First model:

$$u^2=-(p_2+q_2)^2;\quad u_{\min}=Y+m;\quad u_{\max}=W-M;$$
$$E=(u^2+M^2-\mu^2)/2u;\quad \kappa=(E^2-M^2)^{\frac{1}{2}}=(1/2u)[u^4-2(M^2+\mu^2)u^2+(M^2-\mu^2)^2]^{\frac{1}{2}};$$
$$A^2=\mu^2;\quad m_0=M;\quad \nu=\mu;\quad w'=(1/2W)(W^2+M^2-u^2);$$
$$k'=(w'^2-M^2)^{\frac{1}{2}};\quad \Gamma=w'W-2M^2+\mu^2;\quad \alpha=2kk'/\Gamma.$$

Second model:

$$u^2=-(k_2+q_2)^2;\quad u_{\min}=M+m;\quad u_{\max}=W-Y;$$
$$E=(u^2+M^2-m^2)/2u;\quad \kappa=(E^2-M^2)^{\frac{1}{2}}=(1/2u)[u^4-2(M^2+m^2)u^2+(M^2-m^2)^2]^{\frac{1}{2}};$$
$$A^2=m^2-(Y-M)^2;\quad m_0=Y;\quad \nu=m;\quad w'=(1/2W)(W^2+Y^2-u^2);$$
$$k'=(w'^2-Y^2)^{\frac{1}{2}};\quad \Gamma=w'W-Y^2-M^2+m^2;\quad \alpha=2kk'/\Gamma.$$

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Experimental Study of the Magnetic Structure of the Neutron*

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A measurement of the ratio of the magnetic form factor of the neutron to that of the proton has been carried out by comparing large- and small-angle elastic electron-deuteron scattering at constant four-momentum transfers. The experimental result for the average value of the ratio in the range of momentum transfers from 1.6 f^{-1} to 2.25 f^{-1} is $F_{2n}/F_p=(0.91\pm0.05)\pm0.07$; the first error is a standard deviation arising from experimental uncertainties, and the second from theoretical uncertainties in the analysis. Measurements of the ratio of the nucleon isotopic scalar form factors have also been obtained from this experiment. The average value of $F_2{}^s/F_1{}^s$ for the same range of momentum transfers has been found to be $(+0.06\pm0.09)\pm0.13$. The small-angle scattering data have been used to determine the charge form factor of the deuteron in the range of momentum transfers from 0.98 f^{-1} to 2.8 f^{-1}. The results are consistent with a repulsive-core model of the deuteron.

I. INTRODUCTION

A DETAILED knowledge of nucleon structure provides an important test for any meson theory. While measurements of electron-proton scattering[1,2] give direct information about the proton's charge and magnetic moment distributions, the impossibility of studying electron scattering from free neutrons makes it much more difficult to obtain comparable information about the neutron. Until recently, two types of experiments have provided information about the magnetic structure of the neutron. The first was the measurement[3,4] of inelastic electron-deuteron scattering cross sections which by the impulse approximation[5–7] can be related to free neutron and proton cross sections. The second was a measurement[8] of the four-momentum transfer dependence of electroproduction of pions from hydrogen at the (3,3) resonance. With the use of dispersion relations this cross section has been shown[9] to depend on the neutron and proton magnetic form factors. The present experiment uses measurements of a different process to give information about the neutron's magnetic structure. The method consists of comparing elastic electron-deuteron scattering at large and small angles for a constant four-momentum transfer. The ratio of these cross sections, which to a good approximation does not depend on the deuteron model, is a function of the neutron and proton magnetic form factors and thus can provide a comparison of their magnetic structures. In particular, the quantity measured here is the ratio of the neutron and proton anomalous magnetic form factors.

It is also of theoretical interest to study the q de-

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[1] E. E. Chambers and R. Hofstadter, Phys. Rev. **103**, 1454 (1956).
[2] F. Bumiller and R. Hofstadter (to be published).
[3] R. Hofstadter, *Annual Review of Nuclear Science* (Annual Reviews, Inc., Palo Alto, California, 1957), Vol. 7, see especially pp. 267–271.
[4] M. R. Yearian and R. Hofstadter, Phys. Rev. **111**, 934 (1958).
[5] V. Z. Jankus, Phys. Rev. **102**, 1586 (1958).
[6] R. Blankenbecler, Phys. Rev. **111**, 1684 (1958).
[7] A. Goldberg, Phys. Rev. **112**, 618 (1958).
[8] W. K. H. Panofsky and E. A. Allton, Phys. Rev. **110**, 1155 (1958).
[9] S. Fubini, Y. Nambu, and V. Wataghin, Phys. Rev. **111**, 329 (1958).

pendence of combinations[10] of these form factors which correspond to the isotopic scalar and vector parts of a general nucleon form factor. If the four relevant nucleon form factors were all known, the isotopic form factors could be constructed from them. In the absence of such information, it is useful to measure these combinations directly. The present experiment can be analyzed to provide information about the isotopic scalar form factors. In a procedure identical to that for the previous case, the measured cross sections have been used to determine the ratio of the magnetic and electric isotopic scalar form factors.

