yields $p_1/p_4=0.45$, and the Blatt-Weisskopf level densities[3] give $p_1/p_4=0.40$. Thus, the emission of alpha particles is not impeded, so that the preformation factor, f, is near unity (note that $f=p_4/p_1$). At higher excitation energies, f decreases; for example, with $U=100$ Mev and $N=100$, $p_1/p_4=0.63$. It is thus apparent that the reason why alphas are frequently emitted in nuclear reactions although ice crystals never evaporate from water droplets is that nuclei are highly degenerate Fermi systems, whereas a water droplet is a nondegenerate system. The preformation factor is strictly an intuitive concept based on our experience with classical systems, and has no meaning in the nuclear case. The confusion may well serve as a warning against the time honored custom of visualizing a compound nucleus as a classical evaporating liquid drop.

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jj Coupling Model in Odd-Odd Nuclei*

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An analysis has been made, using the *jj* coupling model, of the spins of 76 low-lying levels of odd-odd nuclei with $20<A<120$. Levels have been excluded from the analysis if any ambiguity exists in the assignment of a configuration. Excluding particle-hole configurations we find Nordheim's "strong" rule obeyed in the 22 cases in which it is applicable. Nordheim's "weak" rule is replaced by a rule predicting a ground state of the highest or lowest allowable spin. This revised rule is obeyed in 38 out of 41 cases. This competition between two levels of greatly differing spin results in the frequent occurrence of isomerism. For particle-hole configurations the state of the highest spin minus one is the ground state in 6 out of 13 cases. This agreement with experiment is obtained using proton and neutron configurations which, in 66 of the 76 cases, are found as ground states in the neighboring odd-even nuclei. The three revised coupling rules are predicted by the calculations of Schwartz, in which the residual proton-neutron interaction has a delta-function radial dependence, if the singlet-to-triplet strength is taken as 0.6, independent of mass number. This value is the same as that required to fit the free two-nucleon data. Exceptions to the empirical coupling rules and this theory will be discussed.

I. INTRODUCTION

A STUDY of the low-lying levels in odd-odd nuclei can provide useful information on the nature of the effective interaction between protons and neutrons in nuclear matter. This problem can be conveniently discussed by extending the odd-group model, as normally applied to odd-even nuclei. In the odd-group model the spin and magnetic properties of the nucleus are assumed to be determined by the properties of the odd group of particles. In the extension of this representation to odd-odd nuclei it is assumed that the wave function is a simple vector-coupled product of the wave functions of the two odd groups. A further simplification which permits the use of *jj* coupling, is obtained if it is assumed that the residual interactions are weak compared to the spin-orbit force.[1]

Under these assumptions the levels arising from a given proton and neutron configuration can take on all integral spins between the sum and the difference of the spins of the two odd groups. The degeneracy of these levels is removed by the residual proton-neutron interaction. Furthermore, the low-lying levels in an odd-odd nucleus should result from combinations of the lowest configurations in the adjacent odd-proton and odd-neutron nuclei. If it is then possible to consider only those odd-odd nuclei where the low-lying levels result from a single proton-neutron configuration, a study of the level ordering should provide information about the residual proton-neutron interaction.

The rules governing the coupling of the proton and neutron angular momenta have been studied both empirically and theoretically. In 1950, Nordheim[2] proposed two coupling rules which, with the data available at the time, provided a satisfactory description of the spins of the majority of odd-odd nuclei. However, later empirical studies[3] showed that there were frequent violations of the so-called "weak" rule. These studies,

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[1] For a general discussion of the *jj* coupling model and its application to odd-odd nuclei, see the review article of J. P. Elliott and A. M. Lane, *Handbuch der Physik*, edited by S. Flügge (Springer-Verlag, Berlin, 1957) Vol. 39.

[2] L. W. Nordheim, Phys. Rev. **78**, 294 (1950); Revs. Modern Phys. **23**, 322 (1951).
[3] K. Way, D. N. Kundu, C. L. McGinnis, and R. van Lieshout, *Annual Review of Nuclear Science* (Annual Reviews, Inc., Palo Alto, California, 1956) Vol. 6, p. 129; C. A. Mallman, *Proceedings of the Second United Nations International Conference on the Peaceful Uses of Atomic Energy, Geneva, 1958* (United Nations, Geneva, 1958) Vol. 14, p. 68; C. J. Gallagher, Jr and S. A. Moszkowski, Phys. Rev. **111**, 1282 (1958).

together with some theoretical calculations for specific configurations,[4] have thrown considerable doubt on the general applicability of the model outlined above.

In the present work we have made a further empirical analysis of the spins of the ground states and low-lying isomeric states for odd-odd nuclei with mass numbers in the range $20<A<120$. In this analysis we have considered only those nuclei for which no obvious ambiguity exists in the assignment of the proton and neutron configurations which are obtained from the experimental data on the adjacent odd-even nuclei. Thus, we hope to exclude cases in which configuration mixing has an effect on the level ordering.

We shall show that it is possible, by replacing Nordheim's coupling rules by three revised rules *while still retaining jj coupling*, to obtain excellent agreement between the observed and predicted spins for the nuclei considered.

