machines.[24] The relative errors of the electrostatic generator runs would also include any error arising from degradation of neutron energy through the stack of foils.

The standard deviations of the neutron energies of the Cockcroft-Walton accelerator were obtained by taking $\frac{1}{3}$ of the energy spread at various laboratory angles for thick-target yields[25] except in the cases where the neutron energy spread due to deuteron scattering or sample geometry exceeded the above. Those of the Van de Graaff accelerator were obtained by geometrical relationships of the target and samples.[26] The standard deviations of cross sections used in Table I were those which gave the best fit of the data to a theoretical excitation function based on the statistical model concept of the compound nucleus and reflect the relative consistency of the data for a given function. The absolute errors are difficult to evaluate, but probably the greatest sources of error are in the decay schemes and in estimating γ-counting efficiencies.

Note added in proof. Linear plots of individual curves of the data in Table I will be available in Los Alamos Scientific Laboratory Rept. LA 2493 and will be available at a later date from the Office of Technical Services, U. S. Department of Commerce, Washington 25, D. C.

ACKNOWLEDGMENTS

The authors wish to express their appreciation to Dr. G. A. Cowan and Dr. J. D. Knight for their interest and advice, to R. K. Smith for the Van de Graaff irradiations, to the personnel at the Cockcroft-Walton accelerator, and to T. T. Shull for some of the titanium chemistry.

[24] Code for resolving counting data from more than one activity is a least squares solution, and the Los Alamos Scientific Laboratory designation for this code is J-11-GPF-002. The code for decay corrections during irradiation is designated as J-11-GFW-010 and evaluates the expression

$$(A/A_0)=\sum_{i=1}^{N} F_i[1-\exp(-\lambda\tau_i)\exp(-\lambda T_i)],$$

where A = number of atoms remaining at $T_{(0)}$, A_0 = total number of atoms produced during the N irradiation intervals, T_i = time from any interval to end of irradiation (T_0), and F_i = fraction of the isotope produced during the interval τ_i whose integrated irradiation level is P_i and is given by $F_i=P_i/\Sigma_i P_i$.

[25] Tabulated in Los Alamos Scientific Laboratory Report LAMS-2162 (unpublished).

[26] L. Blumberg and S. I. Schlesinger, Atomic Energy Commission Report AECU-3118, May, 1956 (unpublished).

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Nuclear Structure Studies in the Tin Isotopes with (d,p) and (d,t) Reactions*

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The neutron single-particle states in the odd isotopes of tin are identified by (d,p) angular distribution studies. The cross sections for exciting these states by (d,p) and (d,t) reactions are measured, and the results are analyzed to give values of V_j^2 (in Kisslinger-Sorenson notation), the fraction by which each of the single-particle states is full, for each subshell in each isotope. These are used to calculate ϵ_j, the unperturbed single-particle energies; the results are reasonably consistent. If the observed energies of single-particle states are used to predict the V_j, the agreement is generally good, but some discrepancies are noted and an explanation is offered.

Other weakly excited states are found in the region of the single-particle states. At higher excitation energies, several rather sharp levels are strongly excited in (d,p) reactions. Their energy, cross section, and regularities among the isotopes suggests that these are single-particle levels from the next major shell ($82<N\leq126$); however, their angular distributions cannot be used for identification as they are the same for all levels in this region and show little structure. This last fact is not easily explained.

Some of the two quasi-particle excitation states in the even isotopes of Sn are identified and the apparent pairing energy is thereby measured; it is surprisingly found to vary rapidly with mass number. Spectra from (d,p) and (d,t) reactions in isotonic pairs Cd^{114}-Sn^{116} and Cd^{116}-Sn^{118} are compared to show that the single-particle neutron states are much more radically affected by the addition of two protons than by the addition of two neutrons, contrary to the usual assumption in shell model theory.

Q values for (d,p) and (d,t) reactions on the major isotopes of tin are measured.

I. INTRODUCTION AND THEORY

FROM the simple shell-model viewpoint, the structure of nuclei with more than three particles (or holes) outside of closed shells is extremely complicated, and good theoretical calculations are essentially impossible. However, in the pairing theory approximation,[1,2] the structure becomes simple again provided, at least, that either the neutrons or protons have a closed shell. Such a situation arises among the isotopes

* Work done in the Sarah Mellon Scaife Radiation Laboratory and assisted by the National Science Foundation and the joint program of the Office of Naval Research and the U. S. Atomic Energy Commission.

[1] L. S. Kisslinger and R. A. Sorenson, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. (to be published).
[2] M. Baranger (to be published).

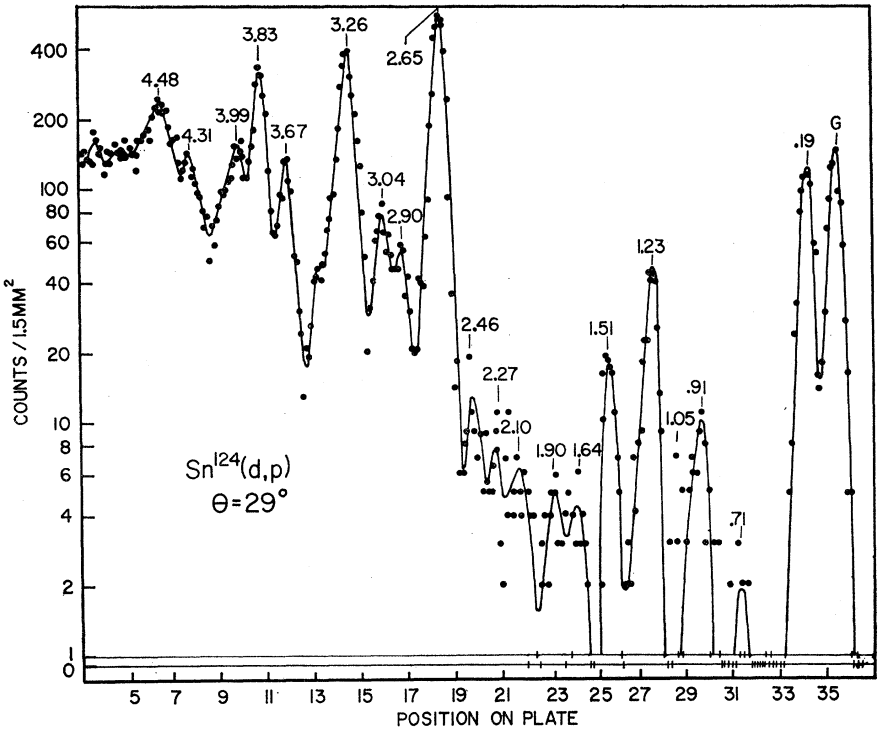

FIG. 1. Typical data for proton spectra from (d,p) reactions. Different symbols for ordinates equal to zero and one have no significance other than legibility. Figures are excitation energy in Mev.

of tin, where there is a closed shell of protons, and the neutron number outside of the closed shell varies from 16 to 24 (the hole number varies from 8 to 16).

The neutron subshells filling in this region are the $d_{5/2}$, $g_{7/2}$, $s_{1/2}$, $d_{3/2}$, and $h_{11/2}$. In the pairing theory approximation, the ground state of an even-even nucleus is characterized by a set of quantities[3] $V_{d5/2}$, $V_{g7/2}$, $V_{s1/2}$, etc., where $(V_{d5/2})^2$ is the fraction by which the $d_{5/2}$ shell is filled, etc. For simplicity, we introduce the notation

$$V_{2j}=(V_{1j})^2 \text{ of reference } 1=(v_{1j})^2 \text{ of reference } 2,$$

so that now the ground state of an even-even nucleus is specified by V_5, V_7, V_1, V_3, and V_{11}. In accordance with pairing theory, these quantities increase slowly and monotonically with mass number, in a manner which can be calculated from the unperturbed single particle level positions. Thus, the addition of two neutrons to an even-even nucleus changes the ground state-configuration only slightly. The low-lying states of odd-neutron isotopes consist essentially of an even-even core plus a particle (or hole) in one of the single-particle states. The spectrum of each of the odd isotopes of tin should thus be quite similar.

An especially useful technique for investigating these single-particle states is a study with the (d,p) and (d,t) stripping and pickup reactions. Firstly, they preferentially excite these single-particle and single-hole states; secondly, measurements of the angular distributions provide an assignment of each level to one of the single-particle states; and thirdly, measurements of the cross sections give determinations of the V's.

The dependence of the cross sections on the V's has been demonstrated rigorously by Yoshida,[4] but the following simple calculation (carried out independently) gives insight into the problem, and yields the correct results. The cross section for a (d,p) and a (d,t) reaction may be expressed as

$$\begin{aligned}\frac{d\sigma}{d\Omega}(d,p)&=\frac{2I_f+1}{2I_i+1}P(l_n,Q,\theta)S(i,f),\\ \frac{d\sigma}{d\Omega}(d,t)&=T(l_n,Q,\theta)S(i,f),\end{aligned}\tag{1}$$

where I_f and I_i are the spins of initial and final nuclei, P and T are functions derivable, in principle, from reaction theory, and $S(i,f)=S(f,i)$ is a quantity derivable from nuclear structure theory which expresses the overlap between the initial and final nuclear states. In all stripping and pickup theories which have been conceived to date, P and T are functions of l_n, the orbital angular momentum of the "stripped" or "picked-up" neutron; Q, the energy release in the reaction; and θ, the angle between the incident deuteron and the emitted proton or triton.

If the initial and final state configurations differ only

[3] Reference 1 uses capital V's while reference 2 uses lower case v's. The former should not be confused with the capital V's used here which are their square.

