$Ti^{46,48}(d,p)Ti^{47,49}$ Reactions and the $1f_{7/2}{}^n$ and $1f_{7/2}{}^{n-1}2p$ Configurations*

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(Received April 25, 1960)

The nuclide Ti^{47} was investigated via a high-resolution spectroscopic study of the reaction $Ti^{46}(d,p)Ti^{47}$. The observed levels, the measured orbital angular momenta of the captured neutrons, and the tentative total angular momenta assigned on the basis of relative intensities were as follows: ground state, 0.16 Mev, $l_n=3$, $J^\pi=\frac{7}{2}^-$; (0.55 Mev, $J^\pi=\frac{3}{2}^-$); 1.56 Mev, $l_n=1$, ($J^\pi=\frac{3}{2}^-$); 1.80 Mev, $l_n=1$, ($J^\pi=\frac{3}{2}^-$); 2.58 Mev, $l_n=1$; 2.83 Mev, $l_n=1$; 3.31 Mev, $l_n=1$, ($J^\pi=\frac{1}{2}^-$ or $\frac{3}{2}^-$); 3.60 Mev, ($l_n=1$, $J^\pi=\frac{1}{2}^-$); 3.71 Mev, ($l_n=1$, $J^\pi=\frac{1}{2}^-$); and 3.95 Mev, ($l_n=1$, $J^\pi=\frac{1}{2}^-$). Similarly for Ti^{49}: ground state, $l_n=3$, $J^\pi=\frac{7}{2}^-$; 1.38 Mev, $l_n=1$, $J^\pi=\frac{3}{2}^-$; 1.72 Mev, $l_n=1$, ($J=\frac{3}{2}^-$); 2.44 Mev; 2.49 Mev; 3.17 Mev, ($l_n=1$, $J^\pi=\frac{1}{2}^-$); and 3.26 Mev, $l_n=1$, ($J^\pi=\frac{1}{2}^-$). It is shown that the ground state of Ti^{49} and the ground-state triplet of Ti^{47} can be accounted for by pure $f_{7/2}{}^n$ configurations, and that most of the observed levels can be associated with $1f_{7/2}{}^{n-1}2p_{\frac{3}{2}}$ and $1f_{7/2}{}^{n-1}2p_{\frac{1}{2}}$ configurations.

I. INTRODUCTION

A STUDY has been made of proton groups from the reactions $Ti^{46}(d,p)Ti^{47}$ and $Ti^{48}(d,p)Ti^{49}$, induced by the 7.8-Mev deuteron beam of the University of Michigan cyclotron and analyzed by means of a high-resolution magnetic spectrometer[1] and nuclear emulsions. Levels up to about 4 Mev in Ti^{47} and 3.3 Mev in Ti^{49} were examined with an over-all resolution of 35 kev; angular distributions were measured for the more intense groups.

The various ways in which a stripping reaction can tell us something about the structure of nuclei have been reviewed in detail by Macfarlane and French.[2] It is shown that, although more sophisticated theories of the stripping process have been developed,[3] it is still appropriate for spectroscopic purposes to analyze the angular distributions of the proton groups in terms of the original simple theory of Butler.[4] The magnitude of the cross section is then employed[2] to extract a reduced width θ^2. The empirical reduced widths so obtained are analyzed with the aid of shell model wave functions for the nuclear states involved. To do this, θ^2 is expressed as a product of two factors:

$$\theta^2 = \mathcal{S}\theta_0{}^2, \tag{1}$$

where $\mathcal{S}$ is the *spectroscopic factor*, depending only on the nuclear wave functions, while the *single-particle reduced width* $\theta_0{}^2$ is, essentially, the reduced width for capture of a nucleon of the appropriate energy and orbital angular momentum by an inert potential well. When dealing with reduced widths by means of the simple Butler theory, $\theta_0{}^2$ is to be regarded as an empirical parameter.

In the present specific cases of $Ti^{46}(d,p)Ti^{47}$ and $Ti^{48}(d,p)Ti^{49}$, several kinds of information are derived from such an analysis. (a) Information is obtained about level spectra of Ti^{47} and Ti^{49} which, while consistent with the results of earlier studies,[5–8] includes the identification of four new levels in Ti^{47}. (b) Spin assignments for several states in Ti^{47} and Ti^{49} are made from angular distributions and reduced widths of the corresponding transitions. (c) Studies in the $f_{7/2}$ shell[9] leave open the possibility that low-lying levels of nuclei in the vicinity of $A=50$ may be reasonably described by wave functions of the configurations $f_{7/2}{}^n$. The reduced widths of transitions to the lowest levels (below 1 Mev) of Ti^{47} and Ti^{49} are pertinent to this question. (d) Transitions to higher excited states reveal single-particle neutron levels in Ti^{47} and Ti^{49}, their positions, spacings, and the manner of their fragmentation by final-state interactions. (See also reference 8.)

Experimental procedures and results—energy level positions, measured angular distributions, and reduced

* Supported in part by the U. S. Atomic Energy Commission. The experimental phase of this work was carried out at the University of Michigan. The work of evaluating and interpreting the data was done at the FOM-Instituut voor Plasma-Fysica, The Netherlands (L. H. Th. R.) and the University of Rochester (O. M. B. and M. H. M.).

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[3] Discussions of the more refined theories of the stripping process, with full lists of references, have been given by W. Tobocman, Technical Report No. 29, Case Institute of Technology, 1956 (unpublished); S. T. Butler, *Nuclear Stripping Reactions* (John Wiley & Sons, Inc., New York, 1957); N. Austern, *Fast Neutron Physics*, edited by J. B. Marion and J. L. Fowler (Interscience Publishers, Inc., New York, 1960), Chap. V; C. R. Lubitz, Ph.D. thesis, University of Michigan, 1960 (unpublished).

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[8] J. E. Robertshaw (private communication).

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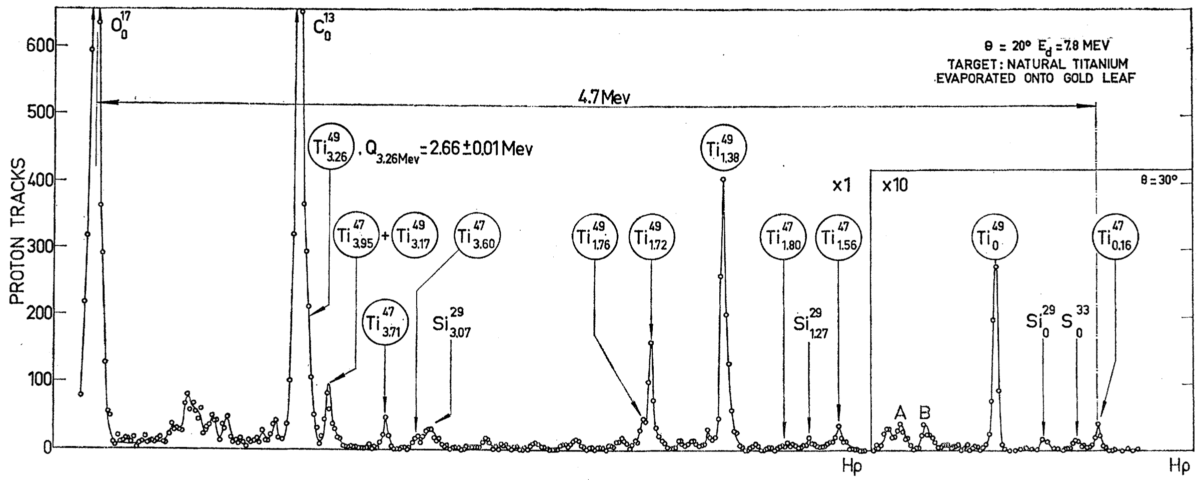

FIG. 1. The (d,p) spectrum from a natural titanium target. This spectrum is a composite of six overlapping partial spectra registered in nuclear emulsions at the image plane of a high resolution magnetic spectrometer. Except for the high-energy portion, for which the Ti^{49} and Ti^{47} $l_n=3$ transitions are stronger at 30°, the spectrum is composed from 20° data. All peaks are labeled by the residual nucleus.

