Modes of Atomic Motions in Liquid Helium by Inelastic Scattering of Neutrons

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The wavelength distribution of 4.03_9 A neutrons scattered inelastically by specimens of liquid helium has been measured at 30 angles of scattering in the range 10° to 140°. At each angle of scattering there is a discrete peak which shows little broadening, and little if any excess scattering outside this peak. The changes in energy and momentum corresponding to the change in wavelength of the scattered neutrons have been calculated for each angle of scattering. The dispersion curve in the liquid is similar in form to that predicted theoretically but there are differences in detail. For momentum changes ($\hbar Q$) in the range $0.26\ \mathrm{A}^{-1} \leq Q \leq 0.6\ \mathrm{A}^{-1}$, the measurements fall on a straight line which has a slope given by the velocity of sound. The maximum and the minimum of the curves occur at 1.10 A^{-1} and 13.7°K and at 1.91 A^{-1} and 8.65°K, respectively. Beyond the minimum the curve starts to rise with a slope equal to or less than the phonon branch and then falls below this, suggesting the possible existence of a second maximum. The relative partial differential cross section for the production of a single phonon excitation is low at low momenta, has a maximum in the region of 2.0 A^{-1} and then decreases rapidly to low values in the region of 2.68 A^{-1}. The position of the maximum may be compared with 2.05 A^{-1}, the maximum in the total differential cross-section curve and 1.91 A^{-1}, the minimum of the dispersion curve, respectively. The widths and mean energy change of neutrons scattered through 80° have been measured at 13 temperatures in the range 1.78°K to 4.21°K. Scattering at this angle corresponds to the production of excitations at the minimum of the dispersion curve. The mean energy change decreases rapidly from a value of 8.6_5°K at 1.1°K to 5.6°K at the λ temperature where there is a marked change in slope and only a much slower decrease above the λ temperature to 4.9°K at 4.2°K. The widths of the spectra increase rapidly from a value of <1°K at 1.1°K to 11°K at the λ temperature where there is also a marked change in slope and only a much slower increase to about 15°K at 4.2°K. The measured widths are compared with theoretically calculated widths.

INTRODUCTION

INFORMATION concerning the atomic arrangements and modes of atomic motions in liquids and solids may be determined from measurements of the scattering of neutrons or x rays. Such determinations are particularly important in the case of liquid helium because at low temperatures the liquid is nearly in its ground state and the details of the λ transition have not been explained. The measured small-angle x-ray[1] and neutron scattering[2] is almost in agreement with that predicted theoretically. Long-wavelength neutrons[2,3] have been used in an unsuccessful search for extra scattering[4] expected to result from the effect of the Bose-Einstein condensation in the He-II region of the transition. Both neutrons[5–7] and x rays[8,9] have been used to determine the atomic distribution in the liquid and have shown[5,6,9] that there is a change in the distribution of scattered neutrons associated with the λ transition, the main diffraction peak being lower below than above it. The interpretation of the neutron diffraction measurements[5,7] has shown that the change in the atomic distribution due to density change caused by temperature alone is different than that caused by pressure above the normal vapor pressure curve. Measurements of the change in wavelength of neutrons inelastically scattered from liquid helium[10–12] with small momentum changes interpreted on the basis of the theory of Cohen and Feynman[13] have confirmed the existence of a dispersion curve in the liquid of a form first proposed by Landau[14] and more recently by Feynman[15] and Brueckner and Sawada.[16] Measurements at large momentum change[5] have shown that the average energy change is like that from free helium atoms. For scattering from liquid helium at low temperatures the broadening of the spectrum of neutrons scattered with the production of a single excitation is small, indicating a long mean free path for the excitations.[10–12] With increasing temperature the spectrum rapidly broadens[10,12] and, above the λ point, exhibits a behavior similar to that expected for a gas.[12]

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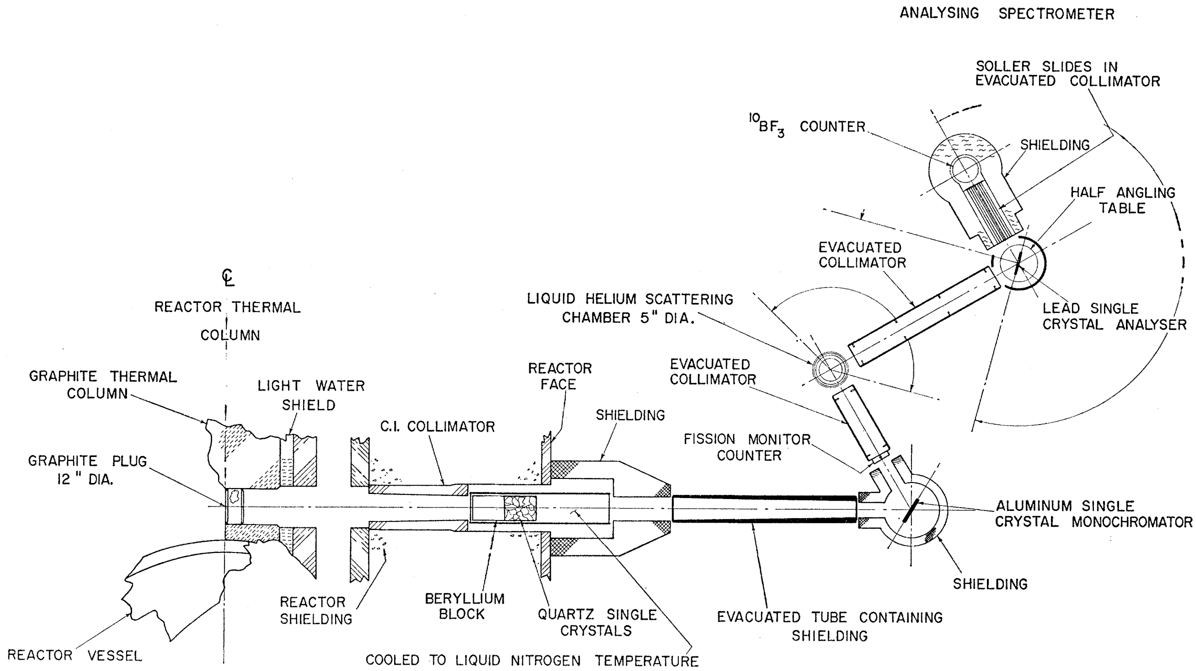

