Measurement of the Gamma-Ray Spectrum from Radiative Muon Decay*†

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The energy spectrum of the gamma rays emitted by inner bremsstrahlung in the muon decay process, $\mu^+ \rightarrow e^+ + \nu + \bar{\nu} + \gamma$, has been measured by means of a large sodium iodide crystal. Only those gamma rays which were time-coincident with the decay electrons were recorded. These decay electrons were detected in a scintillation counter telescope adjoining the crystal counter. The counter geometry permitted detection of events with angles between the electron and gamma ray in the range from 0° to 26°. The muons were obtained by stopping a positive pion beam in a block of material. By measuring the gamma-ray spectra from muons originating first in carbon and then aluminum, a correction could be made for bremsstrahlung externally produced by the decay electrons. The measured spectrum is in agreement with that predicted by the theory with an average ratio of 1.02±0.10.

I. INTRODUCTION

A NUMBER of theoretical calculations of the gamma-ray spectrum to be expected from the reaction

$$\mu^+ \rightarrow e^+ + \nu + \bar{\nu} + \gamma \qquad (1)$$

have been made.[1] The most recent of these[2] emphasizes the fact that if one uses the experimentally well-determined properties of the nonradiative decay mode of the muon, then the gamma-ray spectrum of the reaction of Eq. (1) is completely specified by the theory. However, it should be pointed out that the expected shape of the bremsstrahlung spectrum will be dominated by the electromagnetic nature of the reaction. Hence, the details of the weak interaction producing the muon decay are only slightly reflected in the bremsstrahlung spectrum, and the measurement of this spectrum can only serve as a check on the gross features of the theory. Although the gamma rays in Eq. (1) have been observed by several workers,[3] no careful study of the reaction rate or energy distribution of the gamma rays has been reported. The present experiment is an investigation of both of these points.

II. EXPERIMENTAL METHOD

The experimental arrangement for studying the radiative muon decay mode is shown in Fig. 1. A low-energy π^+ meson beam is brought to rest in a carbon block, oriented at an angle of 45° to the direction of the beam. This carbon block is viewed by an array of three annular scintillation counters and a large sodium iodide crystal, which is protected by a lead collimator and an anticoincidence counter. The pions provide a source of unpolarized muons. The radiative decay of these muons is detected by registering a coincidence between the decay electron in the three ring counters and a gamma ray in the sodium iodide crystal. This coincidence is then used to open a linear gate, which permits the pulse height observed in the sodium iodide counter to be registered in a fifty-channel pulse-height analyzer. The pulse-height spectrum obtained in this way is a measure of the desired gamma-ray energy spectrum. The total number of decay electrons was recorded in the annular counter telescope to determine the rate of muon decays

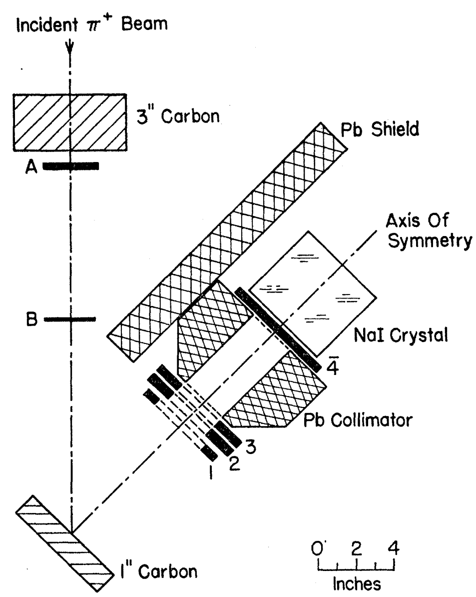

Fig. 1. The geometrical arrangement of the apparatus.

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[1] A. Lenard, Phys. Rev. **90**, 968 (1953); R. E. Behrends, R. J. Finkelstein, and A. Sirlin, Phys. Rev. **101**, 866 (1956). T. Kinoshita and A. Sirlin, Phys. Rev. **107**, 593 (1957); Phys. Rev. **108**, 844 (1957); Phys. Rev. Letters **2**, 177 (1959). B. Ferretti, Nuovo cimento **6**, 999 (1957); R. Gatto and G. Lüders, Nuovo cimento **7**, 806 (1958); N. Tzoar and A. Klein, Nuovo cimento **8**, 482 (1958); R. H. Pratt, Phys. Rev. **111**, 649 (1958); C. Fronsdal and H. Überall, Phys. Rev. **113**, 654 (1959); S. G. Eckstein and R. H. Pratt, Ann. Phys. **8**, 297 (1959).

