Momentum Imparted to Complex Nuclei in High-Energy Interactions*

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The momentum imparted to nuclei in the cascade process has been calculated with the aid of the results of a recent Monte Carlo calculation. Results are presented for 0.46–1.84 Bev protons incident on Ru^{100}, Bi^{209}, and U^{238}. The forward and transverse components of momentum of the residual nucleus exhibit a wide range of possible values and are, on the average, approximately equal. The average forward component of momentum increases linearly with the excitation energy of the residual nucleus. The relation between these two quantities leads to considerably lower values of the average excitation energy associated with experimentally determined values of the forward component of momentum than the relations used previously. The calculated momentum values are in most cases consistent with experimental results.

I. INTRODUCTION

THE initial interaction of a high-energy proton with a complex nucleus is generally considered to consist of an intranuclear cascade initiated by successive interactions of the incident proton with single nucleons. As a result of this initial interaction a number of prompt particles are emitted and the residual nucleus is left with varying amounts of excitation energy and momentum. The cascade process is well suited to calculations employing the Monte Carlo technique. A comprehensive Monte Carlo cascade calculation has been performed recently.[1] This calculation gives information on the emitted particles and on the identity and excitation energy of the residual nucleus for several target nuclei at a number of bombarding energies. The results do not include data on the momentum imparted to the struck nucleus, although this quantity is in principle obtainable in this calculation.

In recent years a number of experiments that provide information on the momentum imparted to the struck nucleus in the cascade process, or that depend for their interpretation on a knowledge of this quantity, have been performed. The recoil properties of fission fragments produced in high-energy fission have been investigated both radiochemically[2–6] and in nuclear emulsions.[7–9] In these studies the average forward component of momentum imparted in the cascade process to the residual nuclei was determined. The interpretation of the above experiments has generally been based on attempts to relate the measured momenta to the excitation energy imparted to the struck nucleus, and then to compare the latter with values obtained from calculations. The relation between momentum and excitation energy has usually been obtained on the basis of highly simplified models for the cascade process, and consequently is of dubious value. A preliminary calculation of the momentum-excitation energy relation for 0.46-Bev protons on bismuth[3] based on the recent Monte Carlo calculations[1] reveals this, in fact, to be the case. A number of experiments with emulsions have yielded information on the momentum imparted to the residual nuclei[10] and on the angular distribution of the latter.[11] In these cases the experiments measure the combined effect of the cascade and evaporation processes. A comparison with a calculation based solely on the cascade process would be of help in unraveling the effects of the two processes.

In view of the large number of experiments related to the momentum imparted to nuclei in the cascade process it was felt that a calculation of this quantity was desirable. The present work is an extension of the recent Monte Carlo calculation.[1] The results of the latter are used for a determination of the forward component of momentum of the residual nucleus, as well as for an approximate determination of the transverse component of momentum. In addition, the calculated momentum values may be related to the previously calculated excitation energies, so that a relation between imparted momentum and excitation energy based on a more sophisticated model becomes available for the interpretation of experimental data. The calculation has been carried out for U^{238}, Bi^{209}, and Ru^{100}

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[1] N. Metropolis, R. Bivins, M. Storm, A. Turkevich, J. M. Miller, and G. Friedlander, Phys. Rev. **110**, 185 (1958); N. Metropolis, R. Bivins, M. Storm, J. M. Miller, G. Friedlander, and A. Turkevich, Phys. Rev. **110**, 204 (1958).
[2] N. Sugarman, M. Campos, and K. Wielgoz, Phys. Rev. **101**, 388 (1956).
[3] N. T. Porile and N. Sugarman, Phys. Rev. **107**, 1410 (1957).
[4] N. T. Porile, Phys. Rev. **108**, 1526 (1957).
[5] C. Baltzinger, University of California Radiation Laboratory Report UCRL-8430, 1958 (unpublished).
[6] J. Alexander and L. Winsberg, University of California Radiation Laboratory Report UCRL-8618 (unpublished).
[7] N. A. Perfilov, N. S. Ivanova, O. V. Lozhkin, V. I. Ostroumov, and V. P. Shamov, *Proceedings of the Conference of the Academy of Sciences U.S.S.R. on the Peaceful Uses of Atomic Energy, July 1, 1955* (Akademia Nauk S.S.S.R., Moscow, 1955), p. 55 [translation by the Consultants Bureau, New York: Atomic Energy Commission Report TR-2435, 1956].
[8] N. S. Ivanova and I. I. Pianov, J. Exptl. Theoret. Phys. (U.S.S.R.) 31, 416 (1956) [translation: Soviet Phys.—JETP 4, 367 (1957)].
[9] A. I. Obukhov, J. Exptl. Theoret. Phys. (U.S.S.R.) **35**, 1042 (1958) [translation: Soviet Phys.—JETP **8**, 727 (1959)].
[10] V. I. Ostroumov, J. Exptl. Theoret. Phys. (U.S.S.R.) **32**, 3 (1957) [translation: Soviet Phys.—JETP **5**, 12 (1957)].
[11] E. W. Baker, S. Katcoff, and C. P. Baker, Phys. Rev. **117**, 1352 (1960).

targets at energies of 0.46, 0.94, and 1.84 Bev. The details of the present calculation are described in Sec. II. The results are given in Sec. III and the comparison with experimental data is presented in Sec. IV.

