$$z=y(2+y^2)^{\frac{1}{2}},$$
$$\alpha_z=(1+z^2)^{\frac{1}{2}}-z, \quad \alpha_1\equiv\alpha_{z1}, \tag{49}$$
$$m_1=\frac{e^{-t(\delta+z_1)}}{1-e^{-t(\delta+z_1)}}.$$

In the very low-momentum region $k\ll k_0$ and for $t\ll 1$, one can show that $\omega_I^{(a)}$ is very much smaller than $\omega_I^{(b)}$. Therefore, we only consider the latter term, which can be written as follows:

$$\omega_I^{(b)}=\frac{k_0^3a}{\sqrt{2}}\int_0^\infty dz_1\left[\frac{z_1^2}{(1+z_1^2)}-(1+z_1^2)^{-\frac{1}{2}}+1\right]^2$$
$$\times m_1(1+m_1)[1+O(y,t)]\underset{y,t\to 0}{\rightarrow}\left(\frac{2\pi^{\frac{3}{2}}}{\pi-2}\right)\left(\frac{k_0}{\lambda_T}\right). \tag{50}$$

We see that in the region $t\ll 1$, $\omega_I^{(b)}/\omega_0\sim(k\lambda_T)^{-1}$ becomes very large for $k\ll k_0$. This means that the half-width of a low-momentum excitation is large compared to the excitation energy itself. The lifetime $\tau=[-2\,\mathrm{Im}\omega(k)]^{-1}$ of the quasi-particles is extremely short for $t\ll 1$, i.e., for temperatures just below the critical temperature, and the excitations are therefore not very well defined. *In a dilute Bose gas of hard spheres, low-momentum quasi-particles can only be well defined in the low-temperature region* $t\gtrsim 1$. The result of Eq. (47) for the real part of $\omega(k)$ becomes less and less physically meaningful as the temperature $T<T_c$ is increased.

It is of interest to compare the qualitative result of Eq. (50) with the experimental results for the inelastic scattering of neutrons in liquid He II. In Fig. 2 of a note by Henshaw[1] are plotted the spectra of inelastically scattered neutrons at a fixed angle and for four different temperatures: $T=1.27$°K, 1.57°K, 2.08°K and 4.21°K. Now, the critical temperature of liquid helium is 2.18°K, and yet one sees from Henshaw's curves that already at 2.08°K the unique energy-momentum relation, which defines the quasi-particles in He II at lower temperatures, has started to wash out. *The experimentally determined quasi-particles of He II are also not very well defined at temperatures just below the critical temperature*, and the result of Eq. (50) is qualitatively compatible with experiment.

ACKNOWLEDGMENT

The stimulation from discussions with Professor T. D. Lee is greatly appreciated.

PHYSICAL REVIEW VOLUME 120, NUMBER 3 NOVEMBER 1, 1960

Thermal Conduction in Rotating Liquid Helium II

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(Received June 30, 1960)

When a heat current flows in a wide channel filled with liquid helium II, the resulting temperature gradient is approximately proportional to the cube of the heat current density. The establishment of this gradient requires a time τ which is a function of heat current, temperature, and past history of the helium. The present experiments concern the effect of uniform slow rotation, about an axis normal to the direction of heat flow, upon grad$T$ and τ. Grad$T$ was measured at 0 and 1.3 rad/sec, the highest angular velocity at which this measurement could conveniently be carried out. No effect of rotation could be observed; however, an approximate calculation suggests that grad$T$ might increase detectably at somewhat higher rates of rotation. τ was measured at a number of angular velocities between 0 and 4 rad/sec; it was found that τ was appreciably reduced by rotation, the effect being greatest at small heat currents and high angular velocities. These results can be explained on the assumption that mutual friction results from turbulence in the superfluid component, taking the form of a tangled mass of vortex line. The delay time τ characterizes the rate of growth of this turbulence when a heat current is switched on; rotation reduces τ by introducing an initial length of vortex line which accelerates this growth.

I. INTRODUCTION

When a heat current flows in liquid helium II, the resulting temperature gradient is approximately proportional to the cube of the heat current density. This fact, together with the results of experiments on fountain pressures, led Gorter and Mellink[1] to postulate the existence of a mutual friction force between the normal and superfluid components given by

$$F_{sn}=A\rho_s\rho_n(|v_s-v_n|-v_0)^3, \tag{1}$$

where v_s and v_n are the superfluid and normal fluid velocities, respectively. A is approximately independent of channel width and is a slowly varying function of temperature; v_0 is of the order of 1 cm/sec, varies somewhat with temperature, and decreases with increasing channel width.

* Operated with support from the U. S. Army, Navy, and Air Force.
[1] C. J. Gorter and J. H. Mellink, Physica **15**, 285 (1949).

Recently Vinen[2–4] (henceforth referred to as V1, V2, V3) has studied the properties of this mutual friction in more detail in wide ($\sim$1 mm) channels. He found that, for values of (v_s-v_n) from 1 to 15 cm/sec, the relation between temperature gradient and heat current density W is of the form

$$\text{grad } T=D(W-W_0)^3, \qquad (2)$$

where D and W_0 are constants at a given temperature. This can be interpreted in terms of the mutual friction force (1), provided

$$A=(\rho_s^3S^4T^3D)/\rho_n \quad \text{and} \quad v_0=W_0/(\rho_sST). \qquad (3)$$

Vinen also discovered that (*i*) second sound propagated across a heat current undergoes a severe excess attenuation proportional to W^2 (V1); (*ii*) there exists a critical heat current W_c below which the mutual friction force abruptly falls to a value much smaller than that given by Eq. (1) (V1–V3); and (*iii*) the equilibrium mutual friction is only established after a time τ, dependent upon the heat current density and the temperature, which varies from about 0.1 sec to about a minute (V2). The equilibrium value of τ is only observed if the helium has been left undisturbed for a considerable time, of the order of two or three minutes; if a heat current is switched off and then on again after a time shorter than this, τ is much smaller and less reproducible. This suggests that the state giving rise to mutual friction, once established, takes a considerable time to decay. Taking into account both these facts and the earlier observation that second sound undergoes excess attenuation in uniformly rotating liquid helium,[5] Vinen proposed that mutual friction is the result of turbulence in the superfluid, and that this turbulence takes the form of a tangled mass of quantized vortex line. The detailed development of this theory made possible an explanation of all the above observations.