[4] S. Yoshida (private communication).

in that one has j^n and the other has j^{n-1}, French[5] has shown that

$$S(n,n-1)=n \quad (n \text{ even}) \qquad =1-(n-1)/(2j+1) \quad (n \text{ odd}). \tag{2}$$

If the target nucleus is even-even, for a (d,t) reaction $S=n_i$, the number of particles in the initial nucleus; for a (d,p) reaction, $S=1-n_i/(2j+1)$. Since $n=(2j+1)V$, $I_i=0$, and $I_f=j$,

$$\frac{d\sigma}{d\Omega}(d,p)=(2j+1)PU_{2j}{}^{(i)},$$
$$\frac{d\sigma}{d\Omega}(d,t)=(2j+1)TV_{2j}{}^{(i)}, \quad \text{(target even)} \tag{3}$$

where

$$U_{2j}=1-V_{2j}, \tag{4}$$

and $V_{2j}{}^{(i)}$ means V_{2j} for the initial (i.e., target) nucleus. When the target nucleus has an odd neutron number, a similar calculation yields, for transitions to the ground state

$$\frac{d\sigma}{d\Omega}(d,p)=PV_{2j}{}^{(f)},$$
$$\frac{d\sigma}{d\Omega}(d,t)=TU_{2j}{}^{(f)}, \quad \text{(target odd)} \tag{5}$$

where $V_{2j}{}^{(f)}$ means V_{2j} for the final nucleus. The results (3) and (5) agree with those of a complete calculation by Yoshida.[4] The methods of obtaining the V's from experiments depends, in general, on forming ratios so that the P's and T's cancel. In Sec. III, the details of this process are described, and the results are discussed and compared with other data.

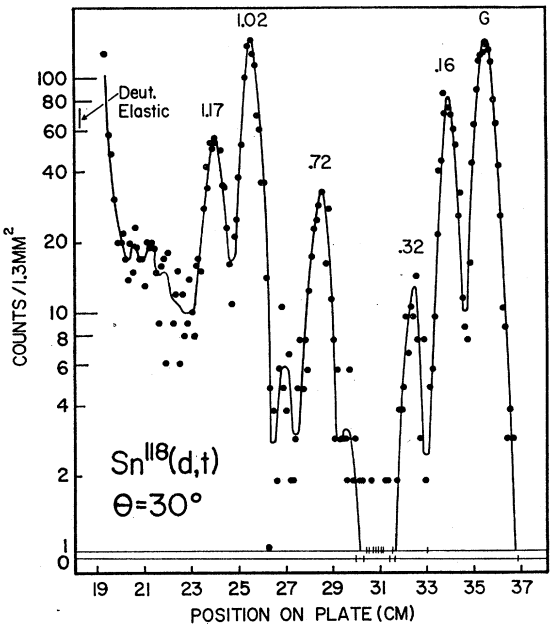

FIG. 2. Typical data for triton spectra from (d,t) reactions. See caption for Fig. 1.

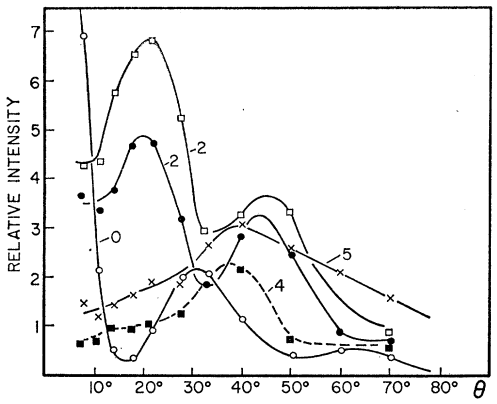

FIG. 3. Angular distributions of protons from $Sn^{116}(d,p)Sn^{117}$ leading to states of Sn^{117} with known spins and parities. Figures attached to the curves are l_n, the orbital angular momentum of the stripped neutron. Of the two curves with $l_n=2$, the upper leads to the $d_{3/2}$ state and the lower to the $d_{5/2}$ state.

In addition to yielding details of the shell being filled, (d,p) reactions leading to higher excited states should give information on the single-particle states in the next (empty) major shell. A considerable effort has been expended in this direction, although the results so far have led to little elucidation. This work is discussed in Sec. IV-A. Other parts of Sec. IV discuss the low-lying non-single-particle states, reactions on the odd isotopes of tin which give a direct measure of the pairing energy and show a strange behavior for the latter, a comparison of the neutron single-particle states in isotopes of Cd and Sn which have the same number of neutrons, and the experimental results for reaction Q values.

II. EXPERIMENTAL

The experimental method has been described previously[6]; it consists of bombarding targets with 15-Mev deuterons from the University of Pittsburgh cyclotron, magnetically analyzing the reaction products with a 60° wedge magnet spectrograph, and allowing them to impinge on a nuclear track photographic emulsion located on its focal plane. After development, the emulsions are scanned under a microscope, and the number of tracks per unit area are counted as a function of position, thus determining intensity as a function of energy. The targets are isotopically enriched foils of each of the major isotopes of tin[7]; typical target

[5] J. B. French, *Nuclear Spectroscopy*, edited by F. Ajzenberg-Selove (Academic Press, Inc., New York, 1960).

[6] B. L. Cohen, J. B. Mead, R. E. Price, K. Quisenberry, and C. Martz, Phys. Rev. **118**, 499 (1960).

[7] The tin isotopes as metal foils were obtained from Stable Isotopes Division, Oak Ridge National Laboratory, Oak Ridge, Tennessee.

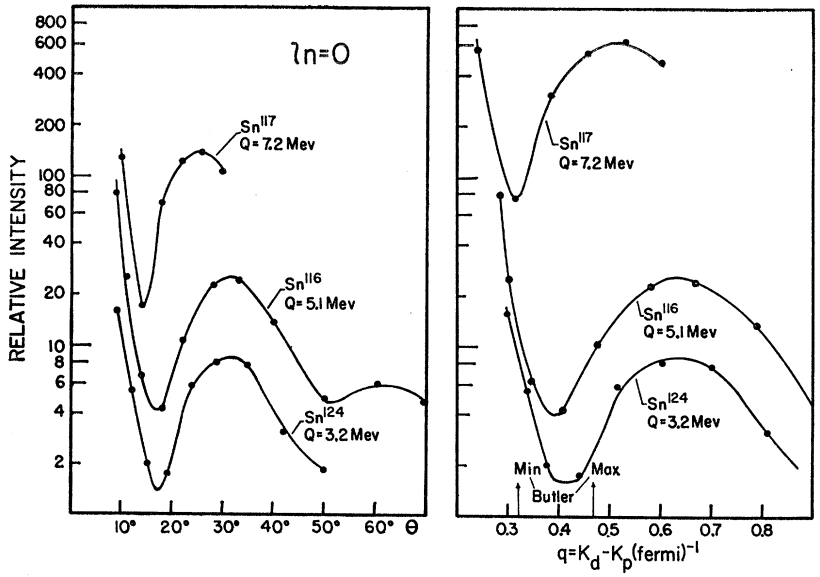

FIG. 4. Angular distributions from (d,p) reactions with $l_n=0$ but differing Q values. The plot on the right is the same data plotted against q, the momentum transferred to the nucleus. If Butler theory were valid, the plots vs q would all have maxima and minima at the same abscissas, namely those labelled "Butler Max" and "Min." A large amount of other data is consistent with that shown here.

thicknesses are 7 mg/cm², but by judicious target orientation,[8] the energy resolution is generally about 80 kev. In runs where protons are detected, the photographic emulsion is covered by sufficient absorber to stop all particles except protons; where tritons are detected, no particle selection is necessary as no other particle has as large a magnetic rigidity as >10-Mev tritons. Below 10 Mev, elastically and inelastically scattered deuterons produce a background which makes triton detection impossible. At small angles, the background from elastically scattered deuterons causes serious difficulty for triton energies up to ~13 Mev.

Typical data are shown in Figs. 1 and 2. For reasons which are not very clear, data taken on the same day are most consistent. In some cases where data are taken several months apart, discrepancies in cross sections as large as 40% have been found. In measuring angular distributions it is thus important to expose plates at all angles on the same day. In deriving results from the data, these problems are taken into account in assigning weights to various experimental determinations. The best results are obtained when a measurement consists of determining the ratio of intensities of two peaks from the same plate exposure. Fortunately, some of the most important results in the experiment are obtained in this way. Portions of the plate used for quantitative data are scanned independently at least twice.

For other reasons which are also not clear, determinations of excitation energies for a given level vary by as much as 4% when plates are exposed at different times. More typical uncertainties in energies are about 1–2%. No extensive effort has yet been made to understand or to correct this difficulty.

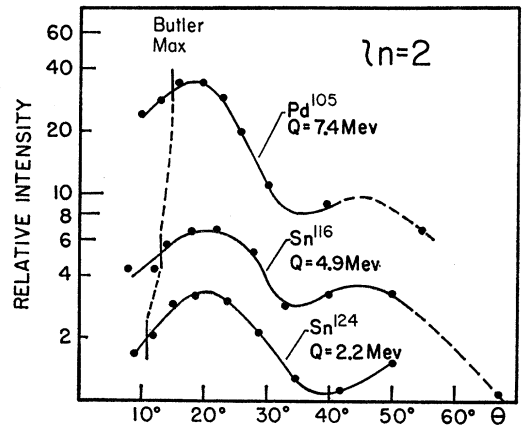

FIG. 5. Angular distributions from (d,p) reactions with $l_n=2$ but differing Q values. The solid vertical lines through each curve connected by dashed lines show the position of the first maximum predicted by Butler theory. A large amount of other data is consistent with that shown here; some is shown in Fig. 11.

[8] B. L. Cohen, Rev. Sci. Instr. **30**, 415 (1959).

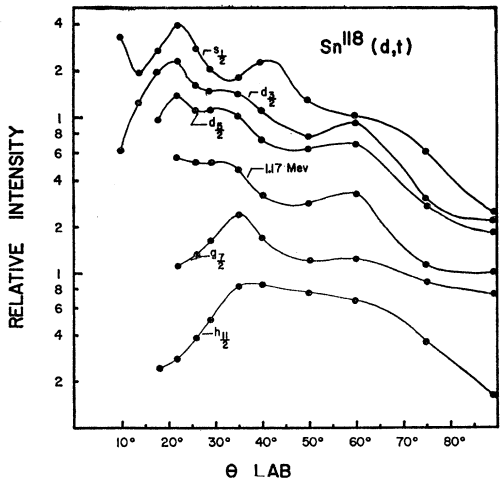

FIG. 6. Angular distributions of tritons from $Sn^{118}(d,t)Sn^{117}$ leading to states of Sn^{117} with known spins and parities designated by single-particle state attached to curves. The low-angle region is difficult to study because of deuteron background. The 1.17-Mev state was found to be $d_{5/2}$ from (d,p) reaction studies.