In addition, the small-angle electron-deuteron scattering data have been used to measure the charge form factor of the deuteron, extending the most recent work[11] on the subject to higher momentum transfers.

II. THEORY AND METHOD

The theoretical basis of this experiment is an impulse approximation calculation by Jankus[5] of the cross section for elastic electron-deuteron scattering. With the omission of a few small terms, which will be discussed below, the expression for the cross section for scattering electrons of incoming energy E_0 through an angle θ in the laboratory system is

$$\frac{d\sigma}{d\Omega}=\left(\frac{e^2}{2E_0}\right)^2\frac{\cos^2(\theta/2)}{\sin^4(\theta/2)}\left[1+\frac{2E_0}{M_dc^2}\sin^2(\theta/2)\right]^{-1}G^2$$
$$=\sigma_{\mathrm{N.S.}}(E_0,\theta)G^2, \tag{1}$$

where

$$G^2=\left[\int_\theta^\infty (u^2+w^2)j_0(qr/2)dr\right]^2 \times\left[1+\frac{2}{3}\frac{q^2}{4M^2c^2}(\mu_p+\mu_n)^2 2\tan^2(\theta/2)+1\right];$$

M is the nucleon mass and M_d is the deuteron mass; μ_p and μ_n are the magnetic moments of the proton and neutron, respectively; u and w are defined by the standard representation of the deuteron wave function,

$$\psi_{dm}=(4\pi)^{-\frac{1}{2}}r^{-1}[u(r)+8^{-\frac{1}{2}}w(r)S_{12}]\chi_1{}^m,$$

where

$$S_{12}=\frac{3(\boldsymbol{\sigma}_1\cdot\mathbf{r})(\boldsymbol{\sigma}_2\cdot\mathbf{r})}{r^2}-\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2,$$

and $\chi_1{}^m$ is the triplet spin function; q is the four-momentum transfer which, for elastic scattering, is given by the expression

$$q=\left[2\frac{E_0}{c}\sin(\theta/2)\right]\left[1+\frac{2E_0}{M_dc^2}\sin^2(\theta/2)\right]^{-\frac{1}{2}}.$$

The quantity $\int_0^\infty (u^2+w^2)j_0(qr/2)dr$ is the form factor of the spherically symmetric part of the deuteron's charge distribution and will henceforth be denoted $F_d(q)$. In the original calculation of the scattering cross section [Eq. (1)], the proton and neutron are treated as point particles; however, the result may be generalized[12] to include nucleon structure, with this modification:

$$G^2=F_d{}^2(q)\Bigg\{[F_{1n}(q)+F_{1p}(q)]^2+\frac{2}{3}\frac{q^2}{4M^2c^2}\times[F_{1n}(q)+F_{1p}(q)+\kappa_nF_{2n}(q)+\kappa_pF_{2p}(q)]^2\times[2\tan^2(\theta/2)+1]\Bigg\},$$

where $F_{1p}(q)$ and $F_{1n}(q)$ are the charge (and Dirac moment) form factors of the proton and neutron, respectively; $F_{2p}(q)$ and $F_{2n}(q)$ are their anomalous magnetic moment form factors; and κ_p and κ_n are their anomalous magnetic moments. In the range of q in which our measurements were carried out, F_{1p} and F_{2p} have been found[1,2] to be equal within the errors of measurement, both in the model-dependent analysis of the data and by direct measurements of their ratio. We will thus take $F_{1p}=F_{2p}=F_p$ where F_p will represent the proton form factor. If we further assume that $F_{1n}=0$, the expression for the cross section can be given in the following simplified form:

$$d\sigma/d\Omega=\sigma_{\mathrm{N.S.}}(E_0,\theta)F_d{}^2(q)F_p{}^2(q)\times\left\{1+\frac{2}{3}\frac{q^2}{4M^2c^2}\left[\mu_p+\mu_n\frac{F_{2n}(q)}{F_p(q)}\right]^2[2\tan^2(\theta/2)+1]\right\}. \tag{2}$$

The sensitivity of the results to these assumptions is discussed in Sec. VII.

It can be seen that small-angle scattering is primarily charge scattering, whereas at large angles there is a significant contribution from magnetic scattering. This difference in angular dependence may be used to separate these two effects.

It is convenient to form the ratio R:

$$R=\left[\frac{d\sigma}{d\Omega}(\theta_L,E_1)\sigma_{\mathrm{N.S.}}(\theta_S,E_2)\right]^{-1}\left[\frac{d\sigma}{d\Omega}(\theta_S,E_2)\sigma_{\mathrm{N.S.}}(\theta_L,E_1)\right],$$

where the cross sections $d\sigma(\theta_L,E_1)/d\Omega$ and $d\sigma(\theta_S,E_2)/d\Omega$ are measured at large and small angles θ_L and θ_S, at incoming energies E_1 and E_2 chosen so both scatterings

[10] S. D. Drell, in *Proceedings of 1958 Annual International Conference on High-Energy Physics*, edited by B. Ferretti (CERN Scientific Information Service, Geneva, 1958), p. 20.

