Angular Correlation Study of the $Mg^{24}(d,p\gamma)Mg^{25}$ Stripping Reaction as a Test of the Distorted-Wave Theory*

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Angular correlations between protons from the $Mg^{24}(d,p)Mg^{25}$ reaction leading to the 3.40-Mev excited state of Mg^{25} and the resulting de-excitation gamma rays were investigated to test the validity of the distorted-wave stripping theory. A natural magnesium target was bombarded by 15-Mev deuterons and proton-gamma coincidences were counted using scintillation detectors in conjunction with conventional fast-slow coincidence circuitry. The correlations were studied at laboratory proton scattering angles of 15° and 45°, each in two mutually perpendicular planes: the reaction plane and the plane perpendicular to it containing the deuteron axis. The angular distribution of protons from this level was also measured and fitted by a Butler stripping curve with $l_n=1$ and $r_0=5.0$ fermis.

The coordinate system used to describe the correlations is defined with the z axis in the $\mathbf{k}_d\times\mathbf{k}_p$ direction and the x axis in the recoil nucleus direction. Correlation functions found by least-squares fits to the experimental data are, for the 15° proton angle, $W(\mathbf{k}_d,\mathbf{k}_p,\frac{1}{2}\pi,\phi)=1-(0.385\pm0.023)\cos^2(\phi-\phi_0)$, with $\phi_0=-27.7°\pm2.9°$, and $W(\mathbf{k}_d,\mathbf{k}_p,\theta,\phi_r)=1+(0.145\pm0.029)\cos^2\theta$, where ϕ_r is the beam direction. The functions found for the 45° proton angle are $W(\mathbf{k}_d,\mathbf{k}_p,\frac{1}{2}\pi,\phi)=1-(0.366\pm0.033)\cos^2(\phi-\phi_0)$, with $\phi_0=-6.8°\pm3.5°$, and $W(\mathbf{k}_d,\mathbf{k}_p,\theta,\phi_r)=1+(0.279\pm0.038)\cos^2\theta$. These observed correlations are in good agreement with the predictions of the distorted wave theory and not with those of the plane wave theory. It is to be noted in particular that the agreement is excellent at 45° indicating that protons scattered at this angle probably arise from the stripping process in spite of the fact that the disagreement between Butler stripping theory and the measured angular distribution is greatest here. This then suggests that protons observed in the entire region beyond the first maximum of a typical angular distribution are due to stripping and might be adequately described by stripping theory if suitably distorted waves are used in the analysis.

INTRODUCTION

PROTON angular distributions from deuteron-induced reactions have been the subject of much investigation, both experimental and theoretical. The deuteron stripping theory proposed by Butler[1] and later recast in the Born approximation by Bhatia *et al.*[2] and by Daitch and French[3] has been significantly successful in describing the shapes of many experimental angular distributions. This theory proposes that a target nucleus captures the neutron from an incident deuteron allowing the proton to continue without interaction, such that the deuteron and proton wave functions are approximated by their plane wave asymptotic limits.

The Butler stripping curve which fits an angular distribution best is usually characteristic of a single value of l_n, the orbital angular momentum transferred to the nucleus by the captured neutron. This sensitivity of the theory to l_n has enabled experimenters to assign parities to states in the product nuclei of stripping reactions, and has allowed them to place the following limits on the spins J_e of these states:

$$|\mathbf{J}_i+\mathbf{l}_n+\mathbf{s}_n|_{\min}\leq J_e\leq J_i+l_n+\tfrac{1}{2}, \quad (1)$$

where $\mathbf{J}_i$ is the spin of the target ground state and $\mathbf{s}_n$ is the intrinsic spin of the captured neutron.

The angular distribution of γ rays measured in coincidence with stripping protons detected at a given angle can add further light to the spin of a state excited in the residual nucleus. Several authors[4-7] have shown that if the simplifying assumptions of the plane wave stripping theory are correct, then the proton-gamma angular correlation can be treated simply as an angular distribution of γ rays resulting from the decay of the excited state following neutron capture in the target nucleus. The angular correlation should then be symmetric about the recoil nucleus (captured neutron) axis, and have in the plane of the reaction an anisotropy uniquely determined by the spins involved. That is to say, the correlation $W(\theta,\phi)$ should describe a surface which is cylindrically symmetric about the polar axis (where the polar axis is in the classical recoil direction). The angular correlation measurement would then distinguish between possible values of J_e deduced from the stripping angular distribution.

It becomes evident upon examining the experimental proton angular distribution data that, although the stripping theory fits the data quite well at the forward angles, making possible in most cases the unique identification of l_n, there is an appreciable discrepancy between the theoretical and measured shapes beyond the first maximum of the distribution. This discrepancy has often been attributed to compound nucleus contribution because it takes the form of a raising of minima in the predicted distribution. Such an explanation seems to sidestep the important fact that the theory is originally based on the simplifying assumption of deuteron and proton plane waves completely undistorted by the

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nuclear potential. In looking for an explanation of this discrepancy between data and theory, Butler *et al.*[8] explain by a semiclassical approach how distortion of the deuteron and proton plane waves by the nuclear optical potential would tend to fill the valleys in the angular distribution and would introduce spin polarization of the scattered protons.