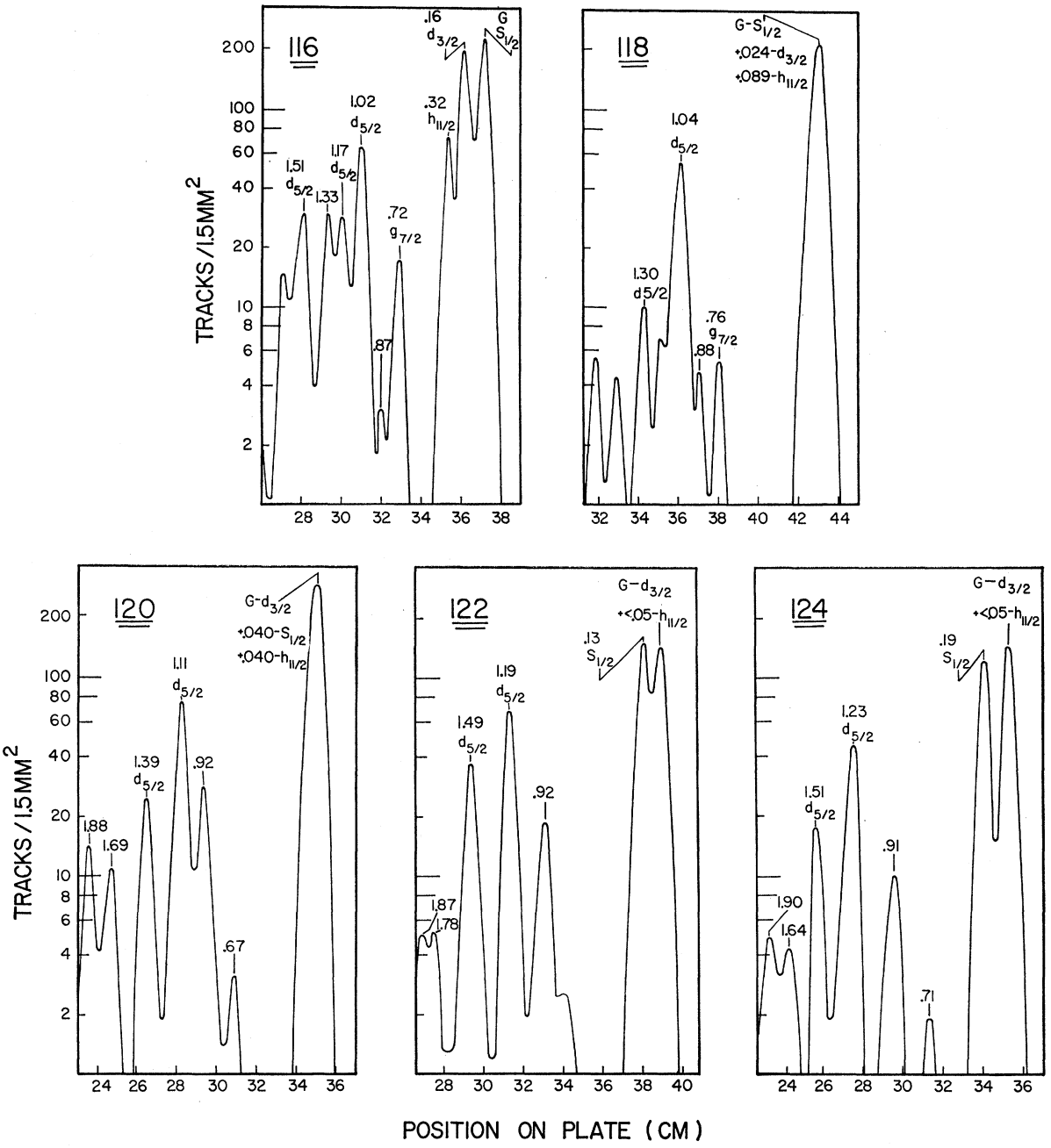

FIG. 7. High-energy portion of energy spectra of protons from (d,p) reactions on even isotopes of Sn (mass number underlined). Detection angle is 29°. Figures are excitation energy in Mev, and spectroscopic notation refers to assignment as single-particle level. Groups not so designated are not single-particle levels as evidenced by angular distribution studies. The energy determination of the $s_{1/2}$ level from $Sn^{120}(d,p)$ was obtained from the difference in apparent energy of the 1.11-Mev state at angles where $l_n=2$ and $l_n=0$ angular distributions have maxima.

III. RESULTS AND CONCLUSIONS ON SINGLE-PARTICLE STATES IN $50<N\leq 82$ SHELL

A. Angular Distributions

In light elements, values of l_n are determined straightforwardly by comparing angular distributions with Butler theory[9]; however, one does not expect Butler theory to be valid in this mass and energy region, so that the first task is to establish a systematics by measuring angular distributions for cases where l_n is known, for various values of Q.

[9] S. T. Butler, *Nuclear Stripping Reactions* (John Wiley & Sons, Inc., New York, 1957).

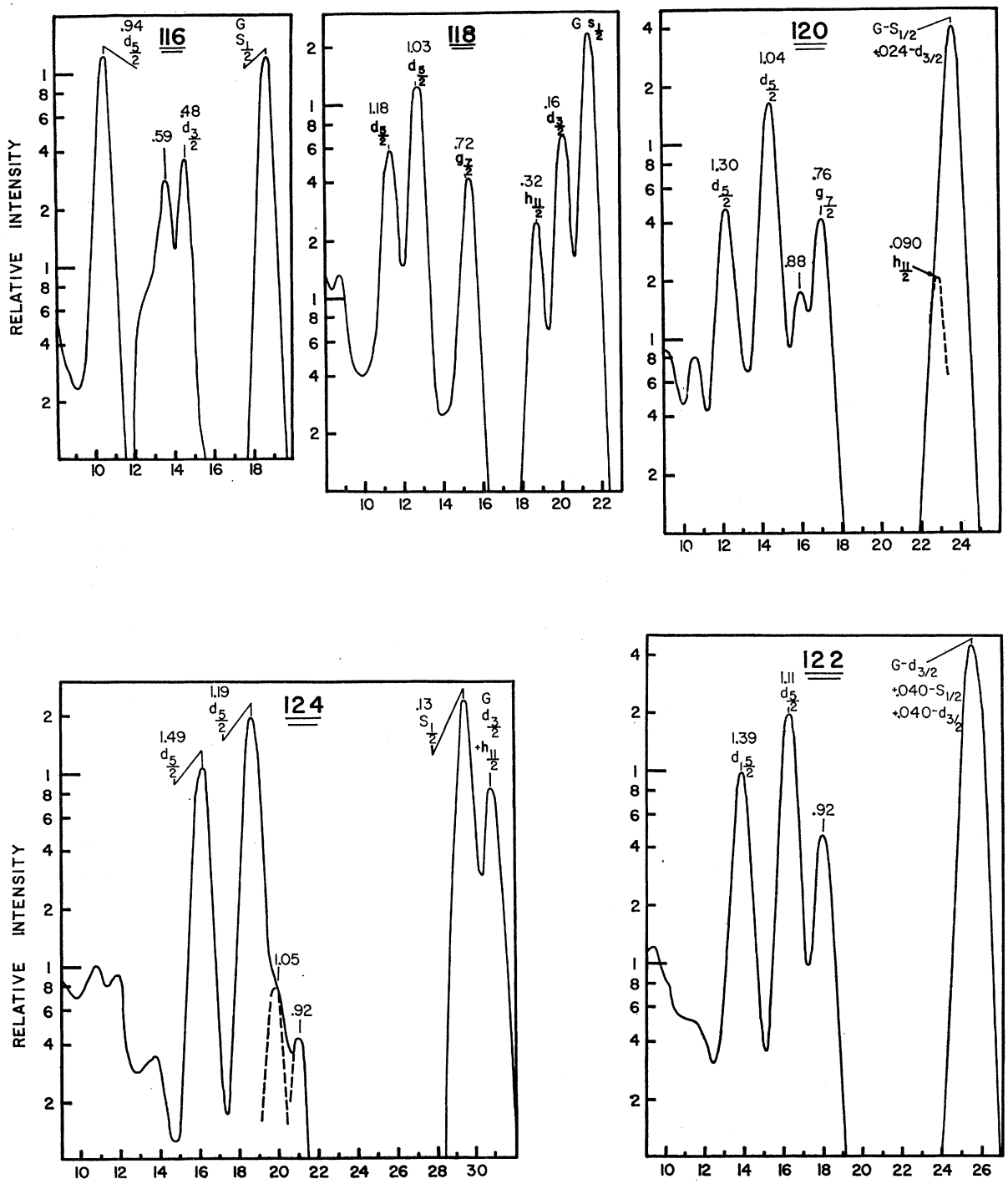

FIG. 8. Energy spectra of tritons form (d,t) reactions on even isotopes of Sn (mass number underlined). See caption for Fig. 7.

Figure 3 shows angular distributions for groups with $l_n=0$, 2, 4, and 5 from the reaction $Sn^{116}(d,p)Sn^{117}$. The shapes of the angular distributions and their variation with l_n fit well with expectations from experience with lighter elements; only the $l_n=0$ has a peak at forward angles, and the angle of the first peak in the other cases increases monotonically with l_n.

The dependence of these angular distributions on Q is shown in Figs. 4 and 5, where comparisons of positions of maxima and minima with Butler theory are also made. The changes in peak angle with Q are very slow, considerably slower than predictions from Butler theory and in the opposite direction. It is thus better to compare angular distributions directly rather than to con-

TABLE I. $d\sigma/d\Omega$ (mb/sr) for (d,p) reactions leading to single-particle states, and to 0.9-Mev states. Also tabulated is the quantity a, defined in Eq. (7).

	30°					45°					19°		
Target mass	116	118	120	122	124	116	118	120	122	124	116	120	124
$s_{1/2}$	2.6	1.6	1.7	1.7	1.4	0.40	~0.4	~0.4	0.40	0.20			
$d_{3/2}$	1.87	1.4	1.3	1.3	1.24	1.08	1.3	1.2	0.85	0.60	4.4	2.5	2.6
$h_{11/2}$	0.63					0.62							
$gf_{/2}$	0.19	0.09				0.10	0.10						
$d_{5/2}$	1.07	0.93	0.86	1.04	0.67	0.48	0.65	0.56	0.54	0.28	1.86	1.42	1.07
a	1.75	1.50	1.5	1.25	1.85	2.25	2.0	2.1	1.6	2.1	2.3	1.8	2.4
0.9 Mev	$\lesssim$0.03	0.04	0.27	0.23	0.10			0.17	0.09	0.06			

vert them to plots of intensity vs momentum transfer, q ($q=K_d-K_p$), as is most convenient when comparing with Butler theory (compare two plots in Fig. 4). It is of some interest to note, however, that angular distributions for the largest Q values do agree roughly with Butler theory. The results shown in Figs. 3, 4, and 5 indicate that angular distribution studies with (d,p) reactions are a fruitful method of determining l_n values.