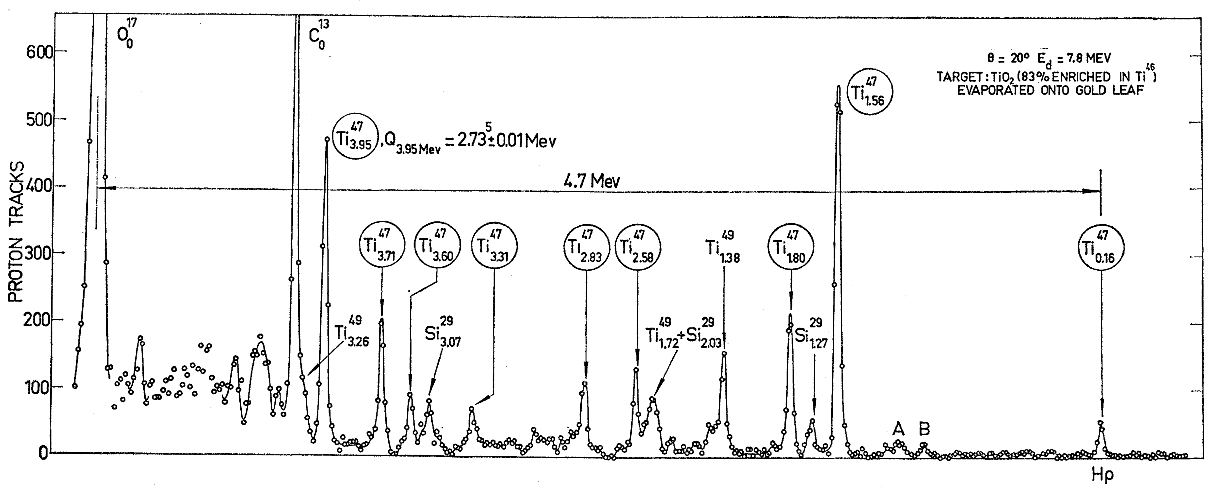

FIG. 2. The 20° (d,p) spectrum from a TiO_2 target enriched to 83% in Ti^{46}. The augmentation of peaks corresponding to Ti^{46} and the diminution of those corresponding to Ti^{48}, as compared with Fig. 1, is commensurate with the isotopic composition of the targets. Such comparison, together with the examination of the kinematic behavior, led to the identification of four new levels in Ti^{47}.

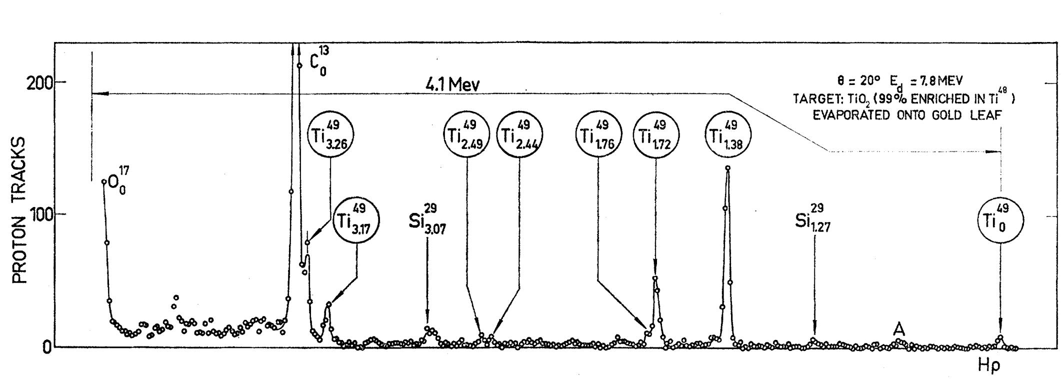

FIG. 3. The (d,p) spectrum at 20° from a TiO_2 target enriched to 99% in Ti^{48}. The large gap between the ground state of Ti^{49} and its first excited state reflects the fact that the $f_{7/2}{}^9$ configuration contains only one state which involves no proton excitation (see Sec. IV). While in Fig. 1 the peaks corresponding to Ti^{47} and Ti^{49} interfere, here the contribution from Ti^{47} is reduced to less than 0.5%, so that the peak is essentially due to Ti^{49} alone.

widths—are presented in Secs. II and III of this paper. A theoretical analysis of these results, after the fashion outlined above, is given in part IV.

II. EXPERIMENTAL PROCEDURE

Targets used in this experiment were obtained by evaporating elemental titanium and titanium dioxides enriched in Ti^{46} and Ti^{48} onto gold leaf. Since both the titanium metal and the dioxide require over 2000°C for evaporation, a carbon boat supported in vacuum by water-cooled copper electrodes and screened by water-cooled shields has been used as crucible. Targets ranged in thickness from 100 to 300 $\mu g/cm^2$ and were capable of withstanding days of bombardment by the deuteron beam (about 1 μa) without sign of wear.

The known relative abundances of the isotopes in the targets (see Table I) provided a direct check and a means for identification of peaks in the respective spectra. Typical proton spectra obtained with the three types of targets are shown in Figs. 1, 2, and 3. Each spectrum covers a range of over 4 Mev in proton energy. Since at a given magnetic field the high-resolution spectrometer[1] registers only over a 1.2-Mev wide energy interval, each of the Figs. 1, 2, and 3 is a composite of six overlapping partial spectra, properly corrected for effects of solid angle.

In addition to identification of the peaks by their relative intensity with regard to isotopic composition of the targets, all titanium groups have also been identified by their kinematic shift with angle and by reference to the ubiquitous groups from $C^{12}(d,p)C^{13}$ (ground state) and $O^{16}(d,p)O^{17}$ (ground state). Peaks due to impurities have been recognized similarly. To double check on weak silicon and sulfur impurity groups, located in the neighborhood of the equally weak $Ti^{46}(d,p)Ti^{47}$ ground-state group, separate exposures of thin silicon and sulfur targets were made and spectra compared as in Fig. 4. The fact that the angular distributions of the $Si^{28}(d,p)Si^{29}$ (ground state) and the $S^{32}(d,p)S^{33}$ (ground state) groups show $l_n=0$ and $l_n=2$ patterns, respectively, provides an additional check on the consistency of the identifications. See Fig. 5.

The angular variation of the stripping cross sections for the stronger levels has been measured in steps of 5°. Except where precluded by interference from extraneous groups, the angular distributions have been followed down to an angle of 10° lab.

