Energy Determination of Heavy Primaries in Nuclear Emulsion*

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The "knock-on electron" method has been used to determine the energy per nucleon of the heavy primary particle. In this method, the energy of the primary particle is determined by measuring the emission angle and the energy of the knock-on electrons. The conditions for the reliable estimate of the primary energy by this method are discussed. This method is applied to 34 flat events, of primary energy between 3 and 30 Bev/nucleon and of charge $Z>4$, which make nuclear interactions and break up into 2 or more α particles. The energy of the primary particle obtained by knock-on electron method is then compared with the energy obtained by (i) opening angle of α particles and by (ii) relative scattering measurements of α particles. The results obtained by knock-on electron method are quite consistent, within the experimental error, with the results obtained by other methods.

I. INTRODUCTION

DETERMINATION of the energy of heavy particles is one of the fundamental questions in cosmic radiation work. The methods that are commonly used for the determination of the energy of the primary particles of known mass and charge are (i) range-energy relation, (ii) change of ionization with range, (iii) multiple scattering method, and (iv) nuclear fragmentation[1] of primary particles into α particles.

There are certain limitations in the applications of all the above methods. While method (i) is useful only for stopping particles, method (ii) is generally used for nonstopping and low-energy ($\ll 0.5$ Bev) particles. In method (iii) the thickness of the tracks of particles gives rise to a large reading error and this method is useful for primary particles of energies up to only 2 or 3 Bev/nucleon. Above this energy, the presence of spurious scattering in emulsion can cause a great error in determining the energy of the primary particle. In method (iv), first, the probability of fragmentation is rather small and secondly, if the cutoff energy per nucleon is not fairly high it will not be possible to make scattering measurements on all the tracks by this method. At low energies the angles between the fragments can be rather large and only very small track length is available for measurements before the separation between the tracks becomes too large. The energy measured from the opening angle of α particles is limited to events in which many α particles are emitted. It gives rise to large fluctuations for an individual event having only a few fragments.

In the present paper we are going to use the "knock-on electron" method for the determination of the primary energy of heavy nuclei. By measuring the energies of the ejected electrons and the angles that they make with the direction of the primary particle, one can determine the energy per nucleon of the heavy particle. The knock-on electron method has been used previously by different authors[2–5] for different purposes and more recently Biswas *et al.*[6] have used this method for finding the energy of the primary particles from 0.2 to 9 Bev/nucleon energy range. In order to check the reliability of this method over the wide range of primary energies we have selected for this experiment only those heavy nuclei which interact in the nuclear emulsion and break up into α particles. We thus compare the energy of heavy nuclei obtained with the energy obtained from the opening angles and from the relative scattering of α particles, produced in nuclear fragmentation.

II. RELATIONSHIP BETWEEN THE ENERGY OF THE PRIMARY PARTICLE AND OF A "KNOCK-ON ELECTRON"

From the application of conservation laws of energy and momentum one can get, with $c=1$,

$$T=2m_e p^2 \cos^2\psi/[(W+m_e)^2-p^2\cos^2\psi], \qquad (1)$$

and

$$T_{\max}=2m_e p^2/[(W+m_e)^2-p^2], \qquad (2)$$

and when $M\gg m_e$ and $p\ll M$ then Eq. (2) reduces to

$$T_{\max}\approx 2m_e(p/M)^2=2m_e\beta^2/(1-\beta^2),$$

where T = kinetic energy of the knock-on electron, m_e = rest mass of the electron, ψ = angle of emission of the knock-on electron with respect to the direction of the primary particle, W = total energy of the primary particle, p = momentum of the primary particle, M = rest mass of the primary particle, β = the velocity of the primary particle, and $T_{\max}$ = maximum kinetic energy of the knock-on electron.

If we consider $m_e\ll M$, Eq. (1) reduces to a more

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simple form:

$$T_1 = 2[E_1^2 - 1]/(1 + E_1^2 \tan^2\psi), \tag{4}$$

$$T_{1\max} = 2[E_1^2 - 1], \tag{5}$$

and

$$T_{1\min} = \frac{2[E_1^2 - 1]}{1 + E_1^2 \tan^2\psi_{\max}}, \tag{6}$$

where $T_{\min}$ = minimum kinetic energy of the knock-on electron which will be used in the text (3 Mev), T_1 = kinetic energy of the electron in units of electron rest mass, and E_1 = total energy per nucleon of the primary particle in units of nucleon rest mass.

The energy relation between the primary particle and the knock-on electron as calculated from Eq. (1) is shown in Fig. 1. We may see that for given energy of electron there is an upper limit to the angle ψ which can be useful in determining the energy of the primary particle. Beyond that angle a small error in the energy value of the electron can cause a great error in the determination of the primary energy. Therefore, one has to be rather careful in making use of this method. We have tried to show by a dashed line the upper limit of the useful angle ψ that one can use for the given energy per nucleon of the primary particle. As long as the angle ψ is smaller than $\psi_{\max}$ (useful angle), one can get a reliable energy value of the primary particle.