We also compare the data with theoretical calculations[5] using a delta-function proton-neutron interaction with arbitrary space and spin dependent strengths. These calculations were previously thought to provide some support for Nordheim's rules on the assumption of a strong spin dependence of the residual interaction. However, the calculations clearly predict the breakdown of Nordheim's "weak" rule for a somewhat weaker spin dependence. We shall show that the theoretical calculations are in agreement with the majority of the observed spins for this weaker spin dependence of the interaction, and that this is just the amount of spin dependence required to fit the low-energy data in the free two-nucleon problem.

II. REVISED COUPLING RULES

In the form originally proposed by Nordheim[2] the coupling rules governing the coupling of the proton and neutron angular momenta reflect the tendency for the proton and neutron spins to line up parallel as found in the deuteron. The two rules specifying the spin of the lowest state are the so-called "strong" rule,

$$J=|J_1-J_2| \quad \text{for} \quad j_1=l_1\pm\tfrac{1}{2} \quad \text{and} \quad j_2=l_2\mp\tfrac{1}{2}, \quad \text{(N1)}$$

and the "weak" rule,

$$|J_1-J_2|<J\leq J_1+J_2 \quad \text{for} \quad j_1=l_1\pm\tfrac{1}{2} \quad \text{and} \quad j_2=l_2\pm\tfrac{1}{2}, \quad \text{(N2)}$$

with the additional comment that in the case of the second rule the resultant spin, J, tends towards the maximum value. Here j_1 and l_1 (or j_2 and l_2) are the single-particle total and orbital angular momenta, obtained from the adjacent odd-A nuclei using standard shell-model assignments. We have distinguished between the observed total angular momentum of the odd-A nucleus, J_1 (or J_2), and the single-particle momenta so as to include cases of high seniority in which $j_1\neq J_1$ (or $j_2\neq J_2$). Such a configuration is found, for example, in Al^{24}, where the three neutrons in the $d_{5/2}$ subshell ($j_2=\frac{5}{2}$) are coupled to a total angular momentum, J_2, of $\frac{3}{2}$.

The three revised coupling rules, which we propose should replace Nordheim's rules, were arrived at on the basis of our empirical analysis of the experimental data, supported by a consideration of the theoretical calculations of Schwartz,[5] which will be discussed in Sec. VI. First, we consider configurations in which both the odd protons and odd neutrons are particles (or holes) in their respective unfilled subshells.[6] For levels of this type we have the coupling rules

$$J=|J_1-J_2| \quad \text{for} \quad j_1=l_1\pm\tfrac{1}{2} \quad \text{and} \quad j_2=l_2\mp\tfrac{1}{2}, \quad (R1)$$

and

$$J=|J_1\pm J_2| \quad \text{for} \quad j_1=l_1\pm\tfrac{1}{2} \quad \text{and} \quad j_2=l_2\pm\tfrac{1}{2}. \quad (R2)$$

For the special case J_1 or J_2 equal to $\frac{1}{2}$ the ambiguity of ($R2$) is removed and we predict $J=J_1+J_2$. We have thus replaced Nordheim's "weak" rule by a much stronger rule, ($R2$), and retained the form of the "strong" rule.

For configurations in which there is a combination of particles and holes, as is the case for Cl^{36}, the prediction is much less certain, although there is a tendency for the resultant spin to be given by

$$J=J_1+J_2-1. \quad (R3)$$

It will also be of interest to test a further prediction of the jj coupling model, that the magnetic moment of a level in an odd-odd nucleus is given by[1]

$$\mu=\frac{J}{2}(g_1+g_2)+(g_1-g_2)\left[\frac{J_1(J_1+1)-J_2(J_2+1)}{2(J+1)}\right], \quad (1)$$

where g_1 and g_2 are the gyromagnetic ratios of the proton and neutron groups.

It was first pointed out by Schwartz,[7] with subsequent theoretical justification by Caine,[8] that a significant improvement in the agreement between the moment calculated using Eq. (1) and the observed moment could be obtained if empirical g factors, obtained from the moments of adjacent odd-even nuclei, were used instead of the single-particle Schmidt values. Moments calculated using both types of g factors are presented in Table I.

III. PRESENTATION OF DATA

In order to make a comparison between the spins predicted by the coupling rules and the observed spins, we have investigated 76 levels found as the ground state or as a long-lived, low-lying isomeric state in nuclei with mass numbers in the range $20<A<120$. The results of this analysis are presented in Table I. The lower limit to the mass number is set to exclude the lighter nuclei for which detailed calculations have shown that the

[4] S. P. Pandya, Phys. Rev. **108**, 1312 (1957).
[5] C. Schwartz, Phys. Rev. **94**, 95 (1954); A. deShalit, Phys. Rev. **91**, 1479 (1953).
[6] In this class we include cases where the number of particles in one of the odd groups is *exactly equal* to half the number of particles required to fill the subshell.
[7] H. M. Schwartz, Phys. Rev. **89**, 1293 (1953).
[8] C. A. Caine, Proc. Phys. Soc. (London) **A69**, 635 (1956).

jj coupling approach is inadequate.[1] The upper limit is essentially an operational one and excludes those nuclei with $Z>50$ for which an unambiguous assignment for the proton and neutron configurations does not appear to be possible, except in a few rare cases. This difficulty arises from the competition, evident in the odd-A nuclei, between several configurations both in the odd-proton and odd-neutron nuclei. This latter region has been studied by Gallagher and Moszkowski,[3] who find excellent agreement between the observed spins and the spins predicted by coupling rules stated in the framework of the collective model.