The situation with (d,t) reactions is much less favorable as is seen from measurements of angular distributions of groups with known l_n shown in Fig. 6. The peaks are not as sharp as in (d,p) reactions, and the differences for different l_n are not as readily apparent. The difficulties are compounded by uncertainties in subtracting backgrounds from elastically scattered deuterons at forward angles. As a result, the use of (d,t) angular distributions for determining l_n values gave few reliable results and was eventually abandoned.

B. Identification of Single-Particle States

The high-energy regions of proton spectra obtained at 30° from the even isotopes of tin are shown in Fig. 7. The excitation energies (in Mev) and the shell model assignments from other work or from angular distribution measurements are also shown. Angular distribution measurements do not, of course, distinguish between $d_{5/2}$ and $d_{3/2}$ states, but the energies of the two are sufficiently well separated to give a clear distinction. It should be pointed out that the single-particle states include all the most strongly excited states; a slight exception to this is the 0.9-Mev state which will be discussed in Sec. IV-B. It, and all other unassigned states in Fig. 7, have angular distributions unlike any of those in Fig. 3.

The spectra of tritons from (d,t) reactions obtained at 30° from the even isotopes of tin are shown in Fig. 8. The single-particle level assignments for the groups from $Sn^{116}(d,t)Sn^{115}$ were not known (except for the ground state); the assignments shown are postulations based on energy and cross-section regularities with the other isotopes. Cross sections for exciting the single-particle levels by (d,p) and (d,t) reactions are listed in Tables I and II. In cases where the $s_{1/2}$, $d_{3/2}$, and $h_{11/2}$ peaks are not resolved, the total cross section for the peak was derived as follows: The $h_{11/2}$ was crudely estimated by extrapolation; since it is relatively small, and its cross section is not used in the analyses, errors in this procedure are unimportant. The ratio of the $s_{1/2}$ to the $d_{3/2}+h_{11/2}$ is measurable in three of the five cases and was found to be relatively slowly varying; this ratio was therefore interpolated to determine the $s_{1/2}$ and $d_{3/2}$ intensities. For most purposes, this procedure does not seem capable of causing large errors in the results.

C. Determination of V's

1. Correction for Q dependence.

Before using (3) and (5), the Q dependence of $P(l_n,Q,\theta)$ and $T(l_n,Q,\theta)$ must be determined. Since the angular distributions are not very dependent on Q (see Figs. 4 and 5), it is assumed that the Q dependence may be separated; in Butler theory,[9] at least, the Q dependence is approximately exponential and equal

TABLE II. $d\sigma/d\Omega$ (mb/sr) for (d,t) reactions leading to single-particle states, and to 0.9-Mev states. Also tabulated is the quantity b, defined in Eq. (8).

	30° Data					45° Data				
Target mass	116	118	120	122	124	116	118	120	122	124
$s_{1/2}$	1.31	1.31	1.6	2.0	2.1	1.27	2.20	3.2	3.4	3.6
$d_{3/2}$	0.50	0.66	1.05	1.4	1.8	0.32	0.71	1.2	1.8	1.8
$h_{11/2}$		0.09					0.22			
$gf_{/2}$		0.27					0.43			
$d_{5/2}$	2.15	1.88	1.94	2.24	2.86	1.27	2.06	2.20	3.00	2.85
b	0.23	0.35	0.54	0.62	0.63	0.25	0.34	0.55	0.60	0.65
0.9 Mev			0.16	0.38	0.06			0.20	0.52	0.07

TABLE III. V_3 and V_5 calculated for various values of A.

	Target mass \ A	116	118	120	122	124
V_3	1.0	0.28	0.40	0.73	0.77	0.94
	1.12	0.26	0.36	0.60	0.66	0.76
	1.25	0.24	0.29	0.49	0.52	0.61
V_5	1.0	0.77	0.78	0.90	0.90	0.98
	1.12	0.78	0.80	0.87	0.87	0.94
	1.25	0.80	0.81	0.86	0.84	0.92

and opposite for P and T. We thus assume

$$P(l_n,Q,\theta)=P_{l_n}'(\theta)A^{-Q},$$
$$T(l_n,Q,\theta)=T_{l_n}'(\theta)A^{Q}, \quad (Q \text{ in Mev}). \tag{6}$$

This approximation is principally justified by the fact that the Q dependence is generally a small correction (with one exception to be noted). It is certainly unreliable if A is much different from unity, or if we compare states with much different Q values. In Butler[9] theory, $A\simeq 1.12$; from the ratios of the $d_{3/2}$ and $d_{5/2}$ states in the $Zr^{90}(d,p)Zr^{91}$ reaction, $A=1.14$; in the Pb isotopes, a recent study[10] gave $A=1.35$. It thus seems reasonable to expect A to be between 1.12 and 1.25; calculations were therefore made for these two values and for $A=1.00$.

2. Determination of V_3 and V_5

Inserting (6) in (3), taking the ratio of cross sections for exciting the $d_{3/2}$ and $d_{5/2}$ states, and dropping superscripts,

$$A^{(Q_3-Q_5)}\left[\frac{d\sigma}{d\Omega}(d,p)\rightarrow d_{3/2}\right]\Big/\left[\frac{d\sigma}{d\Omega}(d,p)\rightarrow d_{5/2}\right]=a=\frac{4}{6}\frac{U_3}{U_5}, \tag{7}$$

where Q_3 and Q_5 designate the Q values for reactions leading to the $d_{3/2}$ and $d_{5/2}$ states, respectively, and a is the experimentally determined quantity on the left side of (7). Similarly,

$$A^{(Q_5-Q_3)}\left[\frac{d\sigma}{d\Omega}(d,t)\rightarrow d_{3/2}\right]\Big/\left[\frac{d\sigma}{d\Omega}(d,t)\rightarrow d_{5/2}\right]=b=\frac{4}{6}\frac{V_3}{V_5}. \tag{8}$$

Experimental values for a and b are listed in Table I. There are some discrepancies in values of a determined at various angles due to differences in angular distributions. Equations (7) and (8) plus the two applicable equations (4) represent four equations in four unknowns (U_3, V_3, U_5, V_5) whose solution is

$$V_5=(a-\tfrac{2}{3})/(a-b),$$
$$V_3=(\tfrac{3}{2}a-1)b/(a-b)=\tfrac{3}{2}bV_5. \tag{9}$$

Values of V_3 and V_5 are listed for the various values of A in Table III.

Some information on the value of A may be obtained if one recognizes that P_2' and T_2' should be approximately constant for all cases at a given θ. Once the U's and V's are determined, P_2' and T_2' may be calculated from (3) and (6); results for $P_2'(30°)$ and $T_2'(30°)$ are listed in Table IV. These calculations are somewhat unreliable as they compare data obtained from different cyclotron runs made over an extended period of time, (in contrast to this, the determinations of V's depend on the relative areas under various peaks on the same photographic plate); they are furthermore quite sensitive to small changes in the experimental results. Nevertheless, they give ample reason to exclude $A=1.00$ and to favor $A=1.25$ over $A=1.12$. Weighting this with the value from the $Zr(d,p)$ reaction, we adopt $A=1.18$ to give our best values of V_3 and V_5 which are listed in Table V in columns labeled "Exp." It should be noted from Table III that V_3 and V_5 are generally not highly sensitive to the value of A. The final values of V_3 and V_5 are probably accurate within ± 0.06. The difference between V_3 and V_5 for successive isotopes are probably accurate within ± 0.03.

3. Determination of V_1

Since the ground states of the odd isotopes Sn^{117} and Sn^{119} are $s_{1/2}$ states, (d,p) and (d,t) reactions on these leading to the ground states of the final even-even nuclei give information on V_1 of the latter by use of (5). For example, V_1 for Sn^{118} can be obtained from (3), (5), and (6) as

$$A^{(Q_{78}-Q_{89})}\frac{d\sigma}{d\Omega}[Sn^{117}(d,p)Sn^{118}]\Big/\frac{d\sigma}{d\Omega}[Sn^{118}(d,p)Sn^{119}]=c=[U_1/2V_1]_{Sn^{118}},$$
$$A^{(Q_{98}-Q_{87})}\frac{d\sigma}{d\Omega}[Sn^{118}(d,t)Sn^{117}]\Big/\frac{d\sigma}{d\Omega}[Sn^{119}(d,t)Sn^{118}]=d=[2V_1/U_1]_{Sn^{118}}, \tag{10}$$

TABLE IV. Values of P_2' and T_2' from 30° data for various values of A (relative units for each A).

	Target mass \ SP state \ A	116 $d_{3/2}$	116 $d_{5/2}$	118 $d_{3/2}$	118 $d_{5/2}$	120 $d_{3/2}$	120 $d_{5/2}$	122 $d_{3/2}$	122 $d_{5/2}$	124 $d_{3/2}$	124 $d_{5/2}$
P_2'	1.0	65	78	54	67	120	144	142	173	560	560
	1.12	70	80	53	67	82	98	94	110	120	142
	1.25	75	86	51	65	66	80	66	74	69	77
T_2'	1.0	44	47	41	40	36	36	46	41	48	49
	1.12	50	49	46	45	43	42	50	48	54	54
	1.25	56	50	59	51	52	48	61	55	62	61

[10] B. L. Cohen, S. Mayo, and R. E. Price, Nuclear Phys. **20**, 360 (1960); and B. L. Cohen, R. E. Price, and S. Mayo, Nuclear Phys. **20**, 370 (1960).

TABLE V. Values of V for various single-particle states in even isotopes of tin. "Exp" denotes experimental values obtained in this work, K-S denotes values from Kisslinger-Sorenson calculation, and E denotes values obtained from observed energies by use of (17) or Fig. 9(b).