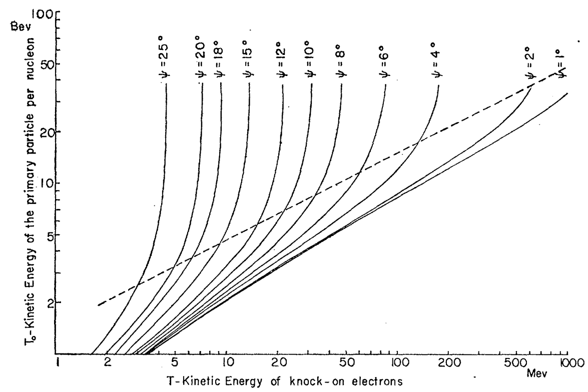

FIG. 1. Relation between the kinetic energy per nucleon (T_0) of the primary particle and the kinetic energy (T) of the knock-on electron for various emission angles (ψ^0). The dashed line shows the upper limit to the useful angle used for a significant measurement of the primary energy in the text.

(a) Frequency of "Knock-On Electrons"

The number of electrons per centimeter with kinetic energies in the interval T to $T+dT$, which are scattered elastically by a primary nucleus of charge Ze is given by Mott[7] as

$$dn(TdT) = \frac{2\pi N Z^2 e^4}{m_e^2 \beta^2} \frac{dT}{T^2}\left[1 - \frac{1-\beta^2}{2}\frac{T}{m_e} + \frac{Z\pi\beta}{137}\left(\frac{1-\beta^2}{2\beta^2}\frac{T}{m_e}\right)^{\frac{1}{2}}\left(1 - \frac{1-\beta^2}{2\beta^2}\frac{T}{m_e}\right)\right], \tag{7}$$

where N is the number of electrons per cm^3 of the stopping material.

In order to make an approximate calculation on the frequency of knock-on electrons for different heavy nuclei, we have used only the first two terms in Eq. (7). In order that the electron may not scatter too much for energy measurements, we have used for lower limit $T_{\min} = 3$ Mev and for upper limit, $T_{\max}$, we used Eq. (3), thus getting

$$n_e = 0.49\frac{Z^2}{\beta^2}\left[0.16 - \frac{1-\beta^2}{2\beta^2} - \frac{1-\beta^2}{2}\ln\left(\frac{1}{6}\frac{2\beta^2}{1-\beta^2}\right)\right]\Big/\text{cm.} \tag{8}$$

In Fig. 2 is shown the number of knock-on electrons per cm track length of different heavy nuclei with different energy per nucleon and these values have been calculated by using Eq. (8). We may notice that for a heavy nucleus of energy greater than 7 Bev/nucleon, the number of knock-on electrons remains almost constant for higher primary energies.

(b) Frequency of Useful "Knock-On Electrons"

Knock-on electrons make different angles with the direction of the primary particle and all of them are not useful electrons for finding out the energy of the primary particle. So, in order to find the number of useful knock-on electrons produced by different heavy nuclei we have modified Eq. (8) by using the values given in Eqs. (5) and (6) for the lower and upper limits of electron energy in terms of the energy of the primary particle. The final expression is given by

$$n_e = 0.49\frac{Z^2}{2\beta^2}\left[\frac{\tan^2\psi_{\max}}{1-(1/E_1^2)} - (1-\beta^2) \times \ln(1 + E_1^2\tan^2\psi_{\max})\right]\Big/\text{cm.} \tag{9}$$

The frequency of useful knock-on electrons as derived from Eq. (9) is shown in Fig. 3 for different energies of the primary particle. From 30 Bev/nucleon the frequency curves are shown by solid lines and below this energy value it is shown by dotted lines. For a given primary nucleus, the number of useful knock-on electrons decreases as its energy increases. From the curves, drawn with the help of Eq. (9), it looks as if for a given heavy nucleus the number of useful knock-on electrons will go on increasing with the decrease in its energy. Actually this is not the case. This is because in Eq. (9) we have only the condition on the angle ψ of the electron and no condition on its energy. If, instead of using the limits of integration from Eqs. (5) and (6), we use the limits from Fig. 1 where the lower energy

[7] N. F. Mott, Proc. Roy. Soc. (London) A124, 425 (1929).

limit is always given by 3 Mev and the upper limit varies for different primary energies per nucleon, we get a complete solid curve as shown in Fig. 3. For energy values of heavy nuclei from 3 Bev/nucleon to 2 Bev/nucleon, all solid curves show less number of knock-on electrons than the dotted curves, and below an energy of 2 Bev/nucleon for the primary particle, the number of knock-on electrons decreases very rapidly.