Several experimenters[9–12] have studied reactions with the $(d,p\gamma)$ method. All of their experiments indicate a certain measure of agreement with the plane wave theory although the strengths of the correlations measured in the reaction plane were always weaker than those predicted by the theory. Allen *et al.*,[9] investigating the $Si^{28}(d,p\gamma)Si^{29*}$ (1.28 Mev) reaction, found an anisotropy in the plane normal to the recoil direction which was as strong as that seen in the reaction plane. These deviations from the plane wave theory are generally indicative of plane wave distortion.

Many authors[13–17] have worked on improving stripping theory by using distorted waves which are eigenfunctions of the optical potential. Unfortunately, the computational effort involved in any such distorted wave calculation is very involved. Huby *et al.*[17] show how experiments can be interpreted to test the distorted wave theory and also to establish the amount of distortion involved in the particular reaction investigated without laborious numerical computations. These experiments are proton-gamma angular correlations and proton spin polarization measurements.

In addition to their general discussion these authors[17] present a special treatment of the case $l_n=1$ for which they give the correlation function,

$$W(\mathbf{k}_p,\mathbf{k}_d,\theta,\phi)\propto 1+A_2^0P_2(\cos\theta)+A_2^2P_2^2(\cos\theta)\cos 2(\phi-\phi_0). \quad (2)$$

The coordinate system is chosen such that the direction $\mathbf{n}=\mathbf{k}_p\times\mathbf{k}_d$ is the polar axis, the recoil direction is along the x axis, and $\mathbf{k}_d$ and $\mathbf{k}_p$ are the deuteron and proton wave vectors, respectively (Fig. 1). The correlation function contains three parameters A_2^0, A_2^2, and ϕ_0 which can be obtained from a correlation experiment. These parameters are related to the theory by the following expressions:

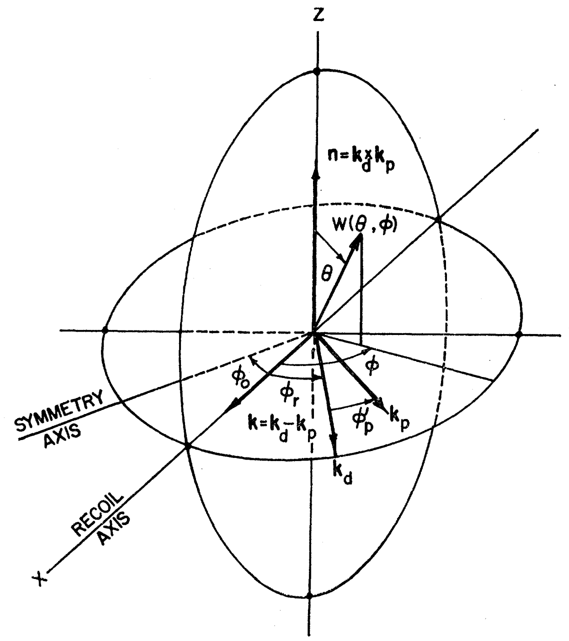

FIG. 1. Angular correlation geometry with x-y plane as the reaction plane, where $\mathbf{k}_d$, $\mathbf{k}_p$, $\mathbf{k}$ are the deuteron, proton, and recoil wave vectors, respectively, and $W(\theta,\phi)$ represents the angular correlation function.

$$A_2^0=-\frac{\eta_2(j_nj_nJ_iJ_e)\sum_{LL'}F_2(LL'J_fJ_e)C_LC_{L'}}{2\sum_{LL'}C_L^2} \quad (3)$$

and

$$-2A_2^2/A_2^0=2[|D|+1/|D|]^{-1}\equiv\lambda, \quad (4)$$

where

$$\phi_0=\tfrac{1}{2}\arg D, \quad (5)$$

$$D\equiv B_{1,-1}/B_{1,1}.$$

The spin of the state to which the product nucleus decays upon emission of the studied γ ray is denoted by J_f, while C_L (or C_L') is the multipole amplitude corresponding to the γ multipolarity L (or L'). The total angular momentum of the captured neutron is denoted by $\mathbf{j}_n$.

According to the distorted wave theory, A_2^0 is independent of the distortion and can be predicted from Eq. (3) using the tables of Satchler[7] for η_2 and of Biedenharn and Rose[18] for F_2. It is the comparison of Eq. (3) with the measured A_2^0 which serves as a principal test of the theory in a correlation experiment.

The symbols $B_{1,-1}$ and $B_{1,1}$ represent, for $l_n=1$, the nuclear overlap integrals B_{lm} defined by Huby *et al.*[17] These are the quantities which require laborious numerical computation to evaluate. The ratio $|D|$ of their amplitude is, from Eq. (4),

$$|D|_1=[1+(1-\lambda^2)^{\frac{1}{2}}]/\lambda,$$

or

$$|D|_2=\lambda/[1+(1-\lambda^2)^{\frac{1}{2}}], \quad (6)$$

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[9] K. W. Allen, B. Collinge, B. Hird, B. C. Maglić, and P. R. Orman, Proc. Phys. Soc. (London) **A69**, 705 (1956).
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and constitutes an experimental result which could be compared with theoretical values of $B_{1,1}$ and $B_{1,-1}$ if available. It can be seen from Eq. (4) that

$$0 \leq \lambda \leq 1. \tag{7}$$

This inequality serves as a subsidiary test of the theory.

