Pion-Pion Interaction in Electromagnetic Processes

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The correction to the photon propagator due to the emission of a virtual pion pair is evaluated taking into account the effect of the strong pion-pion interaction in the $J=1$, $T=1$ state of the pions recently discussed by several authors. Results are given for different sets of parameters describing the pion form factor, and the possibility of investigating the structure of the form factor by means of electron-electron and electron-positron collision experiments is studied.

I. INTRODUCTION

DURING the past years some suggestions have been made of a resonant interaction of pions with pions.[1] In particular, a strong interaction in the $J=1$, $T=1$ state of two pions was introduced by Holladay[2] in studying the nucleon's anomalous magnetic moments and the n-p mass difference in Sachs' model of the nucleon. More recently Frazer and Fulco,[3] following the approach of Chew and Mandelstam, and assuming a resonant interaction in this state of two pions, have derived an electromagnetic form factor for the pion. They were able to show that by appropriate choice of the position and width of the resonance they could resolve previous discrepancies between the experimental data and the dispersion-theoretical treatment of the isotopic vector parts of the nucleon form factors.[4,5] Subsequently this suggestion has been considered in connection with some other strong-interaction processes.[6] In the present work we are primarily studying the effect of the FF pion form factor on certain purely electromagnetic processes[7] in order to derive information about the form factor in a context in which we do not have to simultaneously interpret other strong-interaction effects.

We may expect pions to play an important role in electromagnetic processes only at center-of-mass energies of the order of the pion mass. Experiments at such energies, using clashing electron beams, are now being prepared at Stanford University.[8]

For stydying the pion form factor the most obvious type of experiment is that which leads to the production of real pion pairs. In the case of electron-positron collisions, the annihilation into two pions is of the same order of magnitude as the large-angle Bhabha scattering.[7,9] In fact it occurs already in the same order of perturbation theory as the scattering. In this lowest order of approximation the only possible two-pion final state is the $J=1$, $T=1$ state; therefore the cross section is proportional to $|F_\pi|^2$, the absolute square of the $J=1$, $T=1$ form factor. (It must be borne in mind that "lowest order" statements are approximate owing to radiative corrections, which at these energies are important. However, if $|F_\pi|^2$ is strongly peaked, as FF suggest, the $J=1$, $T=1$ state will be strongly favored near the energy of the peak.) In the case of electron-electron collisions, pion pair creation is also possible, but one may expect the cross section to be smaller by a factor α^2.

Another class of experiments which can give information on the pion form factor involves the effects of virtual pion pairs. These are studied in detail in this work.

In Sec. II we derive the general expression for the photon propagator in terms of a sum over all possible real intermediate states, following the method of Källén.[10] The sum is then restricted to two-pion inter-

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[1] F. J. Dyson, Phys. Rev. **99**, 1037 (1955); G. Takeda, Phys. Rev. **100**, 440 (1955); M. Cini and A. Eberle, Proceedings of International Conference on Mesons and Newly Discovered Particles, Padua-Venice, 1957 (to be published).

[2] W. G. Holladay, Phys. Rev. **101**, 1198 and 1202 (1956).

[3] W. R. Frazer and J. R. Fulco, Phys. Rev. Letters **2**, 365 (1959); Phys. Rev. **117**, 1609 (1960), hereafter referred to as FF.

[4] G. F. Chew, R. Karplus, S. Gasiorowicz, and F. Zachariasen, Phys. Rev. **110**, 265 (1958).

[5] P. Federbush, M. L. Goldberger, and S. B. Treiman, Phys. Rev. **112**, 642 (1958).

[6] M. Gourdin and A. Martin, Nuovo cimento **16**, 78 (1960). J. Bowcock, W. N. Cottingham and D. Lurié, Nuovo cimento **16**, 918 (1960); F. Bonsignori and F. Selleri, Nuovo cimento **15**, 465 (1960); F. Cerulus, Nuovo cimento **14**, 827 (1959).