[11] J. A. McIntyre and G. R. Burleson, Phys. Rev. **112**, 1155 (1958).

[12] D. R. Yennie, M. M. Lévy, and D. G. Ravenhall, Revs. Modern Phys. **29**, 144 (1957).

have the same four-momentum transfer q. Using Eq. (2), R becomes

$$R=\frac{1+\frac{2}{3}(q^2/4M^2c^2)\{\mu_p+\mu_n[F_{2n}(q)/F_p(q)]\}^2[2\tan^2(\theta_L/2)+1]}{1+\frac{2}{3}(q^2/4M^2c^2)\{\mu_p+\mu_n[F_{2n}(q)/F_p(q)]\}^2[2\tan^2(\theta_S/2)+1]}. \tag{3}$$

This ratio is independent of $F_d^2(q)$, and the only unknown that appears is $F_{2n}(q)/F_p(q)$. By measuring R it is possible to measure the magnitude of $F_{2n}(q)$ relative to $F_p(q)$ without having the results depend on the choice of the deuteron model.

The data of the present experiment may also be analyzed to yield measurements of $F_2{}^s/F_1{}^s$ where $F_2{}^s$ is the isotopic scalar form factor of the nucleon associated with its anomalous magnetic moment distribution, and $F_1{}^s$ is the isotopic scalar form factor associated with its charge and Dirac moment distribution. These form factors can be defined in terms of the previous ones in the following way:

$$F_1{}^s=(F_{1p}+F_{1n})/2,$$

and

$$F_2{}^s=(\kappa_pF_{2p}+\kappa_nF_{2n})/2.$$

The expression for R in terms of the isoscalar form factors is the same as (3), except that $[\mu_p+\mu_n(F_{2n}/F_p)]^2$ is replaced by $[1+(F_2{}^s/F_1{}^s)]^2$. It should be pointed out that here the construction of R requires no special assumptions about F_{1n} and no information about the relative magnitudes of F_{2p} and F_p. Thus this experiment can be used to measure $F_2{}^s/F_1{}^s$ without dependence on other experiments.

Measurements of elastic scattering cross sections at 145° have been carried out for q values from 1.6 f^{-1} to 2.25 f^{-1}; they have also been made in the angular range of 43° to 105°, covering the range of q from 0.98 f^{-1} to 2.80 f^{-1}. The latter measurements have been made for comparison with the 145° points in order to study the neutron's magnetic structure. In addition, they have been used to determine a form factor curve for the charge distribution of the deuteron.

III. APPARATUS AND DATA EVALUATION

The experiment was carried out using the electron beam from the Stanford Mark III linear accelerator; the methods and equipment have been described previously.[13] The target assembly had provision for moving liquid hydrogen, liquid deuterium, or solid carbon or polyethylene targets into the beam line.[14] Scattered electrons were momentum-analyzed with a 36-in. magnetic spectrometer and detected with a ten-channel counter array[15] placed along its focal plane. This detector has been used in earlier experiments, and the testing procedures[16] were the same. At intervals of a few hours during the experiment, data were taken at the flat part of the spectrum of electrons inelastically scattered from a carbon target; this information was used to determine the relative efficiency of each channel during the hydrogen and deuterium measurements and to detect improper operation of the equipment.

Absolute cross sections for electron-deuteron scattering were determined by comparison with the scattering from hydrogen; these cross sections have been previously measured.[1,2] Comparison runs were programmed at the same laboratory angle θ and incident electron energy E_0 as the deuteron points, except for the point at $\theta=145°$ and $E_0=260$ Mev. Here comparison runs were made both at $\theta=145°$, $E_0=260$ Mev, and at $\theta=35°$ with E_0 determined so the scattered electron energy was the same as that in the deuterium measurements.

There were uncertainties in the determination of the momentum of the scattered electrons from energy spread in the incident beam, energy loss in the targets and vacuum windows, finite acceptance angles in the magnetic spectrometer, and the finite momentum intervals defined by the detecting counter array. The final momentum resolution $P/\Delta P$ for all the points at $\theta=145°$ was $(1/6)\times10^3$, and for other points about $(1/9)\times10^3$. The final resolution settings were a compromise between obtaining usable counting rates and obtaining high resolution.

Electrons inelastically scattered from deuterium at the threshold for deuteron disintegration have a momentum only 2.2 Mev/c less than electrons scattered elastically. The scattering cross sections at this threshold are appreciable for many of the points measured. Because it was not practical to use higher momentum resolution in the present experiment, this disintegration process contributed some counting rate to a number of measured elastic peaks. These contributions were subtracted by determining experimentally the momentum resolution functions of the apparatus for the parameters of each elastic cross section and using this information to find the corrections to the elastic peak.

