Tunneling Current in Esaki Diodes

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The integral giving the net tunneling current flowing across the junction in an Esaki diode,

$$I=A\int_{E_c}^{E_v}\{f_c(E)-f_v(E)\}Z\rho_c(E)\rho_v(E)dE$$

is evaluated under the normal assumptions that (ξ_c-E_c) and $(E_v-\xi_v)$ are of the order of $2kT$. The resulting expression is

$$I=-A''\frac{(E_v-E_c)^2(1-e^{qV/kT})}{(m+n)e^{a/2}+(1+e^{qV/kT})},$$

where A'' is an arbitrary constant and m, n, and a are functions of the Fermi levels on both sides of the junction, the location of the band edges and the absolute temperature. This expression is plotted as a function of the applied voltage for temperatures of 200°K, 300°K, and 350°K for donor and acceptor concentrations of 10^{19} cm^{-3} and 1.6×10^{19} cm^{-3}, respectively. The resulting curves compare quite favorably with those of Esaki's.

IN his now classic paper on internal field emission in very narrow germanium p-n junctions, Esaki[1] gives an expression for the net current flow when the junction is slightly biased in the forward direction as

$$I=I_{c\to v}-I_{v\to c}$$
$$=A\int_{E_c}^{E_v}\{f_c(E)-f_v(E)\}Z\rho_c(E)\rho_v(E)dE, \quad (1)$$

where $f_c(E)$ and $f_v(E)$ are the Fermi-Dirac distribution functions, $Z=Z_{c\to v}=Z_{v\to c}$ is the probability of penetrating the gap, $\rho_c(E)$ and $\rho_v(E)$ are the energy level densities in the conduction and valence bands, respectively, and A is an appropriate arbitrary constant. If Z may be considered to be almost constant over the range of voltages involved and parabolic energy bands are assumed, this expression reduces to

$$I=A'\int_{E_c}^{E_v}\{f_c(E)-f_v(E)\}(E-E_c)^{\frac{1}{2}}(E_v-E)^{\frac{1}{2}}dE. \quad (2)$$

It has occurred to me and others that nowhere in the literature has there appeared an evaluation of this integral. It is the purpose of this note to present a simple expression for the value of this integral which is valid over a wide temperature range.

Substituting the appropriate expressions for $f_c(E)$ and $f_v(E)$, Eq. (2) reduces to

$$I=A'(E_v-E_c)^2(1-e^{qV/kT})\int_0^1\frac{y^{\frac{1}{2}}(1-y)^{\frac{1}{2}}dy}{me^{ay}+ne^{a(1-y)}+\kappa}, \quad (3)$$

where

$$m=e^{-(\xi_c-E_c)/kT},$$
$$n=e^{-(E_v-\xi_v)/kT},$$
$$a=(E_v-E_c)/kT,$$
$$\kappa=(1+e^{qV/kT}),$$

and all symbols have the same meaning as in Esaki's paper.

The trigonometric substitution $y=\cos^2\varphi$ further reduces the integral to

$$I=-A'(E_v-E_c)^2(1-e^{qV/kT})e^{-a/2}\times\int_0^\pi\frac{\sin^2\varphi d\varphi}{me^{(a/2)\cos\varphi}+ne^{-(a/2)\cos\varphi}+\kappa'}, \quad (4)$$

where $\kappa'=\kappa e^{-a/2}$.

Under the assumption that

$$(\xi_c-E_c)\sim2kT,$$
$$(E_v-\xi_v)\sim2kT,$$

such that m and n are about 0.1 and $a\leqslant10$, then to a first approximation the current can be shown to be equal to

$$I=-A''\frac{(E_v-E_c)^2(1-e^{qV/kT})}{[m+n]e^{a/2}+[1+e^{qV/kT}]}, \quad (5)$$

where A'' is another constant. This checks exactly with the first term of an expansion of the denominator given by Klamkin.[2]

In his paper Esaki gives curves of I vs V for temperatures of 200°K, 300°K, and 350°K for a junction with donor and acceptor concentrations of 10^{19} cm^{-3} and 1.6×10^{19} cm^{-3}. These curves computed by numerical integration, are reproduced in Fig. 1, with Eq. (5)

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[1] Leo Esaki, Phys. Rev. **109**, 603 (1958).
[2] Murray S. Klamkin, AVCO-RAD technical release, August 17, 1960 (unpublished).

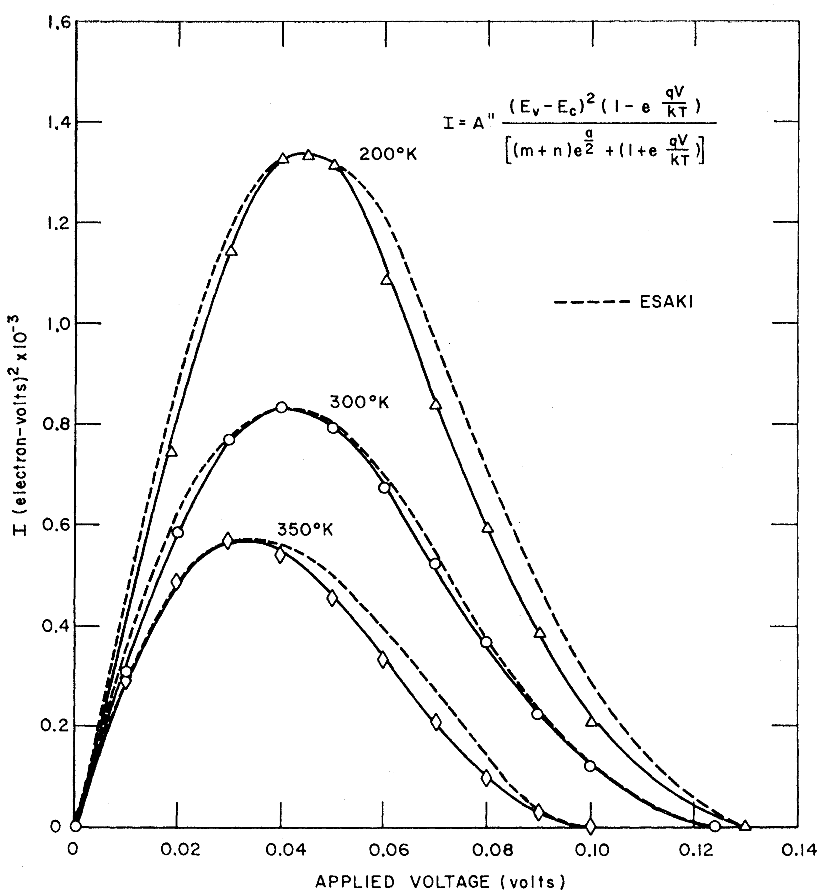

FIG. 1. Comparison of Esaki's curves of I vs V with those obtained by use of Eq. (5).

plotted for comparison. The constant A'' was adjusted so that the maximum value of the current for any two corresponding curves were equal.

It is to be noted that the expression given above agrees quite well with Esaki's results, especially as regards the positions of the maximum currents which are exactly the same as Esaki's.

Since the derivative of this expression with respect to V gives the conductance $G=\partial I/\partial V=f(V,\xi_c,\xi_v)$, one may use this to determine the location of the peak current as a function of V and the Fermi levels on either side of the junction. Mr. Harry Lockwood of the General Telephone and Electronics Laboratories in New York City is now applying this expression to experimental units with known parameters. Preliminary results appear promising.

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