Usefulness of Polarized Targets and the Polarization Transfer Tensor in Reconstruction of the Nucleon-Nucleon Scattering Matrix*

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A method is presented for completely removing the ambiguities arising in the reconstruction of the nucleon-nucleon scattering matrix from data at a given angle and energy due to the bilinear form of expressions for observable quantities. Our method utilizes only the amplitudes defined by Wolfenstein and Ashkin and is more direct and computationally simpler than the methods using unitarity and measurements at all angles or the phase-shift analyses; thus, it would provide an independent means of arriving at the correct set of phase-shift solutions. Our method is based on a knowledge of the polarization transfer tensor $\mathcal{K}_{ik}$, which has a form complementary to the familiar polarization correlation tensor $\mathcal{C}_{ik}$. The $\mathcal{K}_{ik}$ tensor may be obtained from triple scattering experiments on the recoil nucleon similar to those used to determine the familiar depolarization tensor $\mathfrak{D}_{ik}$, or from double scattering measurements on nucleons scattered from a polarized target.

It is also shown that the use of polarized targets would have many experimental advantages: They would permit (1) determination of the depolarization tensor $\mathfrak{D}_{ik}$ for large scattering angles, without requiring the measurement of the polarization of a very slow nucleon; (2) determination of the correlation tensor $\mathcal{C}_{ik}$ by the measurement of the cross section for the scattering of a polarized beam by a polarized target, instead of the difficult simultaneous measurements of the polarizations of both the final nucleons; (3) determination of the "difficult" components A' and R' of the $\mathfrak{D}_{ik}$ and $\mathcal{K}_{ik}$ tensors by the measurement of the polarization of an initially unpolarized beam scattered by a polarized target.

1. INTRODUCTION

In this note we show how ambiguities arising in the reconstruction of the nucleon-nucleon scattering matrix from data at a given angle and energy, due to the bilinear form of expressions for observable quantities, may be removed completely. This is accomplished without using unitarity and measurements at all angles, or even a phase-shift analysis, but instead, by using the more direct and computationally simpler method of working only with the amplitudes defined by Wolfenstein and Ashkin[1,2] (see Sec. 2); thus, we have a means for discriminating between the various phase-shift solutions. Our patently unique reconstruction is carried out (in Sec. 3) by exploiting the complementary relationship between the familiar polarization correlation tensor $\mathcal{C}_{ik}$ and the polarization transfer tensor $\mathcal{K}_{ik}$. The $\mathcal{K}_{ik}$ tensor determines the polarization of the recoil (scattered) nucleon in the scattering of a polarized (unpolarized) beam from an unpolarized (polarized) target.

In addition to their usefulness in determining the $\mathcal{K}_{ik}$ tensor, it is also shown (in Sec. 4) that polarized targets (which have recently been achieved at Saclay) would simplify the determination of the $\mathcal{C}_{ik}$ tensor and of the "difficult" components of the familiar depolarization tensor $\mathfrak{D}_{ik}$ and remove the need for the usual complicated turning of spins in magnetic fields. They would also help to extend present measurements over a broader angular range.

2. NUCLEON-NUCLEON SCATTERING MATRIX

The nucleon-nucleon scattering matrix can be expressed in terms of certain quantities which are invariant with respect to space rotations, space reflections, and time reversal in the familiar form[3]

$$M_T = A_T + B_T\sigma_{1n}\sigma_{2n} + C_T(\sigma_{1n}+\sigma_{2n}) + D_T(\sigma_{1n}-\sigma_{2n}) + E_T\sigma_{1q}\sigma_{2q} + F_T\sigma_{1p}\sigma_{2p}, \quad (1)$$

where σ_1 and σ_2 are the Pauli matrices of the incident and target nucleons, respectively, and $\mathbf{n}$, $\mathbf{p}$, and $\mathbf{q}$ are 3 mutually perpendicular directions, defined by the vectors $\mathbf{n}=\mathbf{k}_{\text{in}}\times\mathbf{k}_{\text{out}}$, $\mathbf{p}=\mathbf{k}_{\text{in}}+\mathbf{k}_{\text{out}}$, $\mathbf{q}=\mathbf{k}_{\text{out}}-\mathbf{k}_{\text{in}}$. $\mathbf{k}$ is the nucleon momentum in the c.m. system, and the subscript T refers to the total isotopic spin of the system.

In the following we are primarily interested in the pure nuclear scattering. Coulomb scattering is usually unimportant, except at small angles, and its effects can be calculated. However, in spite of the small angular range in which Coulombic effects are appreciable and the sizable experimental difficulties in measuring them, measurements in this small-angle region would still be extremely valuable. Their value derives mostly from the interference terms between Coulomb and nuclear scattering which arise when both of these are of the same order of magnitude. The most interesting point is that the interference term permits a determination of the over-all phase of the matrix M. Secondly, this term is linear in the unknown coefficients A to F, and hence allows a much more direct reconstruction of M than the pure nuclear scattering which is bilinear in A to F. Finally, the limit of M for small angles is directly significant for the analysis of the scattering of nucleons by complex nuclei.[3] The usefulness of such measurements in the phase-shift analyses has been discussed by Cromer.[4]

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[1] L. Wolfenstein and J. Ashkin, Phys. Rev. **85**, 947 (1952).
[2] L. Wolfenstein, Phys. Rev. **96**, 1654 (1954).
[3] We follow the notation of H. A. Bethe, Ann. Phys. **3**, 190 (1958), and refer the reader to this paper for a more complete list of references. The basis vectors used here are chosen to agree with those of Wolfenstein, reference 2, and are slightly different from those used in some papers.
[4] A. H. Cromer, thesis, Cornell University, 1960 (unpublished).

The p-p and n-n systems are both pure states of $T=1$, whereas the n-p system is an equal mixture of $T=0$ and 1. Thus, $D_1=0$ because of the identity of the particles, and $D_0=0$ because of the charge symmetry of nuclear forces.

