# Spin and Parity Analysis from Production and Decay of Hyperon Resonant States*

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Processes dominated by the $\Lambda$-$\pi$ resonance state, $Y^*$, should exhibit correlations permitting the determination of the $Y^*$ spin; the relative parities of the $\Lambda$, $\Sigma$, $K$, and $Y^*$; and the ratios of the various hyperon decay asymmetry parameters. The cases of $Y^*$ spin $\frac{1}{2}$ and $\frac{3}{2}$ are examined in detail and a number of exact relations between measurable quantities are given for the various spin and parity assignments. The longitudinal component of polarization of the hyperon from the $Y^*$ decay is, for parity-conserving reactions, the same for all final hyperons, independent of their parities. This allows the ratio of the decay asymmetry parameters for various hyperons to be determined. For a given $Y^*$ spin the transverse hyperon polarization is determined by the longitudinal polarization up to a sign which is fixed by the $Y^*$-hyperon parity. Thus the transverse polarizations for final $\Lambda$ and $\Sigma$ particles will be the same or opposite if the parities are the same or opposite, respectively. For the spin $\frac{3}{2}$ case there are four independent contributions to the longitudinal polarization, each proportional to a different pseudoscalar spherical harmonic of the decay direction. These give four independent experimental determinations of the ratios of the decay asymmetry parameters. Similarly, from the transverse components there are six independent determinations of relative parities.

If the initial and final states of the production process are restricted to $S$ and $P$ waves, all ten of the experimental parameters determining the hyperon polarization are given, except for the aforementioned sign dependence on the $Y^*$-$Y$ relative parity, by a single parameter in a manner fixed by the $K$-$Y^*$ parity.

EVIDENCE for a pion-hyperon resonant state with a mass of about 1.37 Bev has been reported recently.[1] In this note we list some methods for determining from angular correlations in the production and decay of this resonant state, which we call $Y^*$, its spin $S$, and the relative parities of $\Lambda$, $\Sigma$, $Y^*$, and $K$.

Let us consider a two-body production mode of $Y^*$, such as $K+N\rightarrow Y^*+\pi$ or $\pi+N\rightarrow Y^*+K$, and the subsequent decay $Y^*\rightarrow Y+\pi$.[2,3] We let $\mathbf{u}$ and $\mathbf{u}'$ be the unit vectors along the initial and final center-of-mass momenta in the production process, $\mathbf{n}$ be the unit pseudovector along $\mathbf{u}\times\mathbf{u}'$, and $\mathbf{v}$ be the unit vector along the momentum of the final $Y$, measured in the decay rest system. Then for $S=\frac{1}{2}$ the polarization $\mathbf{P}$ of the final $Y$ is related to the polarization $\mathbf{P}^*$ of $Y^*$ [4] by

$$\mathbf{P}=\mathbf{P}^*, \tag{1}$$

for $Y-Y^*$ relative parity, $(Y,Y^*)$, equal to $-1$; or

$$\mathbf{P}=-\mathbf{P}^*+2(\mathbf{P}^*\cdot\mathbf{v})\mathbf{v}, \tag{2}$$

for $(Y,Y^*)$ equal to $+1$. Since $\mathbf{P}^*$ is along $\mathbf{n}$, any polarization of the final $Y$ normal to the plane specified by $\mathbf{n}$ and $\mathbf{v}$ is evidence that $S>\frac{1}{2}$.[5] In the decay of the final $Y$ there can only be an up-down asymmetry with respect to the production plane for $(Y,Y^*)=-1$. But for $(Y,Y^*)=+1$ such an asymmetry must occur together with a forward-backward asymmetry in the ratio fixed by (2).

