Associated Production in Proton-Proton Collisions

E. FERRARI
Istituto di Fisica dell'Università, Roma, Italia, and Istituto Nazionale di Fisica Nucleare, Sezione di Roma, Italia

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The simplest reactions of associated production in proton-proton collisions are investigated with the aid of two simple models, which consider as relevant only the contribution due to the exchange of one intermediate π meson or K meson, respectively. The values of the total cross sections predicted by these models are calculated at various energies up to 3.5 Gev of the incident proton is the lab system. These cross sections are related to the experimental cross sections for simpler processes, namely associated production by pions (first model) and K^+-nucleon scattering (second model). Both models give the same order of magnitude for the cross sections for all reactions (10^{-2} mb at lower energies, 10^{-1} mb at higher energies), except for Σ^0 production, where the results predicted by the two models differ by a factor of 10. Some of the calculated values are affected by large errors, due to the uncertainty of the experimental data which are used.

1

THE simplest reactions of associated production in proton-proton collisions are

$$p+p \to \Lambda^0+K^++p, \tag{1}$$

$$p+p \to \Sigma^++K^0+p, \tag{2}$$

$$p+p \to \Sigma^++K^++n, \tag{3}$$

$$p+p \to \Sigma^0+K^++p. \tag{4}$$

Process (1) has been already investigated[1] with the aid of two simple models, in which it was assumed that the reaction takes place essentially through the exchange of one intermediate boson (π or K meson). Such a process is represented graphically by the diagrams of Fig. 1. A rather intuitive picture of these models can be given by thinking the process to take place through the interaction of the incident nucleon with one meson of the π-meson and K-meson "cloud" of the target nucleon. In such a way the amplitude for any of the processes considered can be related to the physical amplitudes of the "partial" processes involving the particle exchanged: This can be done by means of a procedure of analytic continuation from the physical into the unphysical region, as pointed out by Chew and Low.[2] In the present case the intermediate particle is not on the mass shell and this will be accounted for in the course of the calculations, except for the fact that the form factors in the left-hand side vertex (in the diagram)

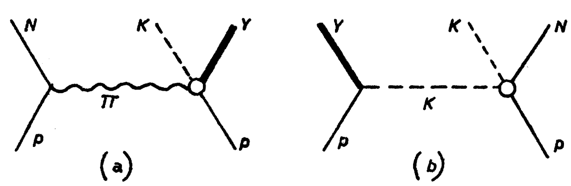

FIG. 1. Diagrams representing any of the processes from (1) to (4), (a) according to the first model; (b) according to the second model. p = incoming protons; N = outgoing nucleon; Y = outgoing hyperon.

as well as in the meson propagator will be put equal to 1 throughout, as if this particle were a real particle. Such an approximation is a good one for low values of the momentum transfer of the vertex: For higher momentum transfer it may not, but upon complete integration over the momentum transfer its influence should be further reduced, if the form factors are supposed not to vary too rapidly. Of course, these models are less reliable at very high energies, in part because of this approximation, in part because of the impossibility of neglecting higher order contributions.

In this note we shall apply the models to all the four reactions from (1) to (4), and we shall calculate the total cross section predicted at various energies in the range from 2.0 to 3.5 Gev of the incident proton in the lab system. The total cross section will be expressed in terms of the total cross sections for simpler processes, namely for associated production from pions, for the model represented by graph 1(a) (which will be called "first" model) and K^+-nucleon elastic scattering for the model represented by graph 1(b) (which will be called "second" model).

The present experimental uncertainty about many of the reactions involved will induce large errors in the numerical computations, and some results will be significant only as order of magnitude evaluations. We shall also give a prescription for a direct evaluation of the branching ratios among processes (2), (3), and (4) in competition. In all the following considerations the hypothesis of charge independence in strong interactions will be supposed to hold.

2

We will first study the first model with the exchange of a π meson. Processes (1), (2), (3), and (4) are then described by graphs (a), (b), (c), and (d) of Fig. 2.