As already indicated, we have included only those levels for which an unambiguous assignment of the configuration could be made. Where no competition between configurations is evident from a study of the neighboring odd-even nuclei, the configuration for the odd-odd nucleus has been taken to be the combination of the ground-state configurations of the adjacent odd-proton and odd-neutron nuclei. The configuration chosen for each odd-odd nucleus is specified in columns 2 and 3 of Table I.

Where competition between two or more configurations existed, three methods were employed to assign the correct configuration to the odd-odd nucleus. In several cases, in the islands of isomerism, it is possible to choose the correct configuration on the basis of the observed parity of the level in question. Such cases are designated by the letter *a* in column 4. A second method, designated by the letter *b*, was to choose the configuration on the basis of the agreement between $\mu_{(\mathrm{emp})}$, the moment calculated from Eq. (1) using the empirical g factors, and the observed magnetic moment, $\mu_{(\mathrm{exp})}$. These moments, together with $\mu_{(\mathrm{sp})}$ calculated from Eq. (1) using the single-particle g factors, are listed in columns 7, 8, and 9. Finally, in the case of three of the indium isotopes labelled by the letter *c*, it was possible to arrive at an unambiguous assignment of the configuration by assuming that the spin of the odd-odd nucleus was restricted by *jj* coupling to lie in the range

$$|J_1-J_2|\leq J\leq J_1+J_2.$$

In column 4 the type of configuration is indicated for each level. If nothing appears in this column this indicates that both the proton and neutron configurations are found at least once as the ground states in the corresponding odd-even nuclei. The symbol (0,1) indicates that the neutron configuration is found only as a first excited state in the corresponding odd-neutron nuclei, while the proton configuration is found as a ground state. This information therefore gives an indication of the validity of the assumption that the lowest state of the odd-odd nucleus is a combination of the lowest states of the two odd groups.

The revised coupling rule appropriate to each case is indicated in column 5. A minus sign [(R1)$-$, (R2)$-$, and (R3)$-$] is used to indicate a failure of a coupling rule. Column 6 contains the observed spins and parities. Spins which have not been measured directly but which have been deduced from spectroscopic data are listed in brackets. Unless otherwise indicated as a footnote the spins and parities are taken from the analysis of Gallagher and Moszkowski,[3] or from other recent compilations.[9] A comprehensive tabulation of the measured magnetic moments, with references, can be found in the paper of Noya, Arima, and Horie.[10]

IV. DISCUSSION OF DATA

The results presented in Table I show that the three revised coupling rules, together with a straightforward choice of configurations, adequately represent the data for the vast majority of cases. A significant improvement over Nordheim's rules has been obtained by considering configurations containing only particles (or holes) separately from those containing both particles and holes. Thus, the spin of K^{40}, which constituted a violation of Nordheim's "strong" rule, is now correctly described by (R3). Indeed, there are no exceptions to the rule (R1) in the 22 cases where it is applicable.

Turning next to rule (R3)[11] we see that in only 6 out of 13 cases is the rule obeyed. It is interesting to note, however, that 3 of the 7 failures are cases where the configuration is one of high seniority and the observed spin is given by $J=J_1+J_2-2$.

The most striking improvement in the validity of the coupling rules is found in rule (R2), which replaces the less definitive "weak" rule of Nordheim. Whereas Nordheim's rule fails in 19 of the 41 cases classified as (R2), the revised rules fail in only three cases. Two of these failures (Mn^{52m} and Rh^{104m_2}) are configurations of high seniority, while in the third failure (Sc^{44}) there is a possibility of some configuration mixing. This latter point will be discussed in Sec. VI.

It is of interest to inquire what general type of configuration we have used in obtaining such excellent agreement between predicted and observed spins. The data of column 4 in Table I show that, in 66 of the 76 levels considered, the configuration is obtained from the ground states of the adjacent odd-even nuclei. Thus, in only 10 cases is it necessary to use a configuration involving excited states of the odd-even nuclei and in 7 of these a combination of ground and first excited states is used. It is also interesting to note that, with the exception of Mn^{52}, the parity or magnetic moment of the level was not needed in the assignment of the odd-odd configuration for the nuclei from Na^{22} to Ga^{70}.

These results contrast with those obtained by Gallagher and Moszkowski,[3] who used a highly distorted potential well to describe the odd-odd nucleus. In their

[9] V. S. Dzepov and L. K. Peker, Atomic Energy of Canada, Limited Report AECL No. 457, 1957 (translation) (unpublished); D. Strominger, J. M. Hollander, and G. T. Seaborg, Revs. Modern Phys. **30**, 585 (1958).

[10] H. Noya, A. Arima, and H. Horie, Suppl. Progr. Theoret. Phys. (Kyoto) **8**, 33 (1958).

[11] See also the analysis of C. J. Gallagher, Jr. and S. A. Moszkowski,[3] in which a rule of this form is also used.

TABLE I. Comparison between the observed spins and magnetic moments and those predicted by the revised coupling rules and Eq. (2). See Sec. III for details.