II. THE CALCULATION

The momentum imparted to the residual nucleus in the cascade process was calculated by use of the output data of the Monte Carlo cascade calculation.[1] The energies of the emitted cascade particles, i.e., neutrons, protons, and pions, are given for each cascade. The component of momentum in the direction of the incident proton, p_X, as well as p_Y, one of the two perpendicular components of momentum, may be obtained for each cascade particle from listed values of the two direction cosines and the energy of the emitted particle. The values for the corresponding components of momentum of the struck nucleus are then obtained straightforwardly from the conservation of linear momentum of the system. The component of momentum in the direction of the incident proton, P_F, is referred to as the forward component of momentum in this paper and it includes those cases where the nucleus recoils backwards. The absolute value of the third direction cosine for each cascade particle may, of course, be obtained from the other two direction cosines. The Monte Carlo calculations have unfortunately not kept track of the sign of this third direction cosine, so that while the magnitude of p_Z may be calculated for each cascade particle it is impossible to determine its sign. Since in general there is more than one cascade particle per interaction, it is impossible to determine the resulting value of P_Z for the struck nucleus.

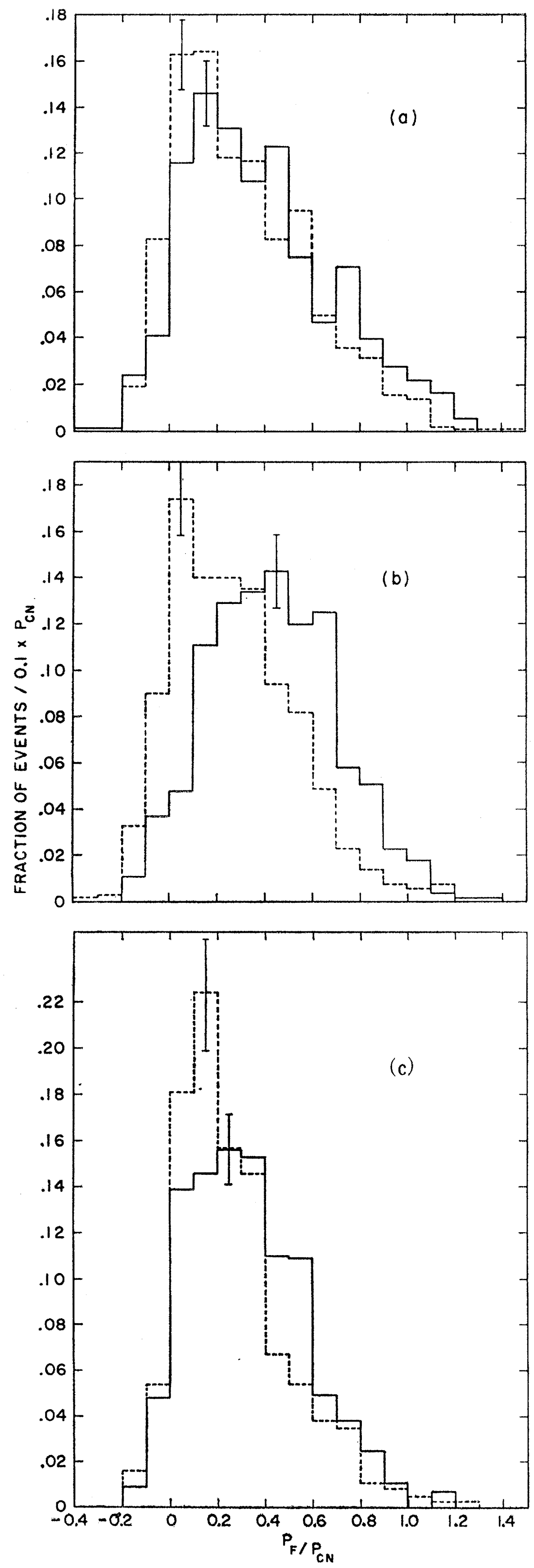
The value of the transverse component of momentum of the struck nucleus was obtained in the present calculation under the assumption that the sign of p_Z for each particle emitted in the cascade had an equal probability of being positive or negative. The actual sign of p_Z was obtained in each case by the choice of a random number. The total transverse component of momentum of the struck nucleus, $P_\perp$, was then obtained through the vectorial addition of P_Y and P_Z. This procedure is based on the assumption that any correlations among the emitted cascade particles can be approximated by a random distribution. A test of this assumption is possible in the case of the $Al^{27}(p,3pn)$ reaction. This reaction has been extensively investigated by the Monte Carlo technique[1] with the purpose of obtaining information on the momentum of the residual nucleus,[12] and the results take account of the correlations among the outgoing particles. This calculation was performed for bombarding energies of 0.36 and 1.84 Bev and includes cascades which lead to Na^{24} directly or which can lead to this product by subsequent evaporation. The momentum calculation was repeated for these cascades by use of the procedure developed for the present study. The resulting values of the transverse component of momentum, $P_\perp$, are considerably different in many cases from the values obtained in the more exact calculation, but the over-all agreement in the angular distribution of the recoils and in the distribution and average value of the transverse momentum is fairly good. While this comparison applies only to a limited variety of cascades, it seems reasonable to assume that the agreement for cascades involving the emission of more than four nucleons should be at least as good. Whatever the deficiencies of the present calculation of the transverse component of momentum may be, the latter appears to be the best procedure for obtaining any information about this quantity from the recent Monte Carlo calculations. While it is true that P_Y may be calculated exactly, it should be pointed out that the y axis does not remain constant from cascade to cascade, but is randomly oriented in the YZ plane. The Monte Carlo calculation thus does not give any explicit information on the projection of the transverse momentum in the XY plane.