Each data point for given E_0 and θ was taken in a number of separate runs. Up to 30 ten-channel profiles of the scattered electron spectra were taken at each E_0 and θ, interspersed with hydrogen comparison and carbon normalization runs. The data were corrected for the relevant channel efficiencies, normalized to a given integrated incident beam flux, and combined by the method of least squares to give the momentum distribution of scattered electrons. The hydrogen comparison runs were evaluated in the same manner. The corrections to these spectra arising from electron

[13] R. Hofstadter, Revs. Modern Phys. **28**, 214 (1956).
[14] S. Sobottka, Stanford University Ph.D. dissertation, 1960 (unpublished).
[15] H. W. Kendall, Trans. Inst. Radio Engrs. **NS-5**, 190 (1958).
[16] G. R. Burleson and H. W. Kendall, Nuclear Phys. (to be published).

bremsstrahlung[17] in the target assembly and from the Schwinger correction[13,18] were made on an IBM-610 digital computer. The correction program used the observed scattered electron momentum distributions and other experimental parameters to develop momentum spectra which would be observed in the absence of radiative effects. Contributions to the elastic peaks from the poorly resolved inelastic spectra were subtracted using an iterative numerical technique to determine the resolution functions appropriate to each set of parameters (E_0,θ). The Jankus theory[5] was used to predict the shapes of the inelastic electron spectra near disintegration threshold. These curves were folded into approximate resolution functions determined from the partially resolved elastic peaks. The folds were normalized to the observed spectra at points about 5–8 Mev away from the elastic peaks so that the corrections would not be dependent on the magnitude of the Jankus cross section, and were then subtracted from the observed spectra. A second approximate resolution function was prepared by averaging the elastic peak resulting from this subtraction with the first approximate resolution functions. The folding and subtraction processes were repeated until the approximate resolution functions no longer changed by more than a few percent per iteration. In practice the procedure converged rapidly and rarely were more than two iterations necessary. From this analysis both the elastic and the inelastic scattering cross sections were obtained for the $\theta=145°$ and 500-Mev forward angle data.

These data plus the data at 190 and 200 Mev were analyzed independently by two different unfolding techniques to determine the elastic scattering cross sections alone. The errors introduced by differences between these results were about $1\frac{1}{2}\%$.

IV. MEASURED CROSS SECTIONS AND DISCUSSION OF ERRORS

The elastic electron-deuteron scattering cross sections as determined in this experiment are given in Table I. The first errors indicated are the total errors arising from all sources; the second are standard deviations arising from counting statistics only. The estimates of the errors are discussed below.

Contributions to the uncertainties in the measured cross sections arise from (1) uncertainty in the primary electron beam energy, (2) uncertainty in the densities of the target materials and possible impurities, (3) multiple scattering by the target material of electrons entering the spectrometer, (4) instability of the electronic equipment, (5) theoretical uncertainties in the radiative corrections.

The errors in the cross sections arising from (1) vary from 1% to 3% in the present experiment. The primary beam energy was known to about $\frac{1}{2}\%$ and was constant during the runs within less than 0.1%.

Table I. The measured cross sections for elastic electron scattering from deuterium. The errors given with the cross sections are standard deviations which include the counting errors and the other uncertainties discussed in the text.

θ (degrees)	E_0 (Mev)	q (f^{-1})	$d\sigma/d\Omega$[a] (10^{-32} cm^2/sr)	Counting error only[a] (10^{-32} cm^2/sr)
43.0	500	1.79	4.60 $\pm$0.45	$\pm$0.12
48.5	500	2.00	1.61 $\pm$0.15	$\pm$0.05
55.0	500	2.22	0.726 $\pm$0.07	$\pm$0.025
61.0	500	2.42	0.217 $\pm$0.02	$\pm$0.009
67.5	500	2.62	0.0947$\pm$0.010	$\pm$0.0041
75.0	500	2.82	0.0335$\pm$0.0337	$\pm$0.0015
60.0	200	0.98	39.2 $\pm$3.7	$\pm$1.4
70.0	190	1.07	20.1 $\pm$1.9	$\pm$0.7
90.0	200	1.37	3.53 $\pm$0.34	$\pm$0.12
105.0	200	1.51	1.08 $\pm$0.10	$\pm$0.05
145.0	179	1.60	0.185 $\pm$0.018	$\pm$0.007
145.0	204	1.80	0.0825$\pm$0.009	$\pm$0.004
145.0	228	2.00	0.0579$\pm$0.006	$\pm$0.003
145.0	260	2.25	0.0192$\pm$0.0023	$\pm$0.0012

[a] Errors quoted are one standard deviation.

Uncertainties in the target densities and the measured impurities contributed much less than 1% uncertainty to the measured cross sections. The liquid hydrogen and deuterium were operated with 3–5 psi pressure above atmospheric pressure.

Each deuterium point and its associated hydrogen normalization were measured at the same primary beam energy. In all cases the electrons passed through approximately 0.02 radiation length of scattering material in reaching the detector; there was no more than 10% variation in this between the hydrogen and deuterium runs. Errors introduced in the measured cross sections by multiple scattering effects were less than 1%.

Instabilities in the electronic equipment were not a serious problem. The determinations of each scattered electron spectrum were interrupted at intervals of about two hours for calibration by inelastic electron scattering from a carbon target. In each calibration run the efficiency of each channel was determined to within $\pm 3\%$. Few variations of these efficiencies were found beyond what was expected from counting statistics.