The present measurements concern the effect of uniform slow rotation, about an axis normal to the direction of heat flow, on the thermal conductivity and the delay time τ. Since according to the theory of Onsager[6] and Feynman[7] rotating superfluid contains an array of vortex lines arranged parallel to the axis of rotation, one might expect these vortex lines to act in a similar manner to those produced by the heat current itself, increasing the mutual friction and reducing τ. The latter conclusion follows from the observation (V2) that τ is greatly reduced by the presence of a small residue of turbulence from a previous heat current; if this residual turbulence consists of vortex lines, the effect of vortex lines produced by rotation should be similar. Possible effects of the different geometrical arrangement of these lines will be discussed in a later section.

τ was measured at a number of angular velocities up to 4 rad/sec; most of these measurements were carried out at 1.41°K, the temperature used by Vinen in much of his work; however, enough measurements were made at other temperatures to establish that the phenomena are qualitatively temperature-independent. Thermal conductivity measurements were made at 0 and 1.3 rad/sec, at a number of temperatures between 1.3°K and the λ point. A number of measurements were, of course, made in the absence of rotation. These are essentially repetitions of parts of Vinen's work, and, except where otherwise noted, agree within experimental error with his results.

II. EXPERIMENTAL METHOD

(a) General

The experimental apparatus is shown schematically in Fig. 1. The inner chamber A contains a wire-wound heater H of nominal resistance 44 ohms, wound on the bakelite former B, and a resistance thermometer T_1. A second thermometer T_2 is mounted in a fixed position in the helium bath. The heat current from H flows down the cylindrical thin-walled stainless steel tube C, of inside diameter 2.62 mm and length 5.16 cm, which opens directly into the helium bath. Leads for the heater H and thermometer T_1 are brought out through this tube and led to rotating joints constructed from modified *BNC* connectors. The apparatus is surrounded by a vacuum jacket D, and is mounted along the diameter of an aluminum rotor (not shown) so that it can be rotated about the vertical axis shown in the figure. The heat flow is therefore radial. The rotor is driven by a variable-speed motor coupled to a drive shaft entering the cryostat through an O-ring seal. Two motors were used in the experiments, providing speeds of 0 to 1.3 rad/sec and 0 to 20 rad/sec. In actual practice speeds above 4 rad/sec were rarely used, because excessive amounts of heat were introduced into the helium bath at higher rates of rotation.

A revolution marker consisting of a battery and *RC* circuit was operated by a microswitch and a cam on the motor shaft. Marker pips from this device were recorded

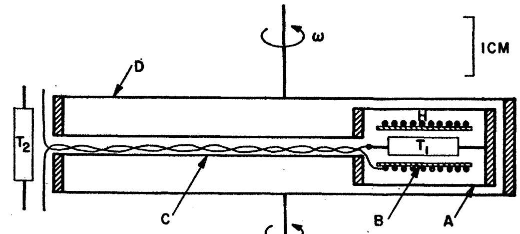

FIG. 1. The apparatus.

[2] W. F. Vinen, Proc. Roy. Soc. (London) **A240**, 114 (1957).
[3] W. F. Vinen, Proc. Roy. Soc. (London) **A240**, 128 (1957).
[4] W. F. Vinen, Proc. Roy. Soc. (London) **A242**, 493 (1957).
[5] H. E. Hall and W. F. Vinen, Proc. Roy. Soc. (London) **A238**, 204 (1956); **A238**, 215 (1956).
[6] L. Onsager, Suppl. Nuovo cimento **6**, 249 (1949).
[7] R. P. Feynman, in *Progress in Low-Temperature Physics*, edited by C. J. Gorter (North Holland Publishing Company, Amsterdam, 1955), Vol. 1, Chap. 2, p. 36.

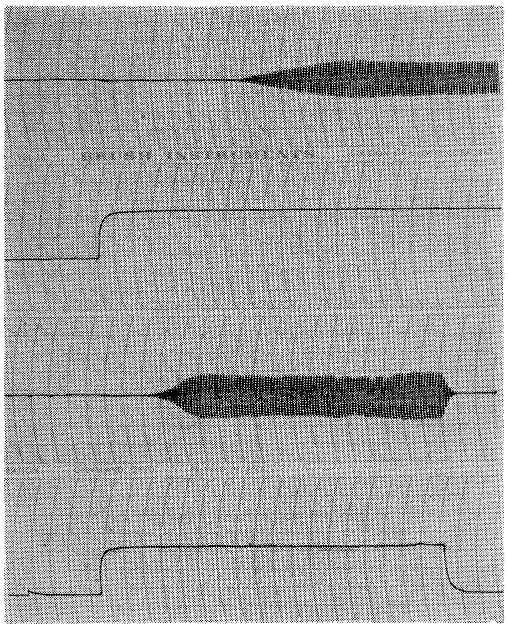

FIG. 2. Typical records of data ($T=1.41°K$; $W=0.081$ watt/cm^2; tape speed 25 mm/sec). Top: $\omega=0$, bottom: $\omega=2.0$ rad/sec. The amplitude of the upper trace of each pair is proportional to the temperature gradient, the upward deflection of the lower trace to the voltage across the heater. Each horizontal division $=0.2$ sec.

on one channel of a dual-channel Brush recorder-amplifier together with the voltage across the heater H. The output of the thermometer bridge (described below) was recorded on the second channel. Delay times were measured directly from the tape records, which were made at a speed of 25 mm/sec. τ was defined, as in V2, as the time required for the temperature gradient to attain one-half its equilibrium value. Two typical records are shown in Fig. 2. The upper record was obtained with the apparatus stationary; the lower while rotating at 2.0 rad/sec. In both cases the heat current density was 0.081 watt/cm^2 and the temperature 1.41°K. One of the revolution marker pips can be seen at the left end of the lower trace. The amplitude of the 33-cycle sine wave is proportional to the temperature difference between the two thermometers.