[7] L. M. Brown and F. Calogero, Phys. Rev. Letters **4**, 315 (1960), hereafter referred to as BC. The expression for $\Pi(p^2)$ contained in BC Eq. (8) is too large by a factor of 2, and as a result so also are the expressions derived from it [Eqs. (9), (12), and (13), as well as the corresponding Figs. 1 and 2]. This error was kindly indicated to the authors by Prof. L. Michel. The same error appears also in older results on the spin-0 vacuum polarization contained, e.g., in reference 13 below—and has been noted by A. I. Akhiezer and V. B. Berestetski in *Quantum Electrodynamics* (Second Edition, State Technico-Theoretical Literature Press, Moscow, 1959).

[8] Barber, Richter, Panofsky, O'Neill, and Gittelman, High-Energy Physics Laboratory, Stanford University Report HEPL-170, June, 1959 (unpublished); W. K. Panofsky, Proceedings of the Ninth Annual International Conference on High-Energy Nuclear Physics, Rochester, 1959 (unpublished); G. K. O'Neill and E. J. Woods, Phys. Rev. **115**, 659 (1959).

[9] N. Cabibbo and R. Gatto, Phys. Rev. Letters **4**, 313 (1960).

[10] G. Källén, in *Handbuch der Physik*, edited by S. Flügge (Springer-Verlag, Berlin, 1958), Vol. 5, Part 1; see especially

mediate states, and the correction to the photon propagator due to the virtual emission of a noninteracting pair of pions is obtained. In Sec. III the FF pion form factor is inserted, obtaining the correction due to a virtual pair of strongly interacting pions. In Sec. IV this result is used to obtain corrections for electron-electron and electron-positron collisions. The influence of ordinary radiative corrections is also discussed.

The results turn out to be small (less than 1%); however, they should be considered in the analysis of experiments designed to look for a failure of quantum electrodynamics. Also, under certain experimental conditions the corrections have a characteristic energy dependence, similar to the derivative of the pion form factor, and could be used to determine precisely the position of the peak as well as to distinguish them from the other radiative corrections of higher order in α which have not yet been calculated but which can reasonably be expected to be monotonic at high energy

II. GENERAL EXPRESSION FOR THE PHOTON PROPAGATOR

In this section we write as a sum over real intermediate states the propagation function of the photon $D_{\mu\nu}^{F'}(x)$, whose definition is

$$D_{\mu\nu}^{F'}(x-x')=i\langle 0|PA_\mu(x')A_\nu(x)|0\rangle. \qquad (1)$$

The Fourier transform of this function can be written[10,11] as the sum of the free-field propagator and an explicitly gauge-invariant correction:

$$D_{\mu\nu}^{F'}(p)=\frac{\delta_{\mu\nu}}{p^2-i\epsilon}+\frac{p^2\delta_{\mu\nu}-p_\mu p_\nu}{p^2}\times\frac{\bar{\Pi}(0)-\bar{\Pi}(p^2)-i\pi\Pi(p^2)}{p^2-i\epsilon}. \qquad (2)$$

In this expression $\Pi(p^2)$ is a sum over all physical states $|z\rangle$ having total energy-momentum $p^{(z)}=p$:

$$\Pi(p^2)=-(V/3p^2)\sum_{p^{(z)}=p}\langle 0|j_\mu(0)|z\rangle\langle z|j_\mu(0)|0\rangle, \qquad (3)$$

where $j_\mu(x)$ is the current operator and V is the normalization volume. $\bar{\Pi}(p^2)$ is also a real quantity, given by

$$\bar{\Pi}(p^2)=\mathrm{P}\int_0^\infty \frac{\Pi(-a)}{p^2+a}da. \qquad (4)$$

One may note that the result actually depends on the renormalized quantity

$$\bar{\Pi}(0)-\bar{\Pi}(p^2)=\mathrm{P}p^2\int_0^\infty \frac{\Pi(-a)}{a(p^2+a)}da, \qquad (5)$$

which is convergent even if $\Pi(-a)$ remains finite for large a.

Equation (2) for the propagator, with the definitions which follow it, is exact. While this important formula appears in the literature,[11] we have not been able to find a derivation. We have thought it worth while to provide one in the Appendix.

