper A.

TABLE III. The screening angle χ_α and the $1/e$ width θ_ω for electron-positron multiple scattering.

μ	Particle	$\chi_0\times10^2$	$\chi_\alpha\times10^2$	ξ	b	B	θ_ω
	Au: E=0.4 Mev, β=0.8279, $\alpha=Z/137$=0.5766						
1.12	e^-	2.6845	2.6926	1.01175	3.3844	5.01	19.52
	e^+		2.6764	0.9882	3.4950	5.11	18.37
1.8	e^-	4.3144	4.374	1.0189	2.3705	3.71	16.24
	e^+		4.253	0.9811	2.5985	3.95	15.54
	Ag: E=0.4 Mev, β=0.8279, α=0.3431						
1.12	e^-	2.259	2.260	1.0059	3.624	5.30	18.50
	e^+		2.257	0.9941	3.676	5.34	17.85
1.8	e^-	3.629	3.649	1.0095	2.6497	4.07	15.76
	e^+		3.609	0.9905	2.7535	4.17	15.37
	Al: E=0.4 Mev, β=0.8279, α=0.09488						
1.12	e^-	1.471	1.4715	1.0001	4.1727	5.957	16.50
	e^+		1.4705	0.9999	4.1747	5.960	16.34
1.8	e^-	2.364	2.3646	1.0002	3.2252	4.7919	14.50
	e^+		2.3634	0.9998	3.2253	4.7927	14.36
	Ag: E=0.59 Mev, β=0.8858, α=0.3431						
1.12	e^-	1.7463	1.7580	1.0029	3.4909	5.131	13.07
	e^+		1.7346	0.9971	3.5464	5.188	12.75
1.8	e^-	2.8066	2.8430	1.0047	2.5219	3.886	11.04
	e^+		2.7702	0.9953	2.5821	3.949	10.80
	Ag: E=1.2 Mev, β=0.9544, α=0.3431						
1.12	e^-	1.0430	1.0519	1.0007	3.3434	4.942	7.02
	e^+		1.0341	0.9993	3.4320	5.050	6.98
1.8	e^-	1.6762	1.7002	1.0011	2.3807	3.6882	5.86
	e^+		1.6523	0.9989	2.4511	3.779	5.84

The results of our calculations based on Eqs. (5) and (7), (8), (9) and data of Table II are presented in Tables III to IV. The calculations are carried out for $\mu=1.12$ and $\mu=1.8$, where μ is the adjustable parameter defined in Eq. (3). In Table IV the observed values are those of Henderson and Scott[5] and refer to percentage difference in widths of the Gaussian part of multiple scattering distributions as predicted by their estimate of $\Delta(\chi_c^2B)$ from the straight-line fit mentioned earlier. We have given the values of $(\theta_\omega^--\theta_\omega^+)/\theta_\omega^-$ as calculated by the full distribution function for multiple scattering, viz., $f(\theta,t)\propto f^{(0)}+(1/B)(f^{(1)\prime}+f^{(1)})$. It is interesting to note from Table III that the difference in $1/e$ widths of the Gaussian part alone, which is equal to $\Delta(\chi_c\sqrt{B})$, is of the wrong sign. This need not worry us since the use of the full distribution function which, in fact, is proportional to the number of electron (or positron) counts in the experiment, gives the correct sign.

TABLE IV. Percentage difference in the $1/e$ widths for electron-positron multiple scattering.

E (Mev)	μ	$(\theta^--\theta^+)/\theta^-$ (%) Au	Ag	Al
0.4	obs	5.79±0.46	4.60±1.04	1.19±0.70
	1.12	5.88	3.47	0.97
	1.8	4.31	2.48	0.97
0.59	1.12		2.41	
	1.8		2.13	
1.2	1.12		0.53	
	1.8		0.31	

IV. DISCUSSION AND RESULT

We find from Tables III and II, that a satisfactory value of the $1/e$ width is obtained for $\mu\sim1.12$, except in the case of Al. The agreement of the calculated difference in $1/e$ width for electron and positron multiple scattering (Table IV) with the experimental results also seems fairly satisfactory for $\mu\sim1.12$, though we are comparing slightly different quantities. The fact that $\mu\sim1.8$, the value found to fit the experimental data of Hanson *et al.*[8] for the $1/e$ width for 15–16 Mev electron multiple scattering by Au and Be in paper A, gives low results for electron scattering around 1 Mev seems to suggest that the screening parameter, λ, is probably somewhat energy dependent.

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Nuclear Quadrupole Absorption in Indium Metal*†

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The nuclear quadrupole resonance spectrum of indium metal has been observed and studied over the temperature range 4°K to 225°K. The four transition frequencies occur in ratios of 1:2:3:4, as expected from the crystal structure of indium. The lowest transition at 1.881 Mc (at 4.2°K) gives a quadrupole coupling constant of (45.19±0.02) Mc.

The root second moment of 18 kc is substantially greater than the 4 kc predicted from magnetic dipolar coupling among the nuclei, is temperature independent, but is somewhat sample dependent. It is shown that for a pure quadrupole resonance, the pseudo-exchange coupling produces an exchange broadening. This mechanism is shown to contribute about 10 kc to the root second moment and predicts a Gaussian line shape, as observed. It is concluded that in some cases magnetic impurities must be present, but it is not known whether or not the narrowest line represents a natural width.

The field gradient is computed assuming point ions and a uniform electron density by a technique that converges rapidly with distance, but the answer is too small by a factor of 3. It is argued that it is important to consider the p valence electrons. The temperature dependence of frequency is very strong (a 23% decrease between 4°K and 220°K), and unexplained. It appears to show that the field gradients cannot be computed assuming point ions and a smooth charge distribution.

A resonance in the superconducting state is reported.

I. INTRODUCTION

PURE quadrupole resonance may be observed in a solid when a nucleus possessing a quadrupole moment is situated in a lattice having lower than cubic symmetry. This paper reports the observation and study of the nuclear quadrupole resonance spectrum in indium metal. During the course of this research, Knight and Hewitt independently observed the same resonance. A preliminary report of their work has been published.[1]

We will center our attention on three principal issues. These are: (1) the interpretation of the shape and breadth of the resonance lines; (2) calculation of the axial field gradient at the nuclear site; (3) the temperature dependence of this field gradient. In addition, an observation of the resonance below the superconducting transition temperature ($T_c=3.39$°K) is reported.

A general theory of line shape for pure quadrupole resonance has not as yet been developed. However, it is well known that nuclear resonance line widths are in general somewhat broader, for heavier elements, than would be expected on the basis of dipolar coupling alone. We account for much of the observed line breadths in indium phenomenologically in terms of the pseudo-exchange interaction.[2,3] On account of the large spin of the indium, we show that we have a case of exchange broadening rather than exchange narrowing. The exchange interaction constant A_{ij} is scaled from the thallium data of Bloembergen and Rowland.[3] To the extent of this approximation, the line shapes are shown to be Gaussian. Pseudo-exchange will lead to broadening in a large-spin, pure quadrupole resonance experiment, since a given neighboring pair of nuclei in general will not be able to conserve energy by a mutual spin flip. In addition, it appears that impurities may in some instances play important roles in determining line breadths.

An analysis of the effect of the magnetic modulation field on the resonance will be given. For the condition that this field is small in comparison with the natural resonance line width, this analysis will lead to a simple interpretation of the second moment and shape of the resonance line. Comparison of the results of this

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