Heater power was supplied from a battery and potentiometer. Separate current and potential leads were used, and both heater voltage and current were measured. When the heater was switched on, a small transient occurred in the thermometer bridge output as a result of direct electrical coupling between the heater and thermometer leads. This was eliminated by placing a condenser across the heater so that the rise time of the heater voltage was increased to about 0.05 sec. This can be observed on the heater voltage records in Fig. 2. It was found that changing the value of this condenser by a factor of 4 or removing it altogether had no effect on the experimental results.

(b) Temperature Measurement

The bath temperature was determined from the vapor pressure of the helium, which was measured on an oil manometer previously calibrated against a mercury manometer. A Wallace and Tiernan absolute pressure gauge was also occasionally used.

Temperature differences were measured with the resistance thermometers T_1 and T_2, which consisted of 100-ohm, one-half watt Allen Bradley carbon resistors. The outer bakelite coating of the resistors was removed to reduce the thermal response time, and they were then coated thinly with glyptal. The thermometers were connected in two adjacent arms of a resistance bridge, and a decade box was placed in series with T_1. The bridge was fed with 33-cycle power from an audio oscillator, and its output was amplified by a 33-cycle amplifier which has been previously described.[8] The resulting signal was fed into the Brush recorder and also displayed on an oscilloscope. The amplifier normally has a bandwidth of less than one cycle, but was so designed that reducing the gain broadens the pass band. All transient measurements were accordingly made with the gain turned all the way down. Under these circumstances the bandwidth was about 20 cycles and the rise time 0.05 sec. Although the thermometer bridge was originally designed to operate with a power dissipation of 10^{-8} watt in each thermometer, this figure was increased to 10^{-5} watt in the present experiments in order to compensate for the reduced gain and to provide the greatest possible sensitivity. This is a factor of 10 smaller than was used in V1–2, and apparently produces no objectionable heating effects. Under favorable conditions temperature differences of one microdegree could be observed with about 10% accuracy with this apparatus.

Since the temperature coefficients of the thermometers differed by a few percent, the balance of the bridge was initially rather sensitive to changes in bath temperature. However, it was found that the ratio of the thermometer resistances was nearly independent of temperature, so that by adjusting the two fixed arms of the bridge in the same ratio this sensitivity could be almost completely eliminated. (The bridge is so arranged that the use of unequal arms does not require any correction in the resistance readings.) When the bridge was properly adjusted, the sensitivity to changes in ambient temperature was less than 0.01% of the sensitivity to temperature differences, and no particular care was needed to hold the bath temperature sufficiently steady during the course of a measurement.

(c) Sources of Error

In V2 transient temperature oscillations, analogous to Helmholtz oscillations of the resonator consisting of the chamber A and the tube C, were observed when the

[8] C. Blake, C. E. Chase, and E. Maxwell, Rev. Sci. Instr. **29**, 715 (1958).

heat current was switched on. These oscillations had a period of about 0.1 sec, and were eliminated by an ingenious system of switching the heat current on in two steps, one-half cycle apart. In the present experiments such oscillations were never detected, possibly because their frequency was too high to be passed by the amplifier (the present apparatus is roughly half as large as Vinen's). No double-switching was therefore used. According to V2, the presence of these oscillations had very little effect upon the value of τ, but only interfered with the observation procedure. Since the present results are in substantial agreement with Vinen's, it is likely that they are also unimportant here.

When the apparatus is rotated, centripetal force causes a pressure gradient to appear along the tube C. It is therefore pertinent to enquire what effect this gradient may have upon the measurements. An elementary calculation shows that this pressure gradient at radius r is given by

$$dp/dr = \rho\omega^2 r, \tag{4}$$

which in the present experiments was never greater than 6% of the vertical gradient produced by the hydrostatic head of the liquid. No relevant effects have been observed in helium II which might be attributed to the hydrostatic pressure; in particular, the results of Vinen's experiments, in which the heat flow was directed upwards, agree with those obtained with the present horizontally oriented apparatus when it was not rotating. It therefore seems likely that the much smaller gradient produced by rotation can be neglected. There will of course be some mass transfer during acceleration, as this gradient is established, but equilibrium was always reached long before any measurements were made.

The bandwidth of the amplifier limits the shortest delay times which can be measured to ≈ 0.1 sec. The fact that delay times as short as this can in certain circumstances be detected shows that no more stringent limitation is imposed by the thermal response time of the thermometers. The shortest delays observed in the present experiments are, with one exception, greater than 0.2 sec, and most of the measurements involve delays of 0.3 to 5 sec. No corrections have been applied for instrumental delays, but it should be borne in mind that they may be significant for small values of τ. The exception mentioned above will be discussed more fully in the following section; in this case it appears likely that $\tau = 0$, and the apparent delay (≈ 0.1 sec) which is actually observed serves to confirm the above estimate of the instrumental delays involved.

