K^--meson capture stars.[5] We are continuing our search and analysis in the hope that this might be confirmed by a trend of ΔE values larger than those expected from free scattering for a given scattering angle in the forward center-of-mass hemisphere.

[5] I. E. MacCarthy and D. J. Prowse, Nuclear Phys. **17**, 96 (1960).

ACKNOWLEDGMENTS

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Demonstration of Quantum Mechanics in the Large*

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An example is given which demonstrates in a straightforward and dramatic manner that when two particles like $(K^0+\bar{K}^0)$ or (2γ) are created simultaneously, the probabilities involved in observing any further events related to their simultaneous creation must be calculated quantum mechanically and are correlated, even for macroscopic distances in absorbing media. In particular, a correlation in the polarization of the two γ rays from positronium annihilation as a function of the thickness of magnetized iron through which they are passed is pointed out by way of a proposed experiment.

RECENTLY, Lee and Yang pointed out a striking effect of quantum mechanics in the large.[1] The effect bears on the objections raised by the Einstein-Rosen-Podolsky paradox,[2] and is yet another facet of the fascinating neutral K-meson system.

We amplify on Lee's remarks as follows. Consider first the creation of a K^0 (or $\bar{K}^0$) meson at some point. If we move some distance from the point of creation such that $\tau_1 \ll t \ll \tau_2$, where τ_1 and τ_2 are the lifetime, respectively, of the $K_1{}^0$ and $K_2{}^0$ meson and t is the time of flight, then we expect that the probability of observing a regenerated K^0 (or $\bar{K}^0$) meson will be $\sim\frac{1}{4}$.[3] Now consider the *simultaneous* creation of two neutral K mesons, say K^0 and $\bar{K}^0$, which move off in opposite directions. Then we ask the question: given that we see a $\bar{K}^0$ meson downstream at a time of flight t_a in one direction, what is the probability of observing in coincidence another $\bar{K}^0$ meson downstream at a time of flight t_b in the opposite direction? One is tempted to answer, according to the ideas of reference 2 (and of classical physics, for large t_a, t_b) that if $\tau_1 \ll t_a, t_b \ll \tau_2$, then again we should see a $\bar{K}^0$ meson at t_b with probability $\sim\frac{1}{4}$. But as Lee points out, this is not so!

When the $(K^0, \bar{K}^0)$ pair are created, they are, in general, created in a mixture of eigenstates of the charge conjugation operator with eigenvalues $C=\pm 1$.[3] Thus, the initial wave function describing one neutral meson moving in direction a, and one in the opposite direction b is (for $C=-1$)

$$\psi_{\text{in}}=\frac{1}{\sqrt{2}}[|K_a{}^0,\bar{K}_b{}^0\rangle-|\bar{K}_a{}^0,K_b{}^0\rangle]. \tag{1}$$

When this wave function is rewritten in terms of the $K_1{}^0$ and $K_2{}^0$ eigenfunctions appropriate for decay, and then allowed to propagate down directions a and b, we get (for $m_{1,2}$ equal to the mass of $K_1{}^0$, $K_2{}^0$)

$$\psi_{\text{fin}}=\frac{i}{\sqrt{2}}[|K_2{}^a,K_1{}^b\rangle\exp(-\lambda_2 t_a-\lambda_1 t_b) - |K_1{}^a,K_2{}^b\rangle\exp(-\lambda_2 t_a-\lambda_2 t_b)], \tag{2}$$

where $\lambda_{1,2}=(2\tau_{1,2})^{-1}-im_{1,2}$. If this is re-expressed in terms of the K^0 and $\bar{K}^0$ eigenfunctions, and then the probabilities of making various coincidence measurements at times t_a and t_b are computed, we find (for $C=-1$)[4]

$$P(K_a{}^0,K_b{}^0)=P(\bar{K}_a{}^0,\bar{K}_b{}^0)=\tfrac{1}{8}|\exp(-\lambda_2 t_a-\lambda_1 t_b)-\exp(-\lambda_1 t_a-\lambda_2 t_b)|^2,$$
$$P(K_a{}^0,\bar{K}_b{}^0)=P(\bar{K}_a{}^0,K_b{}^0)=\tfrac{1}{8}|\exp(-\lambda_2 t_a-\lambda_1 t_b)+\exp(-\lambda_1 t_a-\lambda_2 t_b)|^2. \tag{3}$$

* Supported in part by a National Science Foundation grant.