Nucleus	Configuration Protons	Configuration Neutrons	Type	Coupling rule	Observed spin and parity	Magnetic moment $\mu_{(exp)}$	Magnetic moment $\mu_{(emp)}$	Magnetic moment $\mu_{(sp)}$	Remarks
${}_{11}Na^{22}$	$(d_{5/2}^3)_{3/2}$	$(d_{5/2}^3)_{3/2}$		$(R2)$	3+	1.746	1.55	1.73	
Na^{24}	$(d_{5/2}^3)_{3/2}$	$d_{5/2}^{-1}$		$(R2)$	4+	1.688	1.36	0.96	
Na^{24m}	$(d_{5/2}^3)_{3/2}$	$d_{5/2}^{-1}$		$(R2)$	(1+)				
${}_{13}Al^{24}$	$d_{5/2}^{-1}$	$(d_{5/2}^3)_{3/2}$		$(R2)$	(4+)				
Al^{26}	$d_{5/2}^{-1}$	$d_{5/2}^{-1}$		$(R2)$	(5+)				
Al^{26m}	$d_{5/2}^{-1}$	$d_{5/2}^{-1}$		$(R2)$	(0+)				
Al^{28}	$d_{5/2}^{-1}$	$s_{1/2}$		$(R2)$	(3+)				
${}_{15}P^{30}$	$s_{1/2}$	$s_{1/2}$		$(R2)$	(1+)				
P^{32}	$s_{1/2}$	$d_{3/2}$		$(R1)$	1+	−0.252	−0.03	−0.39	
P^{34}	$s_{1/2}$	$d_{3/2}^{-1}$		$(R1)$	(1+)				
${}_{17}Cl^{34}$	$d_{3/2}$	$d_{3/2}$		$(R2)$	(0+)				
Cl^{34m}	$d_{3/2}$	$d_{3/2}$		$(R2)$	(3+)				
Cl^{36}	$d_{3/2}$	$d_{3/2}^{-1}$		$(R3)$	2+	1.284	1.17	0.85	
Cl^{38}	$d_{3/2}$	$f_{7/2}$		$(R1)$	2−				
Cl^{40}	$d_{3/2}$	$f_{7/2}^3$		$(R1)$	(2−)				
${}_{19}K^{38}$	$d_{3/2}^{-1}$	$d_{3/2}^{-1}$		$(R2)$	(0+)				
K^{38m}	$d_{3/2}^{-1}$	$d_{3/2}^{-1}$		$(R2)$	(3+)				
K^{40}	$d_{3/2}^{-1}$	$f_{7/2}$		$(R3)$	4−	−1.296	≈−1.1	−1.68	a
K^{42}	$d_{3/2}^{-1}$	$f_{7/2}^3$		$(R3)-$	2−	−1.137	≈−1.4	−1.72	a
${}_{21}Sc^{42}$	$f_{7/2}$	$f_{7/2}$		$(R2)$	(0+)				
Sc^{44}	$f_{7/2}$	$f_{7/2}^3$		$(R2)-$	(2+, 3+)				
Sc^{46}	$f_{7/2}$	$(f_{7/2}^{-3})_{5/2}$		$(R3)-$	(4+)				
Sc^{46m}	$f_{7/2}$	$(f_{7/2}^{-3})_{5/2}$		$(R3)-$	(1+, 7+)				
Sc^{48}	$f_{7/2}$	$f_{7/2}^{-1}$		$(R3)$	(6+, 7+)				
Sc^{50}	$f_{7/2}$	$p_{3/2}$		$(R2)$	(2+)				
${}_{23}V^{46}$	$f_{7/2}^3$	$f_{7/2}^3$		$(R2)$	(0+)				
V^{48}	$f_{7/2}^3$	$(f_{7/2}^{-3})_{5/2}$		$(R3)-$	(4+)				
V^{50}	$f_{7/2}^3$	$f_{7/2}^{-1}$		$(R3)-$	6+	3.341	3.51	3.32	
V^{52}	$f_{7/2}^3$	$p_{3/2}$		$(R2)$	(2+)				
${}_{25}Mn^{52}$	$(f_{7/2}^{-3})_{5/2}$	$f_{7/2}^{-1}$	b	$(R2)$	(6+)	±3.08	2.71	2.22	b
Mn^{52m}	$(f_{7/2}^{-3})_{5/2}$	$f_{7/2}^{-1}$		$(R2)-$	(2+)				c
${}_{27}Co^{54}$	$f_{7/2}^{-1}$	$f_{7/2}^{-1}$		$(R2)$	(0+)				
Co^{54m}	$f_{7/2}^{-1}$	$f_{7/2}^{-1}$		$(R2)$	(7+)				d
Co^{56}	$f_{7/2}^{-1}$	$p_{3/2}$		$(R3)$	4+	±3.85	≈4.1	4.28	a
Co^{58}	$f_{7/2}^{-1}$	$p_{3/2}^{-1}$		$(R2)$	2+	3.5	≈4.3	6.23	a
Co^{58m}	$f_{7/2}^{-1}$	$p_{3/2}^{-1}$		$(R2)$	(5+)				
Co^{60}	$f_{7/2}^{-1}$	$p_{3/2}^{-1}$		$(R2)$	5+	±3.8	≈4.5	3.88	a
Co^{60m}	$f_{7/2}^{-1}$	$p_{3/2}^{-1}$		$(R2)$	(2+)				
${}_{29}Cu^{60}$	$p_{3/2}$	$p_{3/2}^{-1}$		$(R3)$	2+				
Cu^{62}	$p_{3/2}$	$p_{3/2}^{-1}$		$(R3)-$	1+				
Cu^{66}	$p_{3/2}$	$f_{5/2}^{-1}$		$(R3)-$	(1+)				
${}_{31}Ga^{64}$	$p_{3/2}^{-1}$	$p_{3/2}^{-1}$		$(R2)$	(0+)				
Ga^{66}	$p_{3/2}^{-1}$	$p_{3/2}^{-1}$		$(R2)$	0+	<0.001	0	0	
Ga^{68}	$p_{3/2}^{-1}$	$f_{5/2}^{-1}$		$(R1)$	1+	±0.05	−0.35	−0.94	
Ga^{70}	$p_{3/2}^{-1}$	$p_{1/2}$		$(R1)$	(1+)				
Ga^{72}	$p_{3/2}^{-1}$	$(g_{9/2}^3)_{7/2}$	ab (0,1)	$(R3)-$	3−	±0.126	−0.47	−1.28	e
${}_{33}As^{72}$	$f_{5/2}$	$g_{9/2}$	a	$(R1)$	(2−)				
As^{74}	$f_{5/2}$	$g_{9/2}$	a	$(R1)$	(2−)				
${}_{35}Br^{76}$	$p_{3/2}^{-1}$	$f_{5/2}^{-1}$	ab	$(R1)$	1+	±0.55	−0.4	−0.94	f
Br^{80}	$p_{3/2}^{-1}$	$p_{1/2}$	a (0,1)	$(R1)$	(1+)				
Br^{80m_1}	$p_{3/2}^{-1}$	$(g_{9/2}^5)_{7/2}$	a	$(R2)$	(2−)				
Br^{80m_2}	$p_{3/2}^{-1}$	$(g_{9/2}^5)_{7/2}$	a	$(R2)$	(5−)				
Br^{82}	$p_{3/2}^{-1}$	$(g_{9/2}^{-3})_{7/2}$	b (0,1)	$(R2)$	5−	±1.626	1.25	2.21	g
${}_{37}Rb^{82}$	$p_{3/2}^{-1}$	$p_{1/2}$	a (0,1)	$(R1)$	(1+)				h
Rb^{82m}	$p_{3/2}^{-1}$	$(g_{9/2}^5)_{7/2}$	ab	$(R2)$	5−	1.50	1.25	2.21	
Rb^{86}	$f_{5/2}^{-1}$	$g_{9/2}^{-1}$		$(R1)$	2−	−1.69	−1.78	−2.13	
${}_{39}Y^{88}$	$p_{1/2}$	$g_{9/2}^{-1}$		$(R1)$	(4−)				
Y^{90}	$p_{1/2}$	$d_{5/2}$		$(R1)$	(2−)				
Y^{92}	$p_{1/2}$	$d_{5/2}^3$		$(R1)$	(2−)				