The theory also relates λ to P, the component of polarization of the proton spin in the direction $\mathbf{n}=\mathbf{k}_d \times \mathbf{k}_p$:

$$P = \pm \tfrac{2}{3}(1-\lambda^2)^{\frac{1}{2}}(2j_n+1)^{-1}. \tag{8}$$

A measurement of P in addition to the angular correlation would provide another test of the theory.

In the plane wave theory $B_{1,1}=B_{1,-1}$ so that $\lambda=1$ corresponds to the plane wave limit. In this limit $2A_2^2=-A_2^0$ and $\phi_0=0$. Equation (2) then reduces to

$$W(\mathbf{k}_p,\mathbf{k}_d,\tfrac{1}{2}\pi,\phi) \propto 1-2A_2^0 P_2(\cos\phi) \tag{9}$$

in the reaction plane and

$$W(\mathbf{k}_p,\mathbf{k}_d,\theta,\tfrac{1}{2}\pi) \propto 1 \tag{10}$$

in the plane perpendicular to the recoil axis. These are exactly the plane wave predictions of Satchler.[7] It can be seen in comparing Eqs. (9) and (10) with (2) that the distortion affects the angular correlation by shifting the symmetry axis away from the recoil axis, introducing anisotropy in the plane perpendicular to this axis, and reducing the anisotropy in the reaction plane. These effects have been pointed out by Newns[19] and by Horowitz and Messiah.[16]

In the present experiment a complete $(d,p\gamma)$ angular correlation was measured for an $l_n=1$ case to test the predictions [Eqs. (3) and (7)] of the distorted wave theory. In order to test the theory properly an experiment must be chosen in which all spins are known so that no adjustable parameters are used in comparing the data with the theory. The reaction

$$\mathrm{Mg}^{24}(d,p)\mathrm{Mg}^{25*}\ (3.40\ \mathrm{Mev})$$

followed by decay of the 3.40-Mev level to either the 0.58-Mev or ground state fulfills these conditions[20] as is illustrated in Fig. 2 and was therefore the reaction studied.

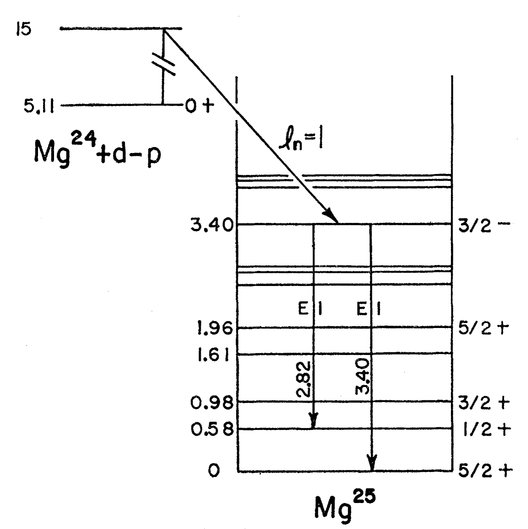

FIG. 2. Partial level scheme for Mg^{25}.

The proton angular distribution was measured, and angular correlations were studied at two proton scattering angles. Angular correlations were obtained at a proton detector angle of $\phi_p'=15°$ which is close to the first stripping maximum. Correlations were measured in both the reaction plane (ϕ dependence) and in the plane containing $\mathbf{k}_d$ and $\mathbf{n}$ (θ dependence). Similar measurements were made at $\phi_p'=45°$, an angle near the predicted stripping minimum and a region where the Butler curve fits the angular distribution poorly. If the correlation fits the distorted wave theory equally well at both proton angles it then suggests that the protons detected where the Butler curve fits poorly are indeed stripping protons and do not result from the formation of a compound nucleus.

EXPERIMENTAL APPARATUS AND PROCEDURE

Fifteen-Mev deuterons from the University of Pittsburgh cyclotron were energy analyzed and focused on a magnesium target by the existing magnetic analysis system.[21] The beam was limited by an aperture effectively $\frac{1}{32}$ in. high by $\frac{1}{32}$ in. wide at a distance of $8\frac{1}{2}$ ft from the target in order to reduce background gamma radiation.

The aluminum scattering chamber used was a cylinder 18 in. in diameter with flat top and bottom cover plates. The detectors were placed inside thin brass wells which pass through the cover plates. Special care was taken to establish a beam path geometry to keep the γ-ray background as low as possible throughout the experiment. For this reason the Faraday cup was mounted at the end of a 6-ft long, 5-in. diameter tube extending from the scattering chamber. The target used was a $\frac{1}{8}$-in. wide strip of natural magnesium ribbon rolled to a thickness of 0.002 in. and suspended on a 4-in. square wire frame. This type of target construction made possible the elimination of all beam defining apertures near the target. Nontarget-induced gamma background was thereby kept to only one-third of the total gamma background in the pertinent energy range.

The proton detector was a thin CsI(Tl) crystal with an aperture chosen to maintain reasonable definition of the reaction plane. For the $\psi_p=\pm 15°$ (Fig. 3) work a $\frac{1}{8}$-in. high by $\frac{5}{16}$-in. wide aperture was used with the crystal $4\frac{1}{8}$ in. from the target. For the $\psi_p=-45°$ work a $\frac{3}{8}$-in. high by $\frac{5}{16}$-in. wide aperture was used with the crystal at $2\frac{5}{8}$ inches. The gamma detector consisted of a

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potted 2-in. by 2-in. cylindrical NaI(Tl) crystal. Both detectors were mounted on RCA–6810A photomultiplier tubes. The gamma detector well was surrounded with $\frac{1}{8}$ in. of lead to attenuate low-energy radiation. Otherwise the amount of matter near the gamma detector was kept to a minimum to reduce the gamma-ray background produced by the intense neutron background present.