Delay times could be measured from the tape records with a precision of about 0.02 sec; this does not include possible instrumental delays of the order of 0.1 sec which were mentioned above. Temperature gradients smaller than 2 microdegrees/cm could be measured to approximately $\pm 2\times 10^{-8}$ degree/cm; larger gradients within $\pm 1\%$. Heat current densities and angular

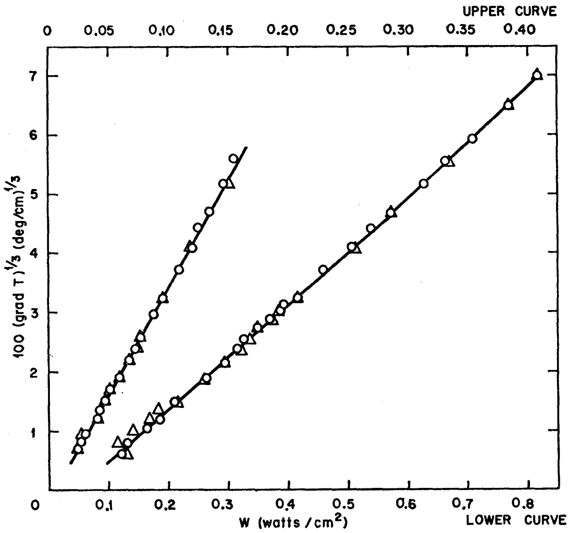

FIG. 3. Thermal conductivity of helium II. Lower curve, 1.993°K; upper curve 1.300°K. ○, $\omega = 0$; △, $\omega = 1.3$ rad/sec.

velocities were both measured to $\pm 2\%$. The absolute temperature could be read to ± 0.002°K, but varied as much as ± 0.005°K during some of the measurements.

III. THE THERMAL CONDUCTIVITY

Thermal conduction measurements were made at a number of temperatures between 1.3°K and the λ point, at angular velocities of 0 and 1.3 rad/sec. Higher angular velocities were not used because the heat generated by rotation caused changes in bath temperature that would have obscured any possible effect of rotation. The results at $\omega = 0$ agree in all respects with V1.[9] In Fig. 3, $(\mathrm{grad}T)^{\frac{1}{3}}$ is plotted against heat current density at two different temperatures. As in V1, the results are straight lines in agreement with Eq. (2) except near the λ point, where small departures from linearity are observed. It is clear that there is no observable difference between the results obtained at $\omega = 0$ (circles) and $\omega = 1.3$ (triangles). It therefore appears that rotation at this angular velocity does not affect the thermal conductivity of liquid helium.

According to Hall and Vinen,[5] the attenuation of second sound in rotating helium is consistent with an excess mutual friction of the form

$$F_{sn} = B(\rho_s\rho_n/\rho)\omega(v_s - v_n), \tag{5}$$

where B is a dimensionless constant varying from ≈ 1.5 at 1.3°K to ≈ 0.8 at 2.0°K, and ω is the angular frequency of rotation. These results were obtained with sound frequencies of 1.5 to 4.5 kc/sec, and were independent of frequency over this range. Vinen (V3) has shown that the same value of B is likely to apply at zero fre-

[9] The author is grateful to Dr. Vinen for communicating the values of entropy, ρ_n, and ρ_s which were used in computing the constant A [Eq. (3)]. A is so sensitive to variations in these quantities that a direct comparison with Vinen's results could only be carried out if identical values were used.

quency, so that if the contributions described by Eqs. (1) and (5) are additive it is possible to write the mutual friction in rotating helium in the form

$$F_{sn}=A\rho_s\rho_n(|v_s-v_n|-v_0)^3+B\frac{\rho_s\rho_n}{\rho}\omega(v_s-v_n). \quad (6)$$

It is possible, of course, that the two mutual frictions are not additive. For example, the turbulent vortex line arrangement giving rise to the Gorter-Mellink force might simply rearrange itself slightly when the system is rotating without any increase in the total length of line per unit volume.[10] Equation (6) therefore probably represents the upper limit of the effect to be expected due to rotation. It is clear that the second term in Eq. (6) will be most important at small values of (v_s-v_n); for an angular velocity of 1.3 rad/sec and the smallest heat currents at which measurements could be made, it amounts to about 5% of the first term at 2°K and 15% at 1.3°K. This is just at the limit of resolution of the present experiment. An increased mutual friction might be observed at angular velocities of the order of 10 rad/sec, but such rates of rotation cannot be obtained with the present apparatus.

Near the end of each experimental run the relation between temperature gradient and heat current abruptly changed in a manner shown in Fig. 4. This curve was obtained at 1.41°K, but the results were similar at other temperatures. At large heat currents the temperature gradient (full curve) was one or two orders of magnitude smaller than in the usual case (shown by the broken curve); at small heat currents the corresponding gradient was larger than usual. The usual cube law was clearly violated. Simultaneously the value of τ fell to $\approx$0.1 sec. Inasmuch as this is comparable to the estimated resolving time of the apparatus, it is likely that in this case τ is actually zero. All these observations were made with the apparatus not rotating. Since the experiment was performed in a metal Dewar, it was impossible to observe the helium level at the onset of this effect. However, the level always fell below the apparatus a few minutes later, as evidenced by rapid warming of the thermometers and a sharp decrease in the apparent thermal conductivity. It therefore seems likely that the above behavior occurs when the helium level is below the top of chamber A and this chamber is not completely filled with liquid. Under these circumstances a net flow of matter into or out of A is possible; in particular, such a flow will occur as the bath level falls. No explanation of these observations is apparent, but it appears that they may be connected with the combined flow of heat and matter in the channel.

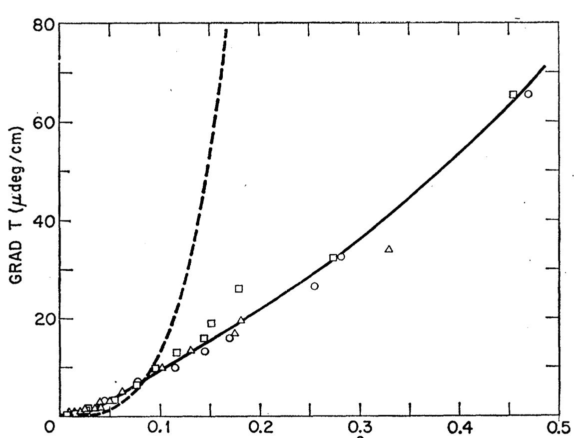

FIG. 4. Thermal conductivity observed when helium level is near apparatus (1.41°K). The broken curve shows the normal behavior; various symbols refer to different runs.