TABLE I.—*Continued.*

Nucleus	Configuration Protons	Configuration Neutrons	Type	Coupling rule	Observed spin and parity	Magnetic moment $\mu_{(exp)}$	Magnetic moment $\mu_{(emp)}$	Magnetic moment $\mu_{(sp)}$	Remarks
$_{45}Rh^{100}$	$p_{1/2}$	$d_{5/2}^n$	a	(R1)	(2−)				i
Rh^{102}	$p_{1/2}$	$d_{5/2}^n$	a	(R1)	(2−)				
Rh^{104}	$(g_{9/2}^5)_{7/2}$	$d_{5/2}^n$	a (1,0)	(R2)	(1+)				
Rh^{104m_1}	$p_{1/2}$	$d_{5/2}^n$	a	(R1)	(2−)				h
Rh^{104m_2}	$(g_{9/2}^5)_{7/2}$	$d_{5/2}^n$	a (1,0)	(R2)−	(5+)				
Rh^{106}	$(g_{9/2}^5)_{7/2}$	$d_{5/2}^n$	a (1,0)	(R2)	1+				j
$_{47}Ag^{108}$	$(g_{9/2}^{-3})_{7/2}$	$d_{5/2}^n$	a	(R2)	(1+)				j
Ag^{110}	$(g_{9/2}^{-3})_{7/2}$	$d_{5/2}^{-1}$	a	(R2)	(1+)				
Ag^{110m}	$(g_{9/2}^{-3})_{7/2}$	$d_{5/2}^{-1}$	a	(R2)	6+				
Ag^{112}	$p_{1/2}$	$d_{5/2}^{-1}$	a	(R1)	(2−)				
$_{49}In^{110}$	$g_{9/2}^{-1}$	$d_{5/2}^{-1}$		(R2)	(2+)				
In^{110m}	$g_{9/2}^{-1}$	$d_{5/2}^{-1}$		(R2)	7(+)				
In^{112}	$g_{9/2}^{-1}$	$g_{7/2}^{-1}$	c (0,3)	(R1)	(1+)				
In^{114}	$g_{9/2}^{-1}$	$g_{7/2}^{-1}$	c (0,3)	(R1)	(1+)				
In^{114m}	$g_{9/2}^{-1}$	$s_{1/2}$	b	(R2)	5+	4.7	4.70	4.88	
In^{116}	$g_{9/2}^{-1}$	$g_{7/2}^{-1}$	c (0,3)	(R1)	(1+)				
In^{116m}	$g_{9/2}^{-1}$	$s_{1/2}$	b	(R2)	5+	4.21	4.50	4.88	