All other contributions are neglected. The partial processes of interest for the evaluation of the cross sections are, respectively,

for (a), $$\pi^0+p \to \Lambda^0+K^+; \tag{5}$$

for (b), $$\pi^0+p \to \Sigma^++K^0; \tag{6}$$

[1] E. Ferrari, Nuovo cimento **15**, 652 (1960).
[2] G. F. Chew and F. E. Low, Phys. Rev. **113**, 1640 (1954).

for (c), $\pi^+ + p \rightarrow \Sigma^+ + K^+$; (7)

for (d), $\pi^0 + p \rightarrow \Sigma^0 + K^+$. (8)

By the charge independence hypothesis, the amplitudes for reactions (5), (6), and (8) can be obtained from the ones for process (7) and for the following reactions

$$\pi^- + p \rightarrow \Lambda^0 + K^0, \quad (9)$$

$$\pi^- + p \rightarrow \Sigma^0 + K^0, \quad (10)$$

$$\pi^- + p \rightarrow \Sigma^- + K^+, \quad (11)$$

about which we have a certain amount of experimental information.

We shall write the differential cross section for any of the processes considered in the form

$$d\sigma = (2\pi/kW)\,|\langle T\rangle|^2 \delta(p_1+k_1-p_2-k_2-q_2)\delta(p_2^2+Y^2) \times \delta(k_2^2+M^2)\delta(q_2^2+m^2)d^4p_2d^4k_2d^4q_2, \quad (12)$$

p_1, k_1, and p_2, k_2, q_2 are the 4-momenta of the incoming protons and of the outgoing hyperon, nucleon, and meson, respectively. $W^2 = -(p_1+k_1)^2$ is the square of the total energy in the center-of-mass system and $k = (\frac{1}{4}W^2 - M^2)^{\frac{1}{2}}$ is the c.m. momentum of the initial protons. M is the nucleon mass, Y the hyperon (Λ or Σ) mass, m the K-meson mass, μ the pion mass, and W is given in terms of the kinetic energy T of the incident proton in the lab system by $W^2 = 4M^2(1+T/2M)$.

The contributions to the matrix element $\langle T\rangle$ come from two graphs of the same type, obtained one from the other by exchanging the momenta of the incoming protons ($p_1 \leftrightarrow k_1$). If we denote by $\langle T_1\rangle$ and $\langle T_2\rangle$ the matrix elements associated to these two diagrams, antisymmetrization of the initial state requires $\langle T\rangle = 2^{-\frac{1}{2}}(\langle T_1\rangle - \langle T_2\rangle)$, i.e., $|\langle T\rangle|^2 = \frac{1}{2}(|\langle T_1\rangle|^2 + |\langle T_2\rangle|^2 - 2\,\mathrm{Re}\langle T_1\rangle\langle T_2\rangle^*)$.

The total cross section will therefore consist of three parts: Upon complete integration the first two parts give the same contribution, which will be denoted by σ_A:

$$\sigma_A = \frac{1}{8\pi}\left(\frac{G^2}{4\pi}\right)\frac{1}{(kW)^2}\int_{u_{\min}}^{u_{\max}} 2\kappa u^2 \sigma_0(u) I(u)\,du. \quad (13)$$

$G^2/4\pi$ is the coupling constant of the pion-nucleon vertex and $\sigma_0(u)$ is the total cross section at total c.m. energy u for the partial process [from (5) to (8)] associated with the reaction considered. The remaining factors, which play the role of a weight function, will be written explicitly at the end of this paper.

The contribution of the interference term can be easily calculated only if it is assumed that reactions from (5) to (8) take place only in one definite state of angular momentum (in other words, only one phase shift is supposed to be relevant, the same one for any isotopic spin state of the same reaction). Under this simplifying hypothesis, the contribution σ_{int} of the interference term can be put in the form

$$\sigma_{\mathrm{int}} = -\frac{1}{8\pi}\left(\frac{G^2}{4\pi}\right)\frac{1}{(kW)^2}\int_{u_{\min}}^{u_{\max}} 2\kappa u^2 \sigma_0(u) I_{\mathrm{int}}(u)\,du, \quad (14)$$

with

$$\sigma_{\mathrm{tot}} = \tfrac{1}{2}(2\sigma_A - \sigma_{\mathrm{int}}). \quad (15)$$

$I_{\mathrm{int}}(u)$ is given at the end of this paper under the assumption that the relevant phase shift is the S phase shift. The expression for I_{int} is somewhat different from the analogous expression of reference 1. This is due to a corrective extra term, which takes into account the fact that the intermediate particle is not on the mass shell, and arises from the calculation of σ_A: it has, however, been included in σ_{int} since it is phase-shift-dependent. In addition, in reference 1 the initial state was not antisymmetrized. Correspondingly, also the numerical values of σ_{tot} given in reference 1 for process (1) turn out to be more or less changed.