The over-all electronic system used conventional fast-slow coincidence circuitry adapted to monitor accidental and true counts simultaneously. The resolution time of the fast coincidence circuit was set at $2\tau=40$ mμsec. One coincidence circuit had in the γ-ray fast pulse line a fixed delay of 86 mμsec (the cyclotron rf period) and thus counted accidental coincidences only.

The proton spectrum observed during the deuteron bombardment of natural magnesium was magnetically analyzed and recorded on photographic emulsions using the system described previously by Moore[22] and by Hamburger.[23] Relative cross sections were determined for laboratory scattering angles between 5° and 90°. Absolute cross sections were obtained from the relative values by comparison with observed relative cross sections of the Mg^{25} ground state whose absolute cross sections at 15° and 25° had been determined previously.[24]

In order to measure the ϕ correlation the horizontal plane containing the beam was established as the reaction plane by placing the proton detector at the proper angle, ψ_p, (Fig. 3), and the gamma detector angle was varied in this same plane. To measure the θ correlation a vertical plane containing the beam was defined as the reaction plane by setting the proton detector at $\psi_p=0$ and lowering it to achieve the desired scattering angle, χ_p. The gamma detector was rotated in the horizontal plane as before.

Gamma rays with a wide range of energies are emitted from the natural magnesium target while it is under deuteron bombardment. Naturally it is important to maintain an energy calibration of this gamma spectrum. To accomplish this a Na^{24} source (decays to Mg^{24}, with gamma-ray energies of 2.75 Mev and 1.36 Mev) was used. These same two γ rays were discernible during the experiment, apparently because levels in Mg^{24} were profusely excited by (d,d') reactions. This made possible a constant check of the energy calibration.

Of interest in the $Mg^{24}(d,p\gamma)Mg^{25}$ angular correlation experiment are the 3.40-Mev and 2.82-Mev γ rays (Fig. 2), the only ones which are emitted in the decay of the 3.40-Mev state. The ideal situation would be to measure the correlation separately for each of these γ rays. Unfortunately the single and double escape peaks of the 3.40-Mev γ ray produce essentially the same pulse heights as the full energy and single escape peaks, respectively, of the 2.82-Mev γ ray. Thus the only peak which could be isolated was the 3.40-Mev full energy peak.

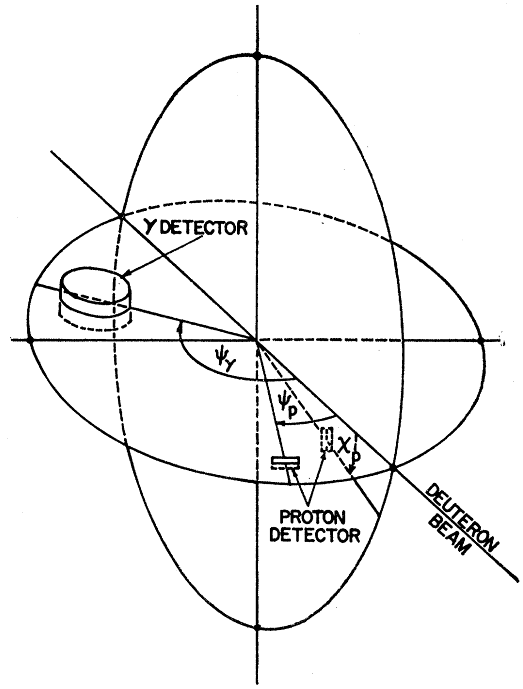

FIG. 3. Laboratory geometry.

There are two reasons which make it more desirable to measure the correlation connected with the 2.82-Mev γ ray than with the 3.40-Mev γ ray. First, the 3.40-Mev level decays preferentially via the 2.82-Mev γ ray, with a branching ratio of about five to one as measured by Campion and Bartholomew.[25] Second, the predicted value of A_2^0 [Eq. (3)] is four times larger for the 2.82-Mev γ ray so the predicted anisotropy is much larger and fewer counts are needed to identify the correlation effects. Because of this the γ-ray energy range was set at 2.0 to 3.0 Mev. Consequently the full energy and single escape peaks of the 2.82-Mev γ ray were counted together with the single and double escape peaks of the 3.40-Mev γ ray.

The net result of this energy selection is that the correlation observed is a mixture of the two separate correlations. This is not serious as long as the branching ratio r_B and the relative detection efficiency r_E within the range used is known for the two γ rays. The product of these two ratios gives the counting ratio a/b for the two γ rays involved. A_2^0 can be predicted for each of the γ rays separately and a weighted A_2^0 can be calculated. Thus,

$$A_2^0=aA_2^0(2.82\ \text{Mev}\gamma)+bA_2^0(3.40\ \text{Mev}\gamma), \quad (11)$$

where $r_Br_E=a/b$ and $a+b=1$. The detection efficiency ratio was determined by studying the pulse-height

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[25] P. J. Campion and G. A. Bartholomew, Can. J. Phys. 35, 1361 (1957).

spectrum from the Na^{24} 2.75-Mev gamma ray. Observation of the relative strengths of the full energy, single, and double escape peaks together with the Compton distribution in this spectrum provided a measure of the relative detection efficiencies of the 2.82- and 3.40-Mev γ rays within the energy range used. The efficiency ratio found (2.82-Mev relative to 3.40-Mev) was $r_E = 0.81 \pm 0.04$. The branching ratio[25] $r_B = 5 \pm 2$ contains by far the largest error in predicting A_2^0.