If $S$ equals $\frac{3}{2}$, the $Y^*$ density matrix after production in a parity conserving two-body reaction with an unpolarized initial nucleon is of the form[6]

$$\begin{aligned}=\tfrac{1}{4}I_0(\theta)+b_1(\theta)T(\mathbf{n})+c_1(\theta)T(\mathbf{u},\mathbf{u})+c_2(\theta)T(\mathbf{w},\mathbf{w})\\+c_3(\theta)T(\mathbf{u},\mathbf{w})+d_1(\theta)T(\mathbf{n},\mathbf{n},\mathbf{n})+d_2(\theta)T(\mathbf{n},\mathbf{u},\mathbf{u})\\+d_3(\theta)T(\mathbf{n},\mathbf{u},\mathbf{w}),\end{aligned} \tag{3}$$

where $\mathbf{w}=\mathbf{n}\times\mathbf{u}$, $I_0$ is the production angular distribution, and $b$, $c$, and $d$ are functions of the production angle $\theta$, determined by the production amplitudes. The quantities $T$ are spin operators defined as in reference 6. Only $b_1$, $d_1$, $d_2$, and $d_3$ appear in the expression for the polarization of the final $Y$. The components of $\mathbf{P}$ along

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[1] M. Alston, L. W. Alvarez, P. Eberhard, M. L. Good, W. Graziano, H. K. Ticho, and S. G. Wojcicki, Proceedings of the Tenth International Rochester Conference on High-Energy Nuclear Physics, 1960 (to be published); Phys. Rev. Letters **5**, 520 (1960). M. Ferroluzzi, J. P. Berge, J. Kirz, J. J. Murray, and A. H. Rosenfeld, University of California Radiation Laboratory Report UCRL-9431, 1960 (unpublished).

[2] We assume spin $\frac{1}{2}$ for $\Lambda$ and $\Sigma$ and spin zero for $K$, and use the notation $Y$ for $\Lambda$ and $\Sigma$. Strong interactions are assumed to conserve parity.

[3] From present data the width of the resonance seems to be smaller than 70 Mev. We assume throughout that processes proceeding through a single resonance state can be isolated.

[4] If $Y^*$ is produced in an $s$ state, it has no vector polarization, only spin tensor alignment, and one has the well-known decay angular distributions 1, $3(\mathbf{u}\cdot\mathbf{v})^2+1$ and $5(\mathbf{u}\cdot\mathbf{v})^4-2(\mathbf{u}\cdot\mathbf{v})^2+1$, for $Y^*$ spin $\frac{1}{2}$, $\frac{3}{2}$, and $\frac{5}{2}$, respectively, independent of parity. The same distributions occur in Adair's analysis for $Y^*$ emitted backward or forward in the production rest frame. R. K. Adair, Phys. Rev. **100**, 1590 (1955).

[5] For $Y^*$ spin $S>\frac{1}{2}$, $\mathbf{P}$ may contain terms originating from tensor polarizations of $Y^*$ proportional to the pseudovectors $(\mathbf{n}\cdot\mathbf{v})\mathbf{u}$ and $(\mathbf{n}\cdot\mathbf{v})\mathbf{u}'$. However, an average of $\mathbf{P}$ over the directions of $\mathbf{v}$, or any weighted average with a scalar weight function, will again be proportional to $\mathbf{n}$.

[6] R. Spitzer and H. P. Stapp, University of California Radiation Laboratory Report UCRL-3796 (unpublished); Phys. Rev. **109**, 540 (1958) (abbreviated version). In the relevant Eqs. (2.20), and (2.13), respectively, of the above papers, the 3 in the factor $5(\mathbf{u}_2{}^K\cdot\mathbf{v})(\mathbf{u}_3{}^K\cdot\mathbf{v})-3(\mathbf{u}_2{}^K-\mathbf{u}_3{}^K)$ should be replaced by 1 when the factor multiplies either of the two terms containing $(\mathbf{u}_1{}^K\times\mathbf{v})$. Using the linearity of the $T$'s in their arguments and identities such as $T(\mathbf{n},\mathbf{n},\mathbf{n})+T(\mathbf{n},\mathbf{u},\mathbf{u})+T(\mathbf{n},\mathbf{w},\mathbf{w})=0$ one can obtain from the corresponding relations of this reference the explicit dependence of $b$, $c$, and $d$ upon the production amplitudes.