[a] The uncertainty in $\mu_{(emp)}$ arises from an uncertainty in the empirical g factor for the neutron group.
[b] $\mu_{(emp)}=3.51$ for the $(f_{7/2}^{-3})$ $(f_{7/2}^{-1})$ configuration.
[c] There is some evidence[9] for a (1+) level close to the (2+) level.
[d] The spin of this level is high, and is probably 7. D. C. Sutton (private communication).
[e] $\mu_{(emp)}=-3.9$ for the $(p_{3/2}^{-1})$ $(g_{9/2}^{3})$ configuration.
[f] $\mu_{(exp)}$ from H. L. Garvin, T. M. Green, E. Lipworth, and W. A. Nierenberg, Phys. Rev. Letters **1**, 293 (1958).
[g] $\mu_{(exp)}$ from H. L. Garvin, T. M. Green, E. Lipworth, and W. A. Nierenberg, Phys. Rev. **116**, 393 (1959).
[h] Spin assignment from *Nuclear Data Sheets*, edited by C. L. McGinnis (National Academy of Sciences—National Research Council, 1958–1959), Vol. 5.
[i] The number of neutrons in the $d_{5/2}$ subshell is uncertain. This does not affect the predictions of the coupling rules since the proton number is always half the number required to fill the proton subshell. (See discussion in Sec. II.)
[j] The isomeric state has been found,[13] but its spin is not known.

work, combinations of first, and even second, excited states are frequently required in order to obtain agreement between the observed spin and the prediction of their coupling rules. Our analysis indicates that, if the jj coupling approach is used, the assumption that the lowest state of the odd-odd nucleus results from a combination of the lowest states of the two odd groups yields a set of coupling rules which adequately account for the observed spins. In Sec. VI we shall see that these coupling rules are supported by calculations based on a delta-function interaction between the protons and neutrons.

A further indication of the validity of jj coupling is found by examining the agreement between the empirically calculated magnetic moment $\mu_{(emp)}$ and the measured value. From Table I we find that in 15 out of 19 cases $\mu_{(emp)}$ lies closer than $\mu_{(sp)}$ to the measured moment. This improved agreement with experiment when the empirical g factors are used, which has been reported previously with more limited experimental data,[1,12] supports the conclusion[8] that most of the effects of configuration mixing on the magnetic moment of an odd-odd nucleus can be accounted for in this way. Detailed calculations,[10] specifically including configuration mixing in the odd-odd nucleus, do not significantly improve the agreement between the calculated and the observed moments.

V. ISOMERISM

In the case of odd-even nuclei, isomerism results from the excitation of a single particle to a configuration different from that of the ground state. In odd-odd nuclei an additional source of isomerism is present due to the possibility of coupling the proton and neutron spins to various resultant spins, without a change in the particle configurations. An example of this type of isomerism is found in Co^{58} and Co^{60}, where the two states of spin 2 and 5 form a closely spaced doublet resulting from a single configuration, $(f_{7/2}^{-1})$ $(p_{3/2}^{-1})$. The success of the revised coupling rules suggests that the occurrence of isomerism due to such recoupling of the proton and neutron angular momenta will depend on the coupling rule which applies to the ground-state configuration.

The success of (R1) indicates that there is, in general, a large energy gap between the ground state and the first excited state in nuclei under this coupling. Isomerism due to angular momentum recoupling is therefore unlikely in these nuclei, and indeed the only case of this type of isomerism in this group occurs in Cl^{38}, where the isomeric state is 672 kev above the ground state. It is also interesting that the large energy gap present under rule (R1) permits the occurrence of isomerism due to the excitation of a single particle in five nuclei in the islands of isomerism (Br^{80}, Rb^{82}, Rh^{104}, In^{114}, In^{116}).

The weakness of (R3) does not permit us to draw any definite conclusions about isomerism in nuclei in which this applies. The only case of isomerism due to angular momentum recoupling under this rule appears to be Sc^{46}.

The most interesting nuclei are those under coupling rule (R2). Here one finds a competition between two states of spin $|J_1 \pm J_2|$, implying that these states form a closely spaced doublet under this coupling. This would lead one to expect angular momentum recoupling to be

[12] R. J. Blin-Stoyle, *Theories of Nuclear Moments* (Oxford University Press, New York, 1957), p. 66.

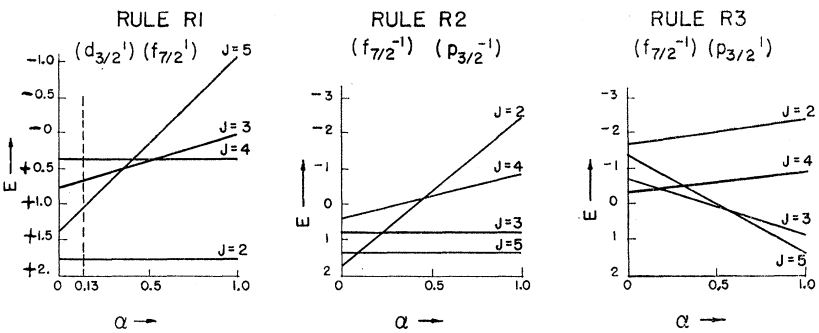

FIG. 1. Theoretical level ordering, as a function of α, for three configurations described by the three revised coupling rules. The energies are given in arbitrary units and are taken from the calculations of Schwartz.[5]

