

## Free Carrier Absorption Arising from Impurities in Semiconductors

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The free carrier absorption due to ionized impurities in semiconductors is essentially the inverse process of bremsstrahlung. The cross section for bremsstrahlung is readily available in the literature and one can calculate the spectral distribution of bremsstrahlung for the carriers in a semiconductor; furthermore, by using Kirchhoff's law of radiation, relating the emission and absorption in the semiconductor, one arrives quite easily at the absorption coefficient. The results so obtained agree with those of previous authors who have used a different method of calculation. The inadequacy of the Born approximation in the calculation of the ionized impurity effects on free carrier absorption is brought out clearly in the present treatment.

### INTRODUCTION

THE free carrier absorption in semiconductors due to ionized impurities has been calculated by various authors,<sup>1-4</sup> as a two-step process involving the electron-photon and electron-ionized impurity interactions. It has also been remarked<sup>2,3</sup> that the free carrier absorption due to ionized impurities can be viewed as the inverse process of bremsstrahlung, but this observation has not been used to its fullest advantage. The cross section for bremsstrahlung has been evaluated exactly by Sommerfeld for low-energy electrons in the presence of a Coulomb field, and suitable approximations to the exact formula are readily available in the literature, so that one can calculate the spectral distribution of bremsstrahlung radiation in a semiconductor. The absorption coefficient can then be obtained with the aid of Kirchhoff's law of radiation, as shown below. In the course of our treatment, we shall find that the Born approximation is inapplicable to the calculation of ionized impurity effects on free-carrier absorption.

### I. SPECTRAL DISTRIBUTION OF BREMSSTRAHLUNG IN A SEMICONDUCTOR

The cross section of collision between a positive ion of charge  $Ze$  and an electron of energy  $\epsilon_0 = \frac{1}{2}mv^2$  is given by the exact expression due to Sommerfeld,<sup>5</sup>

$$\frac{d\sigma_s}{d(h\nu)} = \frac{A}{h\nu} \frac{\eta_0^2}{(e^{2\pi\eta_0} - 1)(1 - e^{-2\pi\eta_f})} x \frac{d}{dx} |F(x)|^2, \quad (1)$$

where

$$A = 8\pi e^2 h / 3m^2 c^3, \quad x = 4\eta_0 \eta_f / (\eta_f - \eta_0)^2,$$

$\eta_{0,f} = Ze^2 / \hbar v_{0,f}$ , and  $F(x)$  is the generalized hypergeometric function,  $F(x) = {}_2F_1(i\eta_0, i\eta_f; 1, -x)$ ;  $m$  is the effective mass of the electron,  $v_0$  and  $v_f$  are the velocities of the electron before and after collision.

For the low energies of interest in a semiconductor, it is difficult to use (1) in all its generality and hence we

replace the exact result by suitable approximations, given by the following expressions, together with their ranges of validity expressed in terms of the interaction parameter  $\eta$ :

$$\frac{d\sigma_B}{d(h\nu)} = \frac{A}{h\nu} \frac{\eta_0^2}{\pi^2} \ln \left( \frac{\eta_f + \eta_0}{\eta_f - \eta_0} \right), \quad \text{valid for } \eta_{0,f} \ll 1, \quad (2)$$

$$\frac{d\sigma_E}{d(h\nu)} = \frac{d\sigma_B}{d(h\nu)} \frac{\eta_f}{\eta_0} \left( \frac{1 - e^{-2\pi\eta_0}}{1 - e^{-2\pi\eta_f}} \right), \quad \text{valid for } \eta_0 \sim 1, \quad (3)$$

but  $\eta_f - \eta_0 \ll 1$ ,

$$\frac{d\sigma_C}{d(h\nu)} = \frac{A}{h\nu} \frac{\eta_0^2}{\pi\sqrt{3}}, \quad \text{valid for } \eta_0 \gg 1 \ll \eta_f - \eta_0, \quad (4)$$

where  $B$  stands for the Born approximation,<sup>6</sup>  $E$  for the Elwert approximation,<sup>7</sup> and  $C$  for the classical limit<sup>8</sup> of (1). Before proceeding further with the calculation, let us note the orders of magnitude of the quantities involved in the cross section. The magnitude of  $\eta$  is large for slow electrons or holes in a semiconductor, and for electrons in an  $n$ -type semiconductor;  $\eta_0$  corresponds to optical energies and is  $\sim 5$  (for  $Z=1$ ,  $\lambda=2$  microns,  $m$ =free electron mass); and  $\eta_f$  corresponds to thermal energies and is  $\sim 20$  ( $Z=1$ , at  $T=300^\circ\text{K}$ ). Therefore we see from the ranges of validity indicated along with the approximations (2-4), that the Born approximation (2) is clearly invalid for the calculation of free carrier absorption and, depending on the energy ranges of interest, (3) and (4) should be applicable to the case of a semiconductor.

In adapting the approximations (2-4) for the cross section to the semiconductor, one must remember that the process is taking place in a medium characterized by a large refractive index  $n$  and dielectric constant  $K$ , i.e., the electron is decelerated in the modified Coulomb field,  $(K\mathcal{r})^{-1}$  of the ionized impurity. This results in a factor  $nK^{-2}$  multiplying the cross section.