Mass No.	116			118			120			122			124		
SP state	Exp	K-S	E	Exp	K-S	E	Exp	K-S	E	Exp	K-S	E	Exp	K-S	E
$s_{1/2}$	0.42	0.37	~0.5	0.50	0.53	~0.5	0.61	0.65	0.57	0.69	0.75	0.69	0.74	0.88	0.74
$d_{3/2}$	0.25	0.25	0.20	0.33	0.39	0.33	0.55	0.53	~0.5	0.59	0.65	~0.5	0.68	0.75	~0.5
$d_{5/2}$	0.79	0.93	0.93	0.80	0.94	0.93	0.87	0.95	0.94	0.86	0.96	0.95	0.93	0.97	0.95
$gf_{/2}$	0.78	0.91	0.89	0.86	0.93	0.90	0.89	0.94	0.90	(0.92)	0.95		(0.95)	0.96	
$h_{11/2}$	0.27	0.11	0.13	0.33	0.19	0.25	0.35	0.27	0.34	0.47	0.38	~0.5	0.55	0.59	~0.5

where Q_{78}, Q_{87}, etc., are Q values for the reactions $Sn^{117}(d,p)Sn^{118}$, $Sn^{118}(d,t)Sn^{117}$, etc.; and c and d are the experimentally determined values of the ratios on the left. These plus the appropriate equation (4) give two independent determinations of V_1; they are listed in Table VI for the three values of A being considered. The results are consistent for $A=1.04$ with $V_1=0.45$.

This is an uncomfortably low value of A, and the whole analysis is somewhat questionable because of the large difference in Q values between reactions being compared (2.8 Mev). There may well be shifts in the angular distributions which render the separation (6) invalid. An alternative procedure was therefore followed: Angular distributions for the two reactions were measured and intensities were compared both at an angle near the peak of the angular distributions of each (27°) and at the peaks themselves; the results are the same from the two comparisons and are listed in Table VI. The results from the (d,p) and (d,t) comparisons are consistent for $A=1.12$ where each gives $V_1=0.50$; this value is adopted. The slight difference between the values of A used here and in the analyses of the $d_{3/2}$ and $d_{5/2}$ states need not be disturbing as the range of Q values covered is quite different, and A might be a function of l_n.

Once the value of A and of V_1 for Sn^{118} is determined, V_1 for the other isotopes may be obtained by assuming P' and T' are the same for all isotopes. In addition, V_1 for Sn^{116} and for Sn^{120} may be determined analogously with (10) from ratios of (d,t) cross sections for Sn^{117} and Sn^{116}, and from ratios of (d,p) cross sections from Sn^{119} and Sn^{120}. All values and weighted averages are listed in Table VII, and the weighted averages are listed in Table V. The values listed are probably accurate within ± 0.08, and the differences between V_1 for successive isotopes is probably accurate within ± 0.05. The data for Sn^{116} are somewhat less consistent than for the others, so that the error on its V_1 is perhaps 50% larger.

TABLE VI. Determinations of V_1 for Sn^{118}.

Method	Reaction	$A=1.00$	$A=1.12$	$A=1.25$
Average of 30° and 45° ratios	(d,p)	0.47	0.40	0.33
	(d,t)	0.43	0.51	0.59
Peaks in angular distribution	(d,p)	0.57	0.49	0.41
	(d,t)	0.42	0.50	0.57

4. *Determination of V_7 and V_{11}*

The methods used for the s and d states are not applicable to the $g_{7/2}$ and $h_{11/2}$, so that another method must be devised. Furthermore, a reasonably good determination of the $h_{11/2}$ cross section is only available for the two reactions leading to Sn^{117}. Since the $g_{7/2}$ cross sections are also well determined in these cases, we may obtain the experimental ratios

$$\left[\frac{d\sigma}{d\Omega}(d,p)\rightarrow g_{7/2}\right]\Big/\left[\frac{d\sigma}{d\Omega}(d,p)\rightarrow h_{11/2}\right]=e,$$

$$\left[\frac{d\sigma}{d\Omega}(d,t)\rightarrow g_{7/2}\right]\Big/\left[\frac{d\sigma}{d\Omega}(d,t)\rightarrow h_{11/2}\right]=f.$$

From (3)

$$\begin{aligned} e&=[(8P_4/12P_5)(U_7/U_{11})]_{Sn^{116}},\\ f&=[(8T_4/12T_5)(V_7/V_{11})]_{Sn^{116}}. \end{aligned} \tag{11}$$

These can be solved for V_7 and V_{11} with the appropriate equations (4) provided we assume V_7 and V_{11} are the same for Sn^{116} and Sn^{118} (the inaccuracy in this is reduced by a perturbation treatment of the final results), and provided we know the ratios P_4/P_5 and T_4/T_5. As in a previous paper,[10] we assume

$$P_4/P_5=T_4/T_5=2^k. \tag{12}$$

In $Pb(d,t)$ reactions,[10] we found $k\simeq 1.0$; in $Pb(d,p)$ reactions,[10] we found $k\simeq 0.7$. From Butler theory,[9] $k\simeq 1-1.5$, and this is in general agreement with experimental evidence in light elements. We thus might expect $k\sim 1$.

The values of V_7 and V_{11} obtained for Sn^{117} (an average between Sn^{116} and Sn^{118}) for various values of k are shown in Table VIII, and the total number of particles in the two states, $8\ V_7+12\ V_{11}$, are also listed.

TABLE VII. Values of V_1 for other isotopes by various methods assuming $V_1(Sn^{118})=0.50$, $A=1.12$.

Target mass	116	120	122	124
(d,p) σ ratios	0.32	0.54	0.54	0.67
(d,t) σ ratios	0.44	0.65	0.76	0.77
From (10)	0.48	0.62		
Weight av	0.42	0.61	0.69	0.74

TABLE VIII. V_7 and V_{11} as determined from 30° and 45° data as a function of k for Sn^{117}. The last row, the number of particles in these satates, should be 10.2.

	From 30° data				From 45° data		
k	0	0.75	1	2	0	1	2
V_7	0.62	0.82	0.86	0.99	0.84	0.96	1.02
V_{11}	0.13	0.30	0.38	0.86	0.28	0.64	1.36
$8\,V_7+12\,V_{11}$	6.6	10.2	11.4	18.2	10.1	15.4	···

However the latter quantity is directly obtainable by subtracting the number of s and d particles, $2\,V_1+V_3+6\,V_5$, from the total number of particles in the shell, 17. The result from Table V is 10.2; this is in agreement with the results in Table VIII for $k=0.75$ at 30°, and for $k=0$ at 45°; in both cases $V_7\simeq 0.83$ and $V_{11}\simeq 0.29$, so that these values are adopted. Incidentally, the low value for k at 45° is not unexpected, as the $l_n=5$ angular distribution is peaked at that angle while the $l_n=4$ is somewhat beyond its peak (see Fig. 3).

The change of V_7 and V_{11} with mass is difficult to obtain from the data because of the sparsity of cases where these levels are resolved. From the (d,p) data, one surmises that U_7 increases by a factor of 1.3 to 2.0 between Sn^{116} and Sn^{118} [the 45° data is somewhat unreliable as the $g_{7/2}$ angular distribution changes rapidly in that angular region (see Fig. 3)]. These two extremes would give $V_7=0.80-0.76$ and $0.85-0.88$ for Sn^{116} and Sn^{118}, respectively. In Table V, we adopt values midway between these two extremes. The ratio of (d,t) cross sections between Sn^{120} and Sn^{118}, with (3) and (6), indicate that V_7 increases by 5% between Sn^{118} and Sn^{120}, but this result is very sensitive to small experimental errors. For want of some better method, it is assumed in Table V that V_7 increases by 0.03 for each succeeding even isotope. This brings V_7 to 0.95 for Sn^{124}, which can hardly be off by more than ± 0.04 if V_7 for Sn^{118} is correct and the theory is applicable. The over-all uncertainties on V_7 are about ± 0.08.

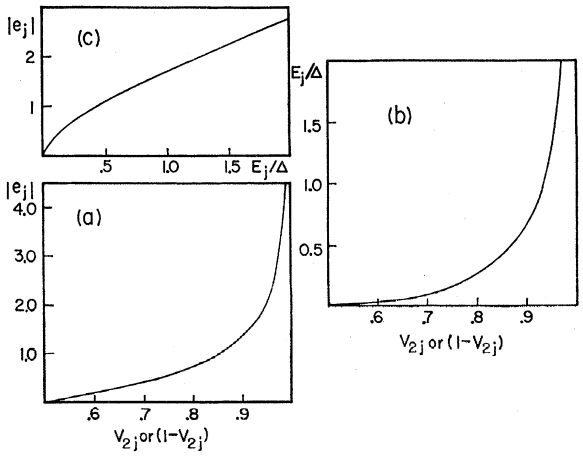

FIG. 9. Plots of theoretical relationships among e_j, the unperturbed single particle energies $\div\Delta$; V_{2j}, the fraction by which the states are filled; and E_j, the observed excitation energy of the single-particle states.

Since V_1, V_3, V_5, and V_7 are now chosen, values of V_{11} can be determined from the condition on the total number of particles. The values so obtained are listed in Table V; the uncertainties are about ± 0.20.

D. Discussion and Conclusions

Pairing theory relates two interesting quantities to the V's: the energies of the single-particle levels, ϵ_j, and the energies of the observed levels, E_j. These relationships are[1]:

$$V_{2j}=\frac{1}{2}\left[1-\frac{e_j}{(1+e_j^2)^{\frac{1}{2}}}\right], \tag{13}$$

where

$$e_j=(\epsilon_j-\lambda)/\Delta, \tag{14}$$

λ is the chemical potential, and Δ is half the energy gap, or about 1.1 Mev; and

$$E_j=\Delta[(1+e_j^2)^{\frac{1}{2}}-(1+e_{j-G}^2)^{\frac{1}{2}}], \tag{15}$$

where e_{j-G} is e_j for the ground state.

Solving (13) for V_{2j} and using (4), we obtain

$$e_j=\frac{U_{2j}-V_{2j}}{2(U_{2j}V_{2j})^{\frac{1}{2}}}. \tag{16}$$

The relationship (16) is plotted in Fig. 9(a). Values of ϵ_j obtained from the V_{2j} of Table V with (4), (14), and (16) are listed in Table IX where they are compared with the values for a Nilsson well.[11] The determinations of the single-particle energies for the various isotopes are reasonably consistent among themselves, although there are rather large fluctuations as might be expected. The agreement with the levels calculated for the Nilsson well is as good as one would expect in view of the uncertainty in the latter.