A, B, C, E, and F are 5 complex functions of the nucleon energy and scattering angle, for each isotopic spin state.[5] Since the over-all complex phase of M remains arbitrary, it can be reconstructed if 9 real quantities are determined experimentally, the magnitudes of the 5 coefficients and their relative phases. However, 9 different scattering experiments are *not* sufficient to determine these coefficients uniquely. When observable quantities are calculated in terms of the scattering matrix coefficients (see Table I below), all the expressions are bilinear in the 9 unknowns; thus, unless more than 9 experimental results are available, ambiguities will arise in the reconstruction of M from data at a given angle.

At the present time not even 9 experiments have been done at any given angle. Nevertheless, it has been possible to perform phase-shift analyses which utilize data taken over a range of angles to compensate for the lack of information at the individual points in the range. More specifically, a phase-shift analysis makes use of (1) unitarity: the unitarity requirement is automatically satisfied in the phase-shift approach; (2) the smoothness of all the observable quantities as a function of angle; and (3) due to the scarcity of information at small angles, a reasonable hypothesis concerning the behavior of the phase shifts for higher l, e.g. that they may be approximated by the one-pion exchange contribution (OPEC). An analysis of the 310-Mev p-p scattering data along these lines has recently been completed by MacGregor, Moravcsik, and Stapp[6]; they have succeeded in reducing the 5 "best" sets of phase-shift solutions of Stapp, Ypsilantis, and Metropolis[7] to 2 sets, with more recent analyses of the energy dependence[8] favoring Solution **1** over Solution 2.

In spite of the apparent success of the phase-shift analyses, they are based on the theoretical considerations listed above. Also, all the solutions are obtained by means of a search through phase-shift space with random sets of initial phase shifts taken as starting points.[6,7] Thus, in spite of the increasing probabilistic evidence to the contrary, there is no *proof* that all regions of the many-dimensional phase-shift space have been completely explored. The somewhat unsatisfactory nature of this situation is well brought out by Smorodinsky's recent statement[9]: "It remains somewhat puzzling why we have only two sets describing the experimental data within (rather large) experimental errors." The uniqueness of the phase-shift solutions can be proved if one has a "complete set" of experiments. Puzikov, Ryndin, and Smorodinsky[10] have shown how this can be done using unitarity and measurements of 5 suitably chosen quantities at *all* angles. In view of the difficulty of performing the complete set of experiments needed for the application of their method,[11] we will now show how this same result may be accomplished with measurements at a single angle and energy.

It is to be noted that our method does not make use of unitarity and thus is applicable at energies at which inelastic processes intrude. Phase-shift analyses may be extended to these higher energies by replacing each $e^{2i\delta}$ by $ae^{2i\delta}$, where a is a real, positive number less than unity; in other words, by making each phase-shift complex. Here $1-a^2$ measures the fraction of particles of the given l, j going into inelastic channels. However, the introduction of a doubles the number of quantities to be determined for each l and j, and remembering the increased number of partial waves contributing, the number of parameters soon becomes very large, rendering this method very difficult and inaccurate. In our method, on the other hand, the number of unknowns remains the same at all energies.

3. RECONSTRUCTION OF M FROM DATA AT ONE ENERGY AND ANGLE

Expressions for various observable quantities in terms of the scattering matrix coefficients are given in Table I. In this section we shall show how the scattering matrix can be unambiguously reconstructed, once certain of these quantities are known at one energy and angle. Here we shall merely define the observables and comment briefly on their significance; the selection of the easiest experiments for determining them is discussed in Sec. 4.

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[5] In the following we shall avoid explicit reference to the isotopic spin variable, whenever this is convenient.

[6] M. H. MacGregor, M. J. Moravcsik, and H. P. Stapp, Phys. Rev. **116**, 1248 (1959).

[7] H. P. Stapp, T. J. Ypsilantis, and N. Metropolis, Phys. Rev. **105**, 302 (1957).

[8] G. Breit *et al.*, Phys. Rev. Letters **5**, 274 (1960); also contributions by P. Noyes and by G. Breit to *Proceedings of the Tenth Annual Rochester Conference on High-Energy Nuclear Physics, 1960* (Interscience Publishers, New York, to be published).

[9] Ya. Smorodinsky, Report to 1959 International Conference on Physics of High-Energy Particles, Kiev, July 1959 (unpublished).

[10] L. Puzikov, R. Ryndin, and Ya. Smorodinsky, J. Exptl. Theoret. Phys. (U.S.S.R.) **32**, 592 (1957) [translation: Soviet Phys.—JETP **5**(32), 489 (1957)].

[11] The unitarity of M imposes 5 conditions, relating the imaginary parts of the coefficients at one angle to integrals of their products over *all* angles.[10] Thus the number of necessary experiments can be reduced, provided the measurements are made at all angles and the nucleon energies are below the meson-production threshold (about 280 Mev for a beam of protons hitting protons at rest; however, the experimental data indicate a practical threshold of about 400 Mev). Five such complete experiments, measuring I_0, P, $\mathfrak{D}_{nn}$, $\mathcal{C}_{nn}$, and $\mathcal{K}_{nn}$, over the angular interval $0\leqslant\theta\leqslant\pi/2$ will determine M_{pp} at a given energy and scattering angle; with the n-p system measurements must be made over the entire interval $0\leqslant\theta\leqslant\pi$. However, in addition to the high-energy limit imposed by loss of unitarity, the method is limited at lower laboratory energies (large θ) by the experimental difficulties of measuring polarization in the medium energy range between about 20 Mev and 100 Mev. Thus, although experiments are now underway in various laboratories, a "normal complete set" needed for the application of unitarity is not likely to be completed soon.[9]

TABLE I. Observable quantities in terms of the coefficients in the scattering matrix.[a, b]