The calculation of the forward and transverse components of momentum of the struck nucleus was performed for an over-all total of approximately 6000 cascades with the aid of an IBM-704 computer. Various collating routines were also programmed in order to facilitate interpretation of the data. The results of the present calculation depend, of course, on the assumptions made in the Monte Carlo cascade calculation. Several of these assumptions are of particular importance in this respect. First, the Monte Carlo calculation does not take account of the possible emission of alpha particles or other complex particles during the intranuclear cascade. The momentum imparted to the residual nucleus would depend strongly on the emission of such heavy particles. Some evidence on the emission of cascade alpha particles from silver and bromine nuclei has been obtained in recent emulsion studies.[13] It appears that the emission of cascade alpha particles may take place in a few percent of the interactions for a bombarding energy of 660 Mev. The forward emission of alpha particles and other fragments in the cascade process would in general lead to lower values for the forward component of momentum of the struck nucleus than those calculated in the present work.

The Monte Carlo cascade calculation gives the energies of the incident and emitted nucleons inside the potential well of the nucleus. The corresponding values in the laboratory system are obtained by subtraction of the total nuclear potential energy. This procedure neglects the possible energy and momentum acquired by the struck nucleus as a result of the sudden effect of the nuclear potential on the incident and emitted nucleons as they traverse the nuclear boundary. There

[12] J. B. Cumming (private communication).

[13] V. I. Ostroumov, N. A. Perfilov, and R. A. Filov, J. Exptl. Theoret. Phys. (U.S.S.R.) **36**, 367 (1959) [translation: Soviet Phys.—JETP **9**, 254 (1959)].



is some question as to whether this type of nuclear recoil is a real effect since it would involve the reaction of the nucleus as a whole in a time comparable to the nuclear transit time. The magnitude of any nuclear recoil due to this effect would furthermore be of uncertain magnitude because of uncertainty principle considerations. An approximate calculation indicates that the net effect of any nuclear recoil due to this source would lead to somewhat larger values of the forward component of momentum than those given in the present work.

The direction cosines obtained in the Monte Carlo calculation have an absolute error of about 0.02. This value includes both random and systematic effects and leads to a variable error in the momentum values obtained in the present calculation. This error is in general largest for cascades involving a small transfer of momentum to the struck nucleus. In these cases the error in the momentum values can amount to about 2% of the momentum of the incident proton. There is no other numerical uncertainty of comparable magnitude in the input information used in the present calculation.

III. RESULTS

A. Forward Momentum of Struck Nucleus

The distribution of forward momenta is given for uranium and ruthenium targets for the three bombarding energies under consideration in Fig. 1. The outstanding feature of these spectra is the wide range in values of the forward momentum imparted to the struck nuclei. This range may be divided into three separate regions. The first region consists of forward momentum values ranging between zero and the momentum corresponding to compound nucleus formation, P_{cn}. This region includes over 80% of the events and the most probable forward momentum generally occurs between one and three tenths of the momentum corresponding to compound nucleus formation.

The second region consists of negative values of the forward momentum, corresponding to backward motion of the struck nucleus. These events occur when the forward momentum of the cascade particles is greater than the momentum of the incident proton, and they reflect the internal motion of the nucleons in the target nucleus. Recoil in the backward direction occurs in about 5 to 13% of the interactions, being most probable for ruthenium and least probable for uranium. No clear trend with bombarding energy is discernible. The magnitude of the forward momentum for events of this type is on the average about 7% of the momentum corresponding to compound nucleus formation. An

FIG. 1. Distribution functions for the forward component of momentum for bombarding energies of (a) 0.46 Bev, (b) 0.94 Bev, and (c) 1.84 Bev. Solid lines—U^{238} target, dashed line—Ru^{100} target. The forward component of momentum is given in terms of the momentum of a compound nucleus.

examination of the cascades leading to these particular events shows that they very often involve the emission of one nucleon of much greater energy than that of the other emitted nucleons and that the number of internal collisions is small. These observations are particularly true at the lower bombarding energies. Cascades with these characteristics are more common for the lighter nuclei since the incident proton has to traverse through less nuclear matter and thus has a greater probability for making only a small number of collisions. These observations explain the observed trend with mass number of the fraction of events in question. The possible occurrence of nuclear recoil due to the discontinuity in potential at the nuclear boundary would have the greatest effect on this particular group of interactions, leading to a decrease in the number of such events.

The momentum distributions in Fig. 1 indicate that in a small fraction of times the struck nucleus receives more forward momentum than would correspond to compound-nucleus formation. The probability for the occurrence of such events decreases with increasing energy and decreasing target mass number, and ranges from 0.05 to 0.01 for uranium, and from 0.02 to 0.01 for ruthenium. On the average events of this type involve the transfer of approximately 10% more momentum than corresponds to compound nucleus formation. Events of this type occur when the net forward momentum of the cascade particles is negative. This process is the result of complicated cascades involving large numbers of internal collisions prior to the emission of all the cascade particles. The variation with mass number of the probability for the occurrence of these events then follows from the greater volume available for this large number of collisions in a heavy nucleus. The variation with energy may also be understood on this basis, since it takes a greater number of collisions to reverse the direction of a 1.8-Bev proton than of a 0.5-Bev proton.

The average forward momentum imparted to the struck nucleus is given as a function of the bombarding energy in Fig. 2 for different target nuclei. It is seen

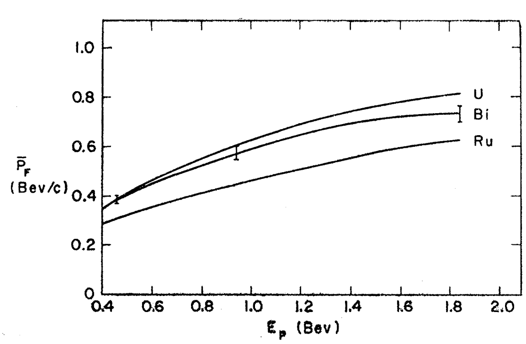


FIG. 2. Variation of the average forward component of momentum with bombarding energy. The magnitude of the statistical error is given for bismuth.