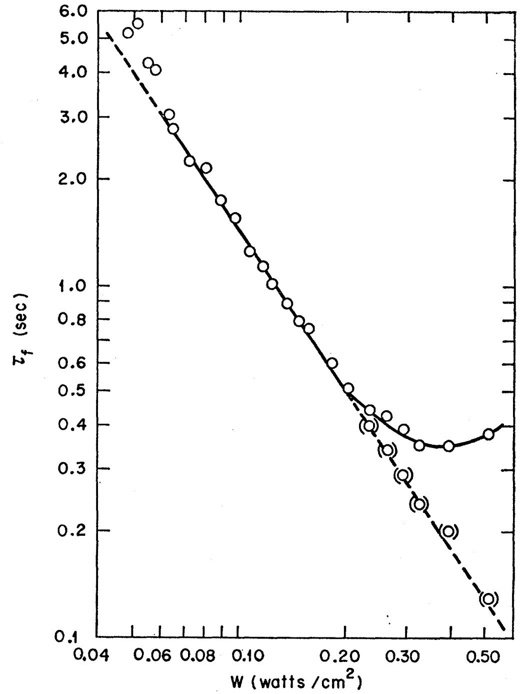

FIG. 5. Dependence of τ on W (1.41°K). Broken line: Eq. (7).

IV. TRANSIENT EFFECTS

(a) Measurements in Nonrotating Helium

Consider the following sequence of operations: A moderately large heat current is switched on for a few seconds, and then switched off. The helium is then left undisturbed for a time t, a heat current W is switched on, and τ is measured. The results of such an experiment are in full agreement with V2: For small values of t, τ fluctuates erratically and is small, while for sufficiently large values of t (greater than 2 to 3 minutes) τ approaches a value τ_f which is fairly reproducible and is unaffected by further increase in t. All the measurements reported below were made with values of t greater than 3 minutes, and will be designated by τ_f. The scatter in

[10] This possibility was suggested by Dr. Vinen.

individual measurements of τ_f is about $\pm 10\%$ when the helium is not rotating, with occasional points deviating by as much as $\pm 20\%$ from the average; in the rotating liquid these figures are increased to $\pm 20\%$ and $\pm 50\%$, respectively. It should be emphasized that this scatter apparently results from a true lack of reproducibility in the rate of buildup of mutual friction, since it is far greater than the limits of resolution of the apparatus and is unaffected by vibration, thermometer power, or any other variable which could be readily changed. A similar situation was reported in V2. In Figs. 7 and 8 all the experimental points are plotted, so that this scatter is evident; in the other figures each data point is the average of two more separate observations.

Figure 5 shows the dependence of τ_f on heat current density W for nonrotating liquid helium at 1.41°K. The broken line is given by the equation

$$\tau_f = aW^{-\frac{3}{2}} \tag{7}$$

as reported in V2. The departures from this behavior at low and high heat currents were also observed in V2. The rise at low values of W was attributed to the proximity to a critical heat current W_c, τ_f presumably becoming infinite at W_c. The behavior at large heat currents is apparently due to the importance of the RC time constant τ_{RC} of the apparatus, consisting of the thermal resistance of the tube C and the total thermal capacity of the chamber A and its contents. It should be noted, however, that an estimate of τ_{RC} for the present apparatus suggests that it is an order of magnitude too small to explain the observed departures from Eq. (7). It is therefore possible that some additional effect may become important at large heat currents. Vinen (V2) was able to study the growth of mutual friction in this region by observing the attenuation of second sound propagated across the heat current, which is practically unaffected by τ_{RC}. In this way, he found that (*i*) Eq. (7) still holds for large W, and (*ii*) at large heat currents mutual friction sets in abruptly after a short delay. Similar observations are not possible with the present apparatus, since no second sound equipment is employed. However, it is evident that a temperature gradient will first appear at that time when mutual friction sets in, since before that time the thermal resistance of the channel C is zero and the capacity of A is not yet charged. If the onset of mutual friction is sufficiently abrupt, it should be possible to estimate τ in this range by measuring to the point at which the temperature gradient first becomes evident. Measurements made in this way are shown in parentheses in Fig. 5, and can be seen to obey Eq. (7) in agreement with (*i*). In all subsequent measurements values of W less than 0.2 watt/cm² were used, so that the complications attendant upon working in the region of high heat currents were avoided.

By making measurements like those shown in Fig. 5 at a number of temperatures, it is possible to determine the temperature dependence of the constant a in Eq. (7). This is shown in Fig. 6 by the circles and full curve. The broken line represents values of the same quantity given in V2. Although agreement is good around 1.3°K, an increasing discrepancy, eventually amounting to about a factor of two, arises at higher temperatures. No combination of experimental errors appears to be large enough to account for this discrepancy, nor has it been possible to think of any deficiency in the present experimental procedure which could result in an apparent *increase* in τ. It is possibly the result of geometrical differences in the size, shape, and orientation of the apparatus, or in the surface state of the tube in which the heat current flows.

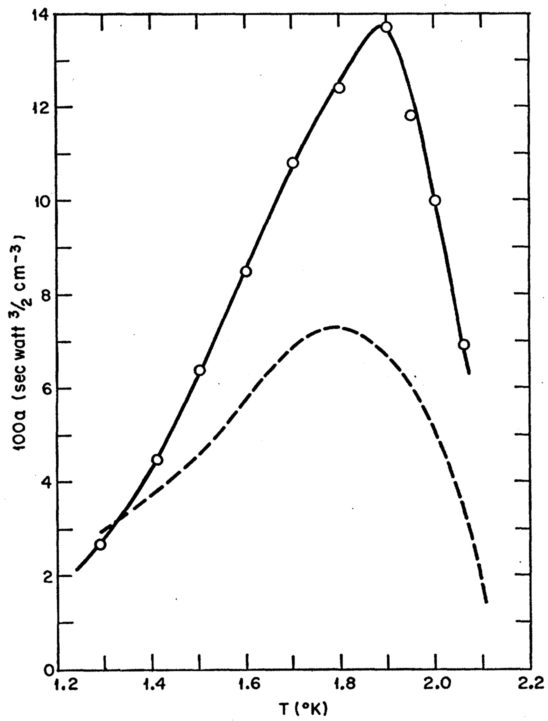

FIG. 6. Temperature dependence of a [Eq. (7)]. ○ present results; ---- V2.