The radiative and bremsstrahlung correction spectra applied to the data were the same within a few percent for each deuterium peak and its associated hydrogen normalization. Although the theoretical approximations used in the derivations of these corrections are expected to be valid, errors in the corrections cancel to first order in the present experiment. The Bethe-Heitler and Schwinger corrections were applied to both the elastic and inelastic electron spectra.

The computer calculations were required to determine, on the average, only 7% of the area of each of the elastic peaks, the remainder of the area being free from inelastic contamination. Uncertainties in the

[17] W. Heitler, *The Quantum Theory of Radiation* (Oxford University Press, New York, 1954), 3rd ed., p. 377.

[18] D. R. Yennie and H. Suura, Phys. Rev. **105**, 1378 (1957).

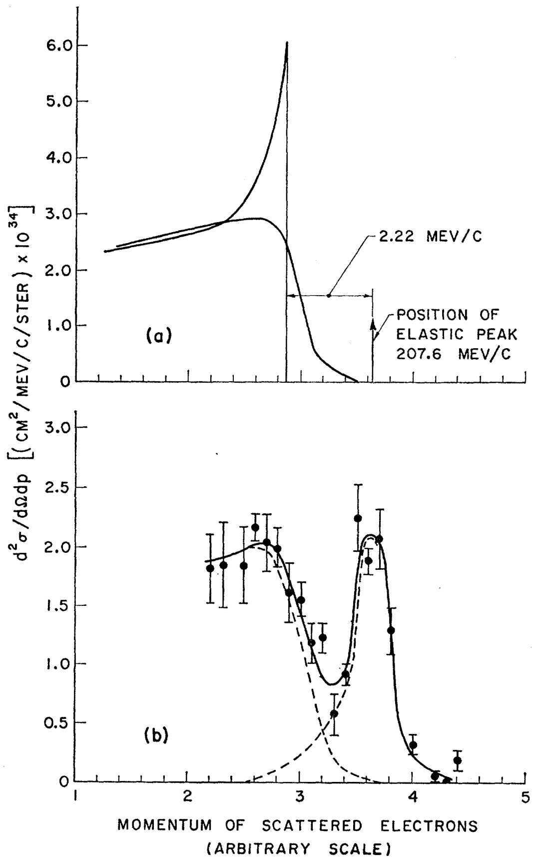

FIG. 1. Momentum spectrum of electrons scattered from deuterium: $E_0=260$ Mev, $\theta=145°$, $q=2.25\ \text{f}^{-1}$. (a) shows the inelastic spectrum predicted by Jankus multiplied by $|F_p|^2$ in the region of electrodisintegration threshold, and the same spectrum folded into the experimental momentum resolution function. (b) shows the observed spectrum, including the elastic scattering peak. The folded curve of (a) has been normalized to fit the experimental points. The errors indicated are standard deviations arising from counting statistics. The elastic cross section as determined from (b) is $\sigma=(1.92\pm0.25)\times10^{-34}$ cm²/sr. The statistical error in this measurement is larger than that in any other measured point.

elastic peak corrections were arbitrarily assumed to be 50% of the value of the corrections, which we feel is a conservative estimate, so the consequent errors in the cross sections were less than $3\frac{1}{2}$%. The data of the point of lowest elastic cross section measured in this experiment are shown in Fig. 1 with the predicted inelastic spectrum and the momentum resolution function.

The Jankus theory was used to predict the shape of the inelastic electron spectra from electron-deuteron scattering near the elastic peak. No use was made of the predicted absolute cross sections as these were found to disagree with the experimental results by as much as 30%. Measurements of these inelastic cross sections are discussed in more detail in a separate paper.[19] Improvements of the Jankus theory are not expected to alter significantly the shape of the inelastic electron distribution within approximately 6 Mev of the elastic peak.

Uncertainties in the measured absolute proton cross sections contribute to the uncertainties in the absolute values of our measured cross sections.[2] These normalization errors approximately cancel out in the evaluation of the ratio R. These errors are included in the total errors of the cross sections listed in Table I.

V. EVALUATION OF RESULTS

The evaluation of F_{2n}/F_p from the ratio R was carried out in an iterative fashion for the most efficient use of the data. The scattering cross sections for the points at angles of 105° or less were first assumed to result from pure charge scattering. This is a good approximation since the average magnetic contribution to these cross sections is only about 5%. A curve of the first approximation of $F_p^2(q)F_d^2(q)$ was then constructed from the charge scattering points using Eq. (2). It was then convenient to introduce a new ratio R' which is defined in the same way as R of Eq. (3) except that the denominator is unity. The ratio R' is constructed from R by replacing $d\sigma(\theta_S,E_2)/d\Omega$ by

$$[\sigma_{\text{N.S.}}(\theta_S,E_2)F_d^2(q)F_p^2(q)].$$

With the use of the 145° points and the approximate curve of $F_d^2(q)F_p^2(q)$, a first set of values of $F_{2n}(q)/F_p(q)$ was calculated from R'. From these values the magnetic contribution in the charge scattering cross sections was determined, a new curve of $F_d^2(q)F_p^2(q)$