Specializing to the contribution to (2) from two-particle states $|k_1k_2\rangle$, we can write

$$\Pi(p^2)=-\frac{V}{3p^2}\sum_{p=k_1+k_2}\langle 0|j_\mu(0)|k_1k_2\rangle\langle k_1k_2|j_\mu(0)|0\rangle, \qquad (6)$$

and, in particular,[4] for two-pion states treated in lowest order in the electromagnetic coupling,

$$\langle 0|j_\mu{}^0(0)|k_1jk_2i\rangle=\frac{ieV^{-1}}{(4\omega_1\omega_2)^{\frac{1}{2}}}(k_1-k_2)_\mu\times(\delta_{j1}\delta_{i2}-\delta_{j2}\delta_{i1})F_\pi(p^2). \qquad (7)$$

Here ω_1, ω_2 are the energies of the pions and i, j are isotopic spin indices. $F_\pi(p^2)$ is the pion form factor which we shall discuss in the next section. From the form of Eq. (7) it is clear that only the $T=1$, $J=1$ state of two pions is produced.[12]

Letting the normalization volume in (6) go to infinity, we replace the restricted sum by an integral over a δ function, and substituting (7), we get

$$\Pi(p^2)=\frac{e^2|F_\pi(p^2)|^2}{6p^2(2\pi)^3}\int d\mathbf{K}_1d\mathbf{K}_2\frac{1+\mathbf{K}_1\cdot\mathbf{K}_2}{\omega_1\omega_2}\times\delta(\mathbf{K}_1+\mathbf{K}_2-\mathbf{p})\delta(\omega_1+\omega_2-p_0) \qquad (8)$$

$$=\frac{\alpha|F_\pi(p^2)|^2}{6\pi p^2}\int_{K_l}^{K_u}\frac{KdK}{|\mathbf{p}|(K^2+1)^{\frac{1}{2}}}\left(2+\frac{p^2}{2}\right) \qquad (9)$$

$$=\frac{\alpha|F_\pi(p^2)|^2}{12\pi}\left(1+\frac{4}{p^2}\right)^{\frac{3}{2}}\theta(-p^2-4), \qquad (10)$$

where

$$\theta(x)=1 \quad \text{for} \quad x>0$$
$$=0 \quad \text{for} \quad x<0,$$

and the momenta are measured in units of the pion mass.

We must evaluate also the real part of the correction $\bar{\Pi}(0)-\bar{\Pi}(p^2)$ and this involves an integral which depends on the detailed form of $|F_\pi(p^2)|^2$. In this section we neglect the pion-pion interaction, taking $|F_\pi|^2=1$, and obtain the well-known results[13]:

$$\Pi(p^2)=(\alpha/12\pi)z^{\frac{3}{2}}\theta(-p^2-4), \qquad (11)$$

$$\bar{\Pi}(0)-\bar{\Pi}(p^2)=-(\alpha/12\pi)f(z), \qquad (12)$$

Sec. 43. We use the notation, and in particular the metric, of Källén; $p^2=-p_0{}^2+\mathbf{p}^2$; $e^2/4\pi\approx 1/137$.

[11] G. Källén, CERN Report 57-43, 1957 (unpublished).

[12] This may be seen also less formally. The two-pion state is connected to one photon, hence, it must have $J=1$. But then the Pauli principle selects the antisymmetric isotopic spin state $T=1$.

[13] R. P. Feynman, Phys. Rev. **76**, 769 (1949).

with $z=1+4/p^2$, and

$$f(z)=\tfrac{2}{3}+2z+z^{\frac{3}{2}}\ln[(z^{\frac{1}{2}}-1)/(z^{\frac{1}{2}}+1)]. \qquad (13)$$

For real positive z, the absolute value of the argument of the logarithm must be taken in $f(z)$. For negative z, one must replace $z^{\frac{1}{2}}\ln[(z^{\frac{1}{2}}-1)/(z^{\frac{1}{2}}+1)]$ by $-2y\,\mathrm{arc\,cot}y$, with $y=(-z)^{\frac{1}{2}}$. For complex values of z (relevant in the next section), $f(z)$ is defined in a plane cut along the negative real axis.

It is interesting to compare Eqs. (11) and (12) with the corresponding quantities calculated for a pair of spin-$\frac{1}{2}$ particles having the same mass. This comparison is displayed in Figs. 1(a) and 1(b). In particular, the dispersive part [Fig. 1(b)], which is the part entering in the lowest order radiative corrections, is very different for the two cases. While both show peaked behavior near the pair threshold, in contrast to the spin one-half result the spin zero peak is about one-tenth as high and has a discontinuity only in the second derivative. This is due to the difference in the two cases of the threshold behavior of the absorptive part $\Pi(p^2)$, arising from the fact that while the boson pair must be produced in a P state, the fermion pair may be produced also in an S state.