A systematic measurement of absolute Q values for the various reactions has not been attempted. However, the Q values for $Ti^{48}(d,p)Ti^{49}$ (3.26-Mev level) and

TABLE I. Isotopic composition of targets used in this experiment.

Target	Ti^{46}	Ti^{47}	Ti^{48}	Ti^{49}	Ti^{50}
Ti	7.99%	7.32	73.99	5.46	5.25
$Ti^{46}O_2$	83.1±0.2	2.6	12.4	1.0	1.0
Ti	0.15	0.32	99.1±0.2	0.22	0.22

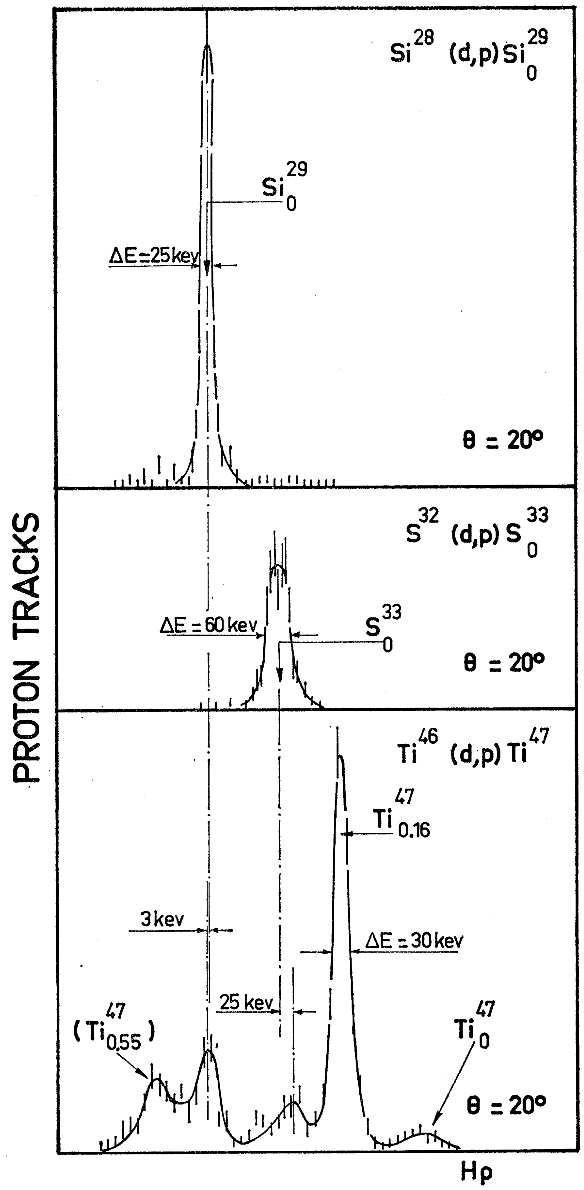

FIG. 4. Comparison of spectra taken under identical conditions off S, Si, and $Ti^{46}O_2$ targets. This juxtaposition served as a double check in the identification of the ground state and the 0.55-Mev state in Ti^{47}; it helped in making a positive identification of the Si^{29} and S^{33} impurity groups in the $Ti^{46}(d,p)Ti^{47}$ spectrum.

$Ti^{46}(d,p)Ti^{47}$ (3.95-Mev level) have been determined with respect to the $C^{12}(d,p)C^{13}$ (ground state) proton group.[10] It was possible to make these determinations without a very precise knowledge of the energy E_d of the deuteron beam because $\partial E_p/\partial E_d$ is equal to 0.99 for Ti^{49} (3.26-Mev) and Ti^{47} (3.95-Mev), while it is equal to 0.96 for C^{13} (ground state). This means that for an uncertainty as large as 100 kev in beam energy, the

[10] R. A. Douglas, J. W. Broer, R. Chiba, D. F. Herring, and E. A. Silverstein, Phys. Rev. 104, 1059 (1956).

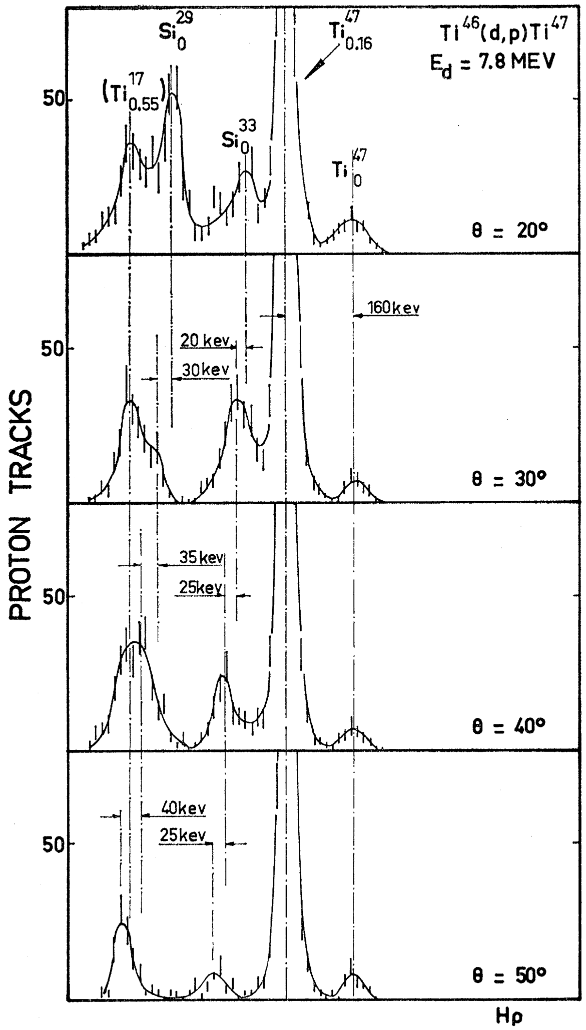

FIG. 5. Kinematic identification, by reference to the stronger and unmistakable Ti^{47} (0.16 Mev) group, of the ground-state and the 0.55-Mev groups of Ti^{47}. Note that while Si^{29} (ground state) and S^{33} (ground state) peaks shift with angle relative to the Ti^{47} (0.16-Mev) group, the energy separation between the Ti^{47} peaks remains the same. The fact that the angular distributions of the $Si^{28}(d,p)Si^{29}$ (ground state) and the $Si^{32}(d,p)S^{33}$ (ground state) show $l_n=0$ and $l_n=2$ patterns, respectively, provides an additional check on the consistency of the identifications.

uncertainty in the Q value will be only 3 kev. Results of these measurements are listed in Table II.

III. EXPERIMENTAL RESULTS

The results of identification of spectral peaks and of the measurement of their energies and relative intensities at $\theta=20°$ (see Figs. 1, 2, and 3) are summarized in Table II. This table contains four previously unreported levels in Ti^{47}, at (0.55), 1.80, 3.60, and 3.71 Mev, and one new level in Ti^{49} at 2.44 Mev. The very weak ground-state transition $Ti^{46}(d,p)Ti^{47}$ (ground state) is reported here for the first time. The identification of this peak has been made by reference to groups from S and Si (Fig. 4) and by its kinematic behavior, which is the same as for the unmistakable $Ti^{47}(d,p)Ti^{47}$ (0.16-Mev) group. See Fig. 5.

TABLE II. Assignments of level energies and Q values for Ti^{47} and Ti^{49} and relative (d,p) cross sections of these levels at 20° lab.