I_0	$=\|A\|^2+\|B\|^2+2\|C\|^2+\|E\|^2+\|F\|^2$	(I.1)
I_0D	$=I_0\mathfrak{D}_{nn}=\|A\|^2+\|B\|^2+2\|C\|^2-\|E\|^2-\|F\|^2$	(I.2)
$X-Y$	$=I_0\mathfrak{D}_{pp}=\|A\|^2-\|B\|^2-\|E\|^2+\|F\|^2$	(I.3)
$X+Y$	$=I_0\mathfrak{D}_{qq}=\|A\|^2-\|B\|^2+\|E\|^2-\|F\|^2$	(I.4)
Z	$=I_0\mathfrak{D}_{pq}=2\,\text{Im}C^*(A-B)=-I_0\mathfrak{D}_{qp}$	(I.5)
I_0P	$=2\,\text{Re}C^*(A+B)$	(I.6)
$I_0\mathfrak{C}_{pq}$	$=2\,\text{Im}C^*(F-E)=\quad I_0\mathfrak{C}_{qp}$	(I.7)
$I_0\mathfrak{K}_{pq}$	$=2\,\text{Im}C^*(F+E)=-I_0\mathfrak{K}_{qp}$	(I.8)
$\frac{1}{2}I_0\mathfrak{C}_{nn}$	$=\text{Re}AB^*+\|C\|^2-\text{Re}EF^*$	(I.9)
$\frac{1}{2}I_0\mathfrak{K}_{nn}$	$=\text{Re}AB^*+\|C\|^2+\text{Re}EF^*$	(I.10)
$\frac{1}{2}I_0\mathfrak{C}_{pp}$	$=\text{Re}AF^*-\text{Re}BE^*$	(I.11)
$\frac{1}{2}I_0\mathfrak{K}_{pp}$	$=\text{Re}AF^*+\text{Re}BE^*$	(I.12)
$\frac{1}{2}I_0\mathfrak{C}_{qq}$	$=\text{Re}AE^*-\text{Re}BF^*$	(I.13)
$\frac{1}{2}I_0\mathfrak{K}_{qq}$	$=\text{Re}AE^*+\text{Re}BF^*$	(I.14)

[a] All the tensor components with subscripts containing only one n are zero.
[b] In reference 3 it is incorrectly stated that $\mathfrak{C}_{pq}=-\mathfrak{C}_{qp}$. The correct relation is given in (I.7).

$I_0=\frac{1}{4}\,\text{Tr}(MM^\dagger)$ is the differential cross section, and $P=\frac{1}{4}\,\text{Tr}(MM^\dagger\sigma_{1n})/I_0$ is the polarization produced (in the **n** direction) in the scattering of an unpolarized beam from an unpolarized target.

$\mathfrak{C}_{ik}$ is the familiar polarization correlation tensor and is defined by

$$I_0\mathfrak{C}_{ik}=\tfrac{1}{4}\,\text{Tr}(MM^\dagger\sigma_{1i}\sigma_{2k}). \tag{2}$$

When an unpolarized beam is scattered from an unpolarized target, the $\mathfrak{C}_{ik}$ tensor gives the expectation value of $P_{1i}'P_{2k}'$, which is the product of the component of the polarization of the scattered particle in direction i, and of that of the recoil particle in direction k.

The correlation tensor also determines the cross section when a beam polarized in direction i is scattered from a target polarized in direction k. Then

$$\sigma_{ik}=I_0(1+P_{1i}P_i+P_{2k}P_k+\mathfrak{C}_{ik}'P_{1i}P_{2k}), \tag{3}$$

where $\mathfrak{C}_{ik}'=\frac{1}{4}\,\text{Tr}(M\sigma_{1i}\sigma_{2k}M^\dagger)/I_0$ may be obtained from $\mathfrak{C}_{ik}$ by interchanging M and $M^\dagger$. [In Eq. (3) the summation convention is not used, though it applies elsewhere in the paper.] Since only the imaginary parts change sign under this transformation, all the components are the same except that $\mathfrak{C}_{pq}'=\mathfrak{C}_{qp}'=-\mathfrak{C}_{pq}$.

$\mathfrak{D}_{ik}$ is the familiar depolarization tensor and is defined by

$$I_0\mathfrak{D}_{ik}=\tfrac{1}{4}\,\text{Tr}(M\sigma_{1i}M^\dagger\sigma_{1k}). \tag{4}$$

When a polarized beam is scattered from an unpolarized target, the polarization of the incident particle in direction i, P_{1i}, is related to the polarization of the scattered particle in direction k, P_{1k}', by means of the $\mathfrak{D}_{ik}$ tensor:

$$P_{1k}'=(P_k+\mathfrak{D}_{ik}P_{1i})/(1+\mathbf{P}\cdot\mathbf{P}_1), \tag{5}$$

where (as above) $\mathbf{P}=P\hat{n}$ is the polarization arising (in the rest system of the scattered nucleon, or in the c.m. system) when an unpolarized beam is used. The repeated index implies summation over all the components of the initial polarization vector $\mathbf{P}_1$.

The depolarization tensor also determines the polarization of the *recoil* nucleon, $\mathbf{P}_2'$, when an unpolarized beam is scattered from a target with initial polarization $\mathbf{P}_2$. Then

$$P_{2k}'=(P_k+\mathfrak{D}_{ik}P_{2i})/(1+\mathbf{P}\cdot\mathbf{P}_2). \tag{6}$$

$\mathfrak{K}_{ik}$ is what we shall call the polarization transfer tensor; it is defined by

$$I_0\mathfrak{K}_{ik}=\tfrac{1}{4}\,\text{Tr}(M\sigma_{1i}M^\dagger\sigma_{2k}). \tag{7}$$

When a polarized beam is scattered from an unpolarized target, the polarization of the *recoil* nucleon in direction k, P_{2k}', is related to the polarization of the incident nucleon in direction i through the $\mathfrak{K}_{ik}$ tensor:

$$P_{2k}'=(P_k+\mathfrak{K}_{ik}P_{1i})/(1+\mathbf{P}\cdot\mathbf{P}_1). \tag{8}$$

The polarization transfer tensor also determines the polarization of the scattered nucleon, when an unpolarized beam is scattered from a polarized target. Then

$$P_{1k}'=(P_k+\mathfrak{K}_{ik}P_{2i})/(1+\mathbf{P}\cdot\mathbf{P}_2). \tag{9}$$

It is convenient to adopt a shorthand notation to refer to the 4 experiments determining the $\mathfrak{D}_{ik}$ and $\mathfrak{K}_{ik}$ tensors. We shall use a letter to indicate whether the beam (B) or target (T) is polarized initially and a number to indicate whether one measures the polarization of the scattered (1) or recoil (2) nucleon. Thus, the $\mathfrak{D}_{ik}$ tensor is determined by experiments $B1$ or $T2$; the $\mathfrak{K}_{ik}$ tensor is determined by experiments $B2$ or $T1$.