(b) The Effects of Rotation

Suppose the helium is left undisturbed for a sufficiently long time ($\approx$3 minutes), and the apparatus is then suddenly set into rotation, at a constant angular velocity ω, at time $t=0$. After a further time t_ω, a heat current W is switched on and τ_f is measured. The dependence of τ_f on t_ω determined in such an experiment is shown in Fig. 7 for three different values of ω. These data were taken at 1.41°K with a heat current density of 0.159 watt/cm²; the behavior at other temperatures and heat currents is similar. From Fig. 7 it is evident that equilibrium is only reached after approximately one minute of rotation, and in subsequent experiments the apparatus was always rotated at least this long

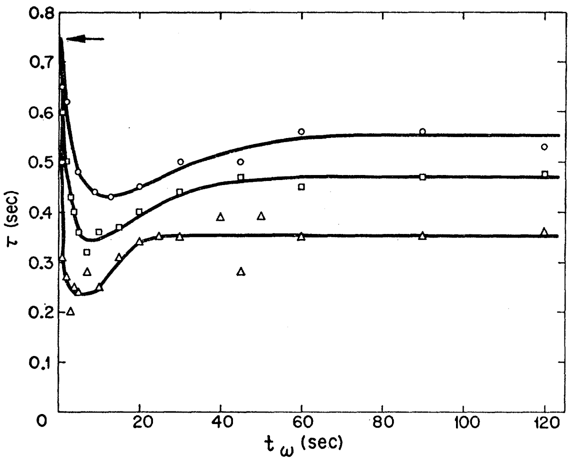

FIG. 7. Dependence of τ on t_ω. ($T=1.41°K$, $W=0.159$ watt/cm^2.) ○, $\omega=1.0$; □, $\omega=2.0$; △, $\omega=4.0$ rad/sec. The arrow indicates the value of τ at $\omega=0$.

before a measurement was made (except in the case of some measurements, shown in Fig. 10, deliberately made near the minima in the curves in Fig. 7). The presence of a minimum in these curves at $t_\omega \approx 5$–10 sec is interesting. If one assumes, as Vinen does, that a decrease in τ corresponds to an increase in the turbulence of the superfluid, it is clear that this turbulence grows rapidly to a maximum a few seconds after the start of rotation and then slowly decays, leaving the liquid in a state of uniform rotation. This is not surprising, since the apparatus lacks rotational symmetry

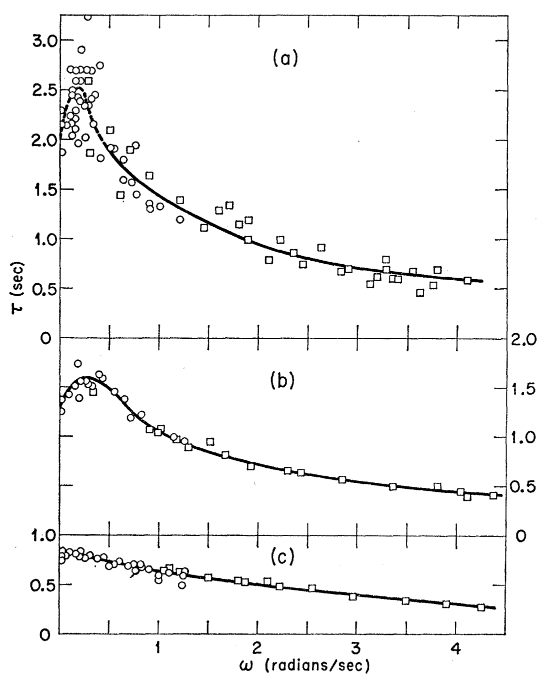

FIG. 8. Dependence of τ on ω (1.41°K). (a), $W=0.081$; (b), $W=0.107$; (c), $W=0.159$ watt/cm^2.

and the entire liquid must accordingly be accelerated in a turbulent manner. It should be noted that, even after this transient has died out, τ is smaller than in the nonrotating liquid. This circumstance, which was anticipated in the introduction, is described in greater detail in the following paragraphs.

Figure 8 shows the dependence of τ on ω for three different heat currents at 1.41°K; different symbols refer to different runs. The magnitude of the scatter is evident from this figure, and it can be seen that it increases with increasing τ (or decreasing W). In general, τ decreases as ω increases, but there is some suggestion of a maximum near $\omega=0.2$ rad/sec at the smaller heat currents. Unfortunately, the scatter is so large that the reality of this maximum cannot be positively affirmed.

Figure 9 shows τ as a function of W at angular velocities of 1.0, 2.0, and 4.0 rad/sec at a temperature of 1.41°K. The straight line without experimental points reproduces the results of Fig. 5 at $\omega=0$; the other curves are theoretical ones and will be discussed below. Each point in this figure (as well as in Fig. 10) is the average of from two to five separate observations.

In Fig. 9, the apparatus was always rotated for at least one minute before observations were begun. Figure 10 shows the corresponding results which are obtained if t_ω is deliberately chosen in the region of the minima of Fig. 7; these values have been designated by $\tau_{\min}$. Comparison of Figs. 9 and 10 shows that for all values of W and ω, $\tau_{\min}$ is less than τ; the significance of this quantity will be discussed in the following section.