[2] Fronsdal and Überall, reference 1.

[3] J. Ashkin, T. Fazzini, G. Fidecaro, N. H. Lipman, A. W. Merrison, and H. Paul, Nuovo cimento **14**, 1266 (1959); R. R. Crittenden, W. D. Walker, and J. Ballam, Bull. Am. Phys. Soc. **4**, 324 (1959); J. Lee and N. P. Samios, Phys. Rev. Letters **3**, 55 (1959).

accompanied by gamma rays relative to the nonradiative muon decay.

The primary source of background in this experiment was "external" bremsstrahlung of the decay electrons as they traverse the carbon block in which they originate. To evaluate this background, the carbon was replaced by aluminum and another gamma-ray spectrum recorded. The difference in the carbon and aluminum spectra is attributed to "external" bremsstrahlung and an appropriate correction was applied to the carbon data before a comparison with the theory was attempted. Although a number of other systematic corrections to the data will be mentioned below, this method of correction for the background is considered to be a fundamental feature of the experimental method.

III. REDUCTION OF THE THEORETICAL GAMMA-RAY SPECTRUM

The reduction of the theoretical spectrum to represent the experimental conditions has been carried out with the aid of the relations given by Fronsdal and Überall.[2] The relative rate of radiative muon decays to nonradiative decays, which is to be compared with the result of the experiment, is

$$R_{\mathrm{theo}}(y')\Delta y' = \frac{e^2\Delta y'}{2^6\pi^2} \frac{\displaystyle\int dy T(y,y')\frac{C(y)}{y}\left\{\sum [\Delta\Omega_\gamma(\theta)]_{\mathrm{eff}} \int_0^{2(1-y)/(2-\Delta y)} dx\epsilon(x) n_V(x,y,\Delta)\right\}}{\displaystyle\int_0^1 dx\epsilon(x)x^2\{3(1-x)+\tfrac{2}{3}(4x/3-1)\}}. \tag{2}$$

Equation (2) is obtained by limiting the type of interaction producing the muon decay to vector and axial vector (V-A) coupling with the two-component neutrino hypothesis, i.e., $\rho=\frac{3}{4}$, $\eta=0$.[2] Here x is the electron energy expressed in units of one-half the rest mass of the muon, and y is the energy of the gamma ray expressed in the same units. θ is the angle between the momentum vectors of the gamma ray and the electron, and $\Delta=1-\cos\theta$. $e^2=1/137$, and $n_V(x,y,\Delta)$ is a function given by the theory.[4] The function $\epsilon(x)$ is the detection efficiency of the ring-counter telescope. For this, the results of the Monte Carlo calculations of Leiss *et al.*[5] were used. It was assumed that, on the average, the electron must traverse 4.25-cm equivalent carbon to be detected in the ring-counter telescope. The solid-angle function $[\Delta\Omega_\gamma(\theta)]_{\mathrm{eff}}$ is shown in Fig. 2, and it is discussed further in the Appendix.

$C(y)$ is a function to correct for the following three factors:

(1) A small increase in the solid angle due to the finite probability of penetration by the gamma rays through the edges of the lead collimator. This correction was calculated by using the cross section calculated with the Born approximation.[6]

(2) The detection efficiency of the sodium iodide counter.[7]

(3) The small gamma-ray losses due to pair creation in the material between the target and counter $\bar{4}$.[6]

$T(y,y')$ is the resolution function of the sodium iodide counter when mounted behind its lead collimator. It has been determined in a different set of experiments and has been assumed to be a "universal" function throughout the energy range of the present experiment.[8]

IV. APPARATUS

A. Counters and Electronics

In Fig. 1, the counters A and B were used to monitor the incident pion beam. The total intensity was approximately 5×10^5 incident particles per minute. The ring-shaped counter No. 1 was three and one-quarter inches inner diameter and five inches outer diameter, while counters No. 2 and No. 3 were three inches inner diameter and six inches outer diameter. These three ring-shaped counters were made of one-quarter-inch plastic scintillator and were enclosed by aluminum light reflectors 0.025 inch thick. Counter No. $\bar{4}$ was a circular disk of one-quarter-inch plastic scintillator, six inches in diameter. The sodium iodide crystal was a cylinder five inches in diameter and four and one-half inches high. It was viewed by a single Dumont 6364 photomultiplier tube. The lead collimator in front of the sodium iodide counter was five inches thick and had a 2.8-inch diameter hole.