FIG. 1. The Chalk River liquid helium double-crystal neutron spectrometer (not to scale). Neutrons from the 12 in. diameter graphite plug at the center line of the N.R.U. thermal column, after transmission through beryllium and quartz (at liquid nitrogen temperature) which act as a band pass filter, are monochromated by reflection from aluminum single crystals to produce a 4-in. by 4-in. monochromated beam. The 4-in. by 4-in. beam of neutrons scattered from the $5\frac{1}{8}$-in. diameter specimen of liquid helium is analyzed by the second (analyzing) spectrometer. The shielding around the lead single crystals at the analyzing table of the second spectrometer is used to reduce the background.

This paper reports a remeasurement and an extension of the existing measurements of the wavelength change of neutrons inelastically scattered from liquid helium to higher precision and statistical accuracy. For these measurements, a new high-resolution double-crystal spectrometer at the Chalk River N.R.U. reactor was used. Analysis of the dispersion curve shows that at low momenta, the measurements are consistent with the theoretically predicted phonon branch. At large momenta corresponding to excitations beyond the minimum, the slope of the curve tends to approach that given by the velocity of sound and then falls below it, suggesting that the curve might exhibit a possible second maximum. The relative partial differential cross sections for the production of a single phonon as a function of momentum change are given. Measurements of the temperature variation of the mean energy change and of the full widths at half maximum of the neutron spectrum at an angle of scattering which corresponds closely to the production of excitations at the minimum of the dispersion curve are reported. The measured widths have been compared with theoretical widths at temperatures above and below the λ temperature.

APPARATUS

The wavelength distribution of scattered neutrons was measured using a new high-resolution double-crystal spectrometer at the Chalk River N.R.U. reactor. This instrument is shown diagrammatically in Fig. 1. Neutrons from a 12-in. diameter graphite plug (used as the neutron source) at the center line of the thermal column of the N.R.U. reactor were filtered[17] by a 10 in. long beryllium block and 10 in. of quartz single crystals at liquid nitrogen temperatures. The transmitted beam was monochromated by diffraction through 120° from the [111] planes of aluminum single crystals to give a 4-in. by 4-in. beam of monochromatic neutrons incident upon the $5\frac{1}{8}$-in. diameter specimens of liquid helium at the axis of the first spectrometer. The beam intensity was monitored using a fission counter located between the first monochromator and the specimens of liquid helium. The wavelength distribution of neutrons in the 4-in. by 4-in. scattered beam was determined using the [111] planes of lead single crystals mounted on the half-angle analyzing table of the second spectrometer. The neutrons were detected after collimation by Soller slits, using a $6\frac{3}{4}$ in. diameter counter filled with $^{10}BF_3$ to a pressure of 21.6 cm Hg. The calculated efficiency of the counter for 4 A neutrons was 55%. The counter was well shielded and

[17] The polycrystalline beryllium acted as a filter which readily transmitted neutrons whose wavelengths were greater than 3.96 A but highly attenuated those whose wavelengths were less than 3.96 A. [H. L. Anderson, E. Fermi, and L. Marshall, Phys. Rev. **70**, 815 (1946)]. The quartz single crystals preferentially scattered higher energy neutrons from the beam thus decreasing the fast neutron contamination and hence the background. [B. N. Brockhouse, Rev. Sci. Instr. **30**, 136 (1959)].