The proton detector had a total of 0.046 in. of aluminum foil before the crystal, assuring that no charged particles from the target except protons above about 14 Mev were detected. The $Mg^{24}(d,p)Mg^{25}$ reaction has a $Q = 1.70$ Mev for the 3.40-Mev level so the protons emitted have a kinetic energy of about 16 Mev at the laboratory scattering angle of 15°. The foil also helped to improve the effective energy resolution of the detector to about 4%. At the two proton angles studied the cross section of the 3.40-Mev level was the largest in the spectrum and the separation from the adjacent levels was sufficient to allow setting the differential pulse-height selector without maintaining an energy calibration.

DATA ACCUMULATION

A standard run consisted of 1.00×10^5 proton counts. Operation of the electronic equipment was monitored and data counts were accumulated on 12 separate scalers connected at various points in the system. Standard checks were made on the circuitry before and during all data runs.

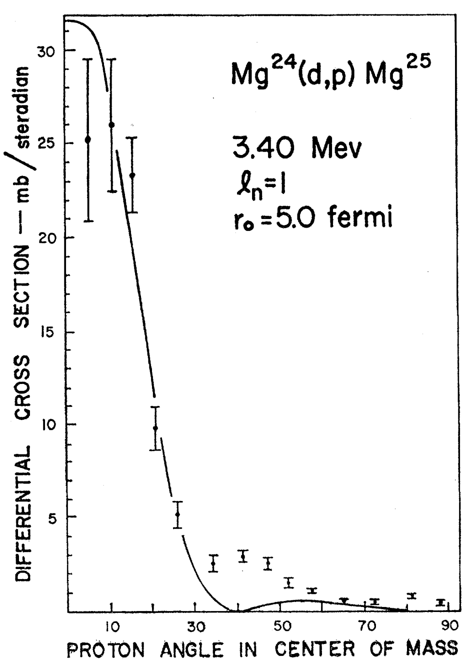

FIG. 4. $Mg^{24}(d,p)Mg^{25}$ (3.40) angular distribution showing the best-fit Butler curve with its corresponding values of l_n and r_0. The error bars shown represent the errors in the relative cross sections only.

The first correlation data taken was at $\psi_p = +15°$. During this set of runs an effort was made to keep the true-to-accidental coincidence ratio high. This ratio averaged about five to one with a deuteron beam of about $0.001 \mu a$, yielding an average of 1.9 true coincidences per minute. The gamma detector angle was changed after each run until fifteen angles each had been repeated three times. Then the roles of the two coincidence units, as accidental monitor and true plus accidental monitor, were reversed and the same fifteen angles were repeated three more times so that coincidences were counted for a total of 6.00×10^5 protons at each angle. Angles with one or more anomalous runs were repeated once to give a little less statistical weight to the individual runs. No data were eliminated in this process.

The beam intensity was increased for the remainder of the experiment because consideration of the manner in which beam intensity relates to statistical accuracy shows that accumulation of data with the same accuracy can proceed at a faster pace with a higher beam intensity even though the true-to-accidental coincidence ratio becomes smaller.

The beam intensity was increased to about 0.005 μa (limited by the maximum reliable counting rate in the γ-ray channel) and the correlation was measured with the proton detector at $\psi_p = -15°$. This angle was studied to verify that the correlation was independent of the laboratory geometry used. The higher beam intensity yielded about 11 coincidences per minute with an average true-to-accidental ratio of 0.9. The role of the two coincidence circuits was reversed after each run in order to cancel possible fluctuations. The gamma detector angle was changed after every four runs with the sequence of angles being chosen randomly. Coincidences were measured at fifteen angles as before with each repeated four different times for a total accumulation of 16.00×10^5 protons at every angle.

At $\chi_p = 15°$ the same procedure was used; but because the anisotropy is weaker here, coincidences were counted at only ten angles; and, to increase statistical accuracy, each angle was repeated six times for a total accumulation of 24.00×10^5 protons at every point. The data at $\psi_p = -45°$ was taken, for a total of 16.00 $\times 10^5$ protons at each point, in the same way as the data at $\psi_p = -15°$ except that coincidences were counted at thirteen angles instead of fifteen. Data accumulation at $\chi_p = 45°$ was performed exactly as at $\chi_p = 15°$ for a total count of 24.00×10^5 protons at each angle.