Excitation energy (Mev)	Q value (Mev)	$\sigma_{rel}(=20°)$ Ti(d,p)	$\sigma_{rel}(=20°)$ $Ti^{46}O_2(d,p)$	$\sigma_{rel}(=20°)$ $Ti^{48}O_2(d,p)$
$Ti^{46}(d,p)Ti^{47}$				
0	6.69 ±0.03[b]		0.4	
0.16	6.53 ±0.03[b]	5[e]	9±2	
(0.55)[a]	6.14 ±0.03[b]		2	
1.56	5.13 +0.03[b]	83±20[e]	100	
1.80[a]	4.89 ±0.03[b]		37±4	
2.58	4.11 ±0.03[b]		27±4	
2.83	3.86 ±0.03[b]		21±4	
3.31	3.38 ±0.03[b]		15±4	
3.60[a]	3.09 ±0.03[b]		18±4	
3.71[a]	2.98 ±0.03[b]		40±6	
3.95±0.03	2.73^5±0.01[c]		83±10	
$Ti^{48}(d,p)Ti^{49}$				
0	5.92 ±0.01[d]	3±1		5±1
1.38	4.54 ±0.01[d]	100	150±35[e]	100
1.72	4.20 ±0.01[d]	38±4		43±4
1.76	4.16 ±0.01[d]			9±3
2.44	3.48 ±0.02[d]			6±3
2.49	3.43 ±0.02[d]			7±3
3.17	2.75 +0.01[d]			27±6
3.26	2.66 +0.01[c]			55±12

[a] New levels.
[b] Q values determined relative to Ti^{47} (3.95 Mev) using level spacings measured in this experiment.
[c] Absolute Q value determined by reference to the $C^{12}(d,p)C^{13}$ (ground state) group. See Sec. II.
[d] Q values determined relative to Ti^{49} (3.26 Mev) using Bartholomew's accurate measurements of level spacings (reference 7) but our data for level identification.
[e] Corrected for isotope abundance.

Noteworthy is the excellent correlation between our results and the γ-ray spectra[7] from $Ti^{48}(n,\gamma)Ti^{49}$. All levels in Ti^{49}, except the 2.44-Mev level (which probably can be identified with the 2.428-Mev level seen by Robertshaw[8]), have been found in both experiments. Reaction Q values listed in Table II were computed on the basis of our level and Q-value determinations described in Sec. II and the very accurate measurements of level spacings by Bartholomew.[6]

Angular distributions have been obtained for the 0.16-, 1.56-, 1.80-Mev levels, and partial angular distributions for the 2.58-, 2.83-, and the 3.31-Mev levels in Ti^{47}, all shown in Fig. 6. All the above angular distributions are best fitted[11] with a theoretical Butler curve corresponding to neutron captures with $l_n=1$, except for the 0.16-Mev state where a reasonable fit is obtained only with $l_n=3$.

Angular distributions for Ti^{49} have been obtained for transitions leading to its ground state and to its 1.38-Mev and 1.72-Mev states. They are shown in Fig. 7. A partial angular distribution corresponding to the 3.26-Mev level in Ti^{49} is also shown in this figure. While the fit with Butler curves is unambiguous for the first three

[11] C. R. Lubitz, *Numerical Table of Butler-Born Approximation Stripping Cross Sections*, University of Michigan, 1957 (unpublished).

TABLE III. Summary of data obtained from the $Ti^{46}(d,p)Ti^{47}$ reaction.

Excitation (Mev)	J^π	l_n	r_0	Relative $(2J+1)\theta^2$	Relative θ^2	Remarks
0	$\frac{5}{2}-$	(3)			($\leq$0.06)	(a) (b)
0.16	$\frac{7}{2}-$	3	5.5	8	1	
0.55	($\frac{3}{2}-$)	(3)			($\leq$0.04)	(c)
1.56	$\frac{3}{2}-$	1	6	6.3	1.6	
1.80	$\frac{3}{2}-$	1	6	2.5		
2.58	($\frac{3}{2}-$)	1	5	1.8		
2.83	($\frac{3}{2}-$)	1	5	1.3		
3.31	($\frac{1}{2}-,\frac{3}{2}-$)	1	5	0.8		
3.60	($\frac{1}{2}-$)	(1)	(5)	(0.9)		(d)
3.71	($\frac{1}{2}-$)	(1)	(5)	(2.0)		(d)
3.95	($\frac{1}{2}-$)	(1)	(5)	(4.2)		(d)

[a] All reduced widths are expressed as multiples of $\theta_{l=3}{}^2$ (0.16 Mev).
[b] The measured relative cross section at $\theta_{\rm lab}=20°$ indicates θ^2 (relative) ≤ 0.06 for a possible $l_n=3$ transition.
[c] The measured relative cross section at $\theta_{\rm lab}=20°$ indicates θ^2 (relative) ≤ 0.04.
[d] Reduced widths are calculated for *assumed* $l_n=1$ transitions, the differential cross section having been measured at only two angles, $\theta_{\rm lab}=20°$ and $\theta_{\rm lab}=35°$. The intensity ratio at these angles is consistent with either $l_n=1$ or $l_n=2$. See Sec. IV for further discussion, leading to selection of $l_n=1$.

experimental distributions in Fig. 7, the fourth, on account of its incompleteness, allows for fits with either $l_n=1$ or $l_n=2$. It is shown in Sec. IV, however, that the $l_n=1$ assignment is to be preferred on theoretical grounds.

The information on Ti^{47} contained in Figs. 2 and 6 is epitomized in Table III in a form suitable for interpretation which follows in Sec. IV. Table IV is a similar epitome of information on Ti^{49} contained in Figs. 3 and 7.

IV. DISCUSSION OF RESULTS

The notation and techniques to be used in this section follow those described by Macfarlane and French.[2] In particular, the symbol

$$[\Psi(J_0)\dot{x}j]_J \tag{2}$$

is used to denote the result of coupling an antisymmetric

TABLE IV. Summary of data obtained from the $Ti^{48}(d,p)Ti^{49}$ reaction.

Excitation (Mev)	J^π	l_n	r_0	Relative $(2J+1)\theta^2$	Relative θ^2	Remarks
0	$\frac{7}{2}-$	3	6	8	1	
1.38	$\frac{3}{2}-$	1	6	14	3.5	
1.72	($\frac{3}{2}-$)	1	6	6.0	1.5	
1.76						
2.44		$\geq$2				(a)
2.49		$\geq$2				(a)
3.17	($\frac{1}{2}-$)	(1)	(5)	2.9	1.5	(b)
3.26	($\frac{1}{2}-$)	(1,2)	(4,7)	6.0	3.0	

[a] Relative cross section has been measured at 20° and 40° lab.
[b] The large cross section at $\theta_{\rm lab}=20°$ suggests an $l_n=1$ transition. The reduced width has been extracted on this assumption, see Sec. IV.