We note that I_0 occurs as a factor in each of the quantities listed in Table I, and hence may be regarded as a scale factor which can conveniently be divided out. This may be an advantage, for example, in the near-forward direction in p-p scattering where the differential cross section increases rapidly. But also at other angles, an *accurate* determination of the differential cross section is often difficult, and inaccurate knowledge has sometimes made phase-shift analysis difficult because the use of unitarity relies heavily on the absolute cross section.

In reference 3 Bethe has discussed the reconstruction of M from data at one angle and energy, using only the information provided by the 10 quantities: I_0, P, and the $\mathfrak{C}_{ik}$ and $\mathfrak{D}_{ik}$ tensors. He found that these were sufficient to determine M except for the ambiguities due to the bilinear forms. The $\mathfrak{K}_{ik}$ tensor, which is very similar in form to the familiar correlation tensor $\mathfrak{C}_{ik}$, can now be used to remove these remaining ambiguities. Both tensors, expressed in terms of the coefficients A to F, are exhibited in Table I and are seen to have complementary forms.[12,13] By measuring both and combining the infor-

[12] There is a simple and direct scheme for constructing $\mathfrak{K}_{ik}$ from the $\mathfrak{C}_{ik}$ tensor, which clarifies the origin of their complementary forms: $\mathfrak{K}_{ik}$ [Eq. (7)] may be obtained directly from $\mathfrak{C}_{ik}$ [Eq. (2)] by first writing M such that the term $C(\sigma_{1n}+\sigma_{2n})\rightarrow C_1\sigma_{1n}+C_2\sigma_{2n}$, and then studying the sign changes suffered by each term in $\mathfrak{C}_{ik}$ in

mation, one finds expressions which are more simply related to the fundamental coefficients A to F, permitting more straightforward schemes for reconstruction of M from the data. Moreover, with 14 (bilinear) equations for 9 unknowns the solution is agreeably overdetermined.

The usefulness of the $\mathcal{K}_{ik}$ tensor is apparent as soon as one tries to solve these equations. A possible scheme is described in the Appendix. It makes extensive use of the components $\mathcal{K}_{nn}$ and $\mathcal{K}_{pq}$. Without the knowledge of these, solution of the equations of Table I is far from straightforward and probably ambiguous.

There are other tensors beyond those listed in Table I, with 3 and 4 indices, relating the polarization of one or both of the initial nucleons to one or both of the final ones, which would provide additional bilinear relations among the unknowns. However, these require more difficult measurements than the above 2-index tensors, and are not actually needed for the determination of M. Lists of the various measurable quantities occurring in all the possible nucleon-nucleon scattering experiments have been given by Puzikov *et al.*[10] and Phillips,[14] in a different notation.

4. SELECTION OF THE EASIEST EXPERIMENTS FOR DETERMINING THE $\mathcal{C}_{ik}$, $\mathcal{D}_{ik}$, AND $\mathcal{K}_{ik}$ TENSORS

Although much attention has previously been given to the correlation experiments and to polarized beam experiments of the type $B1$,[2,3,9,10,15–17] much less attention has been devoted to polarized beam experiments of the type $B2$,[9,10,15,16] and the polarized target experiments have usually been dismissed as unnecessary.[2,3,9,10,15–17] We have shown above that both the $\mathcal{D}_{ik}$ and $\mathcal{K}_{ik}$ tensors are necessary for the unambiguous reconstruction of M from data at one angle. In this section we shall show that polarized targets would simplify the determination of both of these tensors by permitting the replacement of the difficult A' and R' measurements, required to detect the component of polarization along the direction of motion of the scattered particles when only polarized beams are used, with measurements of the simpler R type which have already been done. It is also shown that polarized targets would provide another method for determining these tensors which would be most useful in supplementing present polarized beam experiments when these become difficult because they result in a scattered particle with a laboratory energy at which polarization measurements are difficult. Corresponding simplifications which apply to the measurement of the $\mathcal{C}_{ik}$ tensor are also discussed.

$\mathcal{D}_{ik}$ and $\mathcal{K}_{ik}$ Tensors

The **n**, **p**, **q** coordinate system which we are using is very convenient when the two particles have equal mass and are nonrelativistic because, in the laboratory system, **p** is exactly the direction of motion of the scattered nucleon (1) and (minus) **q** is exactly the direction of motion of the recoil nucleon (2), and these two vectors are perpendicular. Thus **q** and **p** are perpendicular to the direction of motion of nucleons (1) and (2), respectively, and hence P_{1q}' and P_{2p}' are the easily observable components of polarization. From Eqs. (5), (6), (8), and (9) we then have as the easily observable components of the $\mathcal{D}_{ik}$ and $\mathcal{K}_{ik}$ tensors, corresponding to the four types of experiments

$$B1:\quad \mathcal{D}_{iq}P_{1i}=[ZP_{1p}+(X+Y)P_{1q}]/I_0, \tag{10a}$$

$$T2:\quad \mathcal{D}_{ip}P_{2i}=[(X-Y)P_{2p}-ZP_{2q}]/I_0, \tag{10b}$$

$$T1:\quad \mathcal{K}_{iq}P_{2i}=\mathcal{K}_{pq}P_{2p}+\mathcal{K}_{qq}P_{2q}, \tag{10c}$$

$$B2:\quad \mathcal{K}_{ip}P_{1i}=\mathcal{K}_{pp}P_{1p}+\mathcal{K}_{qp}P_{1q}. \tag{10d}$$