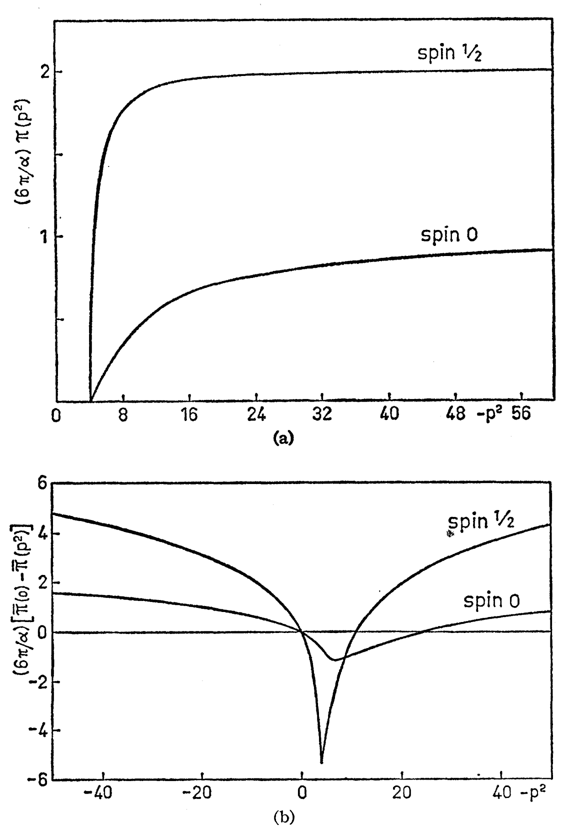

FIG. 1. (a) The imaginary part and (b) the real part of the correction to the photon propagator due to the emission of a noninteracting virtual pair. p is measured in units of the mass of one of the virtual particles. The spin-0 graph is amplified by a factor of 2.

TABLE I. Parameters of Eq. (14) corresponding to the curves of FF, and constants γ and δ of Eq. (18).

Curve	A	B	C	γ	δ
a	115.2	2.65	10.4	0.0185	0.082
b	66.24	1.5	8	0.047	0.106
c	170.5	3.77	12.5	0.014	0.101

III. CORRECTION TO THE PHOTON PROPAGATOR INCLUDING PION-PION INTERACTION

The electromagnetic pion form factor proposed by FF has a rather complicated analytic form. We have found, however, that it can be well approximated by a simple resonance curve, which is very convenient in performing the integration necessary for obtaining the real part of the correction to the photon propagator.

We introduce the function

$$|F_\pi'(p^2)|^2=A[(p^2+C)^2+B^2]^{-1}, \qquad (14)$$

adjusting the three real parameters A, B, C to fit the curves given by FF. This can be done quite satisfactorily over the entire resonance region (which occurs for $p^2<0$), and in particular we satisfy exactly the condition

$$|F_\pi'(0)|^2=1. \qquad (15)$$

For $p^2<p_0^2$, where $p_0^2=2C$ is the point beyond the resonance for which

$$|F_\pi'(p_0^2)|^2=1, \qquad (16)$$

the agreement is less satisfactory, Eq. (14) falling below the FF curve. However, p_0^2 has a value such that for $p^2<p_0^2$ the contribution to the form factor from states containing four or more pions might be of importance, so that even the FF curve becomes unreliable. In this region we have chosen to put the form factor equal to unity, obtaining thus the contribution of a noninteracting pion pair for this part of the integral. That is, we choose

$$\begin{aligned}|F_\pi(p^2)|^2&=|F_\pi'(p^2)|^2, \quad 0>p^2>p_0^2\\ &=1, \qquad\qquad\quad p_0^2>p^2.\end{aligned} \qquad (17)$$

While we believe this choice of asymptotic behavior to be the most reasonable one in the absence of other knowledge, the convergence of the integral Eq. (5) is such that the contribution from the region $a>-p_0^2$ is only of the order of a few percent for any reasonable p^2.