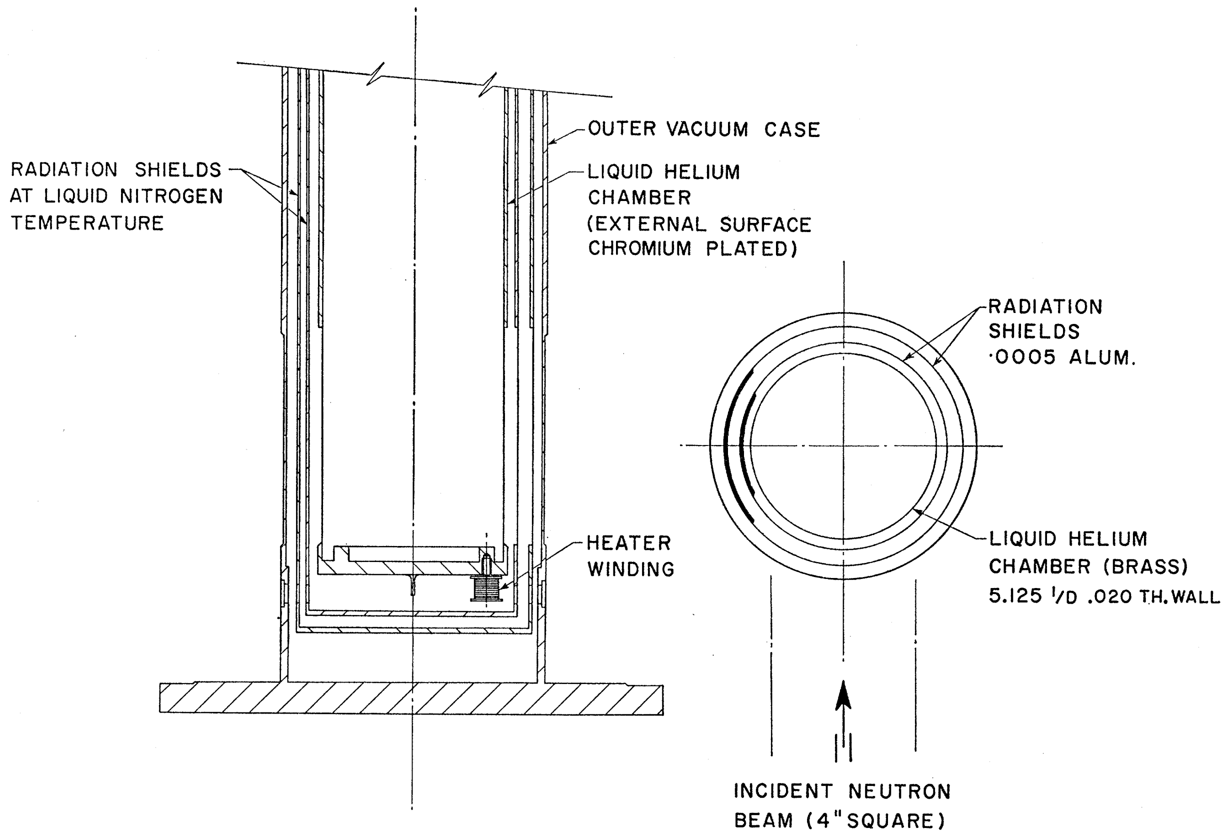

FIG. 2. The specimen holder of the liquid helium cryostat. The $5\frac{1}{8}$ in. diameter scattering chamber has a thin-walled section for a height of 6 in. at the level of the neutron beam. Radiation shields at liquid nitrogen temperature are used as thermal protection for the liquid helium.

mounted on the arm of the analyzing spectrometer which was automatically rotatable in predetermined angular steps.

The specimens of liquid helium were contained in an all-metal cryostat[5] mounted at the axis of the first spectrometer. The scattering chamber shown in Fig. 2 was $5\frac{1}{8}$ in. in diameter and had a thin-walled section for a height of 6 in. at the level of the neutron beam. For the later measurements, 12 horizontal cadmium-coated plates having $\frac{1}{2}$ in. vertical separation were mounted in the liquid helium at the level of the neutron beam to reduce the level of the multiple scattering.[17a] The height of the liquid helium chamber was 20 in. With this arrangement it was possible to make measurements for more than 24 hours and 72 hours with the liquid at 1.1°K and at 4.2°K, respectively, with a single filling of the cryostat. The temperature was recorded, controlled, and measured by means of the liquid helium vapor pressure as previously described.[5] For the measurements at the lowest liquid temperature, an oil manometer was used for the determination of the vapor pressure. The vapor pressure was converted to temperature using the 1949 Cambridge temperature scale.[18] To the accuracy with which the temperature is quoted, there is no significant difference between this and the latter more accurate temperature scales.[19–21]

MEASUREMENTS

Before commencing the measurements the first (monochromating) and second (analyzing) spectrometers were optically aligned and then calibrated with neutrons. The setting corresponding to the zero angle of the second spectrometer was determined by setting the first spectrometer to zero angle and measuring the scattering from cylindrical specimens of powdered lead and aluminum mounted at the axis of the second spectrometer. The zero setting was taken as the mid-position between the corresponding Bragg lines in the parallel and antiparallel rocking positions. The wavelength of the neutrons was deduced from the angle of scattering and the known lattice constants $a_0=4.950$ A

[17a] This suggestion came from G. H. Vineyard (private communication via B. N. Brockhouse).

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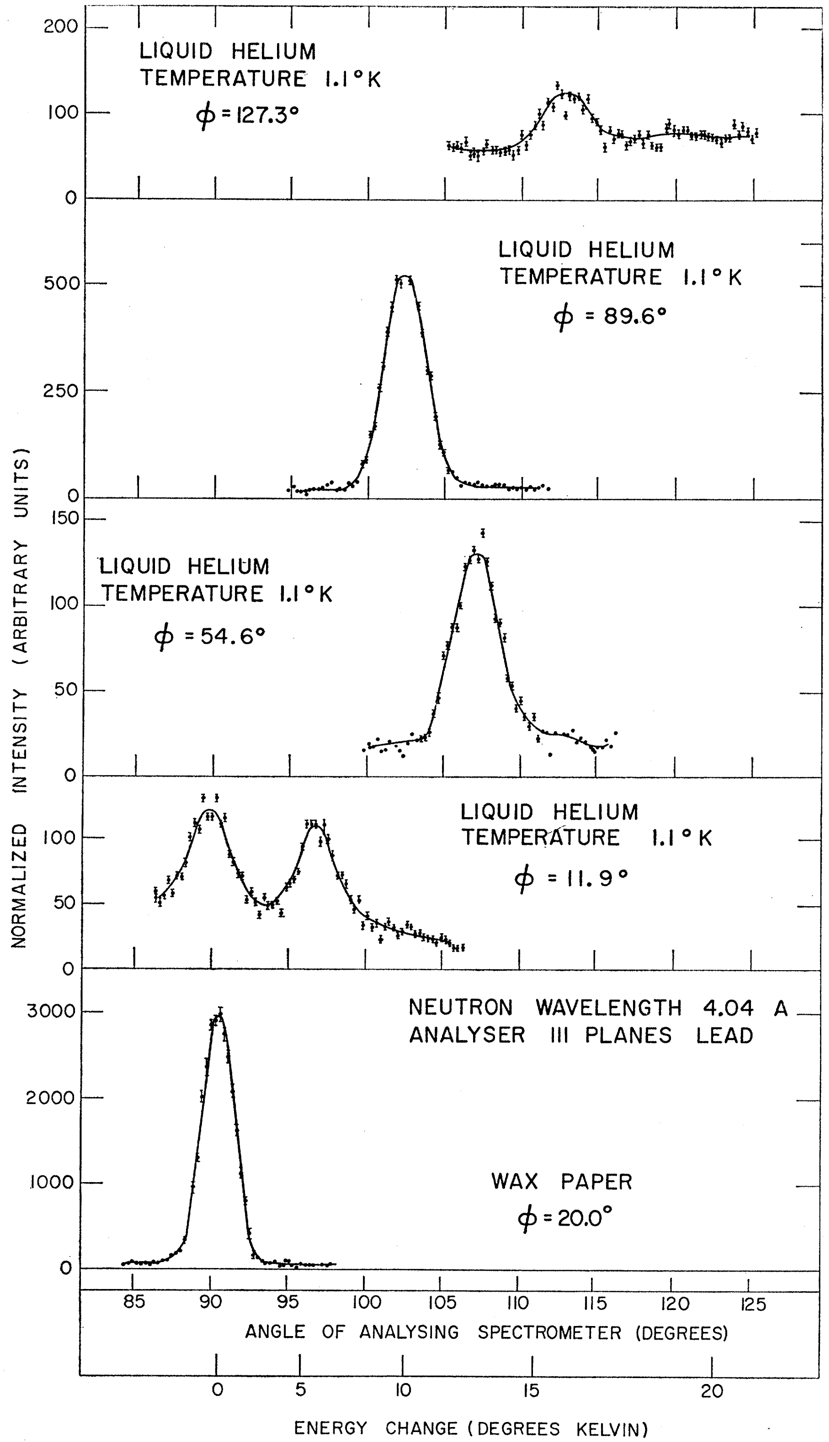