Another approach is to compare the experimental V's with those derived from the Nilsson-well single-particle energies by Kisslinger and Sorenson.[1] This is shown in Table V. The principal discrepancies which are far outside of experimental error are that the calculated

TABLE IX. Values of $\epsilon_j-\epsilon_{d5/2}$ calculated from V's of Table V and compared with the values for the Nilsson well used by Kisslinger and Sorenson. Energies are in Mev and are calculated using $\Delta=1.1$ Mev. The average of the experimental values is listed in the final column. For explanation of rows labeled "E," see text following Eq. (19).

Mass number		116	118	120	122	124	Nilsson K-S	Exp av
$s_{1/2}$	Exp	0.97	0.83	0.97	0.70	1.24	1.90	0.95
	E	1.65	1.72	1.57	1.40	1.37		
$d_{3/2}$	Exp	1.44	1.22	1.10	0.96	1.41	2.20	1.23
	E	2.45	2.12	1.87	1.85	1.94		
$d_{5/2}$	Exp	0	0	0	0	0	0	0
	E	0	0	0	0			
$g_{7/2}$	Exp	0.08	−0.33	−0.22			0.22	−0.16
	E	0.42	0.37	0.22				
$h_{11/2}$	Exp	1.34	1.22	1.54	1.22	1.75	2.80	1.41
	E	2.75	2.32	2.12	2.05	1.94		

[11] S. G. Nilsson, Kgl. Danske Videnskab. Selskab. Mat.-fys. Medd. **29**, 16 (1955).

TABLE X. Energies of observed levels (E) and values of V obtained from them with (17) or Fig. 10. Energies are in Mev; Δ was taken as 1.1 Mev. Also listed are $\Sigma(2j+1)V_{2j}$ and N, the number of neutrons in the major shell.

Mass No.	115		117		119		121		123		125	
SP state	E	V	E	V	E	V	E	V	E	V	E	V
$s_{1/2}$	0	~0.5	0	~0.5	0	~0.5	0.04	0.64	0.13	0.73	0.19	0.76
$d_{3/2}$	0.49	0.14	0.16	0.26	0.024	0.40	0	~0.5	0	~0.5	0	~0.5
$d_{5/2}$	0.97	0.923	1.10	0.935	1.15	0.935	1.20	0.94	1.30	0.95	1.35	0.95
$gf_{/2}$	0.61	0.89	0.72	0.90	0.76	0.90	(1.1)	(0.94)	(1.2)	(0.94)	(1.2)	(0.94)
$h_{11/2}$			0.32	0.18	0.089	0.31	0.04	0.36	~0	~0.5	~0	~0.5
$\Sigma(2j+1)V_{2j}$		$14.2+12V_{11}$		17.0		19.1		20.7		~22.7		~22.7
N		15.0		17.0		19.0		21.0		23.0		25.0

values of V for $d_{5/2}$ and $g_{7/2}$ are too large, especially for the lighter isotopes. This would require that the single-particle energy differences between $d_{5/2}$, $g_{7/2}$, and $s_{1/2}$, $d_{3/2}$, $h_{11/2}$, be reduced from the Nilsson-well values, in agreement with Table IX.

The relationship between the observed energies and the V's may be obtained by solving (15) and (16), which gives

$$E_j=\Delta\left[\frac{1}{2(U_{2j}V_{2j})^{\frac{1}{2}}}-\frac{1}{2(U_GV_G)^{\frac{1}{2}}}\right],$$

where U_G and V_G are U and V for the single-particle level which is the ground state. Since V_G is always close to 0.5, that value may be substituted to give

$$E_j=\Delta\left[\frac{1}{2(U_{2j}V_{2j})^{\frac{1}{2}}}-1\right]. \tag{17}$$

The approximation in (17) is quite good for V_j anywhere between 0.3 and 0.7, which would apply to all cases of interest here. A plot of (17) is shown in Fig. 9(b).

The observed energies of the single-particle states are listed in Table X along with the V's calculated from them. The energies for the $d_{5/2}$ states are taken as the "center of gravity" of the observed levels weighted with the intensity with which they are excited. In the isotopes where the $g_{7/2}$ level is not observed, it is assumed to be under the large $d_{5/2}$ peak. As a check on the V's obtained from the observed energies, the sum $\sum(2j+1)V_{2j}$ is calculated and listed in Table X. This sum should be equal to the number of neutrons in the major shell, N, which is also listed in Table X. The agreement is excellent in all cases except Sn^{125} where it is within the uncertainty.

The V's from Table X for odd isotopes are interpolated for the even isotopes and listed in Table V. In Table V the agreement with Kisslinger-Sorenson values seems to be somewhat better than the agreement with the experimental values obtained from (d,p) and (d,t) cross sections.

On the other hand, the V's calculated from observed energies lead to difficulties when used to calculate single particle energies. This may be done either by using (16) and (17) in combination, or by noting from (16) that the second term in (15) is close to unity so that to a rather good approximation

$$E_j=\Delta[(1+e_j^2)^{\frac{1}{2}}-1]. \tag{18}$$

This may be solved for e_j to give

$$e_j=\pm\left[\left(\frac{E_j}{\Delta}+1\right)^2-1\right]^{\frac{1}{2}}. \tag{19}$$

This functional relationship is shown in Fig. 9(c). The

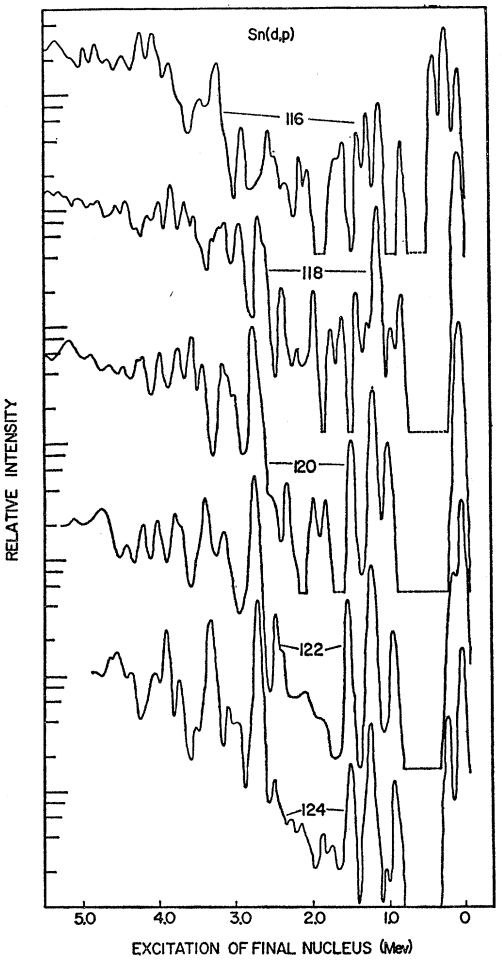

FIG. 10. Energy spectra of protons from (d,p) reactions on even isotopes of tin. Figures denote target mass number. Energy scale is only approximate: best values of energies of various peaks are listed in Table XI. Detection angle is 45°.

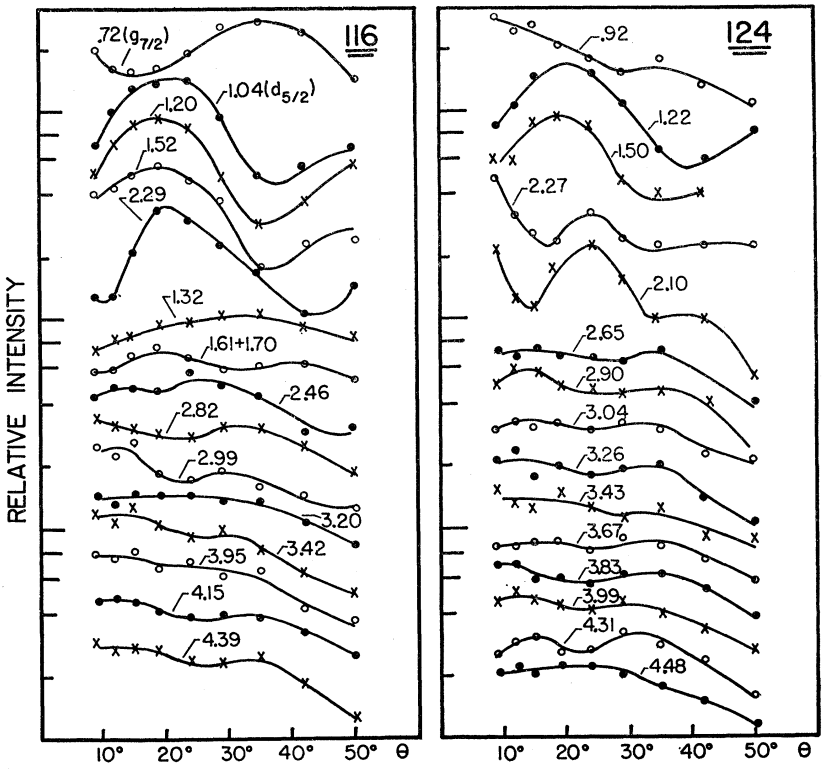

FIG. 11. Angular distributions of protons from (d,p) reactions on Sn^{116} and Sn^{124} leading to various excited states (excitation energy in Mev attached to curves) of final nuclei. Where there are discrepancies between energies shown here and those listed in Table XI, the latter are more accurate.

single particle energies derived from the observed energies of Table X (interpolated between adjacent masses) are listed in Table IX (rows designated "E"). It is seen there that the single particle energies calculated in this way shift rapidly and monotonically as a function of mass number. This behavior is most unexpected; it arises basically from the fact that the observed energy of a level as highly excited as the $d_{5/2}$ is expected from the theory to shift rapidly with mass number, contrary to observation. A similar situation was observed for the $s_{1/2}$ level in the mass 90–100 region. The most probable source of difficulty is that (15) is not completely accurate; there may well be other effects which shift the observed energies appreciably.[11a]

It thus seems most likely that the experimental values of the V's listed in Table V and the single-particle energies derived from these and listed in the last column of Table IX are reasonably accurate. The discrepancies with the Kisslinger-Sorenson values are certainly no larger than expected from their work. On the whole, the agreement between theory and experiment is quite good except for the discrepancy in the observed energies of the $d_{5/2}$ state.

IV. OTHER RESULTS AND CONCLUSIONS

A. Higher Excited States

The energy distributions from (d,p) reactions on the even isotopes of tin are shown in Fig. 10, and the peak energies and intensities at 30° are listed in Table XI. It is clear that the over-all intensity is relatively low at energies above the single-particle levels (~1.5 Mev), until it rises rather suddenly at about 2.6 Mev. The region above 2.6 Mev is characterized by several relatively sharp and very strongly excited levels.