Figure 3 is a block diagram of the electronic circuits used in the experiment. The coincidence of (1+2+3) was registered as an electron count, while the (1+2+3 +$\bar{4}$+"fast" NaI) coincidence sequence was used to trigger a linear gate of 0.1 μsec duration. The time-coincident "slow" pulse in the sodium iodide crystal was transmitted by this gate into a fifty-channel pulse-height analyzer where its pulse height was analyzed.

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[4] $n_V=F_{-1}(\Delta+2/\mu^2x^2)^{-1}+F_0+\Delta F_1+\Delta^2F_2$,

where

$$F_{-1}=8[y^2(3-2y)+6xy(1-y)+2x^2(3-4y)-4x^3],$$
$$F_0=8[-xy(3-y-y^2)-x^2(3-y-4y^2)+2x^3(1+2y)],$$
$$F_1=2[x^2y(6-5y-2y^2)-2x^3y(4+3y)],$$
$$F_2=2x^3y^2(2+y),$$

and μ is the rest mass of muon.

[5] J. E. Leiss, S. Penner, and C. S. Robinson, Phys. Rev. **107**, 1544 (1957), and private communication.

[6] W. Heitler, *Quantum Theory of Radiation* (Oxford University Press, New York, 1954), 3rd ed.

[7] G. R. White, National Bureau of Standards Report No. 1003 (unpublished).

[8] A. S. Penfold and E. L. Garwin, Phys. Rev. **116**, 120 (1959).

"Fast" pulses in the sodium iodide crystal were obtained by means of a small inductance (about 3 μh) attached to the last dynode of the Dumont tube. A special limiter-shaper circuit was used to generate a suitable fast pulse for the coincidence circuit.

The ring-shaped counters were tested to assure equal efficiency of detection at different locations on the ring. The measured inefficiency of the anticoincidence counter $\bar{4}$ was one part in five thousand. This gave a negligible correction to the gamma-ray spectra observed in this experiment.

B. Calibration of the NaI Counter

The pulse-height spectrum produced in the sodium iodide crystal by decay electrons was taken every five hours for calibration purposes. To do this, the gate was opened by the coincidence of counter No. $\bar{4}$ and the "fast" NaI pulse. The end point of this spectrum was assigned to be the highest possible energy of the nonradiative decay electrons, after taking into account the energy loss of a 52.85-Mev electron in the target, in the Al foils, and in counter No. $\bar{4}$, as well as the sodium iodide resolution. The error in assigning the energy of the end point leads to an uncertainty in the number of gamma rays included in a given energy interval of the spectrum. This uncertainty has been combined quadratically with the statistical uncertainty of each point to determine the resultant error. By fixing only two points on the calibration curve, viz., zero and the spectral end point of the muon decay, the tacit assumption is made that the sodium iodide counter has a linear response. This assumption is justified for this counter on the basis of previous experiments with it.[8]

The electronic gate, amplifiers, and pulse-height analyzer were checked with the aid of a precision pulser at the same time the above test was performed. The over-all linearity was very stable and no appreciable drifts in the height of the gating pedestal were observed.

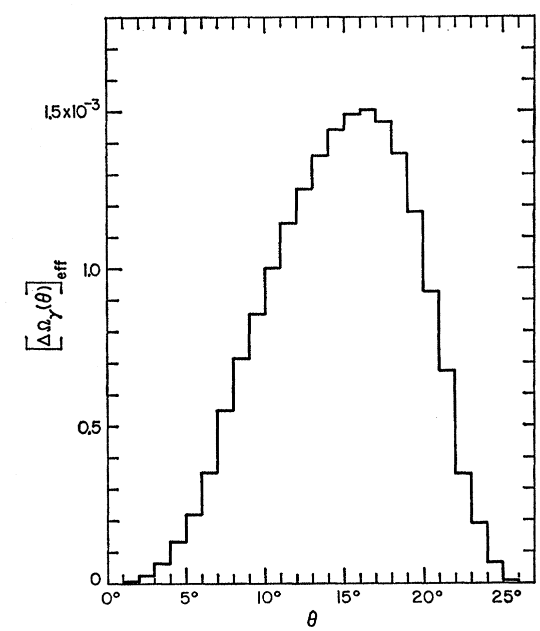

FIG. 2. A plot of the effective solid angle for gamma-ray detection vs θ, the angle between the momentum vectors of the decay electron and the gamma ray.