FF have given, in fact, three curves corresponding to three different sets of resonance parameters, all of which lead to agreement with the experimental data on the nucleon form factors. Our fits for these curves are given in Table I. Inserting Eqs. (17) and (10) into Eq. (5), the integration can be performed analytically and yields the final result,

$$\begin{aligned}\bar{\Pi}(0)-\bar{\Pi}(p^2)=(\alpha/12\pi)\{&(1-|F_\pi'(p^2)|^2)z_0^{\frac{3}{2}}f(z/z_0)\\ &-f(z)+p^2|F_\pi'(p^2)|^2[-\gamma+\delta(C+p^2)/B]\}.\end{aligned} \qquad (18)$$

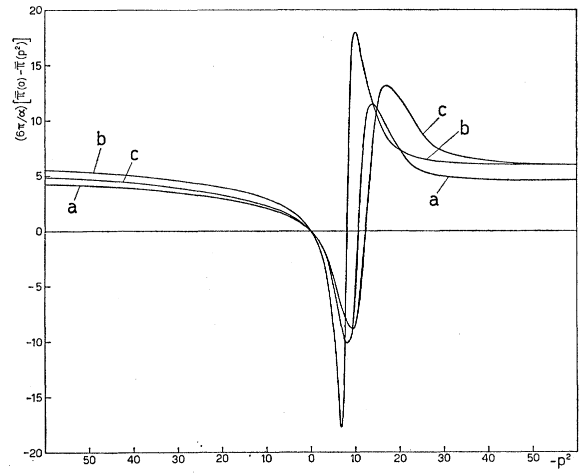

FIG. 2. The real part of the correction to the photon propagator due to the emission of a virtual pair of interacting pions. The curves refer to different sets of parameters describing the pion form factor. p is measured in units of the pion mass. The "6" in parenthesis at the side of the graph should be a "12."

The function $f(z)$ is given by Eq. (13) and $z_0=1+4/p_0^2$.

The constants γ and δ are defined by

$$\gamma+i\delta=\tfrac{1}{4}(1-z')z_0^{\frac{3}{2}}f(z'/z_0), \tag{19}$$

with

$$z'=1-4(C+iB)^{-1};$$

their numerical values are given in Table I. We have plotted Eq. (18) for the three curves of FF in Fig. 2. One notices that in the region around the peak of the form factor, the real part of the correction to the photon propagator resembles the derivative of the resonance curve. This may easily be seen to be a property of the Hilbert transform [Eq. (4)] of a peaked function. This behavior may be useful in the experimental determination of the resonance parameters.

IV. APPLICATION TO ELECTRON-ELECTRON COLLISIONS

The correction to the photon propagator which we have calculated above may be applied immediately to a number of problems of current interest,[14] of which the most important are probably high-energy electron-electron and electron-positron collision cross sections.

Considering the lowest order Møller or Bhabha scattering, we note that only the real part of the correction to the photon propagator enters because of the reality of the lowest order scattering matrix elements. We obtain for the fractional correction

$$K=\frac{2\mathbf{S}(M_1+M_2)(C_1^RM_1+C_2^RM_2)}{\mathbf{S}(M_1+M_2)^2}, \tag{20}$$

where M_1 and M_2 give the contribution of the two lowest-order diagrams, characterized by the momentum transfers p_1 and p_2, and the symbol $\mathbf{S}$ represents a sum over final and an average over initial spins. The correction factors C_i are defined through

$$C_i=C(p_i^2)=\bar{\Pi}(0)-\bar{\Pi}(p_i^2)-i\pi\Pi(p_i^2)$$
$$=C_i^R+iC_i^I. \tag{21}$$

Of the two lowest-order diagrams for Møller scattering, each describes the exchange of a virtual photon having a momentum which is spacelike, and therefore outside of the more interesting region. The correction in this case turns out to be slowly varying and smaller than in the electron-positron case.

Of the Bhabha diagrams, one contains a spacelike photon and the other a timelike photon for which $-p^2$ is equal to the square of the total center-of-mass energy. The contribution of the second diagram is effective at backward c.m. angles, transferring to the cross section the peculiar energy dependence of the real part of the photon propagator exhibited by Fig. 2.