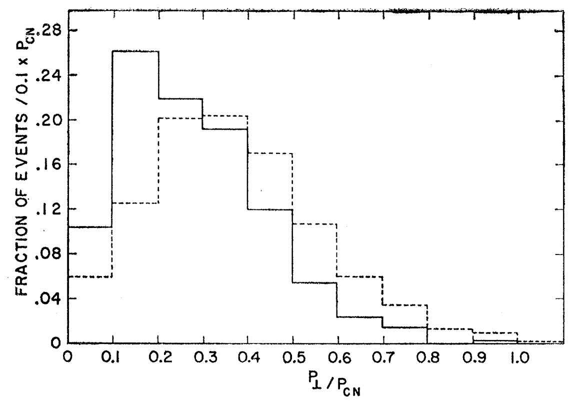


FIG. 3. Distribution functions for the transverse component of momentum. $P_\perp$ is expressed in units of the compound nucleus momentum. Solid curve—bismuth, 1.84 Bev; Dashed curve—bismuth, 0.46 Bev.

that the momentum transferred increases with the mass of the target. This variation is similar to that of the average excitation energy of the residual nucleus, and is due to the decreasing probability for the escape of cascade particles as the size of the nucleus increases. The average forward momentum imparted to the struck nucleus constitutes only a small fraction of the momentum of the incident proton and this fraction decreases with increasing bombarding energy.

B. Perpendicular Momentum of the Struck Nucleus

The spectra of the perpendicular component of momentum imparted to the struck nucleus show similar features for all the cases studied. The curves for bismuth at 0.46 and 1.84 Bev are plotted in Fig. 3. It is interesting to see that substantial transfers of transverse momentum occur in the cascade process. As in the case of the forward component of momentum, a wide range of values is possible. Very large transfers of transverse momentum appear to be less probable than very large transfers of forward momentum. The most probable and average values of the transverse momentum are about equal in magnitude to the corresponding values of the forward momentum for all the cases under consideration.

Although the average values of the two components of momentum are of similar magnitude, the value of $P_\perp$ is, for a given cascade, only slightly dependent on the value of P_F. As a result there is a striking change in the value of $P_\perp/P_F$ as the forward momentum increases, as shown in Fig. 4. The curve is representative of all the targets and bombarding energies studied and the vertical bars indicate the range of values for different targets and bombarding energies. It is seen that the value of $P_\perp/P_F$ shows a rapid increase as the forward momentum approaches zero, while at $P_F=0.2\times P_{cn}$ the curve begins to level off and only decreases relatively lowly thereafter.

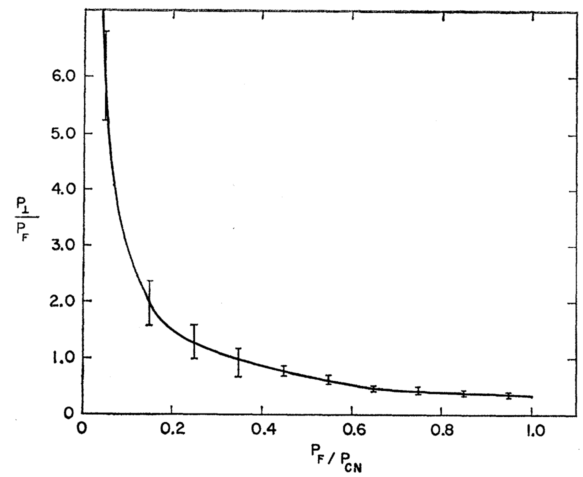


FIG. 4. Variation of the ratio of transverse and forward components of momentum with the forward component of momentum. The curve applies to all targets and bombarding energies within the indicated limits of error.

C. Relation Between the Forward Component of Momentum and the Excitation Energy of the Struck Nucleus

The relationship between the forward component of momentum imparted to the struck nucleus and the residual excitation energy may be investigated in order to facilitate the interpretation of recoil studies. Although, strictly speaking, the quantity obtained in recoil studies is the velocity of the struck nucleus rather than its momentum, there is practically no difference between these two quantities in their variation with the excitation energy because of the small percentage variation in the mass of the residual nucleus. The probability for the occurrence of different $P_F - E^*$ pairs is given as a contour plot in Fig. 5 for 0.94-Bev protons on bismuth and is typical of all the cases studied. The following features of this plot may be noted. First, the most probable value of the excitation energy increases as the forward component of momentum increases. Second, large momentum transfers are seen to be more probable than large excitation energy transfers. While cascades with P_F/P_{cn} of 0.8 thus still occur nearly 0.2 times as often as the most probable cascades, the corresponding cascades with E^*/E_{cn}^* of 0.8 occur less than 0.001 times as often as the most probable events. This fact may also be seen in a comparison of the average forward momentum and the average excitation energy of the struck nucleus. The ratio of these two quantities, each expressed in terms of the corresponding value for a compound nucleus, is about 1.5 for ruthenium, and 1.3 for bismuth and uranium, almost independently of bombarding energy. These observations are related to the fact that the emission of nucleons in a direction perpendicular to that of the incident proton has little effect on the forward momentum of the struck nucleus but does lower the residual excitation energy. Third, the relationship between forward momentum and excitation is not unique. For a given value of the forward momentum there is a fairly wide spread of excitation energies associated with this value. For P_F/P_{cn} of 0.1 to 0.2, 75% of the events thus have excitation energies within a 240-Mev interval. Similarly, for E^*/E_{cn}^* of 0.1–0.2, the same percentage of events has forward momentum values within a 350 Mev/c interval.