[1] T. D. Lee and C. N. Yang, reported by T. D. Lee at Argonne National Laboratory, May, 1960 (unpublished).

[2] A. Einstein, B. Podolsky, and N. Rosen, Phys. Rev. **47**, 777 (1935); D. Bohm and Y. Aharanov, Phys. Rev. **108**, 1070 (1957).

[3] R. H. Dalitz, *Reports on Progress in Physics* (The Physical Society, London, 1957), Vol. 20, p. 229; M. Gell-Mann and A. H. Rosenfeld, *Annual Review of Nuclear Science* (Annual Reviews, Inc., Palo Alto, California, 1957), Vol. 7, p. 407; A. Pais and O. Piccioni, Phys. Rev. **100**, 1487 (1955).

[4] The analysis for $C=+1$ is similar, and we get

$$P(K_a{}^0,K_b{}^0)=P(\bar{K}_a{}^0,\bar{K}_b{}^0)=\tfrac{1}{8}|\exp(-\lambda_1(t_a+t_b))-\exp(-\lambda_2(t_a+t_b))|^2,$$
$$P(K_a{}^0,\bar{K}_b{}^0)=P(\bar{K}_a{}^0,K_b{}^0)=\tfrac{1}{8}|\exp(-\lambda_1(t_a+t_b))+\exp(-\lambda_2(t_a+t_b))|^2.$$

In this case, we get no zero probabilities (except at the origin). As Lee pointed out, this difference in behaviors between $C=+1$ and $C=-1$ may, in principle, be utilized to determine in what mixture of eigenstates the $K^0-\bar{K}^0$ pair is created.

Thus we see the startling answer to the question posed above, that if we see a $\bar{K}^0$ at a time t_a (e.g., by looking for Σ^+ from $\bar{K}^0+p\rightarrow\Sigma^++\pi^0$), the probability of also seeing a $\bar{K}^0$ at time t_b will be strictly zero if $t_a=t_b$, no matter how far apart they are. The conditional probability of observing a K^0 or $\bar{K}^0$ at t_b depends on what, and where, we observe at t_a, and is not at all as it was for the single meson creation. In particular, if we ask for the probability of observing, say, a $\bar{K}^0$ at t_a in coincidence with either K^0 *or* $\bar{K}^0$ at t_a, we find

$$P(\bar{K}_a{}^0,K_b{}^0)+P(\bar{K}_a{}^0,\bar{K}_b{}^0)$$
$$=\tfrac{1}{4}[\exp(-2\lambda_2t_a-2\lambda_1t_b)+\exp(-2\lambda_1t_a-2\lambda_2t_b)]. \quad (4)$$

This depends on t_b in a way entirely different from what would be obtained "classically" by assuming the conditional probabilities to simply factor, i.e.,

$$P(K_a,K_b)=p(K_a)p(K_b).$$

The mixture in K^0 and $\bar{K}^0$ states dictated in the initial wave function by the eigenvalue of $C=\pm1$ determines a correlation between the two events of observation no matter how far apart.

The purpose of this note is to point out an experimental arrangement for demonstrating the above effect using photons rather than K's which seems to be feasible with present day techniques, and which has some additional striking features. The experiment is similar to the measurement of the correlation of the directions of polarization of the two quanta from positronium annihilation, but with magnetic channels added.[5] Thus in the apparatus schematically diagrammed in Fig. 1, S is the source of positronium annihilation radiation (e.g., $N_a{}^{22}$); M_a, M_b are magnetic channels of iron of variable length; (C_{1a}, C_{2a}), (C_{1b}, C_{2b}) are two Compton-effect polarimeters, which are to be put in coincidence with the appropriate delay lines added.