Explicit formulas for K as a function of angle and energy for Møller and Bhabha scattering are given in BC together with illustrative curves for one resonance [parameter set (b)]. For other resonance parameters and other experimental conditions, numerical results are easily obtained from the formulas given in BC and from Fig. 2 of this article.[7]

We may remark that in the annihilation of electron and positron into a pair of particles (not photons), only the diagram containing the timelike photon is present in lowest order, so that the correction to the cross section for this process is simply the factor

$$1+K=1+2C^R(p^2), \tag{22}$$

where $-p^2$ is the square of the total c.m. energy.

We come now to a consideration of the influence of other radiative corrections on our results. In general these are large at high energies, and not only must they be included in any experimental comparison but also their interference with pion-pion interaction effects is relevant. However, these corrections, which have been evaluated in several articles,[15] appear to depend critically on the way the experiment is performed, as has been recently emphasized.[16]

It is well known, in fact, that because any conceivable detection apparatus has a finite momentum resolution, it cannot distinguish elastic scattering from certain bremsstrahlung processes. When one considers radiative corrections to the scattering it is, therefore,

[14] The corrected photon propagator given in BC has been applied by L. Michel and C. Bouchiat to obtain a contribution to the magnetic moment of the muon. We wish to thank Professor Michel for informing us of this result.

[15] G. R. Lomanitz, Cornell University doctoral dissertation, 1950 (unpublished); M. L. G. Redhead, Proc. Roy. Soc. (London) **A220**, 219 (1953); A. I. Akhiezer and R. V. Polovin, Proc. Acad. Sci. U.S.S.R. **90**, 55 (1953); R. V. Polovin, J. Exptl. Theoret. Phys. (U.S.S.R.) **31**, 449 (1956) [translation: Soviet Phys.—JETP **4**, 385 (1957)]; G. Furlan and G. Peressutti, Nuovo cimento **45**, 817 (1960). We wish to thank Dr. Furlan and Dr. Peressutti for helpful communications and data in advance of publication.

[16] Yung Su Tsai, Nuovo cimento **16**, 370 (1960).

necessary to take this finite momentum resolution explicitly into account, thereby eliminating the infrared divergences of the separate results. This is done by adding to the elastic cross section the bremsstrahlung cross section integrated over the photon momentum $\mathbf{k}$ up to a value $|\mathbf{k}|=k_{\max}$, which depends on the energy resolution, as well as the angular resolution, of the electron counters. The assumption of very good energy resolution and isotropic $k_{\max}$ leads to the inclusion of only the soft-photon part of the associated bremsstrahlung processes and to large radiative corrections at large energies. For example, assuming $k_{\max}$ of the order of the electron mass, the radiative corrections at a total c.m. energy of 500 Mev are of the order of 70% at a c.m. angle of 90°.

However, Tsai has remarked[16] that in practice the aforesaid conditions assumed for $k_{\max}$ cannot be satisfied in most high-energy experiments. He has made a calculation of the radiative corrections in the high-energy limit assuming that the electron detectors have *no* energy resolution, an assumption which corresponds to the conditions of the experiment now in progress at Stanford. He was therefore required to include, besides the soft photons, the part of the hard-bremsstrahlung spectrum which is accompanied by both electrons entering the counters' angular apertures. It turns out that in this type of experiment the radiative corrections are much reduced. For example, at 90°, for a total energy of 1000 Mev and with an angular resolution of 7°, all measured in the c.m. system, the correction to the cross section is 9.5%. If, as seems likely, this cancellation between the radiative correction and the relevant part of the hard-bremsstrahlung cross section occurs also for the higher order corrections, we can consider the theoretical cross section to be sufficiently well known for the detection of the effect of pion-pion interaction, and we can neglect the interference of the latter with the ordinary radiative corrections.[16a]

Finally, we discuss the interference of the lowest order radiative corrections with the pion vacuum polarization corrections. As we have stated above, these considerations are relevant to good-resolution experiments, in which case also ordinary higher corrections should be considered. We now define as the fractional correction

$$K'=\frac{2\,\mathrm{Re}\mathbf{S}M^*N}{\mathbf{S}|M|^2}, \tag{23}$$

where $M+N$ is the matrix element for the scattering, N being the pion correction. As before, $\mathbf{S}$ represents a sum over final and an average over initial spins.

[16a] *Note added in proof.*—In considering the interference terms one must bear in mind that when hard bremsstrahlung occurs, it diminishes the energy of the intermediate photon leading to a smearing of the pion correction and a weakening of the cancellation between bremsstrahlung and radiative corrections. An intermediate energy resolution, i.e., much larger than the electron mass but not completely absent, appears therefore as the most convenient.