DISCUSSION OF RESULTS

Proton Angular Distribution

The values of differential cross section measured between zero and ninety degrees for the $Mg^{24}(d,p)Mg^{25*}$ (3.40) reaction are plotted in millibarns per steradian in Fig. 4. A Butler curve with $l_n = 1$, and an effective radius $r_0=5.0$ fermis provided the best fit, in agreement with a previous determination.[26] This curve (Fig. 4) illustrates the general characteristics described in the Introduction concerning the fit of Butler curves to angular distribution data. The data agree reasonably well with the stripping curve on the forward peak but the fit is poor beyond this. The probable errors in the absolute cross-section measurements at the laboratory angles of 15° and 25° were 29%, due to a 25% error in the reference values and a 15% error in the relative measurements. The error bars indicated in Fig. 4 show the relative errors. The methods used in determining these errors as well as the general methods of analyzing the angular distribution data were those described by Moore[22] and by Hamburger.[23]

Proton-Gamma Angular Correlations

As stated in the Introduction, the tests of the distorted wave theory in this experiment consist of checks on the values of A_2^0 and λ as given by Eqs. (3) and (7). Because the correlation is measured in the two planes, $\theta=\pi/2$ and $\phi=\phi_r$, it is convenient to rewrite the correlation function [Eq. (2)] for these two cases. This yields

$$W(\mathbf{k}_p,\mathbf{k}_d,\pi/2,\phi)=1-B'(\phi)\cos^2(\phi-\phi_0), \qquad (12)$$

where

$$B'(\phi)=6\lambda A_2^0/[2+A_2^0(3\lambda-1)], \qquad (13)$$

and

$$W(\mathbf{k}_p,\mathbf{k}_d,\theta,\phi_r)=1+B'(\theta)\cos^2\theta, \qquad (14)$$

where

$$B'(\theta)=\frac{3A_2^0[1+\lambda\cos2(\phi_r-\phi_0)]}{2-A_2^0[1+3\lambda\cos2(\phi_r-\phi_0)]}. \qquad (15)$$

Equations (13) and (15) can be solved for A_2^0 and λ in terms of ϕ_0, $B'(\phi)$, and $B'(\theta)$, which are obtainable directly from the data.

In order to compare the values of A_2^0 deduced from the correlations with the predicted value one must first consider the effect of the combined detection of the two different decay γ rays and find the predicted A_2^0 with the help of Eqs. (3), and (11).

The error in the branching ratio r_B introduces most of the uncertainty in the determination of A_2^0. This branching ratio has been measured several times with varying results.[25,27,20] The value of $r_B=5\pm2$ measured by Campion and Bartholomew[25] is believed by Bartholomew to be fairly reliable since more recent work verifies this result.[28]

The values of A_2^0 calculated separately from Eq. (3) for the two γ rays are A_2^0 $(2.82\gamma)=0.250$ and A_2^0 $(3.40\gamma)=0.050$. Using the ratios $r_B=5\pm2$ and $r_E=0.81\pm0.04$, one obtains a composite $A_2^0=0.210\pm0.016$ from Eq. (11).

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[27] P. M. Endt and J. C. Kluyver, Revs. Modern Phys. **26**, 95 (1954).

[28] G. A. Bartholomew (private communication, 1959).

The data in the $\theta=\pi/2$ (reaction) plane were analyzed by fitting with the method of least squares to the form

$$Y(\psi_\gamma)=A-B\cos^2(\omega+\delta). \qquad (16)$$

A, B, and δ are the parameters evaluated by the least square method and ω is an angular coordinate which is related to the laboratory γ-ray detector angle ψ_γ. The angular coordinate $(\omega+\delta)$ is measured in the same sense as ψ_γ, but is referred to the axis located at the minimum of the angular correlation data which lies closest to the recoil direction. This axis, called the symmetry axis (Fig. 1), is also the zero angle for $(\phi-\phi_0)$ and is displaced from the recoil direction, $\phi=0$, by the angle ϕ_0. The coordinate ω is a guess for the coordinate $(\omega+\delta)$ and is obtained by inspection. The parameter δ evaluated by the least squares analysis is the correction to this guess.

In this way ϕ_0, one of the three experimentally obtainable parameters, was established as the shift between the true symmetry axis and the nuclear recoil axis.

Using $(\omega+\delta)$ as the angular coordinate in the least squares analysis, the coefficients A and B were obtained from which the normalized correlation amplitude $B'(\phi)=B/A$ was determined.

The standard deviations of $B'(\phi)$ and δ were evaluated from the least squares analysis. The error in ϕ_0 was obtained by combining as independent errors the standard deviation for δ, the error in determining the beam direction, and the error in the recoil direction. The error in beam direction was limited by the Faraday cup and amounted to $\pm1.2°$. The error in the recoil direction is directly dependent on the error in the proton direction, which was limited by the horizontal dimension of the proton crystal.

There was a small correction made to the data to account for the finite size of the detectors. A comparison of the angular spread in γ rays, due to the combined effects of the two detectors, with a cosine squared function yields the approximate percentage attenuation in anisotropy of such a function due to the finite size of the detectors. This amounted to about four percent of the amplitude.

Analysis of the data in the plane $\phi=\phi_r$ was very similar to that for the reaction plane. There is expected in this correlation, as can be seen from Eq. (14), a mirror symmetry about the reaction plane which is independent of distortion effects. The form

$$Y(\psi_\gamma)=A+B\cos^2\theta \qquad (17)$$

was used to fit the data, again by least squares analysis, and the normalized correlation amplitude, $B'(\theta)=B/A$, was determined. The angle θ was related to the laboratory gamma detector angle by $\cos\theta=\cos(\psi_\gamma+90°)$ because the Z axis in this case was at $\psi_\gamma=270°$.