$\mathbf{v}$, $\mathbf{v}\times\mathbf{n}$, and $\mathbf{v}\times(\mathbf{n}\times\mathbf{v})$ are given by

$$
\begin{aligned}
(\sqrt{5})I_0 I\mathbf{P}\cdot\mathbf{v} &= b_1 2\cos\Theta-(d_1-\tfrac{1}{2}d_2)(15\cos^3\Theta-9\cos\Theta)\\
&\quad + -d_2(15/2)\cos\Theta\sin^2\Theta\cos 2\Phi\\
&\quad -d_3(15/2)\cos\Theta\sin^2\Theta\sin 2\Phi, \qquad (4)
\end{aligned}
$$

$$
\begin{aligned}
(\sqrt{5})I_0 I\mathbf{P}\cdot(\mathbf{v}\times\mathbf{n}) &= \epsilon d_2 5\cos\Theta\sin^2\Theta\sin 2\Phi-\epsilon d_3 5\cos\Theta\sin^2\Theta\cos 2\Phi, \qquad (4')
\end{aligned}
$$

$$
\begin{aligned}
(\sqrt{5})I_0 I\mathbf{P}\cdot[\mathbf{v}\times(\mathbf{n}\times\mathbf{v})] &= -\epsilon b_1 4\sin^2\Theta+\epsilon(d_1-\tfrac{1}{2}d_2)(15\cos^2\Theta-3)\sin^2\Theta\\
&\quad +\epsilon d_2\tfrac{5}{2}(1-3\cos^2\Theta)\sin^2\Theta\cos 2\Phi\\
&\quad +\epsilon d_3\tfrac{5}{2}(1-3\cos^2\Theta)\sin^2\Theta\sin 2\Phi, \qquad (4'')
\end{aligned}
$$

where the angles $\Theta$ and $\Phi$ are the polar coordinates of $\mathbf{v}$ in a system where $\mathbf{n}$ is the polar axis and $\mathbf{u}$ the $x$ axis,[7] and $I$ is the decay angular distribution. The symbol $\epsilon$ is $+1$ for $(Y,Y^*)=-1$; $-1$ for $(Y,Y^*)=+1$. The coefficients $b_1$, $d_1$, $d_2$, and $d_3$ are given experimentally by certain weighted averages of these components of polarization over the final hyperon direction:

$$\langle I_0 I\mathbf{P}\cdot\mathbf{v}Y_1^0(\Theta,\Phi)\rangle=b_1(\theta)(1/15\pi)^{\frac{1}{2}}, \qquad (5a)$$

$$\langle I_0 I\mathbf{P}\cdot\mathbf{v}Y_3^0(\Theta,\Phi)\rangle=-[d_1(\theta)-\tfrac{1}{2}d_2(\theta)](8/35\pi)^{\frac{1}{2}}, \qquad (5b)$$

$$\langle I_0 I\mathbf{P}\cdot\mathbf{v}Y_3^{\pm 2}(\Theta,\Phi)\rangle=-[d_2(\theta)\pm id_3(\theta)](3/56\pi)^{\frac{1}{2}}, \qquad (5c)$$

$$\langle I_0 I\mathbf{P}\cdot\mathbf{n}\times\mathbf{v}Y_3^{\pm 2}(\Theta,\Phi)\rangle =\epsilon[-d_3(\theta)\pm id_2(\theta)](1/42\pi)^{\frac{1}{2}}, \qquad (6)$$

$$\langle I_0 I\mathbf{P}\cdot\mathbf{v}\times(\mathbf{n}\times\mathbf{v})\rangle=-\epsilon b_1(\theta)(8/3\sqrt{5}), \qquad (7a)$$

$$\langle I_0 I\mathbf{P}\cdot\mathbf{v}\times(\mathbf{n}\times\mathbf{v})(15\cos^3\Theta-3)\rangle =\epsilon[d_1(\theta)-\tfrac{1}{2}d_2(\theta)](48/7\sqrt{5}), \qquad (7b)$$

$$\langle I_0 I\mathbf{P}\cdot\mathbf{v}\times(\mathbf{n}\times\mathbf{v})e^{\pm 2i\Phi}\rangle =\epsilon[d_2(\theta)\pm id_3(\theta)](1/3\sqrt{5}). \qquad (7c)$$