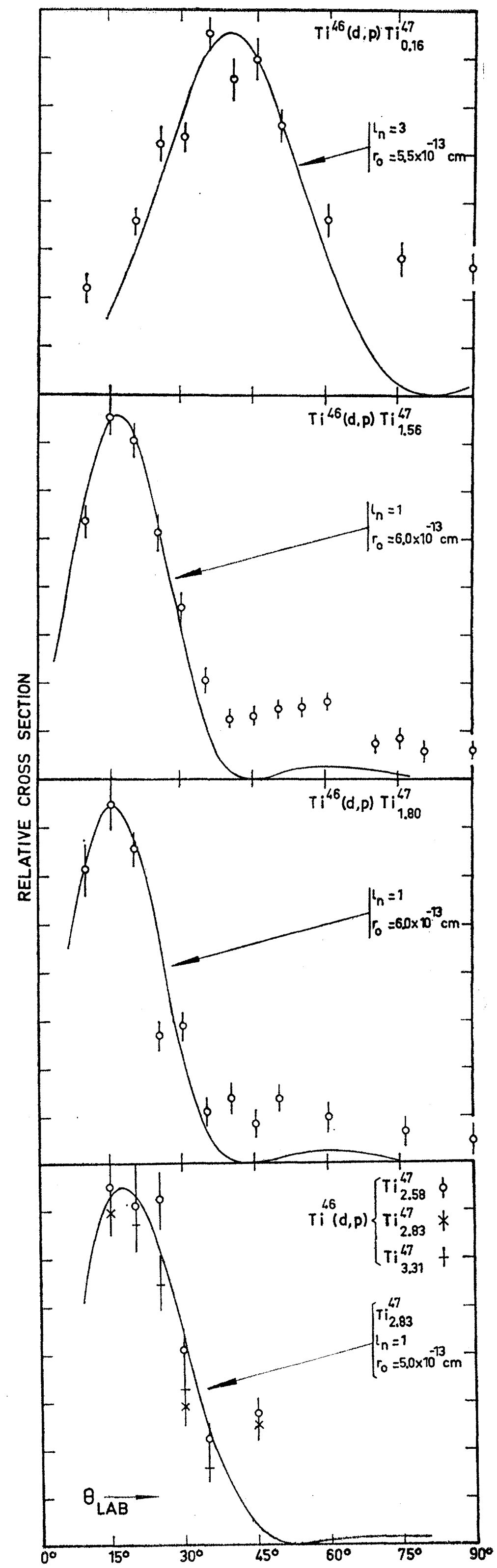

FIG. 6. Angular distributions in the $Ti^{46}(d,p)Ti^{47}$ reaction, leading to the 0.16, 1.56, 1.80, 2.58, 2.83, and 3.31-Mev states in Ti^{47}. Although only partial data are available for the angular distribution corresponding to the 2.58, 2.83, and 3.31-Mev states, an assignment of $l_n=1$ to the captured neutron is unambiguous.

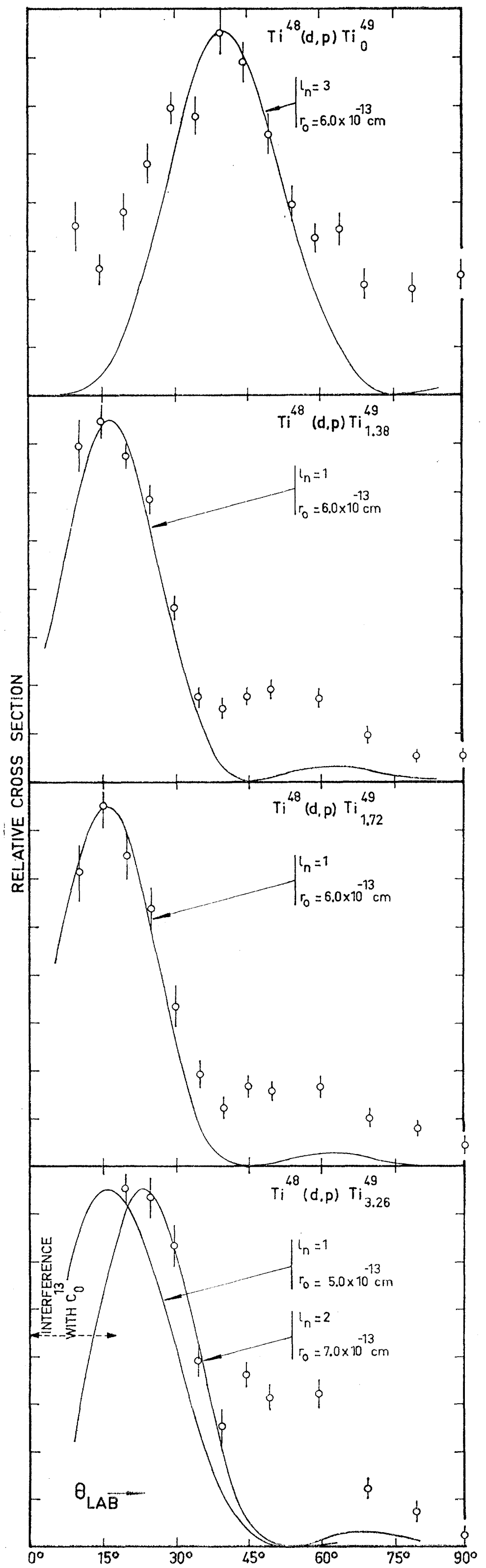

FIG. 7. Angular distributions in the $Ti^{48}(d,p)Ti^{49}$ reaction leading to the 0-, 1.38-, 1.72-, and 3.26-Mev states in Ti^{49}. Because of interference with the strong $C^{12}(d,p)C^{13}$ group, the angular distribution corresponding to the 3.26-Mev state could not be completed at forward angles. While both $l_n=1$ and $l_n=2$ assignments are not inconsistent with this partial angular distribution, $l_n=1$ is preferred on theoretical grounds (see Sec. IV).

$(n-1)$-particle state $\Psi(J_0M_0)$ and a single-particle state $\varphi(jm)$ to a total angular momentum J, and then antisymmetrizing the resulting function in all n particles. We shall use this "weak coupling" approach only when $\Psi(J_0M_0)$ involves no significant contributions from nucleons equivalent to j, since the idea of weak coupling loses much of its special character when applied to equivalent nucleons. Specifically, the final antisymmetrization of the function

$$[\Psi(j^{n-1}J_0)\dot{x}j]_J,$$

will, in general, introduce terms wherein the first $n-1$ nucleons are coupled to spins other than J_0.

In describing the lowest states of the Ti isotopes we shall adopt wave functions belonging to the appropriate[12] configuration $1f_{7/2}{}^n$. We shall, further, treat neutrons and protons on a separate footing, on the assumption that the corresponding angular momenta J_n and J_p are good quantum numbers. The approximations involved are discussed in Sec. V.

States of Ti^{47}

For the wave functions of the lowest states of Ti^{47} we therefore take the symbol shown in Fig. 8(b), where a circular arc denotes antisymmetry and the two $f_{7/2}$ protons are in their lowest state, with $J_p=0$. The appropriateness of this description seems to be borne out by the observed spectrum of Ti^{47}; three levels are found below 0.55 Mev, the next known excited state being 1 Mev higher, at 1.56 Mev. The spins of the ground state and first excited state are known to be $\frac{5}{2}-$ and $\frac{7}{2}-$, respectively. It is therefore probable that the 0.55-Mev second excited state has $J^\pi=\frac{3}{2}-$.