We would point out that since in (14) the experimental cross section σ_0 is again introduced, the hypothesis of the presence of only one phase shift affects only the structure of the weight function I_{int}. We think that in the results which will be given, this source of error is negligible compared to the presumably much larger one due to the experimental uncertainties.

Formulas of the type (13), (14), and (15) hold for all the reactions considered: Only the values of G^2 and σ_0 will in general be different. Denoting, e.g., by $\sigma(1)$ the total cross section for reaction (1), we get

Reaction	Coupling constant to be used	Cross section to be used
(1)	$G^2 = G^2(pp\pi^0)$	$\sigma_0 = \sigma(5) = \frac{1}{2}\sigma(9)$
(2)	$G^2 = G^2(pp\pi^0)$	$\sigma_0 = \sigma(6) = \sigma(10)$
(3)	$G^2 = G^2(pn\pi^+) = 2G^2(pp\pi^0)$	$\sigma_0 = \sigma(7)$
(4)	$G^2 = G^2(pp\pi^0)$	$\sigma_0 = \sigma(8) = \frac{1}{2}[\sigma(7) + \sigma(11) - \sigma(10)]$

The energy behavior of σ_0 is needed in an interval of the energy of the π meson in the lab system from the

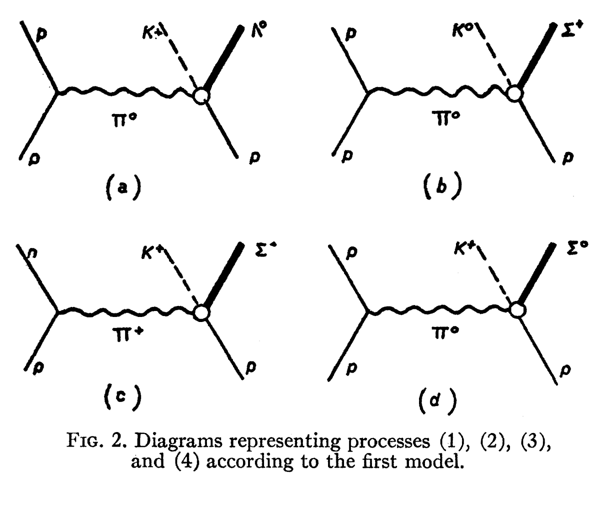

FIG. 2. Diagrams representing processes (1), (2), (3), and (4) according to the first model.

TABLE I. Total cross sections (in mb) predicted by the first model. T=kinetic energy of the incoming proton in the lab system.

T (Mev)	$p+p\to\Lambda^0+K^++p$	$p+p\to\Sigma^++K^0+p$	$p+p\to\Sigma^++K^++n$	$p+p\to\Sigma^0+K^++p$
2000	0.0165±0.0033	0.0070±0.0013	0.011	0.0018
2200	0.029 ±0.005	0.020 ±0.005	0.025	0.0041
2500	0.041 ±0.009	0.036 ±0.011	0.047	0.0078
2850	0.053 ±0.012	0.052 ±0.016	0.069	0.011
3000	0.057 ±0.014	0.062 ±0.019	0.081	0.014

threshold to a maximum energy E_m depending on T.[3]

From the present experimental data about the reactions (7), (9), (10), and (11),[4] we have evaluated the total cross sections given by this model for processes (1), (2), (3), and (4) at lab proton energies of 2.0, 2.2, 2.5, 2.85, and 3.0 Gev. The present experimental information is not sufficient to permit extrapolation to higher energies. In the last paragraph of this paper the calculated values have been collected in Table I. The large errors for processes (1) and (2) are a consequence of the large errors in the experimental cross sections that we have used: For processes (3) and (4) a sensible evaluation of the error is not possible and the values that we give should be taken as order-of-magnitude estimates. From the data listed in the table the branching ratios between the various processes in competition can be deduced: For Σ production a rough direct evaluation for such ratios is also possible. In fact, the variation of the cross sections for reactions (7), (10), and (11) with the energy is rather slow: Also, the weight functions $I(u)$ and $I_{\text{int}}(u)$ are very smooth functions. One can then extract $\sigma_0(u)$ from integrals (13) and (14) and take an average value in the requested interval of energy. One then gets

$$\sigma(3):\sigma(2):\sigma(4)\simeq 2\bar{\sigma}(7):\bar{\sigma}(6):\bar{\sigma}(8) \\ =2\bar{\sigma}(7):\bar{\sigma}(10):\tfrac{1}{2}[\bar{\sigma}(11)+\bar{\sigma}(7)-\bar{\sigma}(10)]. \quad (16)$$