For $S=\frac{1}{2}$ the same formulas are valid with the $d$'s set to zero and with an extra factor of $(-\frac{1}{2})$ on the right-hand side of (7a). In particular, provided only $b_1(\theta)\neq 0$, the ratio of (5a) to (7a) gives an unambiguous distinction between $S=\frac{1}{2}$ and $S=\frac{3}{2}$. For $S=\frac{3}{2}$ the values for $b_1$ and $d_i$ obtained from Eqs. (5) must correspond in magnitude to those given by Eqs. (7). This gives four independent conditions that must be satisfied for $S=\frac{3}{2}$, and Eq. (6) gives two more. The parity is fixed by the relative sign of the averages appearing in Eq. (5) versus Eq. (6) and (7), where there are again six independent determinations. Since the $d_i(\theta)$ and $b_1(\theta)$ are the same for final $\Lambda$ and final $\Sigma$ the averages in (6) and (7) are the same for these two cases except for the relative sign, which is just the $\Lambda$-$\Sigma$ parity.

[7] Relativistically this system is that rest frame of $Y^*$ which is obtained by a direct Lorentz transformation of that production center-of-mass frame with $z$ axis along $n$ and with $x$ axis along $u$. [See H. P. Stapp, Phys. Rev. **103**, 625 (1956), and University of California Radiation Laboratory Report UCRL-8096 (unpublished)].

TABLE I. Relations among the averages for different spin-parity assignments.

(I) $S=\frac{1}{2}$, $Y^*-Y$ parity $=-1$:

$$\rho_1=\rho_2=\tfrac{1}{3}a,\ \rho_3=0,\ \mathcal{P}_1=\mathcal{P}_2=\mathcal{P}_3=\mathcal{P}_4=0,\ a=\mathbf{P}^*\cdot\mathbf{n}.$$

(II) $S=\frac{1}{2}$, $Y^*-Y$ parity $=+1$:

$$\rho_1=\rho_2=\tfrac{3}{5}a,\ \rho_3=0,\ \mathcal{P}_1=\mathcal{P}_2=-\tfrac{2}{5}a,\ \mathcal{P}_3=\mathcal{P}_4=0,\ a=-\tfrac{1}{3}\mathbf{P}^*\cdot\mathbf{n}.$$

(III) $S=\frac{3}{2}$, $Y^*-Y$ parity $=-1$:

$$a=-2b_1,\ \mathcal{P}_1=\frac{2}{15}\left(3b_1-\frac{3}{7}d_1+d_2\right),\ \mathcal{P}_2=\frac{2}{15}\left(3b_1-\frac{3}{7}d_1-\frac{31}{7}d_2\right),\ \mathcal{P}_3=\mathcal{P}_4=-\frac{2}{7}d_3,$$

$$\rho_1=\frac{2}{15}\left(-7b_1-\frac{3}{7}d_1-\frac{31}{7}d_2\right),\ \rho_2=\frac{2}{15}\left(-7b_1-\frac{3}{7}d_1+\frac{2}{7}d_2\right),\ \rho_3=\frac{1}{21}d_3.$$

Relations: $\mathcal{P}_3=\mathcal{P}_4$, $\mathcal{P}_3+\mathcal{P}_4=-12\rho_3$, $\mathcal{P}_1-\mathcal{P}_2=\frac{38}{33}(\rho_1-\rho_2)$, $\frac{1}{2}(\mathcal{P}_1+\mathcal{P}_2)=\frac{17}{33}\rho_1+\frac{16}{33}\rho_2-\frac{2}{3}a$.

(IV) $S=\frac{3}{2}$, $Y^*-Y$ parity $=+1$:

$$a=\frac{10}{3}b_1,\ \mathcal{P}_1=\mathcal{P}_2=\frac{2}{15}(-b_1+3d_1+d_2),\ \mathcal{P}_3=\mathcal{P}_4=0,$$

$$\rho_1=\frac{2}{15}(9b_1+3d_1+d_2),\ \rho_2=\frac{2}{15}(9b_1+3d_1-4d_2),\ \rho_3=-\frac{1}{3}d_3.$$

Relations: $\mathcal{P}_3=\mathcal{P}_4$, $\mathcal{P}_3+\mathcal{P}_4=0$, $\mathcal{P}_1-\mathcal{P}_2=0$, $\frac{1}{2}(\mathcal{P}_1+\mathcal{P}_2)=\rho_1-\frac{2}{5}a$.