The measurement of the recoil properties of a given nuclide has made possible the determination of the average forward component of momentum of the struck nucleus for processes leading to the formation of the nuclide in question. This kind of information has been obtained so far primarily for fission products. It is possible to obtain a value for the excitation energy associated with the measured momentum from either a plot of the average excitation energy vs forward momentum or of the average forward momentum vs excitation energy. Neither plot is entirely correct in view of the fact that a given fission product arises from residual nuclei having a range of momentum and excitation energy values. The present results are given in a plot of the variation of the average forward momentum with excitation energy, since the method for correcting the latter for the range of E^* values associated with a given product has been discussed in a previous work.[14] The results are given for several targets and bombarding energies in Fig. 6. The curves were obtained in each case by dividing the excitation energy range into ten equal bins and calculating the average forward momentum for each bin. It is seen that $\bar{P}_F$ increases practically linearly with increasing

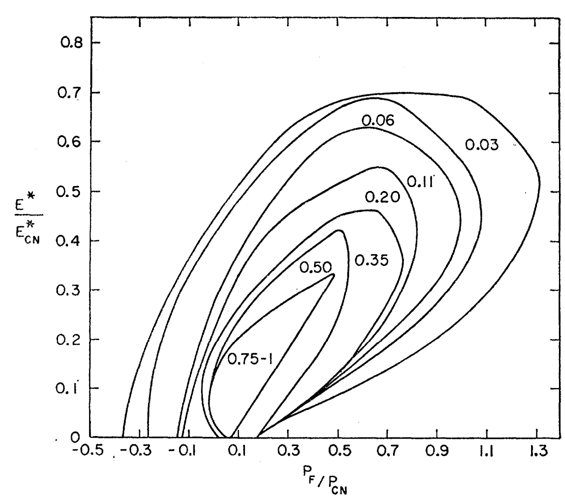


FIG. 5. Contour plot of the probability of occurrence of given $E^* - P_F$ pairs for 0.94 Bev protons on bismuth. The excitation energy and forward momentum are given in terms of the corresponding values for compound nucleus formation. The relative probability for the occurrence of events within the various contours is indicated.

[14] N. T. Porile and N. Sugarman, Phys. Rev. **107**, 1422 (1957).

E^* in all cases. Typical values of the standard deviation associated with the average forward momentum are given in the case of 0.46-Bev protons on bismuth. The rather large values of the standard deviation reflect the spread in momentum values associated with a given excitation energy and are considerably larger than the statistical error. These curves may be used to relate experimental momenta to excitation energies provided a correction is made for the range of excitation energies associated with the formation of a given product, as discussed previously.[14] This correction generally amounts to a few percent at the most.

IV. COMPARISON WITH EXPERIMENT

A number of experimental results may be compared with the present calculation. The recoil properties of the residual nuclides following the interaction of high-energy protons with silver and bromine nuclei in emulsions have been investigated by several groups. Ostroumov[10] has measured the projected length and direction of the recoil prongs formed in the interaction of 460-Mev and 660-Mev protons with silver and bromine. These measurements enable him to obtain the distribution of the forward component of velocity of the recoiling nucleus as well as that of the projection of the transverse component of velocity. Ostroumov's results are based only on the observation of measurable tracks lying within 30° of the plane of observation and require large corrections for tracks having short ranges. These corrections were made on the assumption of a Gaussian shape for the distribution functions. In view of the large uncertainty in the distribution functions introduced by this procedure, it appears more meaningful to compare the average values of the forward and perpendicular components of velocity with those obtained in the present calculation. Table I shows a comparison of the average forward component of velocity, the average perpendicular component of velocity in the plane of the emulsion, and the ratio of forward to backward emission. The calculated values were obtained on the assumption of an equal number of cascades at 460 and 660 Mev, since the actual ratio of events was not stated in Ostroumov's paper. An error in this estimate will affect the calculated values by only a few percent because of the slow variation with energy of the quantities of interest. The values calculated for

TABLE I. Comparison with Ostroumov's results on emulsion recoils.

	Ostroumov[a]	This work
$\bar{v}_F$[b]	9×10^7 cm/sec	11×10^7 cm/sec
$\bar{v}_\perp$[c]	12×10^7 cm/sec	9×10^7 cm/sec
F/B[d]	2.7 ± 0.2	8.1 ± 1.0

[a] See reference 10.
[b] Average forward component of velocity; the statistical error of the velocity values is approximately 3–4% in both cases.
[c] Average of the projections of the perpendicular component of velocity.
[d] Ratio of forward to backward emission.

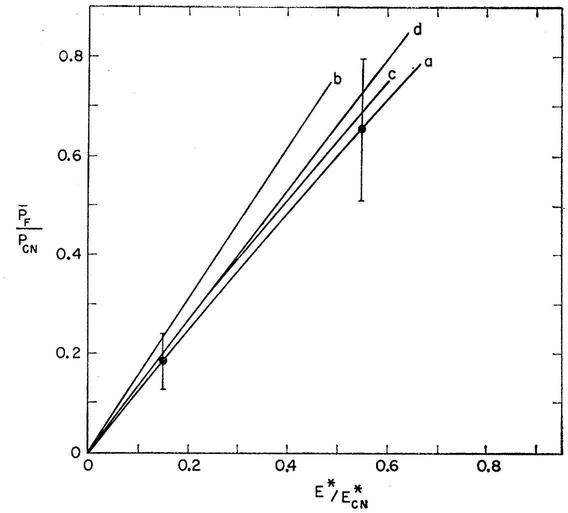


FIG. 6. Variation of the average forward momentum with excitation energy. (a) U or Bi, 0.46 Bev; (b) Bi, 0.94 Bev, (c) Bi, 1.84 Bev; (d) Ru, 0.46 Bev.