the dominant cause of isomerism in such cases. If the levels listed in Table I are examined we find that there are 25 nuclei with configurations classified under rule (*R*2) and for which neither J_1 nor J_2 is equal to one-half. Of these, 12 show isomerism resulting from angular momentum recoupling and in 10 of these cases the spins of the two states are indeed $|J_1 \pm J_2|$. In one further case, Ag^{108}, an isomeric level has been found,[13] but the spin is not yet known. For several of the nuclei where no isomerism is found, the experimental information is scanty and it seems worth while to make a further search for isomerism in these nuclei. It would be of particular interest to determine whether isomerism is present in Al^{24}, Sc^{42}, Sc^{50}, and Br^{82}, since for these nuclei isomerism would be expected by analogy with Na^{24}, Co^{54}, Co^{58}, and Br^{80}, respectively.

VI. THEORETICAL LEVEL ORDERING

We shall confine our discussion primarily to the theoretical work of Schwartz,[5] who extended the calculations of deShalit[5] to include configurations containing an arbitrary number of particles in the unfilled subshells. In these calculations it is assumed that the ordering of the levels arising from a particular proton-neutron configuration is determined by the residual proton-neutron interaction, which is assumed to be of the form

$$V = -V_0[(1-\alpha)+\alpha\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2]\delta(\mathbf{r}_1-\mathbf{r}_2). \qquad (2)$$

Here V_0 is a constant, $\boldsymbol{\sigma}_1$ and $\boldsymbol{\sigma}_2$ are the spin operators for the proton and neutron, $\mathbf{r}_1$ and $\mathbf{r}_2$ the radial coordinates. The strength of the spin-dependent part of the interaction is specified by the parameter α.

Calculations for specific configurations, in which the effect of allowing the potential to be of finite range and of other exchange mixtures, have been made by several other workers.[4,14] There do not appear to be any theoretical calculations for finite-range forces which cover all the configurations in the mass range we have studied in the present work. We shall therefore limit ourselves to the calculations of Schwartz and determine whether it is possible to obtain any information on the parameter α by comparing the theoretical calculations with the experimental data and the revised coupling rules.

In Fig. 1, the theoretical level schemes are shown as functions of α for three configurations corresponding to the coupling rules (*R*1), (*R*2), and (*R*3). These calculations represent the theoretical predictions for Cl^{38}, Co^{58} and Co^{60}, and Co^{56}, respectively. They are typical of the curves obtained for cases of lowest seniority. For cases of high seniority the curves are somewhat modified,[5] and in general the level ordering is changed for any specific value of α. We have not considered cases of high seniority in our comparison with the experimental data.

As is well known, the calculations of Schwartz clearly predict the coupling rule (*R*1) for all values of α. This means that, unless the level order is known experimentally, no information on α can be obtained from configurations of this type. In the case of Cl^{38} the spins and energies of the four levels are known.[15] Also, it has been shown,[16] quite independently of the form of the residual proton-neutron interaction, that the *jj* coupling model is especially successful in this case. For this nucleus the observed level order can be fitted for $\alpha<0.3$, and the level spacing is fitted best for $\alpha=0.13$. However, the actual level spacing is not reproduced very well. It is interesting to note that the calculations of Kurath[14] indicate that the fit becomes worse as the range of the interaction is made finite.

In the case of particle-hole configurations, corresponding to rule (*R*3), the theoretical predictions are much less certain. However, as shown in Fig. 1, there is a tendency for the state of spin (J_1+J_2-1) to be the lowest. In several cases it is possible to place a restriction on the range of α required to obtain agreement with the experimental data. Nuclei for which this is possible are listed in Table II, together with similar information for the other coupling rules.

Nuclei under coupling rule (*R*2) provide the most useful check on the validity of the theoretical calculations. We find that with the exception of those nuclei for which the protons and neutrons are filling the same subshell, a value of α can usually be found for which the theoretical predictions are in agreement with the experi-

[13] M. A. Wahlgren and W. W. Meinke, Phys. Rev. **118**, 181 (1960).
[14] D. Kurath, Phys. Rev. **91**, 1430 (1953); D. M. Brink, Proc. Phys. Soc. (London) **A67**, 757 (1954); N. D. Newby, Jr., and E. J. Konopinski, Phys. Rev. **115**, 434 (1959).
[15] C. H. Paris, W. W. Buechner, and P. M. Endt, Phys. Rev. **100**, 1317 (1955).
[16] S. Goldstein and I. Talmi, Phys. Rev. **102**, 589 (1956); S. P. Pandya, Phys. Rev. **103**, 956 (1956).

mental data presented in Table I. Nuclei for which a determination of α is possible are listed in Table II. The values for α quoted for Co^{58}, Co^{60}, and In^{110} are those for which the two levels $J=|J_1 \pm J_2|$ are degenerate, which is approximately the case for these nuclei in which the energy differences are only 25 kev, 59 kev, and 120 kev, respectively.