The spectral distribution of bremsstrahlung is given

<sup>1</sup> R. Wolfe, Proc. Phys. Soc. (London) **A67**, 74 (1954).

<sup>2</sup> H. Y. Fan, *Reports on Progress in Physics* (The Physical Society, London, 1956), Vol. 14, p. 119.

<sup>3</sup> H. J. Meyer, Phys. Rev. **112**, 298 (1958).

<sup>4</sup> R. Rosenberg and M. Lax, Phys. Rev. **112**, 843 (1958).

<sup>5</sup> A. Sommerfeld, Ann. Physik **11**, 257 (1931).

<sup>6</sup> W. Heitler, *Quantum Theory of Radiation* (Clarendon Press, Oxford, 1944), 2nd ed.

<sup>7</sup> G. Elwert, Ann. Physik **34**, 178 (1939).

<sup>8</sup> H. Bethe and E. Salpeter, *Encyclopedia of Physics*, edited by S. Flügge (Springer-Verlag, Berlin, 1957), Vol. 35, p. 333.

by

$$I(\nu)d\nu = N_i h \int_{\nu}^{\infty} \left( \frac{2\epsilon_0}{m} \right)^{\frac{1}{2}} h\nu \frac{d\sigma}{d(h\nu)} N(\epsilon_0) d\epsilon_0, \quad (5)$$

where  $I(\nu)d\nu$  is the energy of radiation per unit volume per unit time in the frequency range between  $\nu$  and  $\nu+d\nu$ ;  $N_i$  is the number of ionized impurities per unit volume;  $N(\epsilon_0)d\epsilon_0$  is the number of electrons having energies between  $\epsilon_0$  and  $\epsilon_0+d\epsilon_0$ , given for a Maxwellian distribution by

$$N(\epsilon_0)d\epsilon_0 = 4\pi^{-\frac{3}{2}} N_e \left( \frac{m}{2kT} \right)^{\frac{3}{2}} \exp(-m\nu_0^2/2kT) \nu_0^2 d\nu_0; \quad (6)$$

$N_e$  is the number of electrons per unit volume;  $k$  is the Boltzmann constant; and  $T$  is the temperature.

Combining Eqs. (2-4), (5), and (6), we obtain

$$I_B(\nu) = D e^{z/2} K_0(z/2), \quad (7)$$

where

$$D = \frac{64\pi N_i N_e n Z^2 e^6 e^{-z}}{3 m c^3 K^2 (2\pi m k T)^{\frac{3}{2}}}, \quad z = h\nu/kT;$$

$$I_E(\nu) = \text{same as (7) for } z \ll 1, \\ = 2D \text{ for } z \gg 1; \quad (8)$$

$$I_C(\nu) = (\pi/\sqrt{3})D. \quad (9)$$

## II. ABSORPTION COEFFICIENT

If  $\mu(\nu) = k(\nu)/4\pi$  is the absorption coefficient, Kirchoff's law for a medium of refractive index  $n$  gives

$$\frac{I(\nu)}{k(\nu)} = B(\nu) = \frac{2h\nu^3 n^2}{c^2} (e^z - 1)^{-1}.$$

Therefore, the absorption coefficient is given as

$$\mu_B(\nu) = J e^{z/2} K_0(z/2), \quad (11)$$

where

$$J = \frac{8 N_i N_e Z^2 e^6 (1 - e^{-z})}{3 m c h \nu^3 n K^2 (2\pi m k T)^{\frac{3}{2}}}$$

which is the same as the result obtained in references 1 and 2, but corrected for stimulated emission, through the factor  $(1 - e^{-z})$ ;

$$\mu_E(\nu) = \text{same as (11) for } z \ll 1, \\ = 2J \text{ for } z \gg 1, \quad (12)$$

which result is identical with that given in reference 3; finally,

$$\mu_C(\nu) = (\pi/\sqrt{3})J, \quad (13)$$

and this result is the same as the result given by Spitzer<sup>9</sup> for the case of a plasma, where  $n, K$  are  $\sim 1$ , and here again the formula is corrected for induced emission.

## III. DISCUSSION

One sees that the various expressions (11-13) for the absorption coefficient behave as  $\nu^{-2}$  for the limiting case  $z \ll 1$ , and this is precisely what one would expect from semiclassical theory. It was noted earlier that the magnitude of  $\eta_0$  and  $\eta_f$  are quite large in the case of the semiconductor, and hence (12) and (13) are the valid equations for a semiconductor. Furthermore these expressions give the correct limiting behavior at low frequencies and also at the high frequency and give a  $\lambda^3$  behavior of the absorption coefficient, which result seems to be borne out by the recent experimental results of Spitzer and Whelan<sup>10</sup> on the free carrier absorption in heavily doped samples of gallium arsenide.

<sup>9</sup> L. Spitzer, *The Physics of Fully Ionized Gases* (Interscience Publishers, Inc., New York, 1956).

<sup>10</sup> W. G. Spitzer and J. M. Whelan, *Phys. Rev.* **114**, 59 (1959).