660 Mev were obtained by interpolation between values at 460 and 940 Mev. The average perpendicular component in the plane of the emulsion was obtained from the calculated value for the average total perpendicular component of velocity on the assumption that the velocity vector was randomly oriented in the plane perpendicular to the beam. The average value of one component then is $2/\pi$ times the average value of the total transverse component of velocity. The velocities were obtained from the calculated momenta on the assumption that on the average three nucleons were emitted in the cascade.[1] It was assumed that Ru^{100} served as an adequate average for silver and bromine for the purposes of this calculation.

The experimental and calculated values of the average forward component of momentum are directly comparable if it is assumed that the emission of evaporated particles is symmetric about 90°. It is seen that the two values agree to within 20%. This must be considered as very good agreement particularly in view of the fact that the experimental value is somewhat too low due to an overestimate of the fraction of nuclei recoiling backwards. This overestimate follows from Ostroumov's assumption that the shape of the velocity spectrum is Gaussian. It is shown in Fig. 1 that this is a rather poor assumption and one that will overestimate the amount of backward emission. This factor is also partially responsible for the very low forward to backward ratio obtained by Ostroumov. The actual experimental results, based on the observation of measurable tracks lying within 30° of the plane of observation, in fact give a forward to backward ratio of 4.8. This value is still considerably lower than the calculated value and the difference reflects the effect of the evaporation process. The experimental value of the average transverse component of momentum reflects the combined effect of the cascade and evaporation processes and is therefore expected to be larger than the calculated value. The observed difference of about 30% is consistent with the evaporation of 9 or 10 nucleons having a transverse component of momentum randomly oriented in the plane perpendicular to the beam. The number of evaporated nucleons chosen for this comparison follows[15] from the assumption of an initial excitation energy of 125 Mev.[1]

One further comment is pertinent to the present discussion. In the work under consideration Ostroumov attempts to obtain the average forward component of velocity imparted to the struck nucleus in the cascade process and to relate the average velocity to the average excitation energy deposited in the residual nucleus. His procedure entails the assumptions that the observed perpendicular component of velocity is entirely due to the evaporation process and that the forward component of velocity acquired by the residual nucleus in the evaporation process is on the average equal to the perpendicular component. It is seen on the basis of the present calculation that this procedure grossly underestimates the contribution of the cascade process to the velocity of the residual nucleus. Ostroumov subsequently obtains values for the average excitation energy on the basis of the "single fast nucleon" model which will be considered presently. This model overestimates the excitation energy associated with a given value of the forward momentum but, even so, Ostroumov's excitation energies are considerably lower than the values obtained in the recent Monte Carlo calculations.[1] For instance, for a bombarding energy of 660 Mev, Ostroumov obtains a value of 58±4 Mev for the average deposition energy, while the corresponding value obtained in the Monte Carlo calculations is 140 Mev.

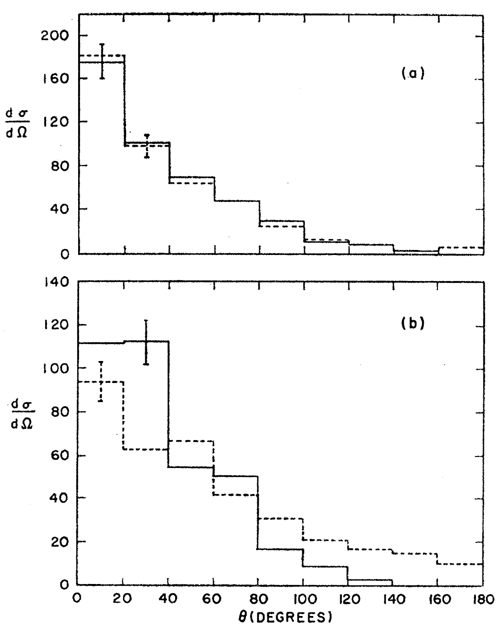


FIG. 7. Angular distribution of heavy emulsion recoils at 1 and 2 Bev; comparison with the experimental results of Baker *et al.*[11,16] (a) 1 Bev, (b) 2 Bev. Solid line—this calculation, dashed line—experimental results.

[15] I. Dostrovsky, P. Rabinowitz, and R. Bivins, Phys. Rev. **111**, 1659 (1958).

Baker *et al.*[11,16] have studied the interaction of the heavy emulsion nuclei with 1–3 Bev protons. The angular distribution of the heavy recoils with respect to the direction of the incident proton is obtained. This study is restricted to events in which at least one alpha particle is emitted. The comparison with the present calculation is restricted to events in which no heavier particles are emitted, so that the perturbation of the angular distribution resulting from the cascade process is kept to a minimum. One would still expect the calculated and experimental angular distributions to be different because the latter includes the effect of the evaporation of approximately two alpha particles and several nucleons.[16] The expected effect of the evaporation process is to weaken the correlation between the direction of the recoil and that of the incident proton. The experimental results should thus give a lower forward-to-backward ratio and a larger average angle with respect to the beam than the calculated values.

TABLE II. Comparison with the results of Baker *et al.* on emulsion recoils.

	Baker *et al.*[a]	This work
	F/B[b]	
1 Bev	5.6±0.9	6.8±0.9
2 Bev	2.8±0.33	13.3±2.7
	$\bar{\phi}$[c]	
1 Bev	40°	37°
2 Bev	57°	37°

[a] See reference 16.
[b] Ratio of forward to backward emission.
[c] Average angle (per unit solid angle) of the recoil with respect to the beam. The values are obtained from the angular distributions in Fig. 7.