In the region from 2.5 to 3.0 Gev we then have (assuming $\bar{\sigma}(7)\simeq 0.15$ mb, $\bar{\sigma}(10)\simeq 0.25$ mb, $\bar{\sigma}(11)\simeq 0.20$ mb)

$$\sigma(3):\sigma(2):\sigma(4)\cong 6:5:1, \quad (17)$$

that is

$$\text{yield }\Sigma^+/\text{yield }\Sigma^0\simeq 11. \quad (18)$$

In practice Σ^0's will not be distinguishable from Λ^0's; for $T\sim 3.0$ Gev we get

$$\text{yield }\Sigma^+/\text{yield }(\Sigma^0+\Lambda^0)\simeq 2, \\ \text{yield }K^+/\text{yield }K^0\simeq 2.5. \quad (19)$$

The evaluation (16) of the branching ratios should practically be insensitive to the approximation of only one phase shift in the interference term.

3

The second model, with the exchange of a K meson, describes the processes (1), (2), (3), and (4) through the graphs (a), (b), (c), and (d) of Fig. 3.

The secondary reactions are

$$\text{for (a), (d),} \quad K^++p\to K^++p; \quad (20)$$

$$\text{for (b),} \quad K^0+p\to K^0+p; \quad (21)$$

$$\text{for (c),} \quad K^0+p\to K^++n. \quad (22)$$

By charge independence considerations, reactions (21) and (22) can be replaced with the following ones:

$$K^++n\to K^++n, \quad (23)$$

$$K^++n\to K^0+p. \quad (24)$$

The mathematical procedure is the same as before: Under the hypotheses discussed, formulas (13), (14), and (15) still hold, and the corresponding expressions for the weight functions are given at the end of this paper. The values of G^2 and σ_0 to be used are the following:

Process	Coupling constant to be used	Cross section to be used
(1)	$G^2=G^2(p\Lambda^0K^+)$	$\sigma_0=\sigma(20)$
(2)	$G^2=G^2(p\Sigma^+K^0)=2G^2(p\Sigma^0K^+)$	$\sigma_0=\sigma(21)=\sigma(23)$
(3)	$G^2=G^2(p\Sigma^+K^0)$	$\sigma_0=\sigma(22)=\sigma(24)$
(4)	$G^2=G^2(p\Sigma^0K^+)$	$\sigma_0=\sigma(20)$

σ_0 must be known over an interval of the energy of the incident K-meson in the lab system from zero to a maximum value E_m dependent on T.[5]

The values of the total cross sections given by this model are listed in Table II, on the basis of the present experimental information.[6] Because of lack of high-

[3] E_m is fixed through the relation $u=W-M$ for both Λ and Σ production. For $T\lesssim 4000$ Mev the dependence of E_m from T is given (within 10 Mev) by the approximate formula $E_m=0.674T-315$ (E_m, T in Mev).

[4] We took the most part of the data from the *1958 Annual International Conference on High-Energy Physics at CERN*, edited by B. Ferretti (CERN Scientific Information Service, Geneva, 1958), p. 148, supplemented with recent work by A. R. Erwin, Jr., J. K. Kopp, and A. M. Shapiro, Phys. Rev. **115**, 669 (1959), and F. S. Crawford, Jr., R. L. Douglas, M. L. Good, G. R. Kalbfleisch, M. L. Stevenson, and H. K. Ticho, Phys. Rev. Letters **3**, 394 (1959). Some values which were in strong disagreement with the others have not been considered.

[5] E_m is fixed through the relation $u=W-Y$, where Y is the hyperon (Λ or Σ) mass. For $T<4000$ Mev the following approximate expressions can be used: $E_m=0.6152T-984$; $E_m=0.5937T-1065$ (E_m, T in Mev) for Λ and Σ production, respectively. These formulas give E_m with an approximation less than 15 Mev: however, they are not to be used just above the threshold.

[6] For K^+-p scattering and some data on K^+-n interactions, we have referred to L. Alvarez, Kiev Conference Report on the Interaction of Strange Particles, 1959 (unpublished). The latest data on K^+-n scattering have been taken from D. Fournet-Davis, N. Kwak, and M. F. Kaplon, Phys. Rev. **117**, 846 (1960). Some

TABLE II. Total cross sections (in mb) predicted by the second model. T=kinetic energy of the incoming proton in the lab system.