FIG. 3. The wavelength distribution of neutrons scattered from liquid helium at 1.1°K and from wax paper plotted as a function of the analyzer setting of the analyzing spectrometer and the energy change of the scattered neutrons. The scattering from wax paper, which is mainly incoherent and elastic, gives the wavelength of the incident neutrons and the instrument resolution at this wavelength. The second elastic peak observed at an angle of scattering of 11.9° is due to elastically scattered neutrons from the walls of the container. The liquid helium curves have been arbitrarily normalized and give the relative intensity for scattering at each angle.

and 4.050 A for lead and aluminum, respectively. The wavelength determined thus was 4.039±0.005 A. This was checked by analyzing[22] the wavelength distribution of neutrons scattered from a 4-in. diameter cylindrical specimen of wax paper mounted at the axis of the first spectrometer. The neutrons scattered from such a specimen are mainly incoherent and elastic. The position of the maximum confirmed the wavelength determination given above. The width of the curve was taken as the resolution of the instrument at the wavelength of the incident neutrons. The full width at half

[22] The [111] planes of lead single crystals mounted on the half-angling table of the second spectrometer were used in the analyzer.

TABLE I. The change in energy and momentum of 4.039 A neutrons scattered from liquid helium at 1.12°K under its normal vapor pressure.

Scattering angle ϕ (degrees)	Helium temperature (degrees)	Momentum change $p/\hbar$ (A^{-1})	Energy change $\Delta E/k$ (°K)
10.0	1.12	0.27 ± 0.01	$5.3\ \pm0.2$
11.9	1.12	0.32 ± 0.01	$5.7\ \pm0.2$
15.0	1.12	0.40 ± 0.01	$7.4\ \pm0.2$
17.0	1.12	0.46 ± 0.01	$8.5_2\pm0.1$
22.5	1.12	0.59 ± 0.01	$10.5_0\pm0.1$
30.0	1.12	0.78 ± 0.01	$12.5_7\pm0.1$
38.0	1.12	0.97 ± 0.01	$13.4_2\pm0.1$
45.0	1.12	1.13 ± 0.01	$13.6_9\pm0.1$
48.1	1.12	1.20 ± 0.01	$13.7_3\pm0.1$
54.6	1.12	1.35 ± 0.01	$13.0_3\pm0.1$
59.9	1.12	1.47 ± 0.01	$12.3_1\pm0.1$
65.1	1.12	1.59 ± 0.01	$11.1_7\pm0.1$
67.9	1.12	1.65 ± 0.01	$10.5_0+0.1$
71.0	1.12	1.73 ± 0.01	$9.9_4\pm0.1$
74.9	1.12	1.81 ± 0.01	$8.9_7\pm0.1$
76.9	1.12	1.86 ± 0.01	$8.8_0\pm0.1$
78.9	1.12	1.89 ± 0.01	$8.6_9\pm0.1$
80.9	1.12	1.94 ± 0.01	$8.6_6\pm0.1$
82.7	1.12	1.97 ± 0.01	$8.8_0\pm0.1$
87.8	1.12	2.06 ± 0.01	$9.5_5\pm0.1$
89.6	1.12	2.09 ± 0.01	$10.1_2\pm0.1$
94.8	1.12	2.17 ± 0.01	$11.1_7\pm0.1$
98.3	1.12	2.23 ± 0.01	$11.8_6\pm0.1$
102.0	1.12	2.27 ± 0.01	$13.2_8\pm0.1$
106.4	1.12	2.34 ± 0.01	$13.7_7\pm0.1$
109.8	1.12	2.37 ± 0.01	$14.6_6\pm0.1$
113.8	1.12	2.42 ± 0.01	$15.3_7\pm0.1$
119.8	1.12	2.49 ± 0.01	$15.9\ \pm0.2$
127.3	1.12	2.57 ± 0.01	$16.7\ \pm0.2$
140.7	1.12	2.68 ± 0.01	$17.1\ \pm0.2$

maximum of this curve corresponds to an energy spread of 2°K at the wavelength of the incident neutrons. The variation of the resolution as a function of the wavelength[23] change of the scattered neutrons has not been measured because of the difficulty of obtaining substances (other than liquid helium itself) which scatter with a small known discrete energy change. The calibration of the angular scale, and the zero of the first spectrometer were checked by measuring the scattering from cylindrical specimens of powdered lead and aluminum mounted at the axis of the first spectrometer and with the second spectrometer set in the zero-angle position. The zero was determined from the known lattice constants for lead and aluminum, the measured wavelength, and the angular positions of the diffracted peaks. The settings for the zero angle determined from the lead and aluminum were consistent within experimental error.