(c) Discussion

In the preceding section it was shown that τ is appreciably reduced by rotation, and that the effect is more pronounced at low heat currents. In order to understand the significance of this result, it is necessary to have a detailed model of the mechanism responsible for the growth of mutual friction. Such a model has been developed by Vinen (V3). As previously mentioned, he assumes that mutual friction is the result of turbulence in the superfluid, taking the form of a tangled mass of vortex line. Detailed calculations by Vinen based on

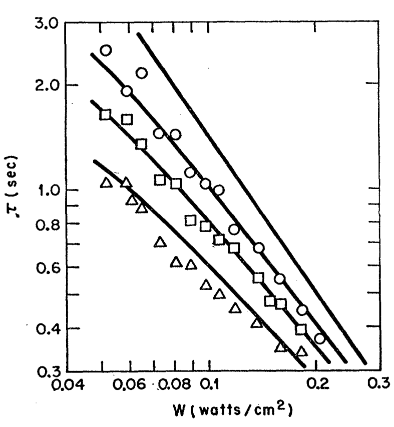

FIG. 9. Dependence of τ on W at various angular velocities. Upper curve, $\omega=0$; ○, $\omega=1.0$; □, $\omega=2.0$; △, $\omega=4.0$ rad/sec. Curves through the experimental points are theoretical ones.

this assumption gave good agreement with the growth curves (grad$T$ against time) measured in V2, and led to the following expression for τ_f:

$$\tau_f=\frac{1}{K_1v^2}\int_0^{\frac{1}{2}}\frac{d\mathfrak{L}}{\mathfrak{L}^{\frac{3}{2}}(1-\mathfrak{L}^{\frac{1}{2}})+\Lambda}, \tag{8}$$

where K_1 is a constant at any given temperature evaluated from Vinen's growth curves; $v=|v_n-v_s|$; $\mathfrak{L}=L/L_0$, the fraction of the equilibrium length of vortex line present at any instant of time; and Λ represents an undefined starting function which is required to initiate the growth of turbulence in the undisturbed liquid. Without the presence of this function, the integral in Eq. (8) would diverge. Λ is a function of temperature and heat current, and was evaluated by Vinen from his experimental data. The upper limit $\frac{1}{2}$ on the integral results from the definition of τ_f as the time required for grad$T$ to reach $\frac{1}{2}$ its final value. In V3 it was further shown that, if there is initially present a small length of vortex line L_i (for example the residue of a previously applied heat current), Eq. (8) is only modified by changing the lower limit of the integral to $\mathfrak{L}_i=L_i/L_0$.

Rotation may require modification of Eq. (8) in two ways: (*i*) The presence of vortex lines produced by rotation may require a nonzero value of the lower limit of the integral, as is the case when residual turbulence is present, and (*ii*) the value of Λ may be changed. However, nothing is known about the mechanism responsible for Λ, so that it is useless to speculate about the possible effects of rotation upon this function. In any case, for sufficiently large values of the lower limit of the integral the effect of Λ on the result is negligible, so that except at very low angular velocities small changes in Λ would be undetectable.

In the following analysis it is assumed that only mechanism (*i*) is operative, that rotation produces a length of vortex line per unit volume

$$L_\omega=2\omega m/h, \tag{9}$$

where m is the mass of the helium atom and h is Planck's constant, and that L_ω has an effect similar to that of

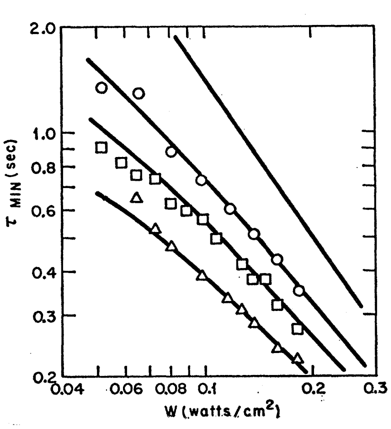

FIG. 10. Dependence of $\tau_{\min}$ on heat current density. Upper curve, $\omega=0$; ○, $\omega=1.0$; □, $\omega=2.0$; △, $\omega=4.0$ rad/sec. Curves through the experimental points are theoretical ones.

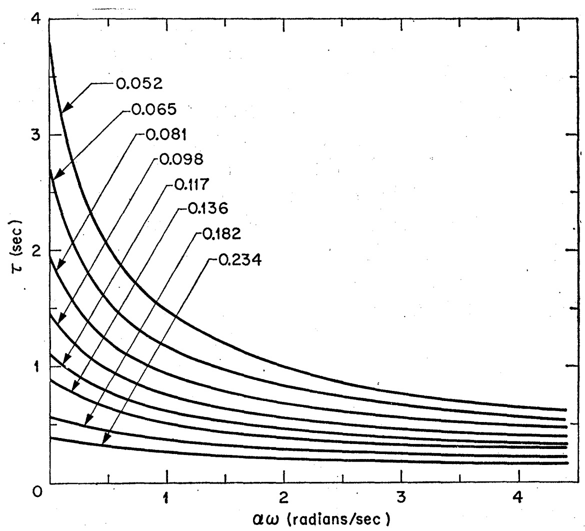

FIG. 11. Calculated dependence of τ on $\alpha\omega$ at various values of W (1.41°K). Numbers attached to the curves give the heat current density in watts/cm^2.

residual turbulence upon τ. In order to take into account possible differences due, for example, to the different geometrical arrangement of the lines, the "effective" length of vortex line L_{eff} will be defined by

$$L_{\text{eff}}=\alpha L_\omega, \tag{10}$$

where α is a dimensionless parameter of order unity; the lower limit of the integral in Eq. (8) will then be replaced by $\mathfrak{L}_i=L_{\text{eff}}/L_0$. The value of α thus indicates the degree to which the different geometry of the vortex lines affects the growth of mutual friction. α should be independent of temperature, heat current, and angular velocity.