Since the initial directions of polarization, P_1 and P_2, can be chosen to our convenience, we have obtained the result that all of the scattering-plane components of the $\mathcal{D}_{ik}$ and $\mathcal{K}_{ik}$ tensors may be obtained from measurements of the easily observable components of polarization after scattering.[18]

This situation is to be contrasted with that occurring in the measurement of the $\mathcal{D}_{ik}$ tensor entirely through polarized beam experiments of the type $B1$. One then introduces the 4 Wolfenstein coefficients, A, R, A', and R', which describe the rotation of the polarization vector in the scattering plane in terms of initial components in the directions $\mathbf{k}=\mathbf{k}_{\text{in}}$ and $\mathbf{s}=\mathbf{n}\times\mathbf{k}$.[2,3] If the polarization before scattering has components P_{1p} and P_{1q} in the **n**, **p**, **q** system, then, using Eq. (5), the polarization after scattering may also be written in terms of Wolfenstein's system:

$$P_{1p}'=(X-Y)P_{1p}-ZP_{1q}=I_0(A'P_{1k}+R'P_{1s}), \tag{11a}$$

$$P_{1q}'=ZP_{1p}+(X+Y)P_{1q}=I_0(AP_{1k}+RP_{1s}). \tag{11b}$$

Now P_{1p}' is the difficult-to-observe component of polarization along the direction of motion, the measurement of which requires a spin-turning magnetic field

bringing $M^\dagger\sigma_{1i}$ to the form $\sigma_{1i}M^\dagger$, for each value of i. In the final expressions, of course, $C_1=C_2=C$.

[13] Use of the variables $G=E+F$ and $H=E-F$ leads to essentially the same relations; the only difference is in the combinations of the *knowns* used to arrive at these relations. The selection of variables will thus depend upon the feasibility (and accuracy) of the experiments which we are proposing.

[14] R. J. N. Phillips, United Kingdom Atomic Energy Authority Report, AERE-R3141 (unpublished). The appendix to this report contains a brief discussion of several of the advantages of polarized targets.

[15] L. Wolfenstein, *Annual Review of Nuclear Science* (Annual Reviews, Inc., Palo Alto, California, 1956), Vol. 6, p. 43. See references therein.

[16] R. Oehme, Phys. Rev. **98**, 147, 216 (1955).

[17] H. Stapp, University of California Radiation Laboratory Report UCRL-3098 (unpublished).

[18] $\mathcal{D}_{nn}$ and $\mathcal{K}_{nn}$ refer to triple scattering in one plane and are even easier to measure. $\mathcal{D}_{nn}$ has been measured at the same energies and over approximately the same angular range as R [defined in Eq. (11b)]; there are already some measurements of $\mathcal{K}_{nn}$ at 635 Mev.[9] However, polarized targets would be useful in measuring $\mathcal{D}_{nn}$ and $\mathcal{K}_{nn}$ when polarized beam experiments result in a scattered particle having a laboratory energy at which polarization measurements are difficult; the solution of this problem is discussed below.

after scattering; also, measurement of A and A' requires that the spin be turned in the direction $\mathbf{k}_{\text{in}}$ *before* scattering. Thus, measurement of R requires no magnetic fields and is easiest; A (or R') requires one spin-turning before (or after) scattering; and A' requires spin-turning *both* before and after scattering.

At small scattering angles the directions $\mathbf{s}$ and $\mathbf{k}$ are essentially $\mathbf{q}$ and $\mathbf{p}$, respectively, and some simplification results. In this region, $X+Y$ may be obtained from the easy R measurement, but Z must be obtained from A or R', and the difficult A' measurement is required for $X-Y$. At larger angles this decoupling no longer exists, and one must measure *both* R and A and solve the pair of linear equations

$$I_0R=(X+Y)\cos\tfrac{1}{2}\theta+Z\sin\tfrac{1}{2}\theta, \qquad (12a)$$

$$I_0A=-(X+Y)\sin\tfrac{1}{2}\theta+Z\cos\tfrac{1}{2}\theta, \qquad (12b)$$

to obtain $X+Y$ and Z. (θ is the c.m. scattering angle.) However, knowing Z, $X-Y$ may be obtained from either of the equations

$$I_0R'=(X-Y)\sin\tfrac{1}{2}\theta-Z\cos\tfrac{1}{2}\theta, \qquad (12c)$$

$$I_0A'=(X-Y)\cos\tfrac{1}{2}\theta+Z\sin\tfrac{1}{2}\theta, \qquad (12d)$$

and thus the difficult A' measurement is needed for $X-Y$ only at small scattering angles.

This situation would be improved considerably if the above $B1$ experiments were supplemented with $T2$ experiments in which, as can be seen from Eq. (10b), $X-Y$ is obtained from a double (not triple) scattering measurement with the target spins initially aligned in the $\mathbf{p}$ direction; Z may be obtained similarly if the target spins are first aligned in the $\mathbf{q}$ direction. This decoupling of $X-Y$ and Z, afforded by the polarized target, persists at all angles. Knowing Z, $X+Y$ may be obtained from a $B1$ measurement of R and Eq. (12a). The $T2$ measurements involve only double scattering and hence presumably much better intensity than triple scattering experiments. All the scattering plane components of the $\mathfrak{D}_{ik}$ tensor could thus be obtained from two double scattering experiments, and one triple scattering experiment of the simple R type.

However, the $T2$ type experiments, like the $B2$ type, suffer, for a wide range of interesting angles and energies, from the fact that nucleon 2, the recoil nucleon, has an energy between about 20 and 100 Mev, an energy range in which the polarization is difficult to measure. On the other hand, for energies below about 20 Mev, the measurement of polarization becomes relatively easy again (by means of a helium analyzer), and these very small recoil energies would be involved in the determination of $X-Y$ for small angles (Coulomb interference region).