C. Beam Scanning Apparatus

To determine the intensity distribution of the incident pions as a function of position on the six-inch square target, a quarter-inch, cubic scintillation counter was used. The beam intensity distribution was measured at three different times during the run, after removing sufficient carbon moderator in front of the counters A and B to compensate for the change in range of the pions. The distributions remained constant during the experiment. The statistical errors with which these intensity distributions were determined were such that less than four percent uncertainty was introduced into the numerical calculation of the solid angle.

FIG. 3. A block diagram of the electronics.

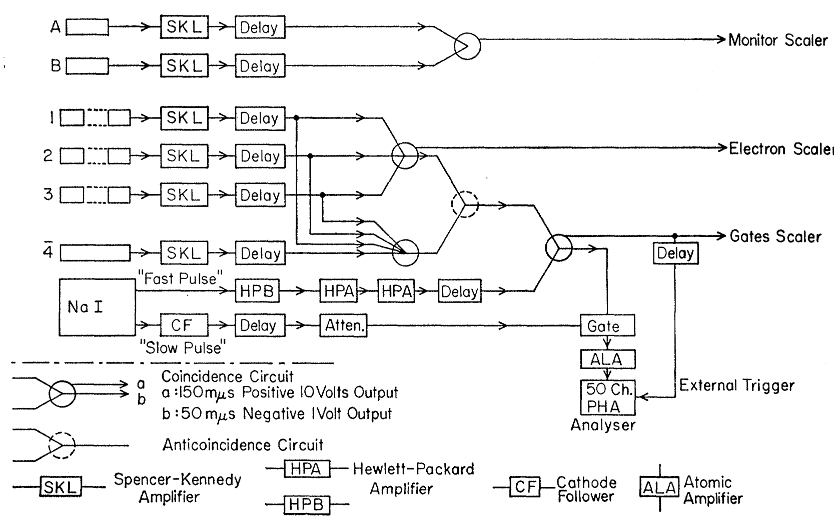

TABLE I. An example of a part of the data and a summary of the final results obtained from all of the data. N_e is the total number of electrons counted in the ring-counter telescope; y' is the interval of the gamma-ray energy; N_{Al}, N_{C}, and N_{BG} are the total number of gamma rays counted when the pions are stopped in aluminum, carbon, and no material, respectively; N_{EB} is the number of gamma rays from the external bremsstrahlung, N_{IB} is the number from the inner bremsstrahlung. $R_{\mathrm{exp}}\Delta y'$ and $R_{\mathrm{theo}}\Delta y'$ are the *total* measured and calculated branching ratios.

				$N_e=(1.70\pm0.01)\times10^6$				
y'		0.125–0.15	0.15–0.25	0.25–0.35	0.35–0.45	0.45–0.55	0.55–0.65	0.65–0.75
	N_{Al}	1252	3446	1262	473	227	39	12
	N_{C}	946	2335	938	358	178	38	9
2.8	N_{BG}	90	263	63	15	20	4	...
	N_{EB}	252	917	268	95	40	0	3
	N_{IB}	604	1155	606	248	118	34	6
$R_{\mathrm{exp}}\Delta y'(\times10^5)$		5.08 ± 0.52[a]	10.5 ± 0.7	4.27 ± 0.44	1.83 ± 0.27	0.888 ± 0.186	0.252 ± 0.087	0.058 ± 0.038
$R_{\mathrm{theo}}\Delta y'(\times10^5)$		4.90	10.2	4.19	1.93	0.831	0.271	0.0550

[a] For errors, see reference 10.