This situation is entirely in line with expectations if

[11a] *Note added in proof.* R. Sorenson (private communication) has fit the observed energies without altering the V's by including the P_2-force in the calculation.

TABLE XI. Energies of levels observed in odd Sn isotopes with (d,p) reactions on even isotopes and their relative cross sections for excitation at 29°.

Sn^{117}		Sn^{119}		Sn^{121}		Sn^{123}		Sn^{125}	
E	I(29°)	E	I(29°)	E	I(29°)	E	I(29°)	E	I(29°)
0	200	0		0	300	0	180	0	140
0.16	190	0.77				0.12	100	0.19	115
0.31	72	0.92				0.91	13	0.91	11
0.72	16	1.08	58	0.92	29	1.20	75	1.23	43
1.02	60	1.35	13	1.11	72	1.50	25	1.51	24
1.18	28	1.57	5	1.39	25	1.79	7	1.64	
1.32	27	1.72	7	1.69	10	1.94	17	1.90	
1.52	30	1.95	7	1.88	13	2.30	15	2.10	7
1.61	10	2.09	10	2.21	16	2.42	20	2.27	8
1.70	10	2.35	16	2.41	22	2.69	400	2.46	14
2.06	40	2.63	60	2.53	···	3.10	150	2.65	520
2.25	18	2.88	60	2.62	250	3.32	300	2.90	54
2.41	43	3.07	50	2.87	73	3.70	200	3.04	78
2.47	35	3.16	50	3.01	140	3.93	180	3.26	380
2.60	~11	3.27	40	3.29	115	4.15	160	3.43	44
2.77	55	3.50	65	3.44	190	4.32	100	3.67	125
2.93	29	3.59	90	3.61	180	4.62	180	3.83	310
3.14	160	3.78	120	3.84	140	4.77	150	3.99	160
3.25	60	3.95	70	4.07	140			4.31	140
3.35	54	4.00	65	4.15	130				
3.52	46	4.36	90	4.35	120				
3.67	88	4.48	80	4.52	110				
3.75	95								
3.87	148								
4.06	185								
4.30	117								
4.48	135								
4.61	165								
4.72	150								

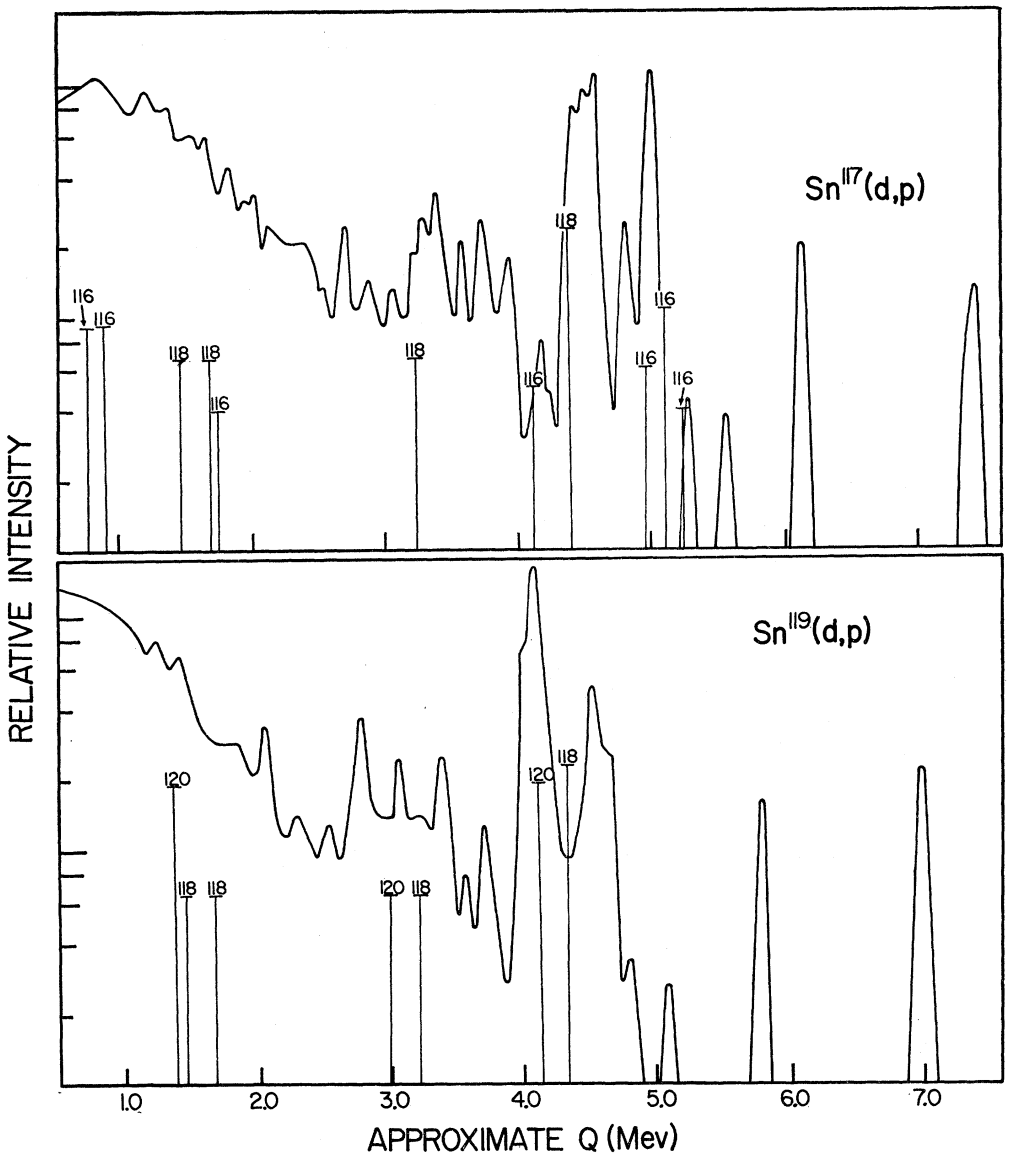

FIG. 12. Energy distributions of protons from (d,p) reactions on Sn^{117} and Sn^{119} at 45°. Vertical lines show positions of principal peaks from neighboring even isotopes (mass numbers stated); their height indicates the relative intensity.

these latter are the single-particle states of the next major shell which fills between neutron numbers 83 and 126. The energies are approximately those expected from the fact that the p-wave neutron giant resonance goes through zero neutron energy [i.e., $Q(d,p)=-2.2$ Mev] at $A=90$; the estimates of Cohen and Price[12] place the p states at an excitation energy of about 3 Mev in this mass region. The cross sections for the large peaks are approximately those expected for these single-particle states if we adopt (6) and (12) with the values of A and k used in the previous section. The regularities in energies and cross sections among the various isotopes is further evidence on this point; several of the peaks can be traced from isotope to isotope with very little imagination, although they become somewhat split up in Sn^{116} and Sn^{118}.

A considerable effort was expended in trying to further identify these levels with angular distribution studies. Some of the results are shown in Fig. 11 along with angular distributions for some of the low-lying levels for comparison. It is very apparent that all groups with excitation energy above 2.5 Mev have essentially the same angular distributions, and that these are almost completely lacking in sharp identifying features such as those possessed by the low-lying single-particle levels (see Figs. 3, 4, and 5).

It is very difficult to believe that this loss of feature in the angular distributions is only due to the fact that

[12] B. L. Cohen and R. E. Price, Nuclear Phys. **17**, 129 (1960).

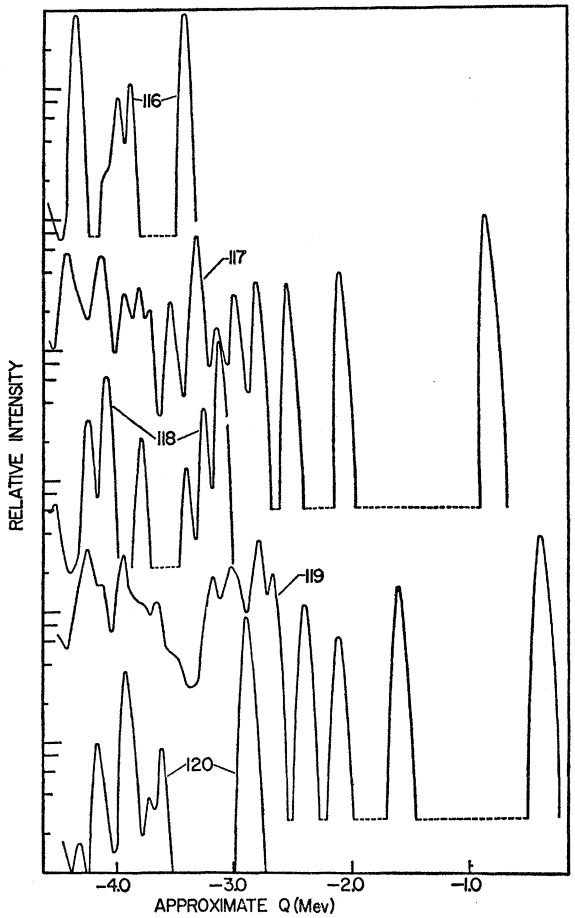

FIG. 13. Energy distributions of tritons from (d,t) reactions on Sn^{117}, Sn^{119}, and neighboring even isotopes (mass number shown). Detection angle is 45°.

their energy is low. The distributions shown in Figs. 4 and 5 show very little sensitivity to energy over a wide range, and indeed the lowest energy case from Fig. 5 has a proton energy corresponding to a 2.9-Mev excited state in Sn^{116}. The authors firmly believe that there is some deeper meaning to this anomaly. A similar situation was found in the Pb isotopes[9,10] although the low energy may have been a factor there.

B. Low-Lying Non-Single-Particle States

In the region below 2.6-Mev excitation energy, there are several states shown in Fig. 10 and listed in Table XI which were not classed as single-particle states. The angular distributions from some of these are shown in Fig. 11.