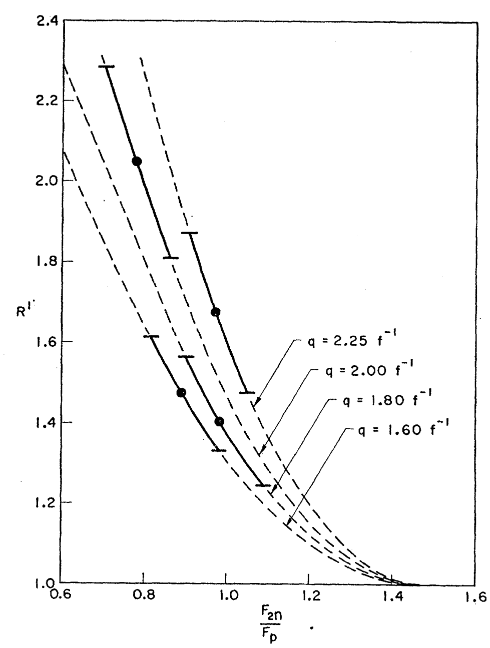

FIG. 2. The quantity R' as a function of F_{2n}/F_p with the four-momentum transfer q as a parameter. R' is defined in the text. The errors in the experimental points are standard deviations arising from counting statistics and are referred to the constant values of q at which the measurements were made.

[19] J. I. Friedman, H. W. Kendall, and P. A. M. Gram (to be published).

constructed, and a new set of values of F_{2n}/F_p calculated. Because of the small original magnetic contamination, this procedure converges very rapidly, and the second set of values was used. In constructing the curves of $F_d^2(q)F_p^2(q)$, linear least-square fits were made to points plotted on semilog paper. It was found that quadratic terms did not increase the goodness of fit. The theoretical predictions in this region of q are themselves approximately straight lines. A family of curves representing R' as a function of F_{2n}/F_p for $\theta_L = 145°$ is shown in Fig. 2. Each curve was calculated for a value of q at which a 145° measurement was made, and the experimental values are shown with their errors. The resulting curve of F_{2n}/F_p as a function of q is shown in Fig. 3. Since there is no statistically significant variation as a function of q, it is possible to form a weighted average of these values for the range of measurement. The average value of F_{2n}/F_p is $(0.91\pm0.05)\pm0.07$ for $1.6\leqq q\leqq 2.25$. The first error is an experimental error resulting from statistical error and the uncertainties discussed in Sec. IV. The additional uncertainty of ±0.07 is due to uncertainties in the analysis; these uncertainties are discussed in Sec. VII.

The values of F_2^s/F_1^s are shown as a function of q in Fig. 4. The average value of F_2^s/F_1^s in this range of q values is $(+0.06\pm0.09)\pm0.13$. The first error results from experimental uncertainties and the second from uncertainties in the analysis. It is of interest to compare the experimental results with the value of F_2^s/F_1^s at $q=0$ which is determined from the static properties of the proton and neutron. Since by definition $F_{2p}(0) = F_{1p}(0) = F_{2n}(0) = 1$, and $F_{1n}(0)=0$, the value of $F_2^s(0)/F_1^s(0) = -0.12$. Thus the results indicate that this ratio remains small up to a q of 2.25 f^{-1}.

VI. CORRECTIONS AND APPROXIMATIONS IN THE ANALYSIS

A. Omission of Terms

In Eq. (1) a number of small terms were omitted which appeared in the original Jankus result for G^2.

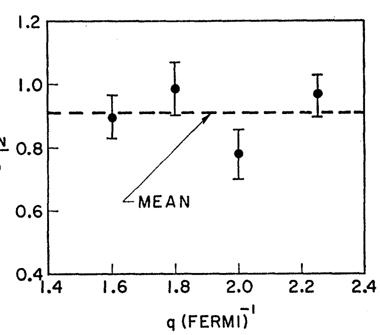

FIG. 3. Experimental values of F_{2n}/F_p as a function of the four-momentum transfer q. The errors are standard deviations from counting statistics. The errors associated with the mean value of F_{2n}/F_p are discussed in the text.

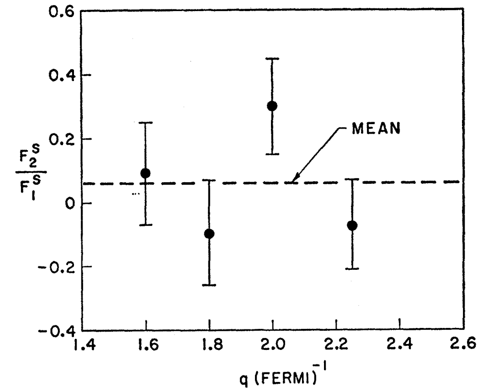

FIG. 4. Experimental values of F_2^s/F_1^s as a function of the four-momentum transfer q. The errors are standard deviations from counting statistics. The errors associated with the mean value of F_2^s/F_1^s are discussed in the text.