IV. EXPERIMENTAL PROCEDURE

The heavy nuclei that we used in this experiment were selected from two stacks of nuclear emulsion. One of the stacks was flown near Minnesota in 1958. It consists of 200 *G*-5 emulsions of size 15×15 cm. The other stack was flown near Guam[8] (Marianna Islands) in 1957.

Thirty-four flat events with primary energy between 3 and 30 Bev/nucleon were selected. All these events have a track length of at least 5 cm before they make a nuclear interaction and break up into two or more α particles. The energy of the primary nuclei was determined from the opening angles of α particles and also from the relative multiple scattering measurement between the α particles, as described in detail previously.[9] In order to get a reliable energy value from the relative scattering measurements, we selected events with track length ≥ 2 cm per plate and with energy ≥ 3 Bev/nucleon. In order to get two or three useful

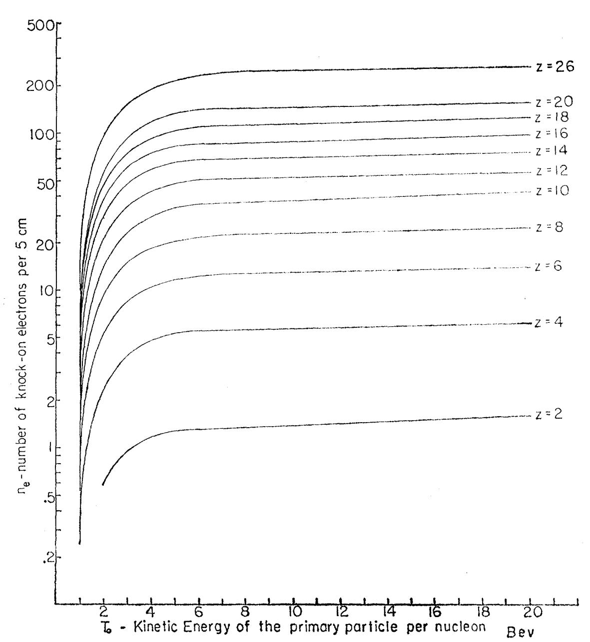

FIG. 2. Relation between the number of knock-on electrons (n_e) and the energy per nucleon (T_0) of the primary particle for different heavy nuclei.

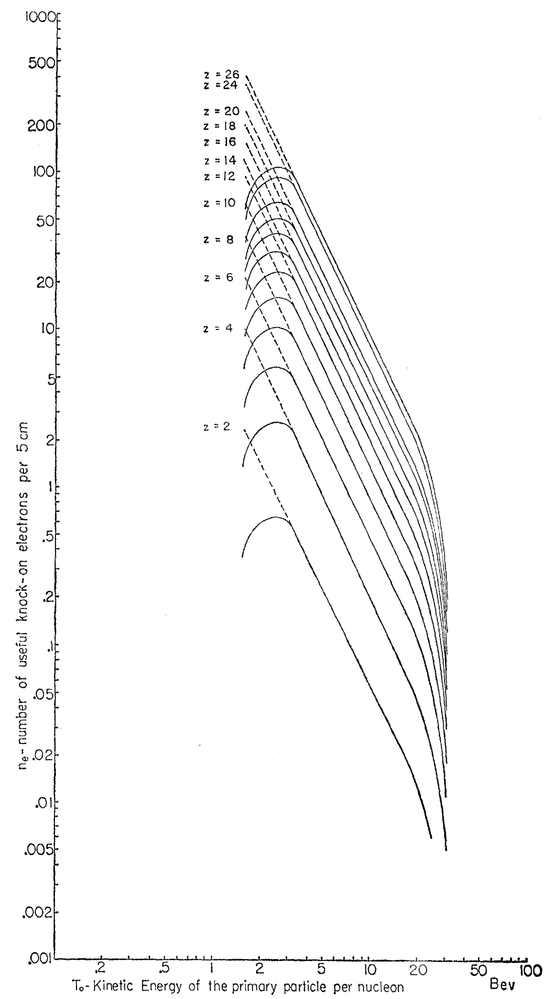

FIG. 3. The frequency of useful knock-on electrons as a function of the primary energy (T_0) for various nuclei. See the text for details.

knock-on electrons per plate and also that a primary heavy particle may fragment into two or more α particles, we used only those heavy primary particles which had $Z>4$.