V. EXPERIMENTAL DATA

The data were taken in short runs which alternated the carbon and aluminum blocks with background runs using no stopping material at all. The result of one set of these runs is summarized in Table I. N_e is the total number of electrons detected in the ring-counter telescope with the carbon stopping block in place. The other numbers in the table give the number of gamma rays in a given energy interval. The total number of gamma rays in a given energy interval from aluminum, N_{Al}, is assumed to be the sum of the true "inner" bremsstrahlung, N_{IB}, the "external" bremsstrahlung, $N_{\mathrm{EB}}(\mathrm{Al})$, and the background, N_{BG}. Similarly, the total number of gamma rays from carbon N_{C} is the sum of N_{IB}, $N_{\mathrm{EB}}(\mathrm{C})$, and N_{BG}. Thus,

$$N_{\mathrm{Al}}=N_{\mathrm{IB}}+N_{\mathrm{EB}}(\mathrm{Al})+N_{\mathrm{BG}}, \tag{3a}$$

$$N_{\mathrm{C}}=N_{\mathrm{IB}}+N_{\mathrm{EB}}(\mathrm{C})+N_{\mathrm{BG}}. \tag{3b}$$

To optimize the statistical accuracy of the data, fewer background runs were taken than those with aluminum and carbon. Table I contains an appropriate factor of 2.8 to compensate for this difference. The dependence of the "external" bremsstrahlung contribution in the above equations upon the gamma-ray energy is the same, but the absolute values depend on the atomic number, Z, of the two materials and their difference in thickness. The Z dependence of the bremsstrahlung cross section can be approximated by $Z(Z+0.75)$.[9] The thicknesses of the carbon and aluminum blocks were chosen with the aid of differential range curves to give the same stopping power for the incident pion beam. The peak of the carbon range curve was contained in 5.90 g/cm² thickness, while that of aluminum was in 6.63 g/cm². If the above Z dependence and difference in thickness are taken into account, $N_{\mathrm{EB}}(\mathrm{Al})=2.21\ N_{\mathrm{EB}}(\mathrm{C})$ at a given value of y. Thus,

$$N_{\mathrm{EB}}(\mathrm{C})=0.826(N_{\mathrm{Al}}-N_{\mathrm{C}}). \tag{4}$$

Finally, Eq. (3a) can be used to determine N_{IB}, the inner bremsstrahlung of the muon decay. The rate, $R_{\mathrm{exp}}\Delta y'=N_{\mathrm{IB}}/[N_e/(\Delta\Omega e)_{\mathrm{eff}}]$, is the number to be compared with the theoretical calculations, $R_{\mathrm{theo}}\Delta y'$, described in Sec. III above.

In quoting the value of N_e, a two percent correction has been included to correct for pions in the primary beam which can scatter from the carbon block into the ring-counter telescope. The magnitude of this correction has been estimated on the basis of calculations which treat the carbon nucleus as a hard sphere of infinite mass. A large error has been assigned to this correction. Table II gives a summary of the various systematic errors introduced both in the calculation of the theoretical spectrum and in the measurements.

The values of $R_{\mathrm{theo}}(y')\Delta y'$ are plotted as an histogram in Fig. 4, while the corresponding values of $R_{\mathrm{exp}}(y')\Delta y'$ are shown as experimental points with the appropriate errors attached. For comparison, the ratio of the

TABLE II. The systematic correction factors and their estimated errors which affect the calculated values of the theoretical spectrum and the determination of the experimental points.

Type of correction	Magnitude and error
Calculated spectrum:	
1. Numerical integration of the spectrum	1.00 ± 0.01
2. Solid angle calculation	1.00 ± 0.04
3. Average correction of solid angle due to gamma-ray penetration of the lead collimator	1.10 ± 0.02
4. Uncertainty in counter telescope efficiency from Monte Carlo calculation	1.00 ± 0.04
5. Uncertainty of the value of ρ used in the $\mu-e$ decay spectrum	1.00 ± 0.03
Resultant	1.10 ± 0.07
Experimental points:	
1. Correction for external bremsstrahlung in the stopping material	1.00 ± 0.04
2. Loss of gamma rays due to conversion between the target and Counter 4	1.00 ± 0.02
3. Contamination of the electron counting rates by scattered π^+	0.98 ± 0.02
Resultant	0.98 ± 0.05

[9] W. Heitler, reference 6, p. 255; H. W. Koch and J. W. Motz, Revs. Modern Phys. **31**, 920 (1959). For the present experiment, the Z dependence of the differential cross section is used.