The experimental angular distributions at 1 and 2 Bev and the calculated values at 0.94 and 1.84 Bev are shown in Fig. 7. The forward-to-backward ratios and the average angle of the recoil to the beam are compared in Table II. The experimental results are based on a total of about 450 and 300 events and the calculated values on a total of 650 and 350 events, at, respectively, 1 and 2 Bev. It is seen that the above expectations are borne out quite well. The experimental results at 2 Bev show a substantial difference from the calculated values in the expected direction. The same trends may be noted at the lower bombarding energy, although in this case the experimental and calculated results are in rather good agreement with each other. This agreement is probably somewhat fortuitous since the multiplicity of evaporated alpha particles increases only

[16] E. W. Baker and S. Katcoff (private communication).

slightly as the bombarding energy is increased from 1 to 2 Bev.[16]

The experimental recoil ranges and forward-to-backward ratios may be used to obtain the value of the average forward component of velocity inparted to the nucleus in the initial interaction, when coupled with a range-velocity relation. Baker *et al.*[11] find that at 2 Bev the value of this quantity is $0.007c$. This result is in excellent agreement with the calculated value of $0.0069c$ obtained at 1.84 Bev in the present work.

A number of groups have studied the fission of uranium through the observation of fission tracks in loaded nuclear emulsions. It has been shown[7] that the angle between two fission fragments is related to the forward component of momentum imparted to the struck nucleus in the intranuclear cascade, and a number of determinations of this quantity have been performed. The results are directly comparable to the results of the present calculation since fission accounts for most of the inelastic cross section of uranium for bombarding energies of a few hundred Mev. In one experiment[9] the average transverse component of momentum has also been determined by the observation of events lying in a plane perpendicular to the beam. These results are also directly comparable with the present calculations. It should be mentioned that in most of these studies the quantity listed is not the average forward component of momentum but the average excitation energy deposited in the struck nucleus. The latter is obtained in all cases from the experimental momentum by use of the "single fast nucleon" model of the cascade. The experimental momentum values quoted below were obtained from the quoted values for the average excitation energy by use of this model. In the study of Obukhov,[9] the measured momentum values are listed and may be compared directly with the present work.

The results of this comparison are summarized in Table III. It is seen that although the experimental data are in rather poor agreement with each other they lead to considerably lower values for the average forward component of momentum than those predicted by the present calculation. The variation of the forward momentum with bombarding energy is furthermore predicted to be much larger than is actually observed. The agreement of the calculated and experimental average transverse components of momentum is, on the other hand, rather good. In view of the approximations involved in the calculation of this quantity this agreement may, however, be fortuitous. The observed discrepancy in the values of the forward component of momentum can be explained if it is assumed that the cascade calculation underestimates the number of events in which small amounts of forward momentum are transferred to the struck nucleus. It has already been demonstrated in a study of (p,pn) reactions[17] that, at the energies under consideration, the Monte Carlo calculations underestimate the number of events in which small amounts of excitation energy are deposited in the struck nucleus by a factor of 2 to 3. In view of the relation between forward momentum and excitation energy, the above assumption thus is reasonable. This explanation would also account for the smaller energy dependence of the average forward momentum since the fraction of interactions involving low energy transfers appears to be rather insensitive to the bombarding energy, as demonstrated by the constancy of the cross section for the (p,pn) reaction at different bombarding energies.[17] In order to bring the experimental and calculated values into agreement, it appears that the deficiency in the calculation of events in which the residual nucleus is left with little forward momentum and excitation energy has to extend to excitation energies of about 30–40 Mev.

TABLE III. Comparison of experimental and calculated values of the average momentum imparted to uranium in the cascade process.

Author	$\bar{P}_{F-460}$[a]	$\bar{P}_{F-660}$[b]	$\bar{P}_{\perp-660}$[c]
Perfilov *et al.*[d]	214 Mev/c	225 Mev/c	
Ivanova and Pianov[e]	268±73	271±88	
Obukhov[f]	···	340±87	430±62 Mev/c
This calculation	386±15	490±20	450±18

[a] Average forward momentum for 460-Mev proton bombardment.
[b] Average forward momentum for 660-Mev proton bombardment.
[c] Average transverse momentum for 660-Mev proton bombardment.
[d] See reference 7.
[e] See reference 8.
[f] See reference 9.

A number of simple models for the nuclear cascade have been used in the past to obtain the excitation energy of the residual nucleus from the value of the forward component of momentum of the latter. The "single fast nucleon" model has been proposed by Perfilov and co-workers[7] and has been used in subsequent Russian reports.[8–10] This model has also been used in the early work of Sugarman.[2] The model assumes that there is only one high-energy nucleon emitted in the cascade and that its direction of motion is along that of the incident proton. According to this model, as used by the Russian investigators, the binding energy of all the nucleons emitted in the cascade is subtracted from the energy left in the residual nucleus in order to arrive at the excitation energy. Turkevich[18] has proposed a model based on the approximation that the cascade may be represented by a single nucleon-nucleon collision between the incident proton and a stationary nucleon. It is assumed that the latter remains in the nucleus and transfers its energy and momentum to the nucleus as a whole. Obukhov[9] has considered a model in which two high-energy nucleons are emitted in the cascade process. One of

[17] S. Markowitz, F. S. Rowland, and G. Friedlander, Phys. Rev. **112**, 1295 (1958).
[18] A. Turkevich as quoted in reference 3.

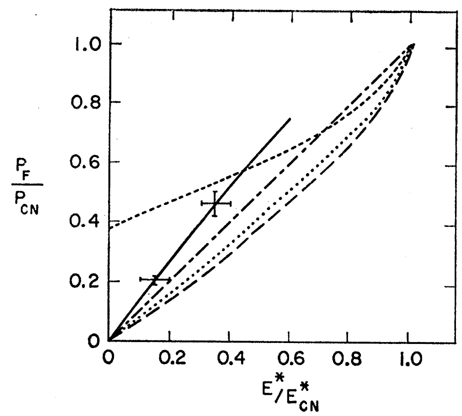


FIG. 8. Comparison of the relation between forward momentum and excitation energy predicted by several simple models with that obtained in the present work for 1.84-Bev protons on bismuth. ——— present calculation; — — "single fast particle" model; ...... "single fast particle model" corrected for binding energy of cascade nucleons; — — — — "two-nucleon collision" model; - - - - - "parallel and perpendicular fast particles" model.

these nucleons is emitted parallel and the other perpendicular to the direction of the incident proton, and both emitted nucleons are assumed to have the same kinetic energy.