We write

$$M=M_1+M_2+M', \tag{24}$$

where M' is the matrix element of the radiative correction including the appropriate bremsstrahlung contribution.

We have

$$\begin{aligned}\mathrm{Re}\mathbf{S}M^*N=\mathbf{S}\{&(M_1+M_2)(C_1^RM_1+C_2^RM_2)\\&+(\mathrm{Re}M')(C_1^RM_1+C_2^RM_2)\\&-(\mathrm{Im}M')(C_1^IM_1+C_2^IM_2)\}.\end{aligned} \tag{25}$$

For Bhabha scattering, where p_1 is spacelike while p_2 is timelike, $C_1^I=0$ but C_2^I is large. Furthermore, C_2^R and C_1^R are generally very different. For these reasons a careful evaluation of Eq. (24) using the high-energy limits of the real and imaginary parts of M' would be required.[17]

For Møller scattering, instead, where p_1 and p_2 are both spacelike, $C_1^I=C_2^I=0$ and C_1^R and C_2^R do not differ greatly, being in fact equal for 90° scattering where the effect is maximum [see Fig. 1(a) in BC]. We may write

$$K'=(K+\bar{C}f)(1+f)^{-1}, \tag{26}$$

where $\bar{C}$ is the average value of C_1^R and C_2^R,

$$\bar{C}=\frac{\mathbf{S}(\mathrm{Re}M')(C_1^RM_1+C_2^RM_2)}{\mathbf{S}(\mathrm{Re}M')(M_1+M_2)}, \tag{27}$$

while f is the fractional ordinary radiative correction

$$f=\frac{2\mathbf{S}(\mathrm{Re}M')(M_1+M_2)}{\mathbf{S}(M_1+M_2)^2}. \tag{28}$$

At 90° we have $C_1=C_2=\bar{C}=\frac{1}{2}K$, and therefore

$$K'=K(1+\tfrac{1}{2}f)(1+f)^{-1}, \tag{29}$$

which for $f=-0.70$, say, gives $K'=2.17K$.

We may expect similar results at other angles.

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[17] While the leading term in the high-energy limit of the real part of the radiative corrections of order α is generally of the form $(\ln\kappa)^2$ where κ is one of the invariants on which the cross section depends, the corresponding imaginary part is expected to behave as $2\pi\ln|\kappa|$ since it arises from $(\ln\kappa)^2=(\ln|\kappa|\pm i\pi)^2$ when κ is negative. Since $\ln(E/m)$ is about 2π for $2E=500$ Mev, we may expect $\mathrm{Im}M'$ and $\mathrm{Re}M'$ to be about equal. Since C_2^I at its peak is several times the maximum of C_2^R, we may guess that the term containing the $\mathrm{Im}M'$ would dominate the term containing $\mathrm{Re}M'$. This would produce an asymmetry of the backward angle curve in Fig. 2 of BC, raising one peak while lowering the other.

APPENDIX

In this Appendix an expression for the vacuum expectation value of the time-ordered product of two photon field operators $A_\mu(x)$, $A_\nu(x')$ is derived. We start from the expression given by Källén[18] for the vacuum expectation value of the commutator of two photon fields:

$$\langle 0|[A_\mu(x),A_\nu(x')]|0\rangle$$
$$=-iD_{\mu\nu}'(x'-x)=-(2\pi)^{-3}\int dp\, e^{ip(x'-x)}\epsilon(p)$$
$$\times\left[\delta_{\mu\nu}\left(\delta(p^2)-\frac{\Pi(p^2)}{p^2}\right)+p_\mu p_\nu\left(\frac{\Pi(p^2)}{(p^2)^2}-2M\delta(p^2)\right)\right], \quad \text{(A.1)}$$

$$M=\frac{1}{2}\int_0^\infty da\frac{\Pi(-a)}{a^2}, \quad \text{(A.2)}$$

which we write as

$$D_{\mu\nu}'(x)=\int_0^\infty da[\delta_{\mu\nu}\rho_1(a)-\partial_\mu\partial_\nu\rho_2(a)]D(x;a), \quad \text{(A.3)}$$

defining

$$D(x;a)=-i(2\pi)^{-3}\int dp\, e^{ipx}\delta(p^2+a)\epsilon(p), \quad \text{(A.4)}$$

and

$$\rho_1(a)=\delta(a)+\frac{\Pi(-a)}{a},$$
$$\rho_2(a)=\frac{\Pi(-a)}{a^2}-2M\delta(a). \quad \text{(A.5)}$$