Basically, the explanation for these states, and for the fact that the $d_{5/2}$ states have more than one component, is that the model we are using is only a *model*, and is not a perfect representation of nuclei. States may occur due to other types of excitation, some of which perhaps have never been considered by nuclear theorists. If these states occur close to single-particle states and have the same spin and parity, they will mix with them; this is very probably the case with the $d_{5/2}$ levels. In other cases, the distance from the single-particle levels would seem to preclude mixing. For example, the angular distributions from the 2.10- and 2.27-Mev states from $Sn^{124}(d,p)$ bear strong similarities to $l_n=0$ distributions except that the minima and maxima occur at smaller angles (~15° vs 18° for the minima, and 24° vs 32° for the maxima). This could be explained by the low Q values, although the trend in Fig. 4 is in the opposite direction.

Perhaps the most interesting of the low-lying non-single-particle states is that occurring at about 0.9 Mev in essentially all the odd isotopes of Sn. The regularity of its occurrence, its low excitation energy, and its relatively large cross section are very suggestive. However, the evidence against its being a single-particle state is quite convincing: the angular distributions are completely untypical (see Fig. 11), and the variation of the cross section with mass is quite different. The evidence for the latter is shown in Tables I and II: The cross sections are much larger in Sn^{120} and Sn^{122} than in the other isotopes, whereas cross sections for single particle states vary monotonically and generally slowly with mass number. A state at 0.90 Mev in Sn^{119} has been reported from Coulomb excitation work,[13] so that this level might perhaps be strictly a collective one. However, it is still difficult to understand why it would not have a typical angular distribution pattern. The excitation of known collective levels in even-even nuclei by (d,p) and (d,t) reactions has been studied previously,[14] and the results are not inconsistent with theory.[1,3]

While there might be considerable interest in the low-lying, non-single-particle states, it should always be kept in mind that their excitation cross sections are low, so that no matter how they are interpreted, they would not appreciably affect the experimental determinations of the V's in Table V. It would seem most reasonable to attempt to study them by some other type of nuclear reaction in which they are strongly excited.

C. Reactions on Odd Isotopes of Tin; the Pairing Energy

The (d,p) and (d,t) reactions on the odd isotopes of tin lead to even-even final nuclei, so that they have large Q values. The low-lying states excited in these reactions have been studied in a previous paper,[14] and the excitation of ground states has been discussed in Sec. III-C-3 above.

One would expect the binding energies of a $d_{5/2}$, $d_{3/2}$, $g_{7/2}$, and $h_{11/2}$ neutron to an odd nucleus to be roughly the same as to an even nucleus, since they cannot form

[13] D. G. Alkhazov, D. S. Andrev, K. I. Erokhina, and I. K. Lemberg, J. Exptl. Theoret. Phys. (U.S.S.R.) **33**, 1346 (1957).
[14] B. L. Cohen and R. E. Price, Phys. Rev. **118**, 1582 (1960).

a pair with the odd nucleon ($s_{1/2}$ in both Sn^{117} and Sn^{119}). One therefore expects strongly excited levels at the same Q value in odd and even isotopes. There is good evidence for this in Figs. 12 and 13 where spectra from (d,p) and (d,t) reactions, respectively, in the odd targets are compared with those in neighboring even targets. The correspondence between groups of levels is readily apparent. The levels excited are principally the $d_{3/2}$ and $d_{5/2}$ forming two quasi-particle states of spin 1^+ and 2^+, and 2^+ and 3^+, respectively; the two groups are readily distinguishable by their energy difference. 0^+ levels in this region should not be excited by (d,p) and (d,t) reactions and this is in agreement with experiment.[14]

Since the $d_{3/2}$ and $s_{1/2}$ levels are very close in the odd nuclei, the separation between the ground state and the $(d_{3/2}s_{1/2})$ states in even nuclei gives an accurate measure of the pairing energy. This is 2.2 Mev in Sn^{116}, 2.6 Mev in Sn^{118}, and 2.9 Mev in Sn^{120} [note that determination for Sn^{118} from $Sn^{117}(d,p)$ and $Sn^{119}(d,t)$ both give 2.6 Mev]. This variation of pairing energy with mass number seems difficult to understand. The above results do not take into account the fact that in Sn^{116}, the $s_{1/2}$ is $\sim$0.2–0.3 Mev below the $d_{3/2}$ without pairing (this is an average between Sn^{115} and Sn^{117}); this would further enlarge the difference in pairing energies between Sn^{116} and the others.

D. Dependence of Neutron Single-Particle States on Coupling with Protons

It is a long-standing assumption of nuclear shell model theory[15] that the neutron single-particle states are essentially unaffected by the protons so long as the proton number is even. Since (d,p) and (d,t) reactions strongly excite neutron single-particle states, one would therefore expect strong similarities between spectra from these reactions when the target nuclei have the same number of neutrons but a different (even) number of protons. Two good cases for testing this are Cd^{114} and Sn^{116}, both of which have 66 neutrons; and Cd^{116} and Sn^{118}, both of which have 68 neutrons.

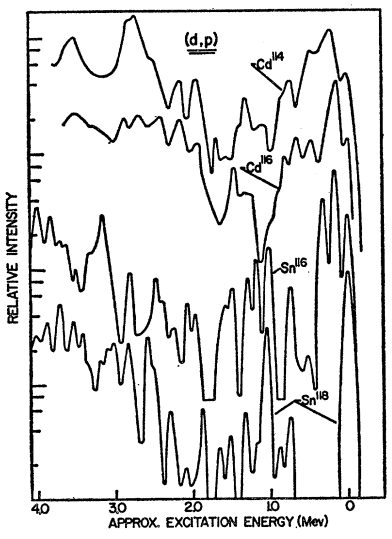

FIG. 14. Energy distribution of protons from (d,p) reactions on isotonic pairs Cd^{114}-Sn^{116} and Cd^{116}-Sn^{118}. Detection angle is 45°.

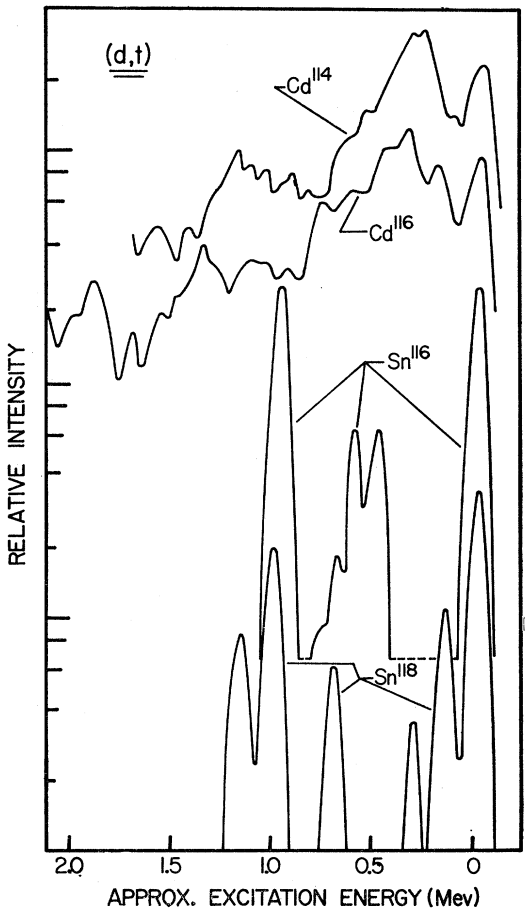

FIG. 15. Energy distribution of tritons from (d,t) reactions on isotonic pairs Cd^{114}-Sn^{116} and Cd^{116}-Sn^{118}. Detection angle is 45°.

The energy spectra from (d,p) and (d,t) reactions on these isotopes are compared in Figs. 14 and 15. It is abundantly clear that the presence or absence of the two protons has a very strong effect on the neutron single-particle states. The energies of the single-particle levels in the Cd isotopes are all within $\sim$0.6 Mev of the ground state, whereas in the Sn isotopes, they extend to about 1.2 Mev; the $s_{1/2}$ state (the ground state in all cases) is relatively weakly excited by (d,p) reactions in Cd whereas it is quite strong in Sn; the multiplicity of levels is far larger in Cd; the intensity rise in (d,p) reactions due to states in the next major shell (discussed in Sec IV-A above) is at a lower energy for Cd ($\sim$2.0 Mev vs 2.6 Mev for Sn). In general, there is a much stronger similarity between the two Cd isotopes and between the two Sn isotopes than between Cd^{114}

[15] M. Mayer and J. H. D. Jensen, *Elementary Theory of Nuclear Shell Structure* (John Wiley & Sons, Inc., New York, 1955).

and Sn^{116}, and between Cd^{116} and Sn^{118}. Thus, the addition of two protons has a much more far reaching effect on the neutron single-particle states than the addition of two neutrons. This is the exact opposite from expectations from shell-model theory; on the other hand, it is not unexpected from pairing theory. It should be pointed out, of course, that the above conclusion is probably not a general one; the two protons added between Cd and Sn are those which close the major shell, which is something of a special case.

E. Ground-State Reaction Q Values

No systematic effort was made in these experiments to obtain accurate Q values, and in fact, there are some little understood experimental difficulties in doing this without apparatus. However, better determinations than any available in the literature are readily available from this work.

In (d,t) reactions, the deuteron elastic scattering peak always appears on the photographic plate at an apparent triton energy of $\frac{2}{3}$ the deuteron energy; thus, the Q value can be determined with little error due to the uncertainty in the energy of the incident deuteron beam. Once (d,t) Q values are determined, the (d,p) Q values for the four cases where initial and final nuclei are reversed in the two reactions may be calculated. Q values for other (d,p) reactions can be determined from positions of isotopic impurity peaks on plates where (d,p) Q values are known. In one case, a check was available from carbon and oxygen impurity peaks.

TABLE XII. Q values for (d,p) and (d,t) reactions on tin isotopes.

Isotope mass	$Q(d,p)$ (Mev) This work	Previous	$-Q(d,t)$ (Mev) This work	Previous
116	4.85	4.97	3.40	3.11
117	7.20	6.98	0.80	0.94
118	4.40	4.56	3.15	2.94
119	7.05	6.73	0.35	0.53
120	4.15	3.93	3.00	2.70
122	3.85	3.70	2.70	2.46
124	3.45	3.51	2.30	2.16

The Q values obtained are listed in Table XII where they are compared with best previously known data.[16] The error in these determinations is very probably less than 0.1 Mev, whereas the uncertainties in previous values[16] were about 0.3 Mev.

[16] V. J. Ashby and H. C. Catron, University of California Radiation Laboratory Report UCRL-5419 (unpublished).