Stripping transitions to $\frac{3}{2}-$ and $\frac{5}{2}-$ levels in Ti^{47} must involve $p_{\frac{3}{2}}$ and $f_{\frac{5}{2}}$ nucleons, respectively. Since the main $2p_{\frac{3}{2}}$ components do not begin to appear until 1.56 Mev and the main $1f_{\frac{5}{2}}$ contributions lie above 4 Mev, the transitions to the ground $(\frac{5}{2}-)$ and second excited $(\frac{3}{2}-)$ states of Ti^{47} are j forbidden. The fact that no stripping is observed in either case supports the assumption of pure $1f_{7/2}{}^7$ wave functions.

The observed upper limit on the reduced width of a possible $l_n=3$ ground-state transition sets a corresponding upper limit on the $1f_{\frac{5}{2}}$ admixture in the ground-state wave function, in fact, if

$$\Psi(Ti^{47},\text{ ground state})=\alpha\Psi(\tfrac{5}{2})+\beta[\Psi(Ti^{46})\dot{x}1f_{\frac{5}{2}}], \quad (3)$$

where $\Psi(\frac{5}{2})$ is given by Eq. (3), it is clear that stripping from Ti^{46} can proceed only to the $1f_{\frac{5}{2}}$ part of the wave function. The reduced width is simply $\beta^2\theta_0{}^2(1f)$ and

[12] B. H. Flowers, Proc. Roy. Soc. (London) **A212**, 248 (1952).

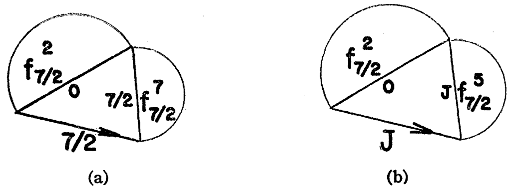

FIG. 8. Symbolic notation delineating (a) the Ti^{49} ground-state wave function, (b) the ground-state triplet of the Ti^{47} wave functions, with $J=\frac{3}{2}$, $\frac{5}{2}$, and $\frac{7}{2}$.

this, according to Table III, is no greater than 0.06 of the $l_n=3$ reduced width of the first excited state. Since the $\mathcal{S}$ value of the latter transition is $\frac{1}{2}$ (see below), we have

$$\beta^2\theta_0^2(1f)/\tfrac{1}{2}\theta_0^2(1f)\leq 0.06,$$

whence $\beta^2\leq 0.03$. In other words, the $1f_{\frac{5}{2}}$ admixture in the Ti^{47} ground state is at most 3% (in intensity).

Proceeding in the same fashion, we find that the $2p_{\frac{3}{2}}$ admixture in the $\frac{3}{2}^-$ second excited state of Ti^{47} is no greater than 4%. In this case, we need to know the ratio $\theta_0^2(2p)/\theta_0^2(1f)$ of single-particle reduced widths. The value 2 used here is that obtained from a number of experiments[2] in the region of mass number $20<A<70$ and is satisfactorily consistent with the value 1.7 derived [Eq. (5), below] from the results of the present experiment.

Above the low-lying states of $f_{7/2}{}^7$ we encounter a succession of levels populated by $l_n=1$ transitions. The first five of these transitions are clearly identified as $l_n=1$ by their angular distributions; the three highest, to levels at 3.60, 3.71, and 3.95 Mev in Ti^{47}, were observed at only two angles ($\theta_{lab}=20°$ and 35°). The intensity ratio at these two angles is consistent with $l_n=1$ or with $l_n=2$ in each case. Since the lowest $2d_{\frac{5}{2}}$ components of any consequence are seen[13] at 4.76 Mev in Ca^{41} and at 4.3 Mev in Ti^{49}, we regard $l_n=2$ as unlikely and therefore tentatively assign $l_n=1$ to the transitions in question and extract reduced widths on this basis.

If the weakly populated level at 3.31 Mev is ignored, for the moment, there is a clearly discernible break in the succession of $l_n=1$ levels between 2.83 and 3.60 Mev. We might therefore suggest that the four $l_n=1$ transitions below 2.83 Mev involve $2p_{\frac{3}{2}}$ nucleons and proceed to fragments[14] of the single-particle state

$$[\Psi(Ti^{46})\dot{x}2p_{\frac{3}{2}}],$$

the three highest transitions involving $2p_{\frac{1}{2}}$ and selecting components of

$$[\Psi(Ti^{46})\dot{x}2p_{\frac{1}{2}}].$$

[13] J. R. Holt and T. N. Marsham, Proc. Phys. Soc. (London) A66, 565 (1953).

[14] Since we are treating the Ti^{46} and Ti^{48} ground states on the basis of pure $f_{7/2}{}^n$ configurations, we may use the weak-coupling approach, mentioned at the beginning of Sec. IV, to discuss $l_n=1$ levels in Ti^{47} and Ti^{49}.

If this interpretation were correct, the total $2p_{\frac{3}{2}}$ and $2p_{\frac{1}{2}}$ intensities $[\sum(2J+1)\theta^2]$ would be in the ratio of the corresponding statistical factors $(2J+1)$, which in this case is 2. The observed ratio is in tolerable agreement with this prediction, being 1.8 or 1.5 according as the 3.31-Mev level is assigned spin $\frac{3}{2}^-$ or $\frac{1}{2}^-$.

Using the fact that the $l_n=3$ transition to the first excited state of Ti^{47} has $\mathcal{S}=\frac{1}{2}$ and again summing over all $l_n=1$ transitions, we have

$$\frac{\sum(2J+1)\theta^2(l=1)}{(2J+1)\theta^2(0.16\text{ Mev}, l=3)}=\frac{4\theta_0^2(2p)+2\theta_0^2(2p)}{8\frac{1}{2}\theta_0^2(1f)}=\frac{3\theta_0^2(2p)}{2\theta_0^2(1f)}. \quad (4)$$

The observed value of this ratio is found, from Table III, to be 2.5, whence (4) yields

$$\theta_0^2(2p)/\theta_0^2(1f)\approx 1.7, \quad (5)$$

in satisfactory agreement with other estimates.[2]

The mean positions of the single-particle $2p_{\frac{3}{2}}$ and $2p_{\frac{1}{2}}$ states can be estimated roughly by weighting the energy of each observed fragment according to its reduced-width amplitude. The resulting p-doublet splitting is

$$E(2p_{\frac{1}{2}})-E(2p_{\frac{3}{2}})\approx 1.7\text{ Mev}, \quad (6)$$

the spin of the 3.31-Mev state being of no importance. Agreement with other observed p-doublet splittings in the mass region[15] $20<A<50$ is excellent.

We therefore conclude that the Ti^{47} levels at 1.56, 1.80, 2.58, and 2.83 Mev probably have spin $\frac{3}{2}^-$, while the levels at 3.60, 3.71, and 3.95 Mev probably have spin $\frac{1}{2}^-$. The 3.31-Mev level may have $J^\pi=\frac{1}{2}^-$ or $\frac{3}{2}^-$.

States of Ti^{49}

Since the configuration $f_{7/2}{}^7\equiv f_{7/2}{}^{-1}$ of identical nucleons contains only one state, the symbol in Fig. 8(a) represents the only state of $f_{7/2}{}^9$ which involves no proton excitation. Since $(f_{7/2}{}^2)_2$ lies about 1.5 Mev above $(f_{7/2}{}^2)_0$, we expect Ti^{49} to have a well-isolated $\frac{7}{2}^-$ ground state; this is indeed found to be the case.