In the first series of measurements, the change in wavelength of 4.039 ± 0.005 A neutrons scattered inelastically from liquid helium at 1.12°K has been measured at 30 angles of scattering in the range 10° to 140°. The analyzing spectrometer recorded the total number of counts for a preset number of monitor counts at each of about 60 equally spaced points separated by 0.28° in the angular range of the neutron group. For the early measurements, the background was also measured at each angular setting by recording the scattering with 0.03 in. of cadmium in the incident beam between the monochromator and the liquid helium specimen. The background thus measured (approximately 4 counts per minute) was due to fast neutrons in the monochromated beam and the general background level in the reactor building. This background was subtracted point by point from the measured curves. In the later measurements the addition of shielding (as shown in Fig. 1) around the analyzing crystal reduced this background sufficiently (to about 1 count per minute) so that it was not necessary to record the background for this series of measurements.

Figure 3 shows the measured distribution of neutrons scattered from liquid helium through 11.9°, 54.6°, 89.6°, and 127.3° and from wax paper at 20.0°. These are plotted as a function of the angular setting of the analyzing spectrometer and the calculated corresponding energy change. The scattering from wax paper is mainly incoherent and elastic. The angle of this maximum is taken as corresponding to the wavelength of the incident neutrons. The form of this curve gives the resolution of the instrument at the wavelength of the incident neutrons. The liquid helium curves are all similar to that for wax paper. They differ in detail, being shifted to larger angles and showing a variation of peak intensity with the angle of scattering, but there is only small change in their form. The measured curve for liquid helium at $\phi=11.9°$ shows two peaks. The peak at the smaller angle is due to elastically scattered background neutrons from the walls of the container. The fact that its maximum does not correspond exactly to the maximum of the scattering from wax paper is presumably owing to a possible asymmetry of the background scattering. The widths of the peaks of the liquid helium curves are similar within experimental error at all angles of scattering. The measured pattern at $\phi=127.3°$ shows little if any increase in the scattering at angles larger than the position of the maximum of the curve.[11]

The angular position of the maximum of each curve has been taken as representing the angle corresponding to the mean wavelength of the scattered neutrons which has been deduced from the measured angle of the analyzing spectrometer and the lattice parameter $a_0=4.950$ A for lead. The corresponding changes in the energy and momentum were computed at each angle using the conservation laws

$$\Delta E=h^2(\lambda_i^{-2}-\lambda_f^{-2})/2m,$$
$$Q^2=(2\pi)^2(\lambda_i^{-2}+\lambda_f^{-2}-2\lambda_i^{-1}\lambda_f^{-1}\cos\phi),$$

where λ_i and λ_f are the incident and scattered neutron wavelengths, ΔE and $\hbar Q$ are the corresponding change in energy and momentum of the scattered neutrons,

[23] Analysis has shown that the wavelength variation of the resolution of the instrument is small for the range over which these measurements were made.

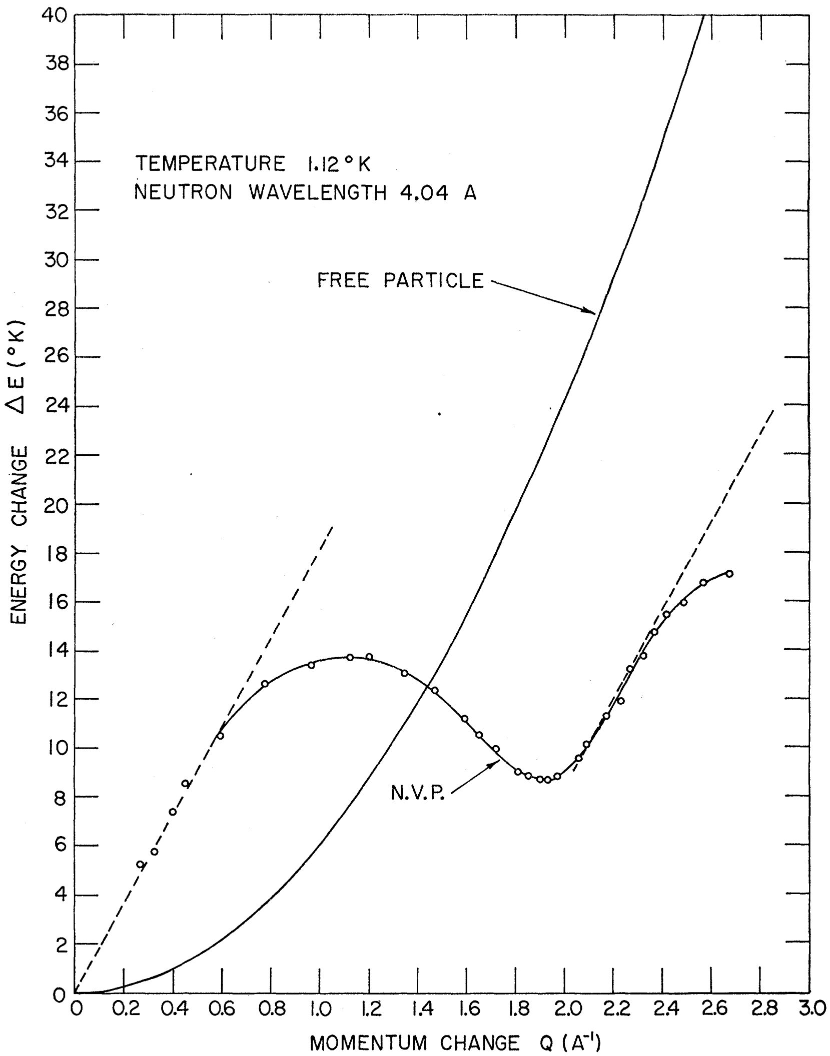

FIG. 4. The dispersion curve for liquid helium at 1.12°K under its normal vapor pressure. The parabolic curve rising from the origin represents the theoretically calculated dispersion curve for free helium atoms at absolute zero. The open circles correspond to the energy and momentum of the measured excitations. A smooth curve has been drawn through the points. The broken curve rising linearly from the origin is the theoretical phonon branch calculated from a velocity of sound of 237 meters $\sec^{-1}$. The dotted curve drawn through the point at 2.27 A^{-1} has been drawn with a slope equal to the velocity of sound.

h is Planck's constant, ϕ is the angle of scattering and m is the neutron mass. The calculated mean changes in energy and momentum of the scattered neutrons are tabulated in Table I for each angle of scattering. The units of energy are temperature where kT is the corresponding energy change. The units of momentum are A^{-1}.