One of these is

$$F_Q{}^2 = \left[\int 2w(u - 8^{-\frac{1}{2}}w)j_2(\tfrac{1}{2}qr)dr\right]^2,$$

which corresponds to scattering from the deuteron's quadrupole moment. This term, which does not cancel out when the ratio R' is formed, is model dependent. For a Yukawa (II) model of the deuteron,[11] the correction resulting from this term decreases the average value of F_{2n}/F_p by 0.9%, whereas for a Gartenhaus model[11,20] of the deuteron there is a 2.5% decrease. Since nucleon-scattering results favor a repulsive-core potential and electron-deuteron scattering results are not inconsistent with this model, the latter correction has been chosen as the proper one.

The corrections due to the small magnetic terms which have been omitted are relatively model independent. For the Gartenhaus model of the deuteron this correction increases the average value of F_{2n}/F_p by 2.8%, and for a Yukawa (II) model[9] by 3.3%. The total correction due to all omitted terms is thus +0.3%, which is negligible compared to the experimental error. However, these terms introduce into the final results a model dependence of about 2% which is included in the total error.

B. Relativistic Corrections

In the Jankus calculation the deuteron is treated nonrelativistically. His expression for the cross section thus contains the three- rather than the four-momentum transfer. Since a completely relativistic result would be a function of the latter, the difference between the two must be regarded as introducing an uncertainty, and it is not clear which is the better quantity to use in the Jankus expression. It is customary to represent the nucleon form factors as functions of the four-momentum transfer, and we thus chose to use the four-momentum

[20] S. Gartenhaus, Phys. Rev. 100, 900 (1955).

transfer in the expression for the cross section also. Since these two quantities differ by only 0.3% at the highest q at which a determination of F_{2n}/F_p was made, this uncertainty is unimportant. There are, however, other uncertainties in the cross section, introduced by relativistic corrections. Blankenbecler has carried out two calculations which demonstrate how these may enter the result. The first[21] was a covariant calculation of elastic scattering from a point particle having the static properties of the deuteron. Terms appear in his expression for G^2 which have no counterpart in the Jankus result. The most significant of these are

$$-(\mu_p+\mu_n-\tfrac{1}{2})\frac{q^2}{4M^2c^2}+\frac{2}{3}\frac{q^2}{4M^2c^2}\times\left[\frac{E_0^2}{2(E_0+Mc^2)^2}\tan^2(\theta/2)\sin^2(\theta/2)\right].$$

The inclusion of these terms in the analysis would increase the average value of F_{2n}/F_p by only about 0.5%; hence they do not constitute a large uncertainty. In his second calculation,[22] Blankenbecler considered scattering from a deuteron model which consists of two bosons. The effects of the relativistic contraction of the final-state wave function and the retardation of the binding potential are taken into account. These effects result in an average correction to the cross section of about -10% for $1.6<q<2.25$. On the other hand, Bernstein,[23] in a calculation using a different model, found a 10% increase of the cross section arising from the retardation of the nuclear force. His calculation omits the contraction of the final-state wave function. The uncertainty in the cross section from these effects is thus estimated to be about $\pm10\%$, and the resulting uncertainty in the average ratio of F_{2n}/F_p is ±0.07 and in F_2^s/F_1^s is ±0.13.

C. Assumption that $F_{1n}=0$

The neutron's charge form factor F_{1n} has not been measured in the range of momentum transfers of the present experiment. Measurements of the low-energy electron-neutron interaction[24,25] indicate that the coefficient of the q^2 term in the expansion of F_{1n} around $q=0$ is very small or zero; and the assumption that $F_{1n}=0$ at higher q values has not led to any inconsistencies in the analyses of electron scattering experiments[26] at high momentum transfers. There is, however, no theoretical justification for this assumption, and the lack of information about F_{1n} introduces some uncertainty in the measured values of F_{2n}/F_p. Schiff,[27] in an analysis of previous measurements of small-angle elastic electron-deuteron scattering, has placed the following limits on F_{1n}:

$$1.0>|F_{1n}/F_{1p}+1|^2>0.8;\quad q<2.9.$$

With these limits, the resulting uncertainty in the average value of F_{2n}/F_p is ±0.02.

The analysis of the data also used the result from electron-proton scattering measurements that $F_{1p}\simeq F_{2p}$; the determination of F_{2n}/F_{2p}, however, is relatively insensitive to a possible error made in setting F_{1p} equal to F_{2p}. These are equal to within about $\pm20\%$,[1,2] and the resulting uncertainty introduced in F_{2n}/F_{2p} is less than ±0.005. As the analysis of this experiment in terms of F_2^s/F_1^s requires no prior information about F_{1n} and F_{2p}/F_{1p}, the determination of F_2^s/F_1^s is of course unaffected by these considerations.

VII. DISCUSSION OF FORM FACTOR RESULTS

The result for the average value of F_{2n}/F_p for $1.6\leqq q\leqq 2.25$ is

$$(0.91\pm0.05)\pm0.07.$$

The first error listed is due to errors in the measurements, and the second is the error introduced by uncertainties in the analysis.

The conclusion that may be drawn from this experiment is essentially the same as that suggested by the previous experiments: that the neutron and proton anomalous magnetic moment distributions are quite similar, but possibly not identical. The present experiment compares the two distributions down to a distance of about 0.5 f, whereas other experiments[3,4,8,28,29] have investigated them down to somewhat smaller distances. It is of great interest to know whether the neutron and proton show any quantitative structure differences. At the present time, however, the uncertainties in all the experiments preclude the possibility of interpreting the deviations observed as being quantitatively significant.