T (Mev)	$p+p\rightarrow\Lambda^0+K^++p$	$p+p\rightarrow\Sigma^++K^0+p$	$p+p\rightarrow\Sigma^++K^++n$	$p+p\rightarrow\Sigma^0+K^++p$
2000	0.021±0.004	0.0024±0.0028	0.0039±0.0012	0.0054±0.0011
2200	0.043±0.006	0.008 ±0.010	0.015 ±0.005	0.0173±0.0032
2500	0.085±0.012	0.011 ±0.024	0.050 ±0.012	0.047 ±0.008
2850	0.145±0.018		0.12	0.096 ±0.013
3000	0.170±0.021		0.16	0.119 ±0.015
3500	0.266±0.024			0.206 ±0.025

energy data (above 300 Mev) and the very large experimental errors in K^+-n scattering (especially elastic scattering),[7] we cannot give definite results for reactions (2) and (3) and even attempt an order-of-magnitude calculation for the higher energies. Here also those data that we report without error are only indicative. In the calculations we assume $G^2(p\Lambda^0K^+)/4\pi = G^2(p\Sigma^0K^+)/4\pi=1.5$; however, these coupling constants only enter as multiplicative factors and any variation of their values can be readily taken into account.

The argument about the evaluation of the branching ratios yields:

$$\sigma(3):\sigma(2):\sigma(4)\simeq\bar{\sigma}(24):\bar{\sigma}(23):\tfrac{1}{2}\bar{\sigma}(20), \qquad (25)$$

the present uncertainty about the experimental data, however, does not allow us to make sensible predictions for the relative yields.

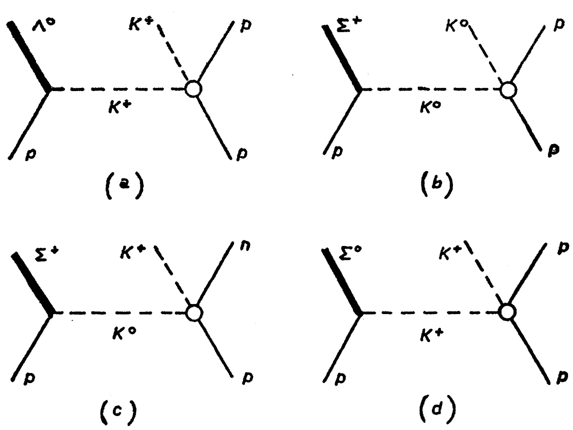

FIG. 3. Diagrams representing processes (1), (2), (3), and (4) according to the second model.

4. DISCUSSION

The calculated numerical values for the total cross section are summarized in Tables I and II.

At low energies the cross sections obtained by the two models are of the same order of magnitude: At higher energies there are some remarkable differences. Both models predict a rapid increase of the cross section with energy: on the average, the second model gives a larger contribution than the first. If we take into account the large experimental errors, we see that only for one reaction [namely reaction (4)] does the result appear to be strongly dependent on the model. Also, for Λ^0 production at energies ~3000 Mev there is a remarkable difference between the two models: In the second the production of neutral hyperons is enhanced.

A better founded theory would perhaps include the contribution of the two types of graphs with both π and K exchanged: We would roughly expect values for the total cross sections equal to the sum of the corresponding two values listed in Tables I and II. This amounts to neglecting the interference terms between different graphs: There are, however, no valid arguments to support this hypothesis. A more rigorous calculation of these terms is impossible at present for lack of experimental knowledge.

ACKNOWLEDGMENTS

We thank Professor R. Gatto for his interest in this work. We are also indebted to Professor A. M. Thorndike for stimulating correspondence which encouraged us to the present investigation.

5. NOTATION AND FORMULAS

$$I(u)=\ln\left(\frac{1+\alpha}{1-\alpha}\right)-\frac{2A^2}{\Gamma}\left(\frac{\alpha}{1-\alpha^2}\right),$$

$$I_{\text{int}}(u)=\frac{1}{2u(E+M)}\bigg\{-12kk'+\frac{1}{\Gamma}[W^2(u+M)(m_0-M) - (u^2-M^2)(m_0^2-M^2)+M(u+m_0)(u^2-m_0^2) + 2\Gamma(m_0-M)^2-\nu^2(4\Gamma-\nu^2)]\ln\left(\frac{1+\alpha}{1-\alpha}\right)\bigg\}.$$

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earlier data on K^+-n interaction for energies larger than 100 Mev, contained in the CERN report, are not consistent with later determinations and have not been considered.