Cohen and Feynman[13] have pointed out that these changes in energy and momentum are equal to the energy and momentum of an excitation produced in the liquid. Since the broadening of the spectra is small at all angles of scattering, each of these changes has been taken as corresponding to the production of a single phonon in the liquid. These changes in energy and momentum listed in Table I are shown plotted in Fig. 4. Each point is shown by an open circle, the size of which nearly represents the upper limit of the experimental error in its determination. The smooth curve through the points has been drawn as a guide to the eye and gives the dispersion curve in the liquid. The parabolic curve which rises from the origin represents the dispersion curve for excitations of free helium atoms at rest. The dotted curve rising linearly from the origin has the slope of the velocity of sound[24] of 237 meters $\sec^{-1}$. Beyond about 0.6 A^{-1}, the curve falls below the phonon branch, has a maximum at 1.10 A^{-1} and 13.7°K then intersects the free particle curve at 1.49 A^{-1} and 12.5°K. The curve exhibits a minimum at 1.91 A^{-1} and 8.65°K beyond which it first starts to rise with a slope slightly less than or equal to the slope of the phonon branch which is shown by the dotted curve through the point at 2.27 A^{-1}. At momenta greater than about 2.4 A^{-1}, the second derivative of the curve becomes negative and the form of the curve suggests the possible existence of a second maximum.

Landau[14] has predicted that the minimum of the curve may be fitted by an equation of the form

$$E=\Delta+(p-p_0)^2/2\mu,$$

[24] K. R. Atkins and C. E. Chase, Proc. Phys. Soc. (London) **A64**, 826 (1951).

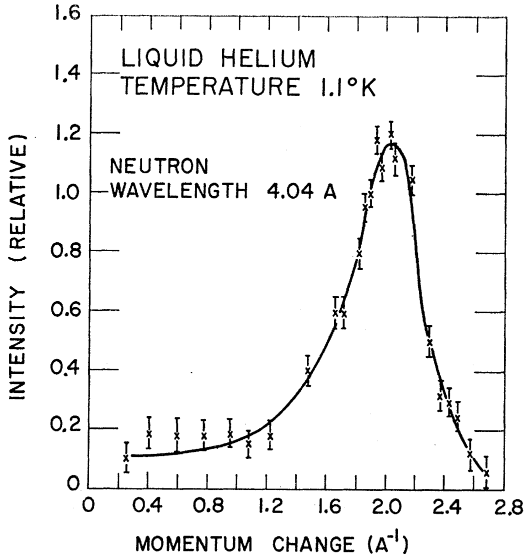

FIG. 5. The relative partial differential cross section for the production of a single-phonon excitation in the liquid.

where Δ and p_0 are the energy and momentum of the minimum and μ is termed an effective mass which is used to define the curvature in the region of the minimum. For a limited range in the region of the minimum the curve may be fitted by

$$\Delta/k = 8.6_5 \pm 0.04°\text{K},$$
$$p_0/\hbar = 1.91 \pm 0.01\ \text{A}^{-1},$$
$$\mu = (0.16) m_{\text{He}}.$$

This formula may be used to fit the measured points over a limited range of momenta within ± 0.15 A^{-1} of the minimum. Beyond this range, the calculated energies lie above the measured values.

Figure 5 gives the relative partial differential cross section for the production of a single phonon excitation in the liquid plotted as a function of the momentum change. These partial differential cross sections have been obtained from measurements of the height of the single-phonon peak at each angle of scattering. In this determination, the broadening of the peaks has been neglected. Analysis has shown that this broadening is small but not negligible. The smooth curve drawn through the points shows low values at small Q, a peak at 2.0_2 A^{-1} beyond which the partial differential cross section decreases rapidly to a low value at the last measured point at 2.68 A^{-1}. No end point[25] in the curve has been observed for measurements out to 2.68 A^{-1}. The position of the main maximum may be compared to 2.05 A^{-1}, the maximum in the differential cross-section curve,[6] and 1.91 A^{-1}, the position of the minimum of the dispersion curve. At small momenta, the measured points suggest the possible existence of a second maximum in the region of 0.5 A^{-1}. This may be due to excess broadening of the neutron groups in the region of 1.0 A^{-1} which would cause the apparent cross section to be lower than the actual cross section. The curve of Fig. 5 shows that the cross section for scattering with the production of a single excitation becomes very small for the larger momentum changes and suggests that the dispersion curve will be difficult to measure at the larger momenta. This is not inconsistent with the measurements[5] reported earlier that the mean energy change of neutrons scattered with large momentum change is that expected for scattering from free helium atoms and corresponds to the scattering with the production of multiple phonons in the liquid.

In the second series of measurements, the wavelength distribution of neutrons scattered through 80° was measured for the liquid at 13 temperatures in the range 1.785°K to 4.21°K. Scattering at this angle corresponded closely to the production of excitations at the minimum of the dispersion curve. With increasing liquid temperature, the position of the maximum of the measured distribution shifted to shorter wavelengths while the distribution broadened and became asymmetrical. The curves were corrected for background, counter sensitivity, and for counter resolution. No correction was applied for the wavelength variation of the crystal reflectivity[26] as this was estimated to be small. The corrected curves were normalized to $N(E)$ and plotted as a function of the energy change E. The curves so plotted were to a good approximation symmetrical in energy about the mean energy change of the scattered neutrons. The full width at half maximum and the mean energy change of the scattered neutrons

TABLE II. The mean energy change, linewidth, and effective mass of single phonon excitation for 4.039 A neutrons scattered through 80° from liquid helium under its normal vapor pressure.