Averages of $\mathbf{P}$ with the scalar weight functions[8] $1,(\mathbf{w}\cdot\mathbf{v})(\mathbf{w}\cdot\mathbf{v})$, $(\mathbf{u}\cdot\mathbf{v})(\mathbf{u}\cdot\mathbf{v})$ and $(\mathbf{w}\cdot\mathbf{v})(\mathbf{u}\cdot\mathbf{v})$ have only a component along $\mathbf{n}$, whereas averages with the pseudoscalar weight functions $(\mathbf{n}\cdot\mathbf{v})(\mathbf{u}\cdot\mathbf{v})$ and $(\mathbf{n}\cdot\mathbf{v})(\mathbf{w}\cdot\mathbf{v})$ have components only along $\mathbf{u}$ and $\mathbf{w}$. We therefore define the following experimentally independent quantities:

$$
\begin{aligned}
a &= (\sqrt{5})(4\pi)\langle I_0 I(\mathbf{P}\cdot\mathbf{n})\rangle = (\sqrt{5})(4\pi)(\text{average polarization}).\\
\mathcal{P}_1 &= (\sqrt{5})(4\pi)\langle I_0 I(\mathbf{P}\cdot\mathbf{w})(\mathbf{n}\cdot\mathbf{v})(\mathbf{w}\cdot\mathbf{v})\rangle,\\
\mathcal{P}_2 &= (\sqrt{5})(4\pi)\langle I_0 I(\mathbf{P}\cdot\mathbf{u})(\mathbf{n}\cdot\mathbf{v})(\mathbf{u}\cdot\mathbf{v})\rangle,\\
\mathcal{P}_3 &= (\sqrt{5})(4\pi)\langle I_0 I(\mathbf{P}\cdot\mathbf{u})(\mathbf{n}\cdot\mathbf{v})(\mathbf{w}\cdot\mathbf{v})\rangle,\\
\mathcal{P}_4 &= (\sqrt{5})(4\pi)\langle I_0 I(\mathbf{P}\cdot\mathbf{w})(\mathbf{n}\cdot\mathbf{v})(\mathbf{u}\cdot\mathbf{v})\rangle,\\
\rho_1 &= (\sqrt{5})(4\pi)\langle I_0 I(\mathbf{P}\cdot\mathbf{n})(\mathbf{w}\cdot\mathbf{v})(\mathbf{w}\cdot\mathbf{v})\rangle,\\
\rho_2 &= (\sqrt{5})(4\pi)\langle I_0 I(\mathbf{P}\cdot\mathbf{n})(\mathbf{u}\cdot\mathbf{v})(\mathbf{u}\cdot\mathbf{v})\rangle,\\
\rho_3 &= (\sqrt{5})(4\pi)\langle I_0 I(\mathbf{P}\cdot\mathbf{n})(\mathbf{u}\cdot\mathbf{v})(\mathbf{w}\cdot\mathbf{v})\rangle.
\end{aligned}
$$

Table I contains the relation of these eight averages to the four unknowns $b_1$ and $d_i$ together with the four independent relations among the averages resulting from eliminating the unknowns. The analogous relations for $S=\frac{1}{2}$ are also included. All these relations are

[8] Weight functions odd in $v$ give zero average if parity is conserved.

linear and are therefore valid not only at a given production angle but also after integration over production angles, or after integration with arbitrary weighting factors, such as $\sin(m\theta)$, for instance. It is apparent from Table I that measurement of these averages can in principle distinguish between $Y^*$ spin $\frac{1}{2}$ and $\frac{3}{2}$ and give the $Y^*$-$\Lambda$ and $Y^*$-$\Sigma$ parities, and thus also the $\Lambda$-$\Sigma$ parity. For example,[9] from Table I, if $\Lambda$ and $\Sigma$ have the same parity the ratio between their average polarizations is 1; if they have opposite parity, it is $-3$, $-\frac{1}{3}$, $-\frac{3}{5}$, or $-5/3$, respectively, for: $S=\frac{1}{2}$, $(Y^*,\Sigma)=+1$; $S=\frac{1}{2}$, $(Y^*,\Sigma)=-1$; $S=\frac{3}{2}$, $(Y^*,\Sigma)=+1$; and $S=\frac{3}{2}$, $(Y^*,\Sigma)=-1$.