Then the analogy with the example given by Lee is complete. (i) The effect depends on the simultaneous production of two particles which are identical except for an internal quantum number; for the neutral K's it is strangeness; for the γ's it is polarization. (ii) The particles are produced in an eigenstate of the relevant operator for the process: for the K's we consider eigenstates of the charge conjugation operator with eigenvalues $C=\pm1$; for γ's it is the space-inversion operator with eigenvalue $\Pi=-1$ (for the decay of the $^1S_0$ ground state of positronium).[6] (iii) The eigenstates of the produced particles are re-expressed in terms of eigen-

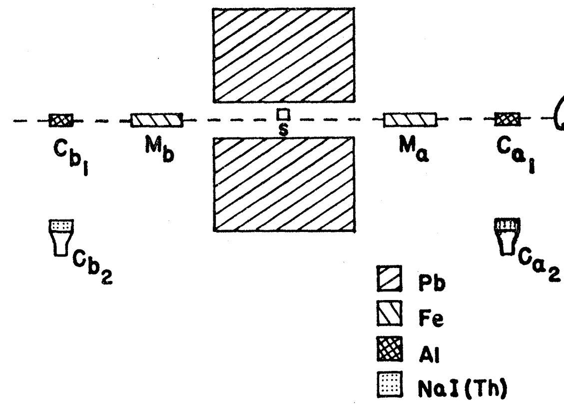

FIG. 1. Schematic diagram of experiment to measure, in coincidence, the polarizations of the two quanta from positronium annihilation in the source S. The photons pass through the magnetic channels M_a and M_b, of variable length, and their polarizations are then measured by the Compton scattering polarimeters (C_{1a}, C_{2a}) and (C_{1b}, C_{2b}). (See Fig. 15 of reference 5.) The Compton polarimeters are put in coincidence, and the counting rate should then be a function, not only of the polarizations as measured by the asymmetry in azimuthal angle ϕ, but also of the lengths of M_a and M_b.

states more appropriate for the absorption by the medium through which they move: for the K's, the new eigenstates are $|K_1{}^0\rangle$ and $|K_2{}^0\rangle$ which are selectively "absorbed" by the vacuum; for the γ's, the eigenstates are of circular polarization which are selectively absorbed by the magnetized iron,[5] i.e.,

$$\psi_{in}=(1/\sqrt{2})[|u_+{}^a,u_+{}^b\rangle-|u_-{}^a,u_-{}^b\rangle],$$

where the u_+ or u_- refers to the state of photon circular polarization parallel or antiparallel, respectively, to the direction of propagation.[6] (iv) Finally, for the observation of the two particles in coincidence, the eigenstates to be used are again the original ones for production, i.e. $|K^0\rangle$, $|\bar{K}^0\rangle$, and plane polarization in directions 1 and 2, $|u_1\rangle$, $|u_2\rangle$ for the γ's. If the same sort of algebra which led to Eq. (3) is followed for photons in magnetized iron, the probability, for example, of observing parallel polarizations at t_a and t_b, (time spent in the iron) when both magnetic fields point to the right ($H_a\equiv+$, $H_b\equiv+$) is

$$P(u_1{}^a,u_1{}^b)=P(u_2{}^a,u_2{}^b)=\tfrac{1}{8}|\exp(-\lambda_-t_a-\lambda_+t_b)$$
$$-\exp(-\lambda_+t_a-\lambda_-t_b)|^2, \quad (5)$$

where[7]

$$\lambda_\pm=\tfrac{1}{2}Nc(Z\sigma_0\pm\nu\sigma_1). \quad (6)$$

Here N= the number of iron atoms per unit volume, c= velocity of light, $Z=26$, and ν= the number of polarized electrons per iron atom (~2). σ_0 and σ_1 are the polarization-spin independent and dependent parts,

[5] See L. W. Fagg and S. S. Hanna, Revs. Modern Phys. **31**, 711 (1959) for a review of experiments in the measurement of such polarizations. We would like to thank Dr. R. W. Detenbeck for bringing this reference to our attention. See also A. S. Wightman, Phys. Rev. **74**, 1813 (1948); S. B. Gunst and L. A. Page, Phys. Rev. **92**, 970 (1953); and C. S. Wu and I. Shaknov, Phys. Rev. **77**, 136 (1950).

[6] J. M. Jauch and F. Rohrlich, *The Theory of Photons and Electrons*, (Addison-Wesley Publishing Company, Inc., Cambridge, Massachusetts, 1955), p. 281. See also G. Wick, *Annual Review of Nuclear Science* (Annual Reviews, Inc., Palo Alto, California, 1958), Vol. 8, p. 28.

[7] The real part of the index of refraction proportional to the real part of the forward elastic scattering amplitude for Compton scattering is ignored here as it is essentially spin-polarization independent. See H. A. Tolhoek, Revs. Modern Phys. **28**, 277 (1956). We would like to thank Professor G. A. Snow for raising this point.