The resulting integral cannot readily be evaluated by analytical methods; the necessary computations were therefore carried out by Gaussian 9-point quadrature on a Royal-McBee LGP-30 computer for various values of heat current and angular velocity at a temperature of 1.41°K. Values of the constants K_1 and Λ were taken from V3. The results of this calculation are shown in Fig. 11, where τ is plotted as a function of $\alpha\omega$ for various values of W. It is clear that these curves have the same general form as the experimental ones shown in Fig. 8, except for the absence of a maximum at small heat currents and low angular velocities. In the light of the above discussion it is tempting to suppose that this maximum is connected with a reduction in Λ by rotation; this would lead to an initial increase in τ, but the effect of $\mathfrak{L}_i$ would dominate the behavior of the integral at large angular velocities leading to a maximum such as is observed. Such a conclusion, however, is hardly justified until more is known about Λ.

The success of this model in explaining the observations is shown by the full curves in Figs. 9 and 10; in each figure the value of α has been adjusted to give the best fit to the experimental points. In Fig. 9, which shows the value of τ after rotational equilibrium has been reached, $\alpha=0.3$; the results of Fig. 10, corresponding to the state of maximum turbulence during acceleration of the liquid, are best represented with $\alpha=1.0$. The general agreement between theory and experiment which is obtained in this way suggests that Vinen's ideas are essentially correct and capable of explaining the behavior of rotating as well as nonrotating liquid helium. The fact that the results during the starting transient are also well fitted by the theory indicates that even this turbulent state can be described in the same way, although the fact that $\alpha=1.0$ in this case is probably coincidental.

V. CONCLUSIONS

The effect of rotation upon the thermal conductivity of liquid helium II and upon the delay time τ associated with the growth of mutual friction has been studied in a wide (2.62-mm diameter) channel of circular cross section. The thermal conductivity was unchanged within experimental error at angular velocities up to 1.3 rad/sec; however, a considerable change in τ was observed. In general, τ decreases as the angular velocity is increased, the effect being largest at small heat currents; however, there is some evidence that τ passes through a maximum near 0.2 rad/sec for small heat currents. In nonrotating helium, all the results obtained agree with those of V1–3, except that the temperature dependence of the delay time is somewhat different. This discrepancy may be connected with differences in the geometry or surface condition of the channel in which the heat current flows. When the helium bath level is near the apparatus a marked change in the thermal conductivity occurs, and τ simultaneously falls to a value comparable to the resolving time of the apparatus; this phenomenon may be connected with the possibility of mass transfer within the channel, but no convincing explanation has been devised.

The effects of rotation upon the delay time can be explained in terms of a semiphenomenological theory proposed by Vinen (V3). This theory postulates that mutual friction is the result of turbulence in the superfluid component, taking the form of a tangled mass of quantized vortex line; the delay time is governed by the mechanisms controlling the growth and decay of this turbulence. In particular, Vinen showed that the presence of a small amount of residual turbulence (resulting, e.g., from a previously applied heat current) greatly reduced the value of τ, and he was able to derive an equation for determining τ under such conditions. In the present case, it was assumed that rotation produces an array of vortex lines parallel to the axis of rotation, and that these lines, apart from a possible geometrical factor, have the same effect upon τ as an equal length of randomly oriented line remaining as the residue of an earlier turbulent state. The geometrical factor was represented by a dimensionless parameter α of order unity, describing the relative effect of the two different arrangements of vortex line upon τ. The results of the present experiments can be adequately explained if $\alpha=0.3$. Some observations of τ were also made during initial acceleration of the liquid, when turbulence was presumably present in both the normal and superfluid components. The minimum value of τ, corresponding to the instant when this turbulence was at a maximum, could also be explained in the above manner with $\alpha=1.0$. The maximum in $\tau(\omega)$ which was observed at low heat currents and small angular velocities cannot be explained by the above theory. However, the expression given in V3 for τ contains a "starting function" Λ of uncertain nature which governs the initiation of turbulence in the undisturbed liquid. One may speculate that Λ is reduced by rotation, giving rise to a maximum such as is observed, but such speculation is of little value until the theory has been developed in greater detail.

It is clear that the above analysis is far from rigorous, and much theoretical work is required before a full quantitative explanation can be attempted. Unfortunately, a detailed theoretical treatment of this problem appears hopelessly difficult at the present time. The general agreement which has been obtained, however, gives further support to Vinen's conclusions, and suggests that his basic ideas are essentially correct.

ACKNOWLEDGMENTS

The author is grateful to Mrs. Ann Sherman for programming and carrying out the calculations described in Sec. IV (*c*), and to Dr. W. F. Vinen for helpful advice and comments upon the manuscript.

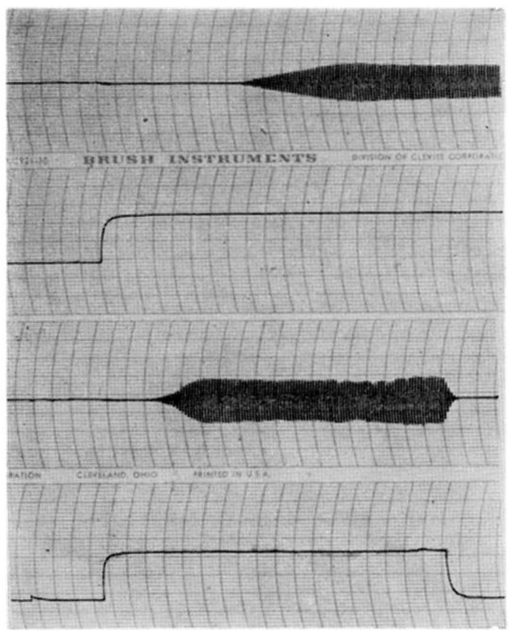

FIG. 2. Typical records of data ($T = 1.41$°K; $W = 0.081$ watt/cm^2; tape speed 25 mm/sec). Top: $\omega = 0$, bottom: $\omega = 2.0$ rad/sec. The amplitude of the upper trace of each pair is proportional to the temperature gradient, the upward deflection of the lower trace to the voltage across the heater. Each horizontal division $= 0.2$ sec.