An analogous situation prevails with the $\mathcal{K}_{ik}$ tensor. Reliance entirely upon experiments of the type $B2$ necessitates R, A, R', and A' measurements on the *recoil* particle, whereas, as can be seen from Eq. (10c), availability of a polarized target would permit $\mathcal{K}_{pq}(=-\mathcal{K}_{qp})$ and $\mathcal{K}_{qq}$ to be obtained from R measurements on the easily observable component of the polarization of the *scattered* particle. For the more elusive $\mathcal{K}_{pp}$ (at small scattering angles) there will, at least, be a choice between an A measurement on the recoil particle in a $B2$ experiment and an R' measurement on the scattered particle in a $T1$ experiment, but only *one* spin-turning magnetic field will be required. If a sufficient number of the other observables are known accurately enough, it may be possible to get along without a measurement of $\mathcal{K}_{pp}$.

At the 1960 Rochester Conference it was reported that a partially polarized hydrogen target has been achieved at Saclay. Although no experience has as yet been obtained in applying it to the above measurements, an examination of the present experimental results reveals a strong need for efforts in this direction: Reference 9 contains a review of the data through July, 1959, at which time R had been measured over a broad range of angles ($\leqslant 90°$) at several energies (140, 210, and 315 Mev) whereas A had only been measured at 316 Mev at the 3 c.m. angles 25°, 50°, and 75°, and A' and R' had not been measured at all. Thus, although we have some information on $X+Y$ and Z, there is no information concerning $X-Y$. Also, although there are now some measurements[9] of $\mathfrak{D}_{nn}$ at 635 Mev between 90° and 126°, which is equal to $\mathcal{K}_{nn}$ between 90° and 54°, there is no information at all concerning the scattering-plane components of the $\mathcal{K}_{ik}$ tensor. There is no evidence that this situation has improved substantially since then.

$\mathcal{C}_{ik}$ Tensor

The use of polarized targets would also permit the replacement of the difficult simultaneous measurements of the final nucleon polarizations, now needed for the determination of the $\mathcal{C}_{ik}$ tensor, with simpler measurements of the cross section for the scattering of a polarized beam by a polarized target, in accordance with Eq. (3). Because of the difficulties in measuring the polarization of the lower laboratory energy recoil particle, the presently used technique is easiest when $\theta=\pi/2$, and, in fact, measurements have only been completed of $\mathcal{C}_{nn}$ and $\mathcal{C}_{qp}$ at this one angle. Since the above problems do not arise in a measurement of the cross section, polarized targets would be most useful in extending our knowledge of the $\mathcal{C}_{ik}$ tensor.

Miscellaneous Experimental Considerations

In applying the above results to the analysis of the *p-p* and *n-p* systems, care must be taken to properly include the effects of the indistinguishability of the two protons and the possible consequences of charge independence. Thus, once the 5 coefficients in M_1 are known for $0\leqslant\theta\leqslant\pi/2$, their behavior for $\theta>\pi/2$ is determined by the Pauli principle. These symmetry considerations have already received an extensive treatment in the literature[10,15–17,19] and thus we shall only mention several points which seem especially relevant here:

A. In the case of 2 indistinguishable protons one defines the scattered particle to be that measured in the (c.m.) angular interval $0\leqslant\theta\leqslant\pi/2$ and the recoil particle to be that in $\pi/2\leqslant\theta\leqslant\pi$. This has the consequence that measurement of the components of the $\mathcal{K}_{ik}$ tensor reduces to extending the measurement of the components of the $\mathfrak{D}_{ik}$ tensor into the angular interval $\theta>\pi/2$. It should be noted, however, that whereas $\mathfrak{D}_{nn}(\pi-\theta)=\mathcal{K}_{nn}(\theta)$, the transformation of the scattering plane components is complicated by the fact that, in the nonrelativistic approximation, the direction parallel to the scattered particle is perpendicular to the recoil one, and thus $\mathfrak{D}_{pp}(\pi-\theta)=\mathcal{K}_{qq}(\theta)$, $\mathfrak{D}_{qq}(\pi-\theta)=\mathcal{K}_{pp}(\theta)$, and $\mathfrak{D}_{pq}(\pi-\theta)=\mathcal{K}_{qp}(\theta)$.

B. Another consequence of this definition of the recoil particle is that it always has less energy in the laboratory system, and in particular, may happen to be in that medium-energy range between about 20 Mev and 100 Mev in which polarization measurements are difficult.[20] This difficulty may be avoided, for example, in polarized beam experiments of type *B*2 by utilizing their equivalence with polarized target experiments of type *T*1; thus, measurements may be made on the scattered nucleon, which has more laboratory energy than the recoil nucleon for the same c.m. energy.

In *n-p* scattering the interval of measurement is doubled to $0\leqslant\theta\leqslant\pi$, and thus at appropriate scattering angles each scattered particle may have energies such that its polarization will be difficult to measure. This situation may likewise be ameliorated if it is possible to supplement polarized beam experiments with the equivalent polarized target experiments, in which the laboratory energy of the nucleon whose polarization is being measured may fall in a more convenient range.[21] Of course, replacing a polarized beam experiment with the equivalent polarized target experiment also means that triple scattering experiments can be replaced with double scattering ones, since an extra scattering is needed to obtain the polarized beam.

[19] B. M. Golovin, V. P. Djelepov, V. S. Nadezhdin, and V. I. Satarov, J. Exptl. Theoret. Phys. (U.S.S.R.) **36**, 433 (1959) [translation: Soviet Phys.—JETP **9**(**36**), 302 (1959)].

[20] We note that, whereas the experimental situation is difficult in this energy range, there are signs of improvement. J. N. Palmieri, A. M. Cormack, N. F. Ramsey, and R. Wilson, Ann. Phys. **5**, 299 (1958), have described polarization measurements at energies as low as 46 Mev.

[21] It has been shown[19] that by using the concept of the charge independence of nuclear forces, and by performing simultaneous analysis of the *p-p* and *n-p* scattering data, one can reduce the number of experiments necessary to reconstruct M. This follows from the fact that 2 experiments measuring the same quantity for the *p-p* and *n-p* systems provide 3 independent combinations of scattering matrix coefficients: two corresponding to nucleon interactions in the states $T=0$ and 1, and one corresponding to interference between these states. Although this discovery will undoubtedly be very useful when more data become available on the *n-p* system, it is more likely to be useful later in removing the ambiguities from M_0 through knowledge of M_1, rather than in solving the converse problem with which we are presently faced.