(a) Scanning Efficiency of "Knock-On Electrons"

The scanning efficiency of knock-on electrons is about 60–80%, depending upon the type of primary nuclei (i.e., *L*, *M*, or *H* nuclei) and also upon the energy of the heavy nuclei. High-energy knock-on electrons are ejected at a very small angle to the primary particle and they travel together for some distance, and hence it is unlikely to miss them unless they are formed just near the surface and move steeply out of the emulsion. As the energy of a knock-on electron decreases, its emission angle and scattering increases. Sometimes it becomes difficult to distinguish a low-energy knock-on electron from the background

[8] E. Lohrmann and M. W. Teucher, Phys. Rev. **115**, 636 (1959).
[9] P. L. Jain, E. Lohrmann, and M. W. Teucher, Phys. Rev. **115**, 643 (1959).

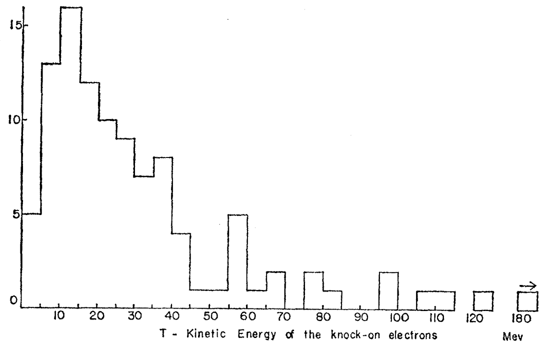

FIG. 4. The energy distribution of the knock-on electrons. Ordinate: number of events.

electron. It is also possible that a relatively low-energy knock-on electron may be missed if it is formed near the surface of the emulsion and is emitted or is scattered so as to move steeply out of the emulsion.

(b) Criteria for the Selection of "Knock-On Electrons"

By following the heavy primary particle we looked for high-energy knock-on electrons. We tried to select about 3 or 4 knock-on electrons per heavy particle which satisfy the following criteria.

(i) Projection angle with the direction of the primary particle less than 15°.

(ii) Dip angle less than 15°.

(iii) The first grain of the minimum track of the knock-on electron should appear within a distance of about 5 to 6 μ from the heavy track.

(iv) The track of the knock-on electron should be long enough to give at least 15 independent cells for multiple scattering measurements.

(v) Electron tracks within 10 μ distance from the top or the bottom of the plate were not accepted.

For a single heavy primary we used the 3 most suitable electrons for its energy measurements. The energy of the electrons was determined by the usual method[10] of multiple Coulomb scattering measurement, in which signal to noise ratio was >2. The length of the cell size used varied from 10 μ to 100 μ, depending upon the energy of the knock-on electron. In a few cases where the electron tracks experienced a large-angle scattering, we did not use the rest of the electron track length available.

The energy distribution of knock-on electrons is shown in Fig. 4. The relative error on the energy determination of electrons was calculated from the relation

$$\frac{\Delta D}{D}=\frac{0.76}{\sqrt{n}}\left[\left(\frac{D_{\text{tot}}}{D_{\text{sig}}}\right)^4+\left(\frac{D_N}{D_{\text{sig}}}\right)^4\right]^{\frac{1}{2}}, \tag{10}$$

[10] P. H. Fowler, Phil. Mag. 41, 169 (1950).

where D_{tot} = measured sagitta, D_N = total noise, D_{sig} = true scattering sagitta, and n = number of independent cells used in the scattering measurement.

(c) Measurement of the Emission Angle of "Knock-On Electrons"

In order to measure the angle ψ which the electron track makes with the direction of the primary particle, we have made use of the optimum conditions under which the errors in the measurement of the dip angles of electrons ϕ and of the primary track ϕ_0, and also the errors in the projection angle θ which the electron track makes with the direction of the primary particle, were minima. Milone *et al.*[5] have discussed these points quite in detail. The error in the alignment of the hair-line of length l to the electron track is given by $k/l^{\frac{3}{2}}$ and the error due to the scattering of the electron track over a length l is given by $\bar{\alpha}(l/300)^{\frac{1}{2}}$, where k is a constant and $\bar{\alpha}$ is the mean angle of scattering per 100 μ expressed in radians, so that the total error in angle θ is given by

$$\Delta\theta=[k^2/l^3+\bar{\alpha}^2(l/300)]^{\frac{1}{2}}. \tag{11}$$

The error $\Delta\theta$ is minimum when $\partial(\Delta\theta)/\partial l=0$ and the optimum length is given by

$$l_{\text{opt}}=5.5(k/\bar{\alpha})^{\frac{1}{2}}, \tag{12}$$

where k has a value of 0.48 radian determined experimentally. In a few cases when the optimum length given by Eq. (12) was very small we used a cell length which permitted a good alignment of the hair line to the electron track. Just like Eq. (12), the expression for the optimum length L_{opt} used for the measurement of the dip angle ϕ is given by[5]

$$L_{\text{opt}}=8.43\{[\rho^2(\Delta L)^2+2s^2(\Delta Z)^2]/\bar{\alpha}^2\}^{\frac{1}{4}}, \tag{13}$$

where $\rho=\tan\phi$, s = shrinkage factor of emulsion, ΔZ = error in reading the depth scale ($\sim 0.5\,\mu$), and $\Delta L = \sim 1\,\mu$.