TABLE I. Values of the probabilities for coincidence observation of positronium annihilation photons of various combinations of polarizations. The photons pass through magnetic channels with the magnetic field directions indicated by + or − below, corresponding to fields pointing to the right or left in Fig. 1.

	$P(u_1^a,u_1^b)=P(u_2^a,u_2^b)$	$P(u_1^a,u_2^b)=P(u_2^a,u_1^b)$
$H_a=H_b=+$, equivalent to $H_a=H_b=-$	$\frac{1}{8}[\exp(-\lambda_-t_a-\lambda_+t_b)-\exp(-\lambda_+t_a-\lambda_-t_b)]^2$	$\frac{1}{8}[\exp(-\lambda_-t_a-\lambda_+t_b)+\exp(-\lambda_+t_a-\lambda_-t_b)]^2$
$H_a=+$, $H_b=-$, equivalent to $H_a=-$, $H_b=+$	$\frac{1}{8}[\exp(-\lambda_-(t_a+t_b))-\exp(-\lambda_+(t_a+t_b))]^2$	$\frac{1}{8}[\exp(-\lambda_-(t_a+t_b))+\exp(-\lambda_+(t_a+t_b))]^2$
$H_a=0$, $H_b=\pm$, equivalent to $H_a=\pm$, $H_b=0$	$\frac{1}{8}e^{-2\lambda t_a}[\exp(-\lambda_\pm t_b)-\exp(-\lambda_\mp t_b)]^2$	$\frac{1}{8}e^{-2\lambda t_a}[\exp(-\lambda_\pm t_b)+\exp(-\lambda_\mp t_b)]^2$
$H_a=0$, $H_b=0$	0	$\frac{1}{2}\exp(-2\lambda(t_a+t_b))$

respectively, of the Compton scattering cross section.[8] Once again we see the striking result that the observations are correlated. The coincidence rate considered above is zero for equal lengths of magnetized iron ($t_a=t_b$) and depends quadratically on (t_a-t_b) for small differences in length.

Not only does the experiment proposed here seem feasible, but in addition the properties of the absorbing medium are under our control. This is in contrast to the K^0-meson case, where the "absorbing" properties of the vacuum are fixed. In particular, one can look at several arrangements for the directions of the magnetic fields in the iron absorbers, and in so doing get quite different results for the coincidence probabilities of Eq. (5). These various possibilities are summarized in Table I, where the direction of the magnetic fields is indicated as + or − for fields pointing right or left in Fig. 1. In the table, $\lambda=\frac{1}{2}Nc\sigma_0Z$. It is seen from Table I that reversing the field in one of the pieces of iron changes the response for the two photons in the same way that starting with $C=+1$ compared to starting with $C=-1$ changes the response for the K^0 case.

It would seem, then, that one could demonstrate in a straightforward and dramatic manner than when two particles like $K^0-\bar{K}^0$ or 2γ are created simultaneously, the probabilities involved in observing any further events related to their simultaneous creation must be calculated quantum-mechanically and are correlated, even for macroscopic distances in absorbing media.

[8] See Fagg and Hanna, reference 5, Eqs. II 8–10, III 5.

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Heavy Nuclei in the Primary Cosmic Radiation at Prince Albert, Canada. II*

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The investigation of the low-energy primary cosmic radiation has been extended to include the heavier elements of $Z\geq 9$: Sec. 1; the light elements of Li, Be, and B: Sec. 2; and α particles: Sec. 3. The results of a previous paper on carbon, nitrogen, and oxygen have also been confirmed with better statistics. The energy spectra of all these components show a general similarity in shape. A possible deviation of the light-element spectrum from this similarity is discussed. The abundances of various elements in the low-energy region of 200 to 700 Mev per nucleon are essentially the same as observed in the higher energy region.

1. THE HEAVIER ELEMENTS OF $Z\geq 9$

THE charges and energies of the heavy nuclei in the primary cosmic radiation have been determined from the analysis of the tracks recorded by them in the stack of photographic emulsions.

A detailed description of the assembly, exposure, and processing of the stack has been given in Sec. 2 of Part I.[1]

The scanning for the heavier elements has been per-

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‖ This paper was completed after the death of Professor Marcel Schein. Therefore, the responsibility for any possible error in this paper rests on the other authors.

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