5. CONCLUSIONS

1. A unique reconstruction of the nucleon-nucleon scattering matrix is possible, based on a knowledge of a sufficient number of the quantities: I_0, P, and the $\mathcal{C}_{ik}$, $\mathfrak{D}_{ik}$, and $\mathcal{K}_{ik}$ tensors, at one energy and angle. Our method utilizes only the amplitudes defined by Wolfenstein and Ashkin[1,2] and is more direct and computationally simpler than either the methods using unitarity and measurements at all angles or the phase-shift analyses; thus, it would provide an independent means of arriving at the correct set of phase-shift solutions. The method should be particularly useful at high energies where inelastic processes (pion production) make phase-shift analysis difficult.

2. Our method is based on a knowledge of the polarization transfer tensor $\mathcal{K}_{ik}$, which has a form complementary to the familiar correlation tensor $\mathcal{C}_{ik}$, but may be obtained from triple scattering experiments on the recoil particle similar to those already used to determine the familiar depolarization tensor $\mathfrak{D}_{ik}$.

3. The utilization of polarized targets would simplify the determination of both the $\mathfrak{D}_{ik}$ and $\mathcal{K}_{ik}$ tensors, by permitting the replacement of the difficult A' and R' (triple scattering with spin-turning magnetic fields) measurements, required to detect the component of polarization along the direction of motion of the scattered particles when polarized beams are used, with double scattering measurements.

4. Polarized targets would also help to extend measurements of the $\mathfrak{D}_{ik}$ and $\mathcal{K}_{ik}$ tensors over a broader angular range: When the laboratory energy of one of the scattered particles falls into a range in which polarization measurements are difficult, it would be possible to utilize the equivalence of polarized beam and polarized target experiments to transfer measurements to the other scattered particle, whose laboratory energy can be quite different for the same c.m. energy.

5. Polarized targets would also permit the replacement of the difficult simultaneous measurements of the final nucleon polarizations, now needed for the determination of the $\mathcal{C}_{ik}$ tensor, with simpler measurements of the cross section for the scattering of a polarized beam by a polarized target.

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APPENDIX. SOLUTION OF THE EQUATIONS OF TABLE I

If a number of experimental quantities in Table I are measured *with high accuracy*, the coefficients A to F may be determined by a straightforward procedure. For simplicity, we shall assume that C is real—which, in general, it is not; but as is well known, all coefficients may be multiplied by a factor $e^{i\alpha}$ with arbitrary complex phase α, without changing the observable quantities in Table I. Further, we shall assume that *all* equations in Table I have been divided by $\frac{1}{4}I_0$ so that we are really talking about $2AI_0^{-\frac{1}{2}}$, etc., but we shall still denote these quantities by A, B, etc. Further, A_r and A_i denote, respectively, the real and imaginary parts of A. The equations of Table I will be referred to by their numbers.

With these notations, the first two equations of Table I are:

$$4=|A|^2+|B|^2+2|C|^2+|E|^2+|F|^2, \quad (A.1)$$

$$4\mathfrak{D}_{nn}=|A|^2+|B|^2+2|C|^2-|E|^2-|F|^2, \quad (A.2)$$

and so on for the others. Subtracting these θ, we have

$$|E|^2+|F|^2=2(1-\mathfrak{D}_{nn}). \quad (A.3)$$

Similarly, subtracting (I.4) from (I.3) gives

$$|F|^2-|E|^2=2(\mathfrak{D}_{pp}-\mathfrak{D}_{qq}), \quad (A.4)$$

so that $|E|$ and $|F|$ are given explicitly by

$$|E|^2=1-\mathfrak{D}_{nn}-\mathfrak{D}_{pp}+\mathfrak{D}_{qq}, \quad (A.5)$$

$$|F|^2=1-\mathfrak{D}_{nn}+\mathfrak{D}_{pp}-\mathfrak{D}_{qq}. \quad (A.6)$$

Assuming for the moment that C is known, (I.7) and (I.8) give

$$E_i=(\mathcal{K}_{pq}-\mathcal{C}_{pq})/C\equiv N/C, \quad (A.7)$$

$$F_i=(\mathcal{K}_{pq}+\mathcal{C}_{pq})/C\equiv M/C. \quad (A.8)$$

Note the abbreviations M and N introduced in (A.7) and (A.8).

Now subtracting (I.9) from (I.10) gives

$$\mathrm{Re}EF^*\equiv E_rF_r+E_iF_i=\mathcal{K}_{nn}-\mathcal{C}_{nn}\equiv L. \quad (A.9)$$

Clearly we must have

$$|L|=|\mathrm{Re}EF^*|\leqslant|E|\,|F|, \quad (A.10)$$

which, with (A.5) and (A.6), gives an inequality which the experimental quantities have to fulfill. Inserting in (A.9) E_i and F_i from (A.7) and (A.8), and E_r and F_r from (A.5) to (A.8) yields an equation for C which, after some algebra, reduces to

$$C^2=\frac{|E|^2M^2+|F|^2N^2-2LMN}{|E|^2|F|^2-L^2}. \quad (A.11)$$

Due to the inequality (A.10), both numerator and denominator of (A.11) are positive so that C^2 is positive as it must be. For some purposes it may be useful to rewrite (A.11) in the form

$$C^2=\frac{(|E|M-|F|N)^2}{|E|^2|F|^2-L^2}+\frac{2MN}{|E|\,|F|+L}, \quad (A.12)$$

which puts the positive definite nature of C^2 directly in evidence. Whether C can be determined accurately from (A.11) depends not only on the accuracy of the experiments but also on the question whether $|L|$ is nearly equal to its upper limit $|E|\,|F|$ or not.