When the projection angle θ was $\leq 5°$ we calculated the angle ψ by measuring the difference of dips and the horizontal separation of the two tracks (i.e., the primary and the electron) at a distance of optimum track length l of an electron but when θ was $>5°$ then the angle ψ was calculated from the projection angle and from the dip angles of the two tracks. In either case the total error in the measurement of an angle ψ was considered to be $\sim 1\,\mu/L$ radian. In most cases we took two readings for an angle ψ to check the consistency in their values.

(d) Energy Determination of the Primary Particle

From the emission angle and from the energy of the knock-on electron, one can determine the energy per nucleon of the primary particle from Fig. 1. The limits

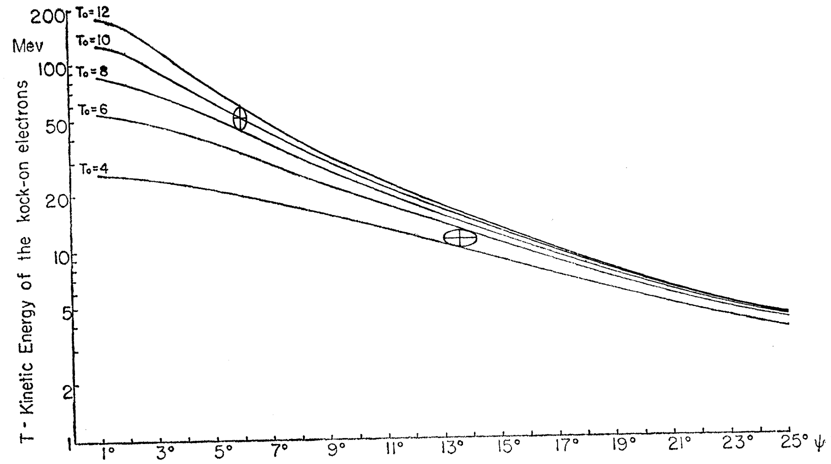

FIG. 5. Relation between the kinetic energy (T) of the knock-on electron and the emission angle ψ for various primary energies (T_0). Axes of the ellipse show the limits of errors in the measurement of energy and the emission angle of the knock-on electron.

of error in the energy value of heavy nuclei are determined the way it is shown in Fig. 5, where T_0 represents the kinetic energy in Bev/nucleon of the primary particle. The error increases with the increase in the angle of emission of the knock-on electron.

Out of 3 or 4 knock-on electrons by a heavy nucleus we accepted those electrons which gave the energy of the primary particle which did not differ from each other by more than 2 standard deviations. Then the weighted mean of the energy value of the accepted events was considered to be the correct value of the energy per nucleon of the primary particle.

V. RESULTS AND DISCUSSION

On the average we used three suitable electrons for the energy determination of a single primary particle. The energy values of the incoming particle as determined from these three knock-on electrons must be consistent with each other within the experimental errors. If the values of the primary energy as given by one of these three electrons differ from each other by more than two standard deviations, then we generally rejected this electron. The final value of primary energy was accepted from the weighted mean of the estimated values which were consistent with one another (within two standard deviations).

Thus the energy value of each heavy primary particle as obtained from the knock-on electron method was then compared with their energy values as obtained from the relative scattering method and from the opening angle of α-particle fragmentation.

In order to check the reliability of the knock-on electron method for determining the energy of a primary particle, we have selected a number of knock-on electrons from the same heavy primary and have thus calculated its energy from the emission angle and from the kinetic energy of the electron. Table I shows the consistency in the value of the primary energy as obtained from different knock-on electrons from the same primary particle of three heavy nuclei of different masses and of different energies. [The number of α particles in event number $6(M)$ and $30(H)$ is two, while in $9(H)$ there are three.] The weighted mean of the primary energy as obtained from (i) knock-on electrons method is then compared with primary energy per nucleon as obtained from (ii) direct scattering measurement of the heavy primary, from (iii) opening angle of the α fragmentation, and from (iv) the relative scattering of α particles from nuclear fragmentation. The following general results are summarized from Table I. (a) Energy values of the primary particle, as obtained from different knock-on electrons, are consistent with one another, within the experimental errors. (b) In general, the error in the primary energy increases with the increase in the angle of emission of the knock-on electron. (c) The weighted mean value agrees fairly well with the value obtained from the relative scattering measurement. (d) The reliability of the primary energy, as obtained from the direct scattering measurement of the heavy nuclei, decreases with the increase in the mass number and with the primary energy. (e) Fluctuations around the true energy value in method (ii) and (iii) are of the same order and they are much higher than the fluctuations in methods (i) and (iv).