Liquid temperature (°K)	Mean energy change (°K)	Effective mass of excitation (helium mass units)	Full width at half maximum of neutron group (°K)
1.1	8.65±0.11	2.55	≲1°K
1.78	8.1 ±0.3	2.73	2.6±0.5
1.89	8.1 ±0.3	2.73	3.4±0.5
2.02	7.5 ±0.5	2.95	4.5±0.5
2.09	7.4 ±0.5	2.99	5.8±0.5
2.10	7.0 ±0.5	3.16	5.0±0.5
2.13	5.9 ±0.5	3.76	8.0±0.5
2.16	7.0 ±0.5	3.16	10.6±0.5
2.17	5.6 ±0.5	3.95	11.0±0.5
2.20	5.4 ±0.5	4.09	11.0±0.5
2.29	5.3 ±0.5	4.17	11.3±0.5
2.51	4.9 ±0.5	4.51	12.8±0.5
3.03	5.2 ±0.5	4.25	12.7±0.5
4.21	4.9 ±0.5	4.51	14.8±0.5

[25] L. P. Pitaevskii has recently predicted [Soviet Phys.—JETP **9**, 830 (1959)] that an end point in the spectrum will occur for some value of $p = p_c$ and that one of the possibilities for the shape of the spectrum of this point is $E = 2\Delta - a \exp[-\alpha(p - p_c)]$ (where $p_c < 2p_0$) which has zero slope and a limiting value of 2Δ, in agreement with these measurements. Measurements at high pressure do not, however, agree with this limiting value of 2Δ (see reference 12).

[26] See J. W. Knowles, Can. J. Phys. **37**, 203 (1958).

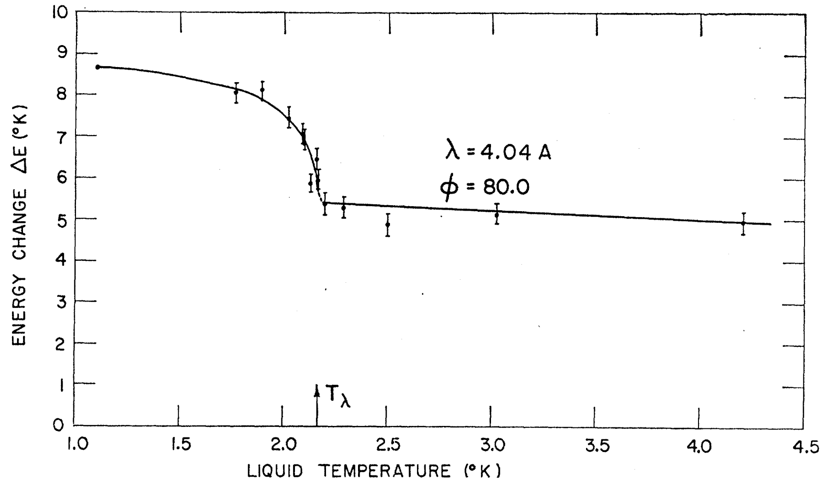

FIG. 6. The temperature variation of the mean energy change of 4.039 A neutrons scattered through 80° from liquid helium. This corresponds closely to the energy of the Landau roton minimum. The smooth curve is drawn through the points as a guide to the eye.

together with the effective mass[27] of an excitation has been deduced for each temperature at which the patterns were recorded and are listed in Table II. Figure 6 shows the mean energy change plotted as a function of temperature. This change corresponds to the variation of the minimum energy of the dispersion curve and shows that the energy decreases rapidly from about 8.6$_5$°K to 5.5°K for temperature changes from 1.1°K to the λ tempeature where there is a marked change in slope and above which there is a slow uniform decrease to 4.9°K at 4.2°K.

Figure 7 shows the variation of the effective mass of an excitation plotted as a function of liquid temperature. The curve shows that the effective mass increases rapidly from a value of about 2.8 to 4.2 helium masses for temperature change from 1.1°K to the λ temperature above which temperature it increases more slowly to about 4.6 mass units at 4.2°K. This curve again shows a marked change in slope at the λ transition.

Figure 8 shows the full width at half maximum of the measured curves plotted as a function of the liquid temperature. The width increases rapidly from 2.5°K to 11.0°K for temperature change from 1.78°K to the λ temperature. Above this temperature, the width increases more slowly to 15°K at 4.2°K. This curve also shows a marked change in slope at the λ temperature. The broken curve gives the theoretical full width at half maximum for scattering from a gas of atoms having a mass equal to the effective mass given in the curve of Fig. 7. This curve was calculated using a formula due to Spiers[28] and is based on the assumption of thermal motion in the atomic system. The dotted curve out to 2.2°K represents the widths calculated on the basis of the Landau-Khalatnikov theory[10] which in the present paper is modified to take into account the measured variation of Δ with liquid temperature. This theory gives

$$\delta T=\frac{\hbar T\rho_{nr}}{5\mu\eta_r}=\frac{2}{15}\frac{p_0^4\cdot\sqrt{T}}{(2\pi)^{\frac{3}{2}}\eta_r\hbar^2(k\mu)^{\frac{1}{2}}}e^{-\Delta/kT},$$

where μ is the effective roton mass which defines the curvature in the region of the minimum, η_r is the roton portion of the viscosity, ρ_{nr} is the roton part of the normal fluid density, p_0 is the momentum of the minimum, h is Planck's constant, k is Boltzman's constant, and Δ is the energy corresponding to the minimum of the curve.