As is well known, a similar spectral representation will hold for all the D' functions,[19] in particular,[20]

$$D_{\mu\nu}^{F\prime}(x)=\int_0^\infty da[\delta_{\mu\nu}\rho_1(a)-\partial_\mu\partial_\nu\rho_2(a)]D_F(x;a), \quad \text{(A.6)}$$

where

$$D_F(x;a)=(2\pi)^{-4}\int dp\, e^{ipx}[p^2+a-i\epsilon]^{-1}. \quad \text{(A.7)}$$

Introducing the explicit form of $\rho_1(a)$, $\rho_2(a)$ we get, for the Fourier transform of $D_{\mu\nu}^{F\prime}(x)$,

$$D_{\mu\nu}^{F\prime}(p)=\int_0^\infty da[p^2+a-i\epsilon]^{-1}\left(\delta_{\mu\nu}\left[\delta(a)+\frac{\Pi(-a)}{a}\right]+p_\mu p_\nu\left[\frac{\Pi(-a)}{a^2}-2M\delta(a)\right]\right). \quad \text{(A.8)}$$

Now noticing that

$$\int_0^\infty da[a(p^2+a-i\epsilon)]^{-1}\Pi(-a)$$
$$=\int_0^\infty da[p^2-i\epsilon]^{-1}\left(\mathrm{P}\frac{1}{a}-\frac{1}{p^2+a-i\epsilon}\right)\Pi(-a), \quad \text{(A.9)}$$

and using the well-known expression

$$\frac{1}{p^2+a-i\epsilon}=\mathrm{P}\frac{1}{p^2+a}+i\pi\delta(p^2+a),$$

where P stands for principal value, we get

$$\int_0^\infty [a(p^2+a-i\epsilon)]^{-1}\Pi(-a)da$$
$$=[p^2-i\epsilon]^{-1}\int_0^\infty da\left(\mathrm{P}\frac{1}{a}-\mathrm{P}\frac{1}{p^2+a}-i\pi\delta(p^2+a)\right)\Pi(-a)$$
$$=[p^2-i\epsilon]^{-1}[\bar{\Pi}(0)-\bar{\Pi}(p^2)-i\pi\Pi(p^2)], \quad \text{(A.10)}$$

with

$$\bar{\Pi}(p^2)=\mathrm{P}\int_0^\infty da[p^2+a]^{-1}\Pi(-a). \quad \text{(A.11)}$$

In the same way we also get

$$\int_0^\infty da[a^2(p^2+a-i\epsilon)]^{-1}\Pi(-a)=[p^2-i\epsilon]^{-1}2M$$
$$-[p^2-i\epsilon]^{-1}\int_0^\infty da[a(p^2+a-i\epsilon)]^{-1}\Pi(-a). \quad \text{(A.12)}$$

Introducing (A.12) and (A.10) into (A.8), the desired expression for $D_{\mu\nu}^{F\prime}(p)$ is obtained.

[18] G. Källén, reference 10, p. 348, Eq. (43.26).

[19] H. Lehmann, Nuovo cimento **11**, 342 (1954). Lehmann treats the scalar and spinor fields; the generalization to include a vector field is straightforward.

[20] The validity of this spectral representation, which is evident for the D' functions which are solutions of the homogeneous Klein-Gordon equation and which are connected by linear relations with constant coefficients, follows also for

$$D_{\mu\nu}^{F\prime}(x)=\tfrac{1}{2}[iD_{\mu\nu}^{(1)\prime}(x)-\epsilon(x)D_{\mu\nu}'(x)].$$

[where $D_{\mu\nu}^{(1)\prime}(x)$ and $D_{\mu\nu}'(x)$ are defined so as to agree, for free fields, with the definitions used by Källén, reference 10, p. 190 ff.], from $\partial_\mu\epsilon(x)=\delta_{\mu 0}\delta(0)$ and $D_{\mu\nu}'(0)=0$.