Thus we see that there is a fair amount of agreement between the results obtained from knock-on electron method, and from other methods mentioned above.

We know that the determination of the primary energy from the opening angle of α fragmentation is limited to events in which very many α particles are emitted. It is expected to give rise to large fluctuation for an individual event having only a few fragments. In Fig. 6(a) we have plotted the distribution of E_{sc}/E_{fr} where E_{sc} and E_{fr} represents the energy per nucleon of the primary particle obtained by relative scattering measurements and by opening angle measurements, respectively. The distribution of individual measurements is asymmetric, as is to be expected from the mechanism of the evaporation process and as was

TABLE I. Comparison of the determination of the primary energy obtained from knock-on electron method and from the other standard methods.

Event number	Emission angle of electron (ψ^0)	Kinetic energy of the electron T (Mev)	Primary energy/nucleon, T_0 (Bev) (i) From knock-on electron: Individual values	(i) From knock-on electron: Weighted mean	(ii) From direct scattering of the primary	(iii) From the opening angle of the α fragment	(iv) From relative scattering measurement
	9.0±1.9	30.1± 3.5	10.1±2.4				
	4.2±1.2	84.7±14.5	11.0±1.5	10.58±1.1	5.2±3.0	12.5	9.1±2.1
6(M)	5.0±0.5	62.1±11.3	9.9±2.2				
	11.0±2.0	22.5± 3.3	10.8±3.8				
	10.2±0.6	15.4± 4.0	4.8±1.2				
	8.5±1.2	21.5± 2.6	5.4±1.5				
30(H)	8.0±1.3	24.2± 4.2	5.8±1.3	5.12±0.56	8.5±4.1	7.3	5.9±1.2
	5.0±1.2	28.2± 3.8	4.9±0.8				
	15.3±2.3	8.6± 2.6	4.2±1.5				
	9.5±1.0	24.2± 4.0	7.8±2.3				
	6.0±1.9	40.4± 5.8	7.2±1.9				
19(H)	10.2±2.0	20.5± 3.5	7.0±2.4	7.7 ±0.82	5.0±2.2	6.5	8.9±1.9
	5.1±1.2	55.7± 6.6	8.4±1.6				
	15.1±1.9	11.0± 2.5	6.5±4.3				
	4.0±1.5	57.7±11.5	7.5±1.5				

already shown by Kaplon *et al.*[1] In Fig. 6(b) we have plotted the distribution of $\langle E_e\rangle/E_{\text{fr}}$, where $\langle E_e\rangle$ represents the weighted mean energy per nucleon of the primary particle obtained by knock-on electron. This distribution like the $E_{\text{sc}}/E_{\text{fr}}$ distribution is also asymmetric, showing that the distribution in the primary energy obtained by knock-on electron method behaves the same way as obtained by relative scattering method, although the distribution function may not necessarily be the same in both the cases. In Fig. 7 is shown the distribution E_e/E_{sc}, where E_e represents the energy per nucleon of the primary particle obtained by an individual knock-on electron. In this distribution all the knock-on electrons were used including even those which were not used in finding the weighted mean energy value of the primary particle. The distribution curve is quite symmetrical and has a sharp peak at the center and long tails at the edges. The long tails are due to the electrons which underestimated or overestimated the primary energy by a factor of up to 10 in the measured energy range. In Fig. 8 we have plotted the energy values of 34 primary individual events along with their experimental errors as calculated by relative scattering measurement and by knock-on electron method. We may note that the agreement between the energy values as given by the two methods is fairly good up to a primary energy of 15 Bev/nucleon and beyond that the uncertainties in the energy determination increase with the increase in the primary energy.

In about 70–75% of the cases the events along with their experimental errors lie on a straight line with a slope of unity. On the average the error of E_{sc} was 25%. In 3 events we found that although the primary energy as given by two or three knock-on electrons by the same primary particle was consistent with each other, within their experimental errors, but their weighted mean value was definitely not consistent with the values obtained from relative scattering measurements. This is due to the fact that although we carried out track-to-track scattering measurements, the influence of spurious scattering and of emulsion distortions cannot be disregarded completely which results in giving a wrong energy value of the primary particle. In spite of the fact that all the knock-on electrons which were selected for this experiment lay within the range of useful angle, about 10–12% of the total number of electrons gave quite different values of the primary energy. These energy values did not agree with the values obtained from the other useful electrons ejected from the same primary particle, and also with the values obtained from the scattering measurements. This may be explained as due to one of the following reasons:

(i) An event may be a background electron and may accidentally originate from the primary track.