The measured average value of F_2^s/F_1^s for $1.6\leqq q\leqq 2.25$ from the present experiment is

$$(+0.06\pm0.09)\pm0.13,$$

where the errors have the same meanings as those given above. The results indicate that F_2^s/F_1^s remains small at q values as high as 2.25 f^{-1}.

VIII. CHARGE FORM FACTOR OF THE DEUTERON

A number of measurements have been made by McIntyre *et al.*[30] of the deuteron's charge form factor.

[21] This calculation is discussed in reference 3, p. 259.
[22] R. Blankenbecler, Stanford University Ph.D. dissertation, 1958 (unpublished).
[23] J. Bernstein, Phys. Rev. **104**, 249 (1956).
[24] D. J. Hughes, J. A. Harvey, M. D. Goldberg, and M. J. Stafne, Phys. Rev. **90**, 497 (1953).
[25] E. Melkonian, B. M. Rustad, and W. W. Havens, Jr., Bull. Am. Phys. Soc. **1**, 62 (1956).
[26] R. Hofstadter, F. Bumiller, and M. R. Yearian, Revs. Modern Phys. **30**, 482 (1958).
[27] L. I. Schiff, Revs. Modern Phys. **30**, 462 (1958).
[28] S. Sobottka, Phys. Rev. **118**, 831 (1960).
[29] G. Ohlsen (to be published).
[30] References to previous measurements are given in reference 11.

The present experiment provides an additional measurement, made with improved equipment, and extends the latest work on the subject to higher momentum transfers. From the definition of this form factor F_d given in Sec. II, it can be seen that these measurements can yield information about the deuteron's wave function and consequently about the neutron-proton potential. With the use of the previously measured[1,2] values of the proton's form factor F_p, the measurements of $F_d^2F_p^2$ discussed in Sec. VI can be compared with the predictions of various deuteron models. This comparison is shown in Fig. 5, in which are also shown the theoretical curves calculated from three models which have been extensively discussed in the previous work. The present results, which extend to a four-momentum transfer of 2.8 f^{-1}, favor a Gartenhaus potential. This is in agreement with the results of McIntyre and Burleson,[11] who have carried out measurements up to a q of 2.4 f^{-1}. Blankenbecler,[22] however, has pointed out that relativistic corrections affect the scattering in much the same way as a static repulsive core. His covariant two-boson model, which uses a Hulthén wave function, agrees quite well with the static nonrelativistic repulsive-core model in predicting electron-deuteron scattering. Thus the results indicate that a static nonrelativistic treatment of the problem requires a repulsive core, but it not clear at the present time to what extent the repulsive core appears in the actual force or results from relativistic corrections.

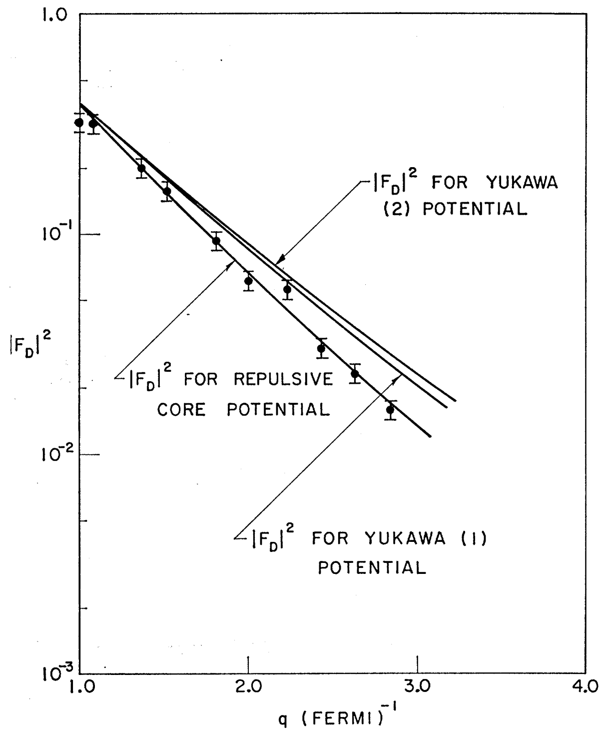

FIG. 5. Experimental values of $|F_d|^2$ as a function of the four-momentum transfer q, where F_d is the charge form factor of the deuteron. The solid curves are predictions for a repulsive core and for two Yukawa potentials describing the bound n-p system. The errors in the experimental points from counting statistics and other uncertainties have been estimated to be about $\pm 10\%$; a further discussion of the errors is included in the text. In the calculation of the experimental values of the deuteron's charge form factor, no attempt was made to eliminate the contribution from the quadrupole moment since it is model dependent. Consequently, the theoretical curves also contain this contribution and do not strictly represent the form factor of the symmetrical part of the deuteron's charge distribution. The quadrupole contaminations are given in J. A. McIntyre and S. Dhar, Phys. Rev. **106**, 1074 (1957).

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