With the exception of K^{42}, Cu^{66}, and V^{52} the values of α listed in Table II are approximately $\frac{1}{10}$. This value is less than that of $\frac{1}{4}$ to $\frac{1}{6}$ deduced by Schwartz[5] on the basis of the limited amount of data available at the time of his analysis. The major difference is that we now consider K^{42} as an exception to the theory, whereas Schwartz included it in his determination of α. This nucleus appears to be a case in which there is a considerable admixture of configurations of higher seniority. Indeed, Goldstein and Talmi[17] found that the predicted ground-state spin and the magnetic moment were in agreement with the experimental values if a considerable admixture of the $(d_{3/2}^{-1})\ (f_{7/2}^{3})_{5/2}$ configuration was included in the ground-state wave function. It is possible that a large admixture of this type in the $f_{7/2}^{3}$ configuration could explain the small value of α needed in the case of V^{52} and also the fact that the observed spin of 2 or 3 for Sc^{44} cannot be obtained for any value of α. It is also possible that for the other exceptional case, Cu^{66}, there is a considerable amount of configuration mixing arising from the competition between the $p_{3/2}^{-1}$ and $f_{5/2}^{-1}$ neutron configurations.

Returning to those nuclei in which the protons and neutrons are filling the same subshell, we note that the calculations of Schwartz specifically exclude such cases. If, however, these calculations are applied to these nuclei, one finds for $\alpha \approx 0.1$ a close competition between the levels of spin 0, 1, and $2J_1$. (Note that $J_1=J_2$ for these nuclei.) With the exception of Na^{22} the experimental results indicate only the states of spin 0 and $2J_1$ in close competition. Stripping data on Al^{26} and Cl^{34} show that the spin-one level is approximately 1 Mev above the other two states.[18]

The calculations of Kurath[14] on Cl^{34}, using a Serber space-exchange mixture and a spin-exchange mixture corresponding to $\alpha=0.1$, show that the spin-1 level is raised relative to the $2J_1$ state as the range of the interaction is increased. However, the spin-zero level is raised even further, in contradiction with experiment. Agreement with the observed level order can only be obtained, for the Serber force, for $\alpha \approx 0$.

TABLE II. Determination of α from a comparison of the calculations of Schwartz[5] and the experimental data presented in Table I. The nuclei are classified according to the empirical coupling rules. The value (or range) of α is shown in brackets after each nuclei.

Coupling rule	Nucleus	
($R1$)	$Cl^{38}(0.13)$	
($R2$)	$Sc^{50}(\lesssim 0.10)$;	$V^{52}(\lesssim 0.05)$;
	$Co^{58}(0.10)$;	$Co^{60}(0.10)$;
	$In^{110}(0.14)$.	
($R3$)	$K^{40}(\lesssim 0.3)$;	$K^{42}(\gtrsim 0.25)$;
	$Co^{56}(\lesssim 0.16)$;	$Cu^{66}(\gtrsim 0.5)$.

[a] See reference 5.

[17] S. Goldstein and I. Talmi, Phys. Rev. **105**, 995 (1957).

[18] P. M. Endt and C. M. Braams, Revs. Modern Phys. **29**, 683 (1957).

VII. DISCUSSION

In our analysis of the spins of the low-lying levels in odd-odd nuclei with mass numbers in the range $20<A<120$ we have found that the majority of the spins are predicted by simple coupling rules in conjunction with a straightforward choice of the appropriate configuration involving only the configurations of the states of lowest energy in the neighboring odd-A nuclei. Further, it is apparent that, with the exception of certain of the self-conjugate nuclei, the theoretical calculations of Schwartz reproduce these coupling rules.

The breakdown of Nordheim's "weak" rule appears to be a consequence of the occurrence of a doublet, consisting of the states $J=|J_1 \pm J_2|$, at a value of α of approximately $\frac{1}{10}$. There is no evidence for a variation of α with mass number or with changes in the proton and neutron configurations. It is tempting to ascribe the rather small changes in α for the "pure" configurations to the effects of small amounts of configuration mixing. Recently, Newby[19] has suggested that the tensor force will tend to break down Nordheim's "weak" rule.

It is desirable that further calculations be made, using forces which are more realistic than the delta-function force. Of particular interest are nuclei for which several energy levels have been found. Cl^{38} and K^{40} appear to be potentially valuable for providing information on the form of the residual proton-neutron interaction since jj coupling is apparently an excellent approximation in these nuclei.[16] In this connection it would be of great interest to establish the validity of jj coupling in other cases.

Nuclei containing neutrons and protons in the same subshell appear to require more extensive theoretical investigation. It is probable that the assumption that the two odd groups are weakly interacting, thus retaining their separate identity, is not valid since there is the maximum overlap in their wave functions.

We would also like to emphasize the need for further experimental investigations of the energy levels in odd-odd nuclei. It would be of interest to extend the stripping experiments to nuclei of higher mass number and to investigate nuclei away from the lines of stability using β-decay spectroscopy.

Finally, we note that the value of $\frac{1}{10}$ for α found in the present work corresponds to a singlet-to-triplet strength of 0.6. This is just the relative strength required to fit the free two-nucleon data. Although the delta-function interaction is clearly not realistic, this result suggests that the relative strength of the space and spin dependent parts of the interaction is not markedly affected by the presence of other nucleons.

[19] N. D. Newby, Jr., Phys. Rev. **119**, 747 (1960).