Using the generally accepted values for the roton viscosity, which has been assumed constant independent of temperature, and taking the value of $p_0=1.91$ A^{-1} and $\mu=0.16m_{\rm He}$, the expression reduces to

$$\delta E\simeq 94\sqrt{T}e^{-\Delta/kT}.$$

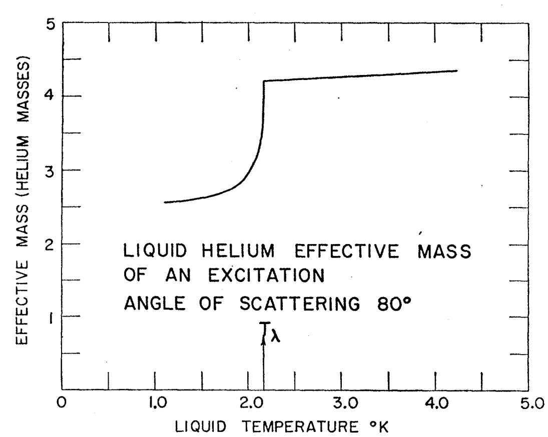

FIG. 7. The temperature variation of the effective mass of an excitation corresponding to the curve of Fig. 6.

[27] The effective mass defined above and used subsequently is the number of helium masses which would give the same energy change for neutron scattering from free particles as that observed in the liquid. This expression is not to be confused with the term μ in the Landau roton expression used to define the curvature in the region of the minimum of the dispersion curve.

[28] See B. N. Brockhouse and D. G. Hurst, Phys. Rev. 88, 542 (1952).

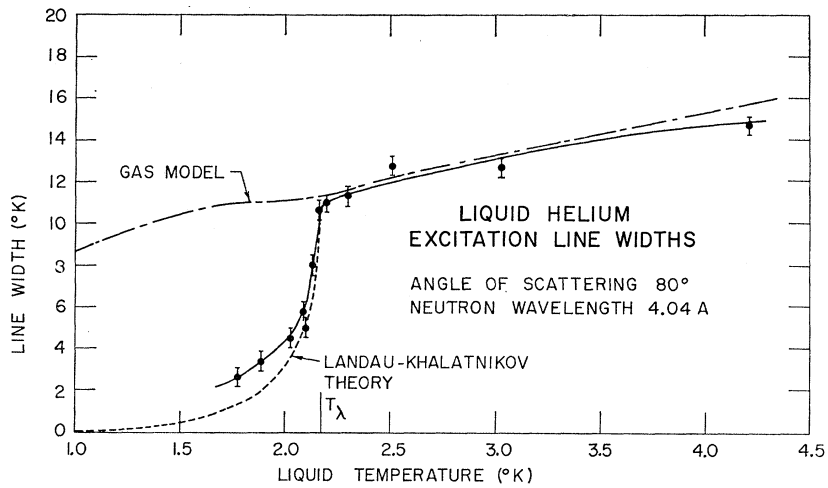

FIG. 8. The temperature variation of the full width at half maximum of the single phonon peak. The solid curve has been drawn through the experimental points as a guide to the eye. The broken curve gives the theoretically calculated widths for a gas with the mass of atoms as given in Fig. 7. The dotted curve represents the theoretically calculated widths on the basis of the Landau-Khalatnikov theory.

The values of the parameter Δ were taken from the measurement listed in Table II and shown in Fig. 6.

DISCUSSION

The spectrum of neutrons scattered from liquid helium at 1.1°K showed a discrete peak with only little broadening at all angles of scattering which was taken as indicating that the observed scattering was with single-phonon production having a long mean free path. Only little, if any, excess scattering was observed over the range of wavelengths studied at wavelengths longer than the single-phonon peak even for scattering at large angles.[11] The cross section for the production of a single phonon is small at low momenta, rises rapidly to a maximum in the region of 2.0 A^{-1} beyond which it decreases rapidly, but continuously, at larger momenta changes and becomes very small in comparison to the total differential cross section.[5] Since no scattering was observed at wavelengths less than the single-phonon peak for the range of wavelengths measured, these results are not inconsistent with the hypothesis that most of the scattering at large momenta change is with multiphonon production in the liquid corresponding to an average energy change like that from free helium atoms as has been observed in other earlier measurements.[5] This indicates that the single-phonon excitations at large momenta will be difficult to measure. No endpoint[25] in the dispersion curve was observed for momenta out to 2.68 A^{-1}. The dispersion curve has been measured over the range 0.26 A^{-1} to 2.68 A^{-1}. The measurements in the range $0.26\ A^{-1} \leq Q \leq 0.6\ A^{-1}$ have a slope which agrees with the measured velocity of sound of 237 meters sec^{-1}. For momenta greater than 2.1 A^{-1}, the maximum slope is equal to or less than that given by the velocity of sound[11] and at larger momenta the curve bends over in such a manner as to suggest the possible existence of a second maximum and does not approach the free-particle curve. In the range 0.26 A^{-1} to 2.3 A^{-1}, the curve is similar in form to that predicted theoretically but there are differences in details. The position of the minimum for the liquid temperature of 1.1°K is at $1.91 \pm 0.01\ A^{-1}$ and 8.65 ± 0.1°K in agreement with the measurements of Yarnell *et al.*[11]

The temperature variation of the energy Δ/k of the Landau roton minimum decreases rapidly for temperatures in the range 1.8°K to the λ temperature where there is a marked change in slope and only a slight further decrease for temperatures in the He-I temperature range. The corresponding widths of these spectra of scattered neutrons increase rapidly with increasing temperature toward the λ temperature where they also show a marked change in slope and only a slight further increase in the He-I temperature range. The measured widths are in agreement with the theoretical widths calculated for a gas of free particles using the measured effective mass of the excitations at temperatures above the λ temperature but fall well below these widths at temperatures below the Lambda point. For this temperature region the measured widths lie slightly above the widths calculated using the Landau-Khalatinkov theory modified to allow for the measured temperature variation in Δ/k. This behavior is similar to that observed by Larsson and Otnes.[10] It is possible that the deviations of the measurements from the theoretically predicted widths may in part be due to the temperature variation of the roton portion of the viscosity. Thus these results suggest that at temperatures above the λ temperature the motions of assembly of atoms are gaslike in nature while below the λ temperature the motions become ordered and the widths of the spectra are owing to the mean free paths of the excitations.

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