It is of interest to compare the relation between the forward component of momentum and the excitation energy predicted by these simple models with the relation obtained in the present calculation. The variation of forward momentum with excitation energy is given for the case of 1.84-Bev protons on bismuth in Fig. 8. The calculated curve was obtained in the manner described previously and the magnitude of the statistical error is shown for a few of the excitation energy intervals. It is seen that none of the models is in very good over-all agreement with the results of the present calculation although the model proposed by Turkevich is in fair agreement, particularly for low excitation energies. The situation represented in Fig. 8 is typical of all the targets and bombarding energies under consideration.

A number of deposition energy values obtained with the aid of the "single fast nucleon" model have been reported in the literature. These values are based on the results of recoil studies on fission fragments. It seems worth while to obtain more realistic values for the excitation energies associated with these processes with the aid of the Monte Carlo results. This may be done by calculating the momenta associated with the published excitation energies by use of the simple model in question and then obtaining the excitation energies by use of Fig. 6. A correction is made for the fact that a given process occurs over a range of excitation energies, as described elsewhere.[14] The effect of this correction is small, leading to an increase in the calculated excitation energies by some 5 to 15 Mev. The results of this recalculation are given in Table IV.

TABLE IV. Recalculation of average excitation energies associated with particular reactions by use of Monte Carlo calculation results.

Process	Bombarding energy (Mev)	$\bar{E}^*$ old[a] (Mev)	$\bar{E}^*$ new[b] (Mev)
$Bi+p \rightarrow$ fission[c]	460	190	122
$W+p \rightarrow$ fission[c]	460	340	227
$Bi+p \rightarrow$ fission[c]	660	230	142
$W+p \rightarrow$ fission[c]	660	440	257
$Bi+p \rightarrow Sr^{91}$ [d]	2200	400	246
$Bi+p \rightarrow Ba^{129,133m}$ [d]	2200	660	408

[a] Average excitation energy values in the literature.
[b] Recalculated average excitation energies.
[c] From reference 7.
[d] From reference 2.

It is seen that in all cases the corrected values are lower than the old values, as would be expected from the comparison given in Fig. 8. The discrepancy is greatest for large momentum transfers. Thus in the fission of tungsten with 660-Mev protons the correction amounts to about 70%.

In addition to the results given in Table IV, a large number of average excitation energies have been obtained for the formation of specific nuclides in the fission of bismuth and tantalum with 450-Mev protons.[3] These results were based on a preliminary version of the present calculation based on about 25% of the available cascades for bismuth, and the assumption that the momentum-excitation energy relation for tantalum was the same as that for bismuth. The results of the present calculation indicate that the average excitation energies quoted in the earlier work were on the average too low by 9% in the case of bismuth and by 4% in the case of tantalum.

The angular distribution of fission fragments in high-energy fission generally involves a slight preferential emission along the beam direction.[3] In the case of processes involving rather low excitation energies, such as the formation of Ba^{139} and Ba^{140} in the fission of uranium with 450-Mev protons, preferential emission in a direction perpendicular to that of the incident proton has been observed.[19] Halpern[20] has advanced considerations to account for these anomalous anisotropies. His main point is that a substantial fraction of the interactions at high energies involve grazing collisions in which the transfer of transverse momentum to the struck nucleus is substantial while the transfer of forward momentum is small. Since the excitation energy of the residual nucleus after such interactions is low, the latter may be considered as almost equivalent to interactions with low-energy nucleons incident at 90° to the direction of the primary beam. The preferential emission of fission fragments along the direction of these secondary nucleons then leads to the observed angular distribution.

[19] N. Sugarman (private communication).
[20] I. Halpern, Nuclear Phys. **11**, 522 (1959).

The present calculation lends quantitative support to Halpern's considerations. In particular it is seen in Fig. 4 that the value of $P_\perp/P_F$ is large for small values of P_F, while in Fig. 6 it is seen that interactions of this type lead to low deposition energies. Figure 1 indicates that the fraction of events involving small transfers of forward momentum is appreciable.

The results given in Fig. 4 are of course applicable to spallation, as well as fission, reactions. The cascade process is thus expected to lead to preferential transverse motion of fragments with small forward momentum and excitation energy and to preferential forward motion of fragments with large forward momentum and excitation energy. The experimental results on spallation reactions are too meager for a meaningful comparison with this prediction.

V. CONCLUSIONS

The following are considered to be the main conclusions of this calculation:

1. The forward component of momentum imparted to the struck nucleus in the cascade can range from negative values to values greater than those expected for compound nucleus formation, with the most probable value ranging from one to three tenths of the compound nucleus momentum.
2. The transverse momentum imparted to the struck nucleus in the cascade is on the average approximately equal to the forward component, and exhibits a wide range of possible values.
3. The transverse component of momentum increases only slightly as the forward component increases from zero to its maximum value.
4. The average forward component of momentum of the struck nucleus increases linearly with the excitation energy but there is a fairly large spread in the values of the forward momentum associated with a given excitation energy.
5. The calculated momentum values are in most cases consistent with the pertinent experimental results. The main exception occurs in the fission of uranium, where the calculation appears to underestimate the occurrence of events with small transfer of forward momentum to the residual nucleus.
6. The forward momentum-excitation energy relation obtained in the present work leads to much lower average excitation energies associated with high-energy fission than the previously used relation.

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