Using C from (A.11), E_i and F_i are then determined. Also, E_r, F_r are, except for sign, using (A.5) to (A.8). But also the relative sign of these two quantities is determined, from (A.9). Indeed (A.9) gives the product E_rF_r by direct substitution. It is, however, useful to substitute (A.11) back into (A.7) and (A.8); then after some algebra it turns out that

$$\text{Sign of } E_rF_r=\text{sign of }[L(M|E|-N|F|)^2 -MN(|E|\,|F|-L)^2]. \quad (A.13)$$

The signs of E_r and F_r individually cannot be determined with the information thus far used.

Next we determine A and B. Adding (I.1) and (I.2) gives

$$|A|^2+|B|^2=2(1+\mathfrak{D}_{nn}-C^2). \quad (A.14)$$

Adding (I.3) and (I.4),

$$|A|^2-|B|^2=2(\mathfrak{D}_{pp}+\mathfrak{D}_{qq}). \quad (A.15)$$

Since C is known from (A.11), this yields $|A|$ and $|B|$. Further, adding (I.10) and (I.11)

$$\mathrm{Re}AB^*=\mathcal{C}_{nn}+\mathcal{K}_{nn}-C^2. \quad (A.16)$$

We could proceed to solve for A and B; however, it is in this case more convenient to use

$$U=\tfrac{1}{2}(A+B),\quad V=\tfrac{1}{2}(A-B). \quad (A.17)$$

From (A.14) and (A.16) we obtain immediately

$$2|U|^2=1+\mathfrak{D}_{nn}+\mathcal{C}_{nn}+\mathcal{K}_{nn}-C^2, \quad (A.18)$$

$$2|V|^2=1+\mathfrak{D}_{nn}-\mathcal{C}_{nn}-\mathcal{K}_{nn}, \quad (A.19)$$

and from (A.15)

$$\mathrm{Re}UV^*=U_rV_r+U_iV_i=\tfrac{1}{2}(\mathfrak{D}_{pp}+\mathfrak{D}_{qq}). \quad (A.20)$$

Further, (I.5) and (I.6) now give, respectively,

$$U_r=P/C, \quad (A.21)$$

$$V_i=\mathfrak{D}_{pq}/C. \quad (A.22)$$

Except for sign, U_i and V_r can be obtained by combining (A.18) with (A.21), and (A.19) with (A.22). Then (A.20) provides in addition a linear relation between U_i and V_r. If the experimental data are accurate, there will in general only be one solution for the signs of U_i and V_r. In addition, a check on all quantities will be provided.

Thus, with accurate data, U and V, hence A and B, will be completely determined, while in the determination of E, F above one sign remained ambiguous. The difference is due to the fact that (A.21) and (A.22) give the real part of U and the imaginary part of V, while (A.7) and (A.8) give the imaginary parts of both quantities, E and F. The remaining ambiguity in sign of E_r, F_r can now be resolved by using *any* of the last four equations (I.11) to (I.14). In addition, valuable checks of the correctness of the obtained solution can be deduced from the last 4 quantities.

Our primary solution relies on the first ten quantities of Table I which on the whole include the easier types of experiments. As discussed in Sec. 4, experiment 14 tends to be somewhat easier than 11 to 13.

We have not investigated the problem of solving the set of equations (I.1) to (I.14) when the experiments are rather inaccurate, as in practice they tend to be. In this case, the use of all 14 experiments is probably desirable.

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High-Energy Nucleon-Nucleon Collisions*

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The single virtual boson exchange interaction model is applied to high-energy inelastic nucleon-nucleon collisions over an energy range of several orders of magnitude. The phase space is discussed simply in terms of three "natural" phase-space variables, and a simple, exact formula is given for the "upper" boundary of these variables. The probability for a particular final-state configuration is then discussed in terms of the available phase space, the magnitude of the phase-space factor, and the magnitude of the average total "cross-section" factors that occur in this model. Qualitative features of experimental data for incident nucleon laboratory energies of 10, 10^2, and 10^3 Bev can be understood on the basis of this model.

EXPERIMENTAL evidence that many high-energy inelastic nucleon-nucleon collisions occur with large impact parameters[1–3] suggests the importance of single π-meson exchange graphs.[4,5] A recently given field-theoretical description of general binary collisions dominated by single-boson exchange graphs[6] is applied in this note to high-energy nucleon-nucleon collisions.[7,8] This model leads naturally to the two "independent" groups of final-state particles that are observed. It is shown that if the plausible assumption is made that the "scattering" cross sections of the exchanged "almost real" pion with the incident nucleons are close to the real cross sections at high energies, then this model leads to qualitative understanding of certain features observed in inelastic nucleon-nucleon scattering all the way from incident nucleon laboratory energy, $E_{iL}\sim 10$ Bev, up to and including ultrarelativistic energies.

The pertinent graph is shown in Fig. 1. Nucleons N and N', with four-momenta p_i and p_i' exchange a π meson with four-momentum Δ_i, leading to two groups of particles, C and C', with total four-momenta P and P'. The nucleon rest mass is M, and the metric is chosen so that $p_i^2=p_i'^2=-M^2$. The "rest masses" W and W' of C and C' are defined by $P^2=-W^2$ and $P'^2=-W'^2$. The rest (barycentric) system of the group of particles C is denoted by (W) and that of the group C' by (W'). For the case considered (at least one pion in each group C, C'), the minimum value of W, and of W', is $m_\pi+M$. The over-all barycentric system is

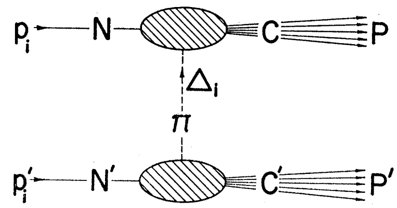

FIG. 1. A general inelastic collision in which nucleons N and N', with four-momenta p_i and p_i', interact by the exchange of a single π-meson π, with four-momentum Δ_i, leading to two groups of final state particles, C, with total four-momentum P, and C', with total four-momentum P'. Each group contains at least one π meson.

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