Representing the lowest $\frac{7}{2}^-$ states of Ti^{47} and Ti^{49} by symbols of Fig. 8, and using similar wave functions for the ground states of Ti^{46} and Ti^{48}, we find, for the respective $l_n=3$ transitions,

$$\begin{aligned}\mathcal{S}(Ti^{47}, 0.16\text{ Mev})&=\tfrac{1}{2},\\ \mathcal{S}(Ti^{49}, \text{ground state})&=\tfrac{1}{4}.\end{aligned} \quad (7)$$

The corresponding ratio of empirical reduced widths obtained from the present experiment is

$$\theta^2(Ti^{47}, 0.16\text{ Mev})/\theta^2(Ti^{49}, \text{ground state})=2.2\pm0.7,$$

in good agreement with the ratio 2 given by Eq. (7).

[15] R. H. Nussbaum, Revs. Modern Phys. 28, 423 (1956).

The $l_n=1$ transitions to the levels at 1.38 and 1.72 Mev clearly exhaust the major portion of the $2p_{\frac{3}{2}}$ single-particle state. If we proceed as we did in writing down Eq. (4) and use the second of Eqs. (7), the observed reduced widths of the first three levels of Ti^{49} yield

$$\theta_0^2(2p)/\theta_0^2(1f)\approx 1.3, \tag{8}$$

a considerably smaller ratio than is found elsewhere. (Compare reference 6.)

The associated strong $l_n=1$ transition indicates that the 3.26-Mev level contains a large fragment of $[\Psi(Ti^{48})\dot{x}2p_{\frac{1}{2}}]$. The nearby 3.17-Mev level, for which no angular distribution is available, probably does so as well. In contrast to the situation in Ti^{47}, the total $2p_{\frac{1}{2}}$ intensity in Ti^{49} is rather less than half the $2p_{\frac{3}{2}}$ intensity. This circumstance suggests that sizeable $2p_{\frac{1}{2}}$ fragments may lie above the highest energy attained in the present experiment. Nevertheless, in view of the wide gap between 1.72 and 3.17 Mev devoid of any strong transitions, we can assign spin $\frac{3}{2}^-$ to the 1.38- and 1.72-Mev levels and spin $\frac{1}{2}^-$ to the 3.26 and probably also to the 3.17-Mev level, with some confidence.

Schiffer, Lee, and Zeidman,[6] who studied gross structure in the proton spectra from $Ti^{48}(d,p)Ti^{49}$, found results consistent with those presented here, where the two experiments overlap. In particular, they identified $1f_{\frac{5}{2}}$ components around 2.5 Mev in Ti^{49}. Levels observed at 2.44 and 2.49 Mev in this study may contain such components since comparison of 20° and 40° data for these transitions show definitely that $l\neq 1$.

The density of known levels of Ti^{47} and Ti^{49} up to about 3 Mev is noticeably less than in the corresponding[16] energy range in Ca^{41}, Ca^{43}, and Ca^{45}. It is likely that other, as yet unknown, levels, excited very weakly in (d,p) reactions, actually exist in the relevant parts of the spectra of Ti^{47} and Ti^{49}.

A study[17] of circular polarization of γ rays from the capture of polarized neutrons by Ti^{48} has been interpreted[17,18] as indicating spins of $\frac{3}{2}^-$ and $\frac{1}{2}^-$, respectively, for the 1.38-Mev and 1.72-Mev levels of Ti^{49}. Similar assignments have been based on γ-γ angular correlation measurements.[19] The assignment of $J=\frac{1}{2}$ to the 1.72-Mev level of Ti^{49} is in conflict with our interpretation of the stripping data. In view of the consistency of stripping data and since the circular polarization and γ-γ correlation measurements are very delicate, we have decided to treat the stripping results on their own merits; we assign $l_n=1$ to the 3.26-Mev level and a probable spin of $\frac{3}{2}^-$ to the 1.72-Mev level in Ti^{49}.

The implications of assigning a spin of $\frac{1}{2}^-$ instead of $\frac{3}{2}^-$ to the 1.72-Mev level are twofold. (a) If we keep the assignment for the 3.26-Mev state of $l_n=1$ and of negative parity, we are faced with the phenomenon of two gross-structure peaks in the proton spectrum, separated by 1.7 Mev, both corresponding to $l_n=1$ capture, but neither belonging to pure $2p_{\frac{3}{2}}$ or $2p_{\frac{1}{2}}$ single-particle states. It is clear that such an interpretation is incompatible with the work of Schiffer, Lee, and Zeidman,[6] wherein each gross-structure peak is identified with just one single-particle state. Any alternative explanation for the observed consistent[6] gross structure is hard to conceive. (b) The 3.26-Mev level might have positive parity and the transition to it involve $l_n=2$ rather than $l_n=1$ capture. This possibility is suggested by the fact that difficulty has frequently been encountered in distinguishing $l_n=1$ from $l_n=2$ in stripping experiments[2,16,20,21] with 6- to 10-Mev deuterons on nuclei with $A>40$. See also Fig. 7. In the present instance, however, the $l_n=2$ assignment would lead to a $2p$-doublet splitting of only 340 kev, in comparison with 1.5 to 2 Mev in nearby nuclei (Ca^{41}, Ca^{43}, Ca^{45}, Ca^{49}) and would imply a value of $\theta_0^2(2p)/\theta_0^2(1f)$ markedly smaller than that of Eq. (8), which is already disquietingly low.

V. CONCLUDING REMARKS

Instead of regarding neutrons and protons as different particles, we might have used the isotopic spin formalism. If all possible excited states of the separate neutron and proton groups are taken into account, the two approaches are, of course, equivalent. However, we do not wish to carry out an extensive calculation of energy levels and wave functions; rather, we seek approximations to provide simple and reasonable descriptions of those few states of $f_{7/2}{}^n$ which lie lowest in energy. The approximations which seem most natural in the two representations are, in general, different. Separate treatment of neutrons and protons leads to wave functions such as in Fig. 8. With the isotopic-spin formalism, the lowest states may reasonably be supposed[12] to be those with the smallest possible isotopic spin T and the lowest symplectic symmetry σ.

To return to cases considered in Sec. IV, neither symbol of Fig. 8 is an eigenfunction of T^2, nor are the similar wave functions of the Ti^{46} and Ti^{48} ground states. If, however, we were to choose some reasonable effective two-body interaction between nucleons, and then perform a calculation of energy levels and wave functions, we would expect to find that the lowest states so obtained would have large overlaps with symbols of Fig. 8. Thus, although J_n and J_p are probably farther from being good quantum numbers than is T, the errors involved in our treatment of the very lowest levels are probably small. In fact, using the isotopic-spin formalism and assuming lowest T and symplectic symmetry, we again obtain the $\mathcal{S}$ values of Eq. (7) for the two $l_n=3$

[16] C. K. Bockelman and W. W. Buechner, Phys. Rev. **107**, 1366 (1957); C. K. Bockelman, C. M. Braams, C. P. Browne, W. W. Buechner, R. D. Sharp, and A. Sperduto, Phys. Rev. **107**, 176 (1957); W. R. Cobb and D. B. Guthe, Phys. Rev. **107**, 181 (1957).

[17] G. Trumpy, Nuclear Phys. **2**, 664 (1957).