[7] After examination of all data relative to reactions (23) and (24) we have chosen the following determinations for $\sigma(23)$ and $\sigma(24)$:

$\sigma(24)$		$\sigma(23)$	
0–100 Mev	5.0±1.5 mb	0–100 Mev	3.0±3.5 mb
100–200 Mev	6.0±2.0 mb	100–200 Mev	3.4±3.9 mb
200–300 Mev	9.0±2.0 mb	200–250 Mev	1.2±4.5 mb

First model:

$$u^2=-(p_2+q_2)^2;\quad u_{\min}=Y+m;\quad u_{\max}=W-M;$$
$$E=(u^2+M^2-\mu^2)/2u;\quad \kappa=(E^2-M^2)^{\frac{1}{2}}=(1/2u)[u^4-2(M^2+\mu^2)u^2+(M^2-\mu^2)^2]^{\frac{1}{2}};$$
$$A^2=\mu^2;\quad m_0=M;\quad \nu=\mu;\quad w'=(1/2W)(W^2+M^2-u^2);$$
$$k'=(w'^2-M^2)^{\frac{1}{2}};\quad \Gamma=w'W-2M^2+\mu^2;\quad \alpha=2kk'/\Gamma.$$

Second model:

$$u^2=-(k_2+q_2)^2;\quad u_{\min}=M+m;\quad u_{\max}=W-Y;$$
$$E=(u^2+M^2-m^2)/2u;\quad \kappa=(E^2-M^2)^{\frac{1}{2}}=(1/2u)[u^4-2(M^2+m^2)u^2+(M^2-m^2)^2]^{\frac{1}{2}};$$
$$A^2=m^2-(Y-M)^2;\quad m_0=Y;\quad \nu=m;\quad w'=(1/2W)(W^2+Y^2-u^2);$$
$$k'=(w'^2-Y^2)^{\frac{1}{2}};\quad \Gamma=w'W-Y^2-M^2+m^2;\quad \alpha=2kk'/\Gamma.$$

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Experimental Study of the Magnetic Structure of the Neutron*

J. I. FRIEDMAN, H. W. KENDALL, AND P. A. M. GRAM
High-Energy Physics Laboratory, Stanford University, Stanford, California

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A measurement of the ratio of the magnetic form factor of the neutron to that of the proton has been carried out by comparing large- and small-angle elastic electron-deuteron scattering at constant four-momentum transfers. The experimental result for the average value of the ratio in the range of momentum transfers from 1.6 f^{-1} to 2.25 f^{-1} is $F_{2n}/F_p=(0.91\pm0.05)\pm0.07$; the first error is a standard deviation arising from experimental uncertainties, and the second from theoretical uncertainties in the analysis. Measurements of the ratio of the nucleon isotopic scalar form factors have also been obtained from this experiment. The average value of $F_2{}^s/F_1{}^s$ for the same range of momentum transfers has been found to be $(+0.06\pm0.09)$ ±0.13. The small-angle scattering data have been used to determine the charge form factor of the deuteron in the range of momentum transfers from 0.98 f^{-1} to 2.8 f^{-1}. The results are consistent with a repulsive-core model of the deuteron.

I. INTRODUCTION

A DETAILED knowledge of nucleon structure provides an important test for any meson theory. While measurements of electron-proton scattering[1,2] give direct information about the proton's charge and magnetic moment distributions, the impossibility of studying electron scattering from free neutrons makes it much more difficult to obtain comparable information about the neutron. Until recently, two types of experiments have provided information about the magnetic structure of the neutron. The first was the measurement[3,4] of inelastic electron-deuteron scattering cross sections which by the impulse approximation[5–7] can be related to free neutron and proton cross sections. The second was a measurement[8] of the four-momentum transfer dependence of electroproduction of pions from hydrogen at the (3,3) resonance. With the use of dispersion relations this cross section has been shown[9] to depend on the neutron and proton magnetic form factors. The present experiment uses measurements of a different process to give information about the neutron's magnetic structure. The method consists of comparing elastic electron-deuteron scattering at large and small angles for a constant four-momentum transfer. The ratio of these cross sections, which to a good approximation does not depend on the deuteron model, is a function of the neutron and proton magnetic form factors and thus can provide a comparison of their magnetic structures. In particular, the quantity measured here is the ratio of the neutron and proton anomalous magnetic form factors.

It is also of theoretical interest to study the q de-

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