The correlation data are shown in Figs. 5, 6, and 7 and the results summarized in Table I. The data corre-

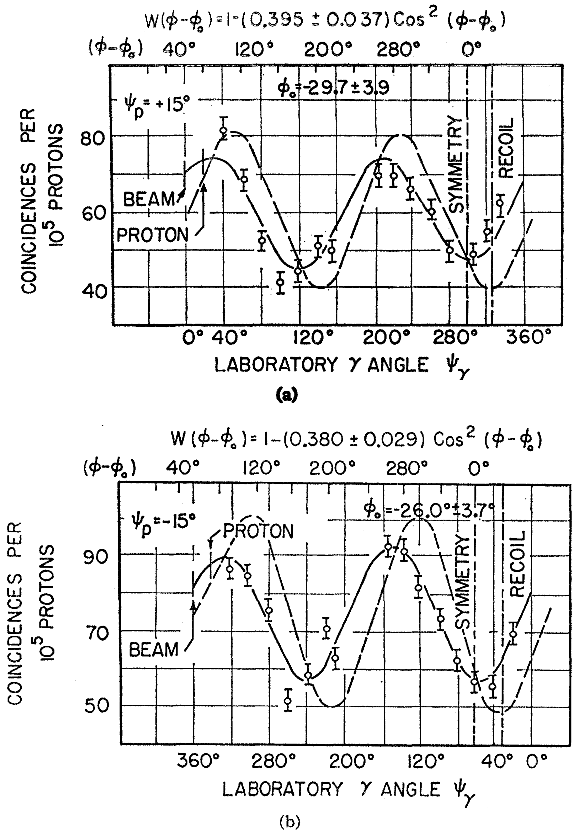

FIG. 5. Angular correlations in the reaction plane with the proton detector at $\psi_p=15°$ (top) and $\psi_p=-15°$ (bottom). The error bars indicate the probable statistical errors given by $[(T+A)+A]^{\frac{1}{2}}$. The equations for the solid curves shown, which are the best fits to the experimental points, are given above the graphs. The dashed curves are the plane wave predictions.

sponding to the proton scattering angle $\phi_p'=15°$ are illustrated in Figs. 5 and 6 together with the results predicted using the plane wave theory. This scattering angle is close to the maximum of the proton angular distribution, as is shown in Fig. 4. The correlations obtained with the proton detector on either side of the

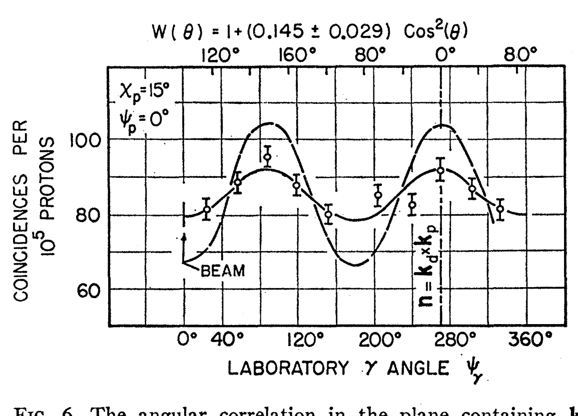

FIG. 6. The angular correlation in the plane containing $\mathbf{k}_d$ and $\mathbf{n}$ with the proton detector at $\chi_p=15°$. (See caption of Fig. 5 for notation.)

deuteron beam ($\psi_p=15°$ and $\psi_p=-15°$) are in essential agreement so that the values in Table I for $B'(\phi)$ and ϕ_0 at $\phi_p'=15°$ are the averages of the two sets of data, weighted according to the inverse squares of their standard deviations.

The correlation measured with the proton detector at $\chi_p=15°$ shown in Fig. 6 shows a decided anisotropy and illustrates the expected symmetry about the reaction plane. The plane wave prediction is shown also, and it differs only in amplitude, as is to be expected.

The data at the proton angle $\phi_p'=45°$ are shown in Fig. 7. This angle is close to the predicted minimum of the Butler curve but, as can be seen from Fig. 4, it actually corresponds to a second maximum in the measured distribution. Visual comparison of the plane wave predictions with the experimental results in Fig. 7 seems to show less discrepancy between data and plane wave theory than was evident in the $\phi_p'=15°$ work shown in Figs. 5 and 6; whereas the discrepancy in the angular distribution between data and plane wave theory (Fig. 4) is greater at $\phi_p'=45°$ than at $\phi_p'=15°$. It appears, however, that the distortion effect is greater at $\phi_p'=45°$ than at $\phi_p'=15°$ since the value of $|D|$ (see Table I) obtained here is further from the plane wave value, $|D|=1$. The smaller discrepancy between data and plane wave theory observed in the angular correlation seems therefore to be simply an accident of the planes chosen for the correlation measurements. Satchler and Tobocman[29] have recently made some calculations using an optical potential wave Born approximation for several nuclei which showed that the distortion effects are not always equally obvious in the angular distributions and angular correlations, and that these effects varied greatly with the direction of the outgoing proton.

It is evident from Table I that the condition of Eq. (7), viz., $0\leq\lambda\leq1$, is fulfilled by both sets of correlations. Both cases show significant distortion effects since the values of $|D|$ which were found differ appreciably from the plane wave value.