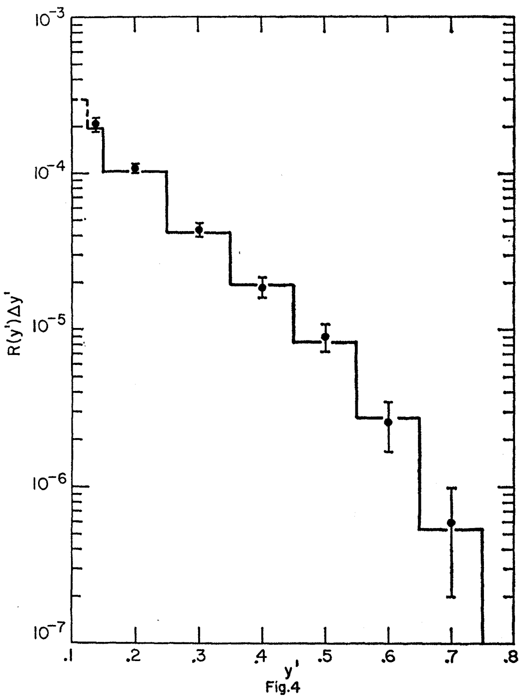

FIG. 4. A plot of the calculated spectrum as an histogram with the experimental spectrum as experimental points with associated errors. The first 2 bins are increased by a factor of 4 to normalize to the value for $\Delta y'=0.10$. The errors are from a combination of the statistical error and the systematic error due to the uncertainty of the energy calibration.

measured spectral values to the theoretical ones gives a weighted average of 1.02 ± 0.10.

VI. DISCUSSION

It is clear that the results of the present experiment are in good agreement with the predictions of the theory. An additional check on the internal consistency of the agreement can be made. If the experimental data and the calculated spectral values are used to evaluate the Michel parameter ρ by retaining only the $n_V(x,y,\Delta)$ term and the ρ dependence of the numerator and denominator, then $\rho=0.725\pm0.056$.[10] This is in excellent agreement with the value 0.75 given by the theory. However, it does not constitute a measurement of ρ. In order to do that, the spectral calculation must include the other types of coupling for the weak interaction. This, in turn, introduces two new functions,[2] $n_S(x,y,\Delta)$ and $n_T(x,y,\Delta)$, together with their appropriate coupling constants into Eq. (3). This introduction of new parameters makes it impossible to obtain an accurate value of ρ from the present data. In fact, the result of such a determination is $\rho=0.64\pm0.32$.[10]

It is of interest to investigate the possibility of observing effects on the shape of the gamma-ray spectrum due to the presence of an intermediate vector boson in the decay process of Eq. (1). In general, it can be shown that this contribution to any one spectral point can be of the order of 4% or less.[11] Because the uncertainty of the present data is 7% or more for each point, it is not possible to observe such small effects.

VII. ACKNOWLEDGMENTS

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APPENDIX

The effective solid angle of the electron counter telescope, $(\Delta\Omega_e)_{\text{eff}}$, was 0.148 sr. The calculation of this number required that the beam distribution on the target be taken into account. Since the angular correlation between the elementary solid angle of the electron, $d\Omega_e$, and that of the gamma ray, $d\Omega_\gamma$, becomes very critical in the calculations of the expected spectrum, all of the elements $(d\Omega_e/4\pi)_{\text{eff}}$ and $d\Omega_\gamma$ which had the same angle, θ, of separation were summed in intervals of 1°. This procedure was carried out with the aid of an analog computer which reproduced the experimental geometry. By making use of the fact that the decay electrons were isotropically distributed, the effective solid angle of the sodium iodide counter, for a given θ, was written

$$[\Delta\Omega_\gamma(\theta)]_{\text{eff}}=\sum\left(\frac{d\Omega_e}{4\pi}\right)_{\text{eff}} d\Omega_\gamma(\theta)\bigg/\left(\frac{\Delta\Omega_e}{4\pi}\right)_{\text{eff}}. \quad (6)$$

The total effective solid angle of the sodium iodide counter in the experimental geometry, $(\Delta\Omega_\gamma)_{\text{eff}}$, was

$$(\Delta\Omega_\gamma)_{\text{eff}}=\sum_{\theta=0°}^{26°}[\Delta\Omega_\gamma(\theta)]_{\text{eff}}=1.83\times10^{-2}\text{ sr.}$$

In the determination of this value of $(\Delta\Omega_\gamma)_{\text{eff}}$, the small extension of solid angle due to the finite probability of penetration by gamma rays through the lead collimator was neglected. A suitable correction, taking account of the energy variation of the gamma-ray penetration, was applied to the calculated spectrum to allow for this effect as mentioned in the text.

[10] The uncertainty quoted above results from a quadratic combination of the statistical error and the error due to the uncertainty in the energy calibration.

[11] The authors wish to thank Mrs. S. Eckstein for her help in carrying out these calculations.