The decay distribution in $Y^*\to Y+\pi$ is independent of the parity. It is isotropic for $S=\frac{1}{2}$ and for $S=\frac{3}{2}$ it is given by

$$I_0I=I_0(\theta)-c_1(\theta)[3(\mathbf{u}\cdot\mathbf{v})^2-1]-c_2(\theta)[3(\mathbf{w}\cdot\mathbf{v})^2-1]-c_3(\theta)[3(\mathbf{u}\cdot\mathbf{v})(\mathbf{w}\cdot\mathbf{v})].$$

One finds

$$\langle I_0I(3\cos^2\Theta_w-1)\rangle=-\tfrac{4}{5}[c_2(\theta)-\tfrac{1}{2}c_1(\theta)],$$

$$\langle I_0I\sin^2\Theta_w\cos 2\Phi_w\rangle=\tfrac{2}{5}c_1(\theta),$$

and

$$\langle I_0I\sin 2\Theta_w\sin\Phi_w\rangle=-\tfrac{2}{5}c_3(\theta),$$

for the choice of $\mathbf{w}$ as polar axis and $\mathbf{n}$ as azimuthal axis, and similar equations when $\mathbf{n}$ and $\mathbf{u}$ are chosen as polar axes. These independent determinations of the $c_i$ must agree if $S=\frac{3}{2}$. Also, the limiting angles for Adair's analysis, which depends on the fact that $c_1+\frac{1}{2}I_0=c_2=c_3=0$ at $\theta=0°$ or $180°$, may be experimentally determined by using these independent determinations of the $c_i$.

If in $K^-+p\to Y^*+\pi$ at sufficiently low energy only $s$ and $p$ states occur initially and finally,[10] the production angular distributions are $A+B\cos\theta$, $A+B\cos\theta+C\cos^2\theta$, $A+B\cos\theta$, and $A+C\cos^2\theta$, respectively, for $S=\frac{1}{2}$ and $(K,Y^*)=+1$, $S=\frac{1}{2}$ and $(K,Y^*)=-1$, $S=\frac{3}{2}$ and $(K,Y^*)=+1$, $S=\frac{3}{2}$ and $(K,Y^*)=-1$. Furthermore the hyperon polarization $\mathbf{P}$ has in all cases a unique form depending only on the $Y^*$ spin, the $K$-$Y^*$ parity, and the $Y$-$Y^*$ parity. Equations (1) and (2) apply for $S=\frac{1}{2}$, and for $S=\frac{3}{2}$ Eqs. (4) apply with $d_3=d_3\tan\theta=6b_1\tan\theta=(\text{constant})\sin^2\theta$, $d_1=0$ for $K$-$Y^*$ parity $-1$; or with $b_1=(\text{constant})\sin\theta$, $d_1=d_2=d_3=0$ for $K$-$Y^*$ parity $+1$.

In $K^-$ capture by nuclei, $Y^*$ production may occur on many nucleons without real meson emission. If these processes occur frequently enough, angular correlations in, for instance, $K^-+\mathrm{He}^4\to\mathrm{He}^3+Y^{*-}$, $Y^{*-}\to\Lambda+\pi^-$ could measure $S$. For capture at rest the decay angular distributions are 1, $3\cos^2\alpha+1$, and $5\cos^4\alpha-2\cos^2\alpha+1$ for spin $\frac{1}{2}$, $\frac{3}{2}$, and $\frac{5}{2}$, respectively, where $\cos\alpha=(\mathbf{u}'\cdot\mathbf{v})$.

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[9] This result has been noted by Ph. Mayer, J. Prentki, and Y. Yamaguchi.

[10] The longest range force is expected to have a range of the order of the inverse of $m_K+m_\pi$. Of course our assumptions[8] may not be valid at such low energies.