(ii) Large-angle scattering of the electrons within a short distance of 10 to 15 μ from its origin.

(iii) A high-energy electron may lose energy by bremsstrahlung processes very near its origin.

On the whole we see that the knock-on electron method as used in this experiment for determining the energy of heavy primary particles is a good and a reliable method.

VI. CONCLUSIONS

The energy per nucleon of the primary particle has been determined by measuring the emission angles and the energies of the knock-on electrons which are produced in elastic collision by the heavy primary particle in its passage through nuclear emulsion. In order that a knock-on electron may give a reliable energy value of the primary particle, it has to satisfy certain necessary conditions which have been discussed in Sec. IV, part (b) quite in detail. The frequency of these useful knock-on electrons has been considered in Sec. III, part (b) for different primary nuclei with different energies.

One should not use only one knock-on electron per heavy primary to determine the energy per nucleon of the incoming particle. If this particular electron happened to suffer from one of the sources of error mentioned in Sec. V, then it will give an energy value of the primary particle which will be quite different from its true value. Thus it is important to use three or more knock-on electrons by a single primary particle such that the energy value of the primary particle given by two of these electrons does not differ from one another by more than 2 standard deviations. It is found that if the proper precautions are taken for the selections of suitable knock-on electrons, then the energy by "knock-on electron" method is quite consistent with that determined by other well-known methods, up to an energy of 15 to 20 Bev/nucleon.

If the total energy of the incoming particle is known then from Eq. (1) one can find the mass of that particle, provided the energy and the emission angle of the knock-on electrons which have been ejected by the primary particle in its passage through emulsion is known. This technique may be good in studying the isotopic configuration of different nuclei present in the cosmic radiation.

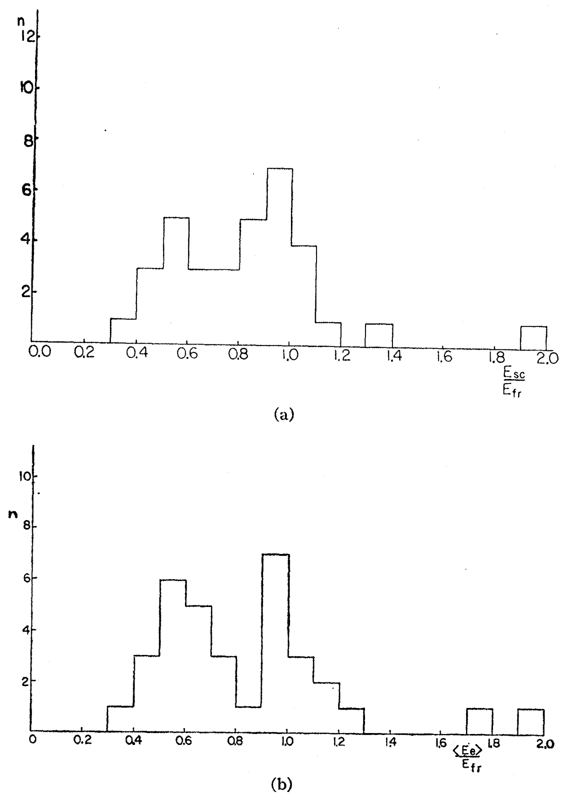

FIG. 6. (a) Comparison between energy E_{sc} obtained by relative scattering measurements and energy E_{fr} obtained from the opening angle of fragments. Ordinate: number of events. (b) Comparison between energy $\langle E_e \rangle$ obtained by knock-on electron and energy E_{fr} obtained from the opening angle of fragments. Ordinate: number of events.

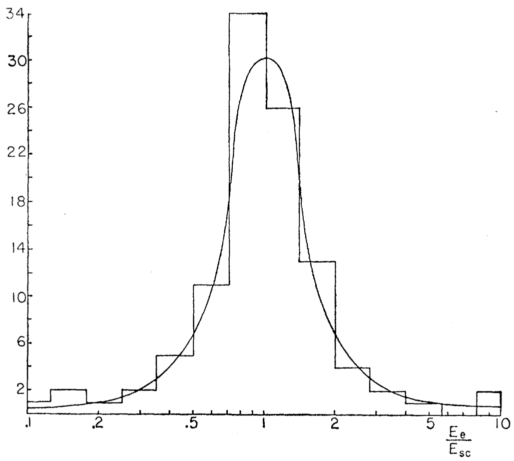

FIG. 7. Comparison between energy E_e obtained by individual knock-on electron and energy E_{sc} obtained by relative scattering measurements. Ordinate: number of events.