[18] L. W. Fagg and S. S. Hanna, Revs. Modern Phys. **31**, 711 (1959).

[19] J. W. Knowles, G. Manning, G. A. Bartholomew, and P. J. Campion, Phys. Rev. **114**, 1065 (1959).

[20] A. W. Dalton, A. Kirk, G. Parry, and H. D. Scott, Proc. Phys. Soc. (London) **75**, 95 (1960).

[21] F. B. Shull and A. J. Elwyn, Phys. Rev. **112**, 1667 (1959).

transitions of interest, in spite of the fact that the wave functions involved are not identical.

In Sec. IV we were able to set upper limits on $1f_{\frac{5}{2}}$ and $2p_{\frac{3}{2}}$ contributions to the states represented by the symbol of Fig. 8(b), with $J=\frac{5}{2}$ and $\frac{3}{2}$, respectively. The smallness of these upper limits and the good agreement between the observed ratio of $l_n=3$ reduced widths in Ti^{47} and Ti^{49} and the predictions of our simple model suggest that the lowest states of the Ti isotopes may be described quite well by pure $1f_{7/2}{}^n$ wave functions. There are, however, certain kinds of admixtures about which the experiments under consideration give no direct information. It is worthwhile to list the more important of these admixtures together with suggested ways of investigating them experimentally. (a) $(2p_{\frac{3}{2}}{}^2)_0$ and $(1f_{\frac{5}{2}}{}^2)_0$ admixtures in the Ti^{46} and Ti^{48} ground states. The presence of sizable components of this kind would be revealed by $l_n=1$ and $l_n=3$ transitions to known $2p_{\frac{3}{2}}$ and $1f_{\frac{5}{2}}$ single-particle states in Ti^{45} and Ti^{47} in the appropriate (d,t) or (p,d) pickup reactions. (b) Excitation of particles from $1d_{\frac{3}{2}}$ and $2s_{\frac{1}{2}}$ shells in the ground states of the Ti isotopes. Such admixtures again lend themselves to investigation by the appropriate pickup reaction; they might be seen through weak $l_n=0$ and $l_n=2$ transitions, of which the former are more likely to be observable.

A final summary of the present work is given in graphical form in Fig. 9, in which energy level diagrams of Ti^{47} and Ti^{49} are shown as they are now known. The levels at (0.55), 1.80, 3.60, and 3.71 Mev in Ti^{47} have not been previously reported. The very weak ground-state transition in $Ti^{46}(d,p)Ti^{47}$ has been identified here for the first time, consequently, the $p_{\frac{3}{2}}$ single-particle state reported until now[5,22] as lying at 1.40 Mev is placed at an excitation of 1.56 Mev.

It is clear that experimental studies of (d,t) or (p,d) reactions on the Ti isotopes would be profitable. Apart from the points mentioned above, the $Ti^{47}(d,t)Ti^{46}$ reaction would be particularly interesting because it involves the anomalous[23] ground-state of Ti^{47} as target, and accordingly [together with $Ti^{47}(d,p)Ti^{48}$] provides an excellent way of probing its structure. Absolute cross-section measurements in both stripping and pickup reactions would also be valuable. There is some evidence[2] that $\theta_0{}^2(1f)$ and $\theta_0{}^2(2p)$ decrease by a factor of two or four between $A=40$ and $A=55$. In view of the strategic position of the Ti isotopes, the absolute (d,p) cross section would provide valuable insight into the manner in which this decrease takes place. Comparison of the (d,p) and (d,t) cross sections would yield a determination of the empirical normalization factor in the (d,t) differential cross section.[24] Existing determinations of this normalization factor[2,25] are obtained entirely from reactions on light nuclei ($A<25$). Since many (d,t) experiments[26,27] are now being undertaken on nuclei with $A\geq 50$, it is desirable to extend these determinations to heavier nuclei.

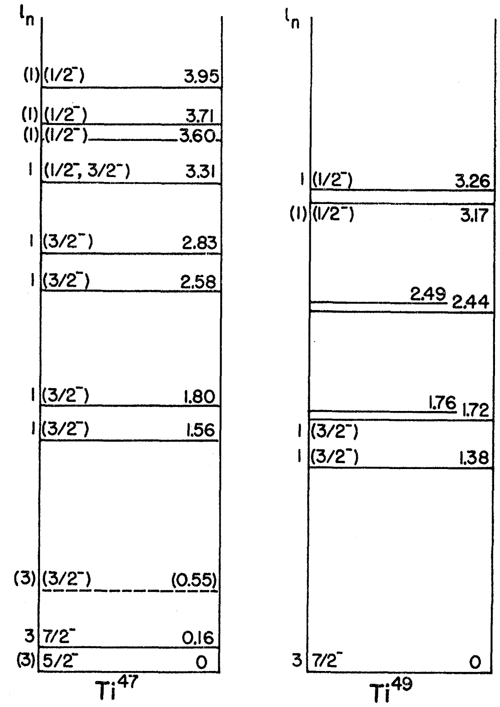

FIG. 9. Energy level diagrams for Ti^{47} and Ti^{49}. Levels at (0.55), 1.80, 3.60, and 3.71 Mev in Ti^{47} and at 2.44 Mev in Ti^{49} have not been reported previously. Known spins, and those assigned in Sec. IV of this paper, are shown next to the corresponding levels.

ACKNOWLEDGMENTS

It is a pleasure to thank Dr. B. J. Raz for suggesting this problem and to Professor W. C. Parkinson for his advice and encouragement throughout its experimental phase. We are also grateful to Dr. S. S. Hanna for a discussion of the Ti^{49} circular-polarization measurements and to Dr. F. E. Throw for his help in preparing the manuscript.

The expert and willing assistance of the cyclotron engineers W. E. Downer and R. Pittman is sincerely acknowledged. Since this paper is based on information derived from nuclear emulsions, particular recognition is given to Mrs. Lorraine Graaf for her meticulously accurate track counting. Professor J. B. French's influence is patent to cognoscenti.

[22] L. L. Lee and W. Rall, Phys. Rev. **99**, 1384 (1955).

[23] The word "anomalous" in this context is, perhaps, something of a misnomer. The lowest $\frac{5}{2}^-$ and $\frac{7}{2}^-$ states in Ca^{43} and Ca^{45} lie within a few hundred kilovolts of each other. It is clearly not difficult to produce the inversion of order which is observed in Ti^{47}, Mn^{53}, and Mn^{55} and which seems to be a feature of the configuration $f_{7/2}{}^5$ of identical nucleons. D. Kurath [Phys. Rev. **91**, 1430 (1953)] and B. H. Flowers [Phil. Mag. **45**, 329 (1954)] have shown that an inversion of order in the lowest $\frac{5}{2}^-$ and $\frac{7}{2}^-$ states of such configurations can be produced by an effective two-body interaction of suitable range and exchange dependence.

[24] A. I. Hamburger, Phys. Rev. **118**, 1271 (1960).

[25] B. H. Flowers, Phil. Mag. **45**, 329 (1954).

[26] B. Zeidman, B. J. Raz, and J. L. Yntema, Bull. Am. Phys. Soc. **5**, 77 (1960) and Phys. Rev. (to be published).

[27] B. L. Cohen and R. E. Price, Phys. Rev. **118**, 1582 (1960).