The agreement between the prediction for A_2^0 and the result from the $\phi_p'=45°$ measurements is excellent (Table I), strengthening the assertion that the protons measured at 45° are due to the stripping process, with some distortion of the deuteron and proton plane waves by the nuclear optical potential. The agreement between the predicted and experimentally determined values of A_2^0 in the $\phi_p'=15°$ case is not as good, although it is just within the limits suggested by the probable errors. The effect of a contribution from the 4.27-Mev level in Mg^{25} was considered as a possible source of error, since this level has a spin of $\frac{1}{2}$ emitting a 3.29-Mev γ ray.[25] This is, however, unlikely because the relative intensity of this level is greater at $\phi_p'=45°$ where the agreement is excellent.

[29] G. R. Satchler and W. Tobocman, Bull. Am. Phys. Soc. 5, 30 (1960).

TABLE I. Summary of results. The parameters $B'(\phi)$ and ϕ_0 are obtained from the correlations in the reaction plane while $B'(\theta)$ is the amplitude of the correlation in the plane $\phi=\phi_r$. These values are corrected for the finite detector geometry. The values of λ, $|D|$, and A_2^0 (exp.) are derived from these three parameters using Eqs. (13), (15), and (6) while Eqs. (3) and (11) yield A_2^0 (theory). The magnitude of the proton spin polarization predicted by this experiment using Eq. (8) is shown in the last column.

ϕ_p'	$B'(\phi)$	$B'(\theta)$	ϕ_0	λ	$\|D\|$	A_2^0(exp)	A_2^0 (theory)	$\|P\|$ (predict)
15°	0.385±0.023	0.145±0.029	−27.7°±2.9°	0.82±0.11	1.91±0.74	0.177±0.021	0.210±0.016	0.096±0.026
45°	0.366±0.033	0.279±0.038	− 6.8°±3.5°	0.63±0.11	2.82±0.99	0.211±0.027	0.210±0.016	0.101±0.019

CONCLUSION

The results of this experiment show that the distorted wave theory can adequately describe the characteristics of a proton-gamma angular correlation following a deuteron stripping reaction. The correlation provides a test of the theory in the relationships (3) and (7) and also yields a measure, $|D|$, ϕ_0, of the amount of distortion. The values of $|D|$ and ϕ_0 provide a guide for calculating the nuclear overlap integrals $B_{1,\pm 1}$. The values of λ are used to predict the proton spin polarization expected from the reaction. Direct measurement of these polarizations would then provide another test of the theory.

The comparison between the observed results and the plane wave predictions (Figs. 5, 6, and 7) suggest that it is not always possible to detect the effect of wave distortion in a correlation experiment using only one geometrical configuration, but that enough data must be obtained to actually evaluate the parameters of interest numerically. The measurements in the plane containing $\mathbf{k}_d$ and $\mathbf{n}$ are much less sensitive an indicator of distorted waves than are those taken in the reaction plane.

The agreement between theory and experiment suggests that the $(d,p\gamma)$ method might be effectively used for nuclear spectroscopy measurements, although the interpretation is not quite so simple as for the plane wave theory. The $\phi_p'=45°$ measurements suggest further that it may be possible to obtain meaningful spectroscopic information by studying an angle away from the first stripping maximum in situations for which it is impossible to study at the maximum. It is significant that the correlation studied at $\phi_p'=45°$ agrees well with the distorted wave stripping theory since this is an angle where the Butler fit to the proton angular distribution is poorest. This indicates that the poor fit characteristic of angles beyond the first stripping maximum probably is not due to compound nucleus contribution, as has often been suggested, but due to the inadequacy of the plane wave theory. It would be interesting to test this further by measuring angular correlations at proton scattering angles greater than ninety degrees.

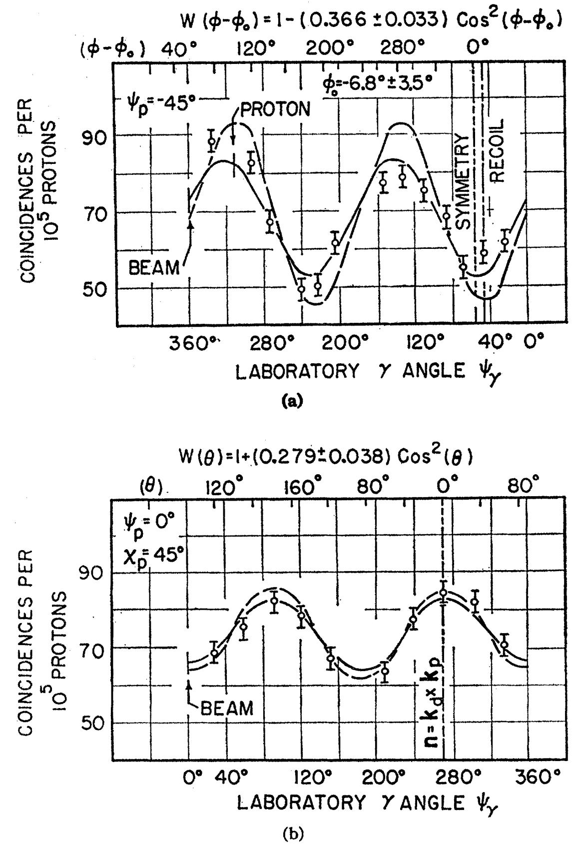

FIG. 7. Angular correlations with the proton detector at a scattering angle $\phi_{p'}=45°$ in the reaction plane (top) and in the plane containing $\mathbf{k}_d$ and $\mathbf{n}$ (bottom). (See caption of Fig. 5 for notation.)

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