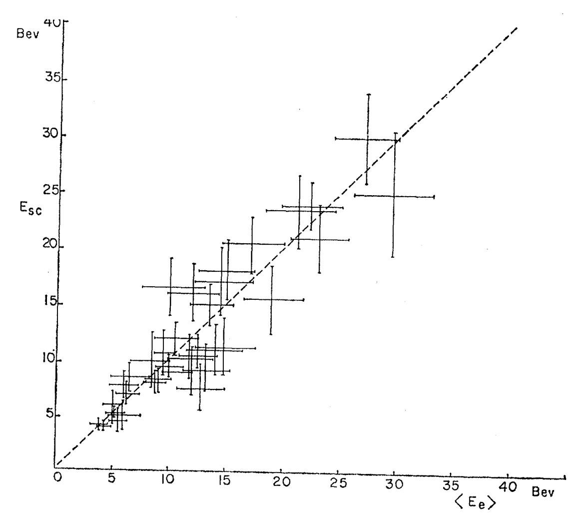

FIG. 8. Comparison between energy $\langle E_e \rangle$ obtained from knock-on electron and energy E_{sc} obtained from relative scattering measurements along with their experimental errors. Dashed line makes 45° with either axis.

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Influence of Bose-Einstein Statistics on the Antiproton-Proton Annihilation Process*

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Recent observations of angular distributions of π mesons in $\bar{p}$-p annihilation indicate a deviation from the predictions of the usual Fermi statistical model. In order to shed light on these phenomena, a modification of the statistical model is studied. We retain the assumption that the transition rate into a given final state is proportional to the probability of finding N free π mesons in the reaction volume, but express this probability in terms of wave functions symmetrized with respect to particles of like charge. The justification of this assumption is discussed. The model reproduces the experimental results qualitatively, provided the radius of the interaction volume is between one-half and three-fourths of the pion Compton wavelength; the dependence of angular correlation effects on the value of the radius is rather sensitive. Quantitatively, there seems to remain some discrepancy, but we cannot say whether this is due to experimental uncertainties or to some other dynamic effects. In the absence of information on π-π interactions and of a fully satisfactory explanation of the mean pion multiplicity for annihilation, we wish to emphasize the preliminary nature of our results. We consider them, however, as an indication that the symmetrization effects discussed here may well play a major role in the analysis of angular distributions. It is pointed out that in this respect the energy dependence of the angular correlations may provide valuable clues for the validity of our model.

I. INTRODUCTION

RECENTLY a study has been made[1] in a propane bubble chamber of "hydrogenlike" annihilations of antiprotons of 1.05-Bev/c laboratory-system momentum, corresponding to an energy release of 2.1 Bev in the center-of-mass system. A hydrogenlike event is defined as one in which equal numbers of π^+ and π^- mesons are produced and in which no visible evaporation prongs appear.[2] The experiment indicates[1] that the distribution of the angle between pairs of pions (in the c.m.-system of $\bar{p}$-p) deviates from the prediction of the conventional statistical model. In particular it was found that there is a clear difference between the angular distribution for pion pairs of like charge and that for pairs of unlike charge. In the statistical model in its usual sense, there is no room for distinctions of this kind.

It is the purpose of this paper to indicate a simple refinement of the statistical model which could possibly explain the bulk of the effect, and which consists of taking into account the influence of the Bose-Einstein (BE) statistics for pions of like charge. As we show in what follows, such an interpretation appears to reproduce the experimental results qualitatively—provided, however, that the radius of the volume of strong interactions is about $\frac{3}{4}$ times the π Compton wavelength, which is a physically reasonable order of magnitude. The dependence of the angular effects on the interaction radius appears to be a sensitive one. Hence, it would seem that such effects may provide valuable information on the annihilation mechanism.

It should be stressed from the outset, however, that results of this study should not be construed to imply that detailed dynamical effects (such as, for example, π-π interactions) are definitely negligible in the discussion of the kind of phenomena considered here. The present stage of both our experimental and our theoretical knowledge of the annihilation process seems to us to be far too early to make such categorical statements. In the concluding remarks (Sec. IV), we briefly discuss the dependence of the BE effect on the available energy for annihilation. This gives one instance of how further experimental study may reveal whether or not the present considerations provide substantially the correct approach to the problem. It may directly be noted, however, that the symmetrization effects which we shall now outline are relevant regardless of whether π-π interactions are large or small.

For the statement of our ideas, it is helpful to recall first what the assumptions of the usual statistical model (SM) are. For definiteness, consider the system

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[1] G. Goldhaber, W. B. Fowler, S. Goldhaber, T. F. Hoang, T. E. Kalogeropoulos, and W. M. Powell, Phys. Rev. Letters 3, 181 (1959).

[2] All center-of-mass transformations were made on the assumption that the struck proton is at rest. From the known annihilation cross sections in carbon and hydrogen and from the π-multiplicity distribution, it was deduced that about 85% of the hydrogenlike events correspond to annihilations on hydrogen.