Possible Method for Determining the Parity of the Cascade Hyperon

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A study of the reaction $\bar{\Xi}^+ + p \to K^+ + K^+$ is suggested as a possible means of determining the parity of the cascade hyperon relative to the nucleon.

I. INTRODUCTION

WITH the advent of intense and very energetic beams of antiprotons, it will likely become possible to produce with a reasonable frequency hyperon-antihyperon pairs via two-body reactions of the following kind: $\bar{p}+p \to Y+\bar{Y}$.[1] Here Y and $\bar{Y}$ denote a hyperon and an antihyperon, respectively. Such reactions may be the most ready source of such particles as the cascade and anticascade hyperons. A particular reaction producing these particles would be

$$\bar{p}+p \to \Xi^- + \bar{\Xi}^+ \qquad (1)$$

with a threshold at $\sim$1.8 Bev antiproton laboratory kinetic energy. In the present note, and in a subsequent note,[2] we examine two particular reactions that may be initiated by the $\bar{\Xi}^+$ interacting with hydrogen. We study here the annihilation reaction,

$$\bar{\Xi}^+ + p \to K^+ + K^+. \qquad (2)$$

We note that we may be able to infer the relative cascade-nucleon parity from the features of this reaction. A study of the interactions of Ξ^- in hydrogen to obtain information on this relative parity has also been carried out.[3]

II. ANALYSIS

In Table I we list the possible transitions involved when reaction (2) is initiated by low-energy $\bar{\Xi}^+$ incident upon hydrogen, for the two cases of even and odd relative cascade-nucleon parity. The center-of-mass angular distribution characteristic of each transition is also given. It is assumed that the cascade hyperon has spin $\frac{1}{2}$. It is further assumed that the two K^+ mesons are identical, spin-zero bosons. This latter assumption allows us to conclude that the Bose statistics rules out the presence of final states with odd orbital angular momentum. Indeed, a test of the identity of the two K^+ mesons is provided by an experimental search for terms proportional to an odd power of $\cos\theta$ in the center-of-mass angular distribution of the final state.

We now note that, if the $\bar{\Xi}^+$ are of very low kinetic energy, say $\lesssim 5$ Mev, we might expect the initial S-wave interaction to predominate. Then the combined conservation of angular momentum and parity forbids reaction (2) if the cascade-nucleon relative parity is even. For odd relative parity the reaction will proceed with an isotropic angular distribution in the center-of-mass system. It is difficult to estimate the frequency of the annihilation mode (2) relative to the modes with one or more pions emitted in addition to the two K^+ mesons. We only note that the mode (2) takes up into the two K^+ masses a fraction of the available mass which is quite comparable to that fraction taken up by the 5 or 6 pions emitted in the dominant annihilation modes of low-energy antinucleons incident upon nucleons.[4] This distinction between the reaction proceeding at very low energies in the case of odd relative parity and being forbidden in the case of even relative parity provides, in principle, the cleanest determination of the parity of the cascade hyperon.

In practice, even with low-energy $\bar{\Xi}^+$, we should allow for the possibility of higher orbital angular momenta in the incident state. We note from Table I that if there are S and P waves in the incident state, reaction (2) will proceed for either relative parity. However, in the case of odd relative parity, the center-of-mass angular distribution, $F(x)$, will be isotropic; in the case of even relative parity the angular distribution will, in general, have the form $F(x)=a+bx^2$. Here, the coefficients a and b are related to the amplitudes A and B, describing transitions from the $^3P_0$ and $^3P_2$ states, respectively, by the equations

$$\begin{aligned} a&=18|A|^2+|B|^2+6\sqrt{2}\,\mathrm{Re}A^*B,\\ b&=3|B|^2-18\sqrt{2}\,\mathrm{Re}A^*B. \end{aligned} \qquad (3)$$

TABLE I. Possible transitions and angular distributions for the reaction $\bar{\Xi}^+ + p \to K^+ + K^+$.

Relative $\Xi-p$ parity	Initial state	Final orbital angular momentum	$F(x)$ = center-of-mass angular distribution[a]
$-$	$^1S_0$	0	1
$-$	$^1D_2$	2	$1-6x^2+9x^4$
$-$	$^3D_2$	2	x^2-x^4
$+$	$^3P_0$	0	1
$+$	$^3P_2$	2	$1+3x^2$

[a] $x=\cos\theta$, where θ is the center-of-mass angle of the relative momentum of the two K^+ mesons measured relative to the incident $\bar{\Xi}^+$ momentum.

[1] S. Barshay, Phys. Rev. **113**, 349 (1959). In this paper there is a misprint. The first two terms on the right-hand side of the third equation in (4a) and the third equation in (4b) should be multiplied by R and R', respectively.

[2] S. Barshay, following paper [Phys. Rev. **120**, 267 (1960)].

[3] S. Treiman, Phys. Rev. **113**, 355 (1959).

[4] W. H. Barkas *et al.*, Phys. Rev. **105**, 1037 (1957).

Thus, except in the case $b=0$, it is possible to make a clean distinction between even and odd relative parity by a study of the angular distribution in reaction (2). Observation of an anisotropic distribution would indicate even relative parity (under the assumption of only S and P incident waves). Observation of an isotropic distribution would suggest odd relative parity, or if even relative parity, then $b=0$. In order to rule out the latter possibility, we would have to observe the variation in the frequency of reaction (2) as the $\bar{\Xi}^+$ kinetic energy is lowered to zero. For odd relative parity there should be essentially no variation over a small energy interval. For even relative parity, the frequency of (2) should go to zero as the selection rule forbidding the reaction from incident S waves comes into play.

Finally, we consider the possibility that reaction (2) is observed to go with an anisotropic angular distribution at $\bar{\Xi}^+$ kinetic energies such that it is possible that incident D waves are contributing. In the case of odd relative parity reaction (2) would proceed from a mixture of incident states, ${}^1S_0$, ${}^1D_2$, and ${}^3D_2$, with amplitudes C, D, and E, respectively. The center-of-mass angular distribution will, in general, have the form, $F(x)=c+dx^2+ex^4$, where the coefficients, c, d, and e are given in terms of the above amplitudes by

$$\begin{aligned} c&=36|C|^2+|D|^2-12\,\mathrm{Re}C^*D,\\ d&=-6|D|^2+6|E|^2+36\,\mathrm{Re}C^*D,\\ e&=9|D|^2-6|E|^2. \end{aligned} \tag{4}$$

A nonzero value for the coefficient of x^4 would rule out the case of even parity, for which $F(x)=a+bx^2$. For example, a negative coefficient for x^4 would allow us to infer that the relative cascade-nucleon parity is odd, and further, that there is a ${}^3D_2$ interaction in the $\bar{\Xi}^+-p$ system which is greater than the ${}^1D_2$ interaction. In the event of a near-zero value for the coefficient of x^4, we would infer that the relative parity is even, provided that we could obtain some confirmation of the assumption that incident D-wave interactions are small. This confirmation might come from a study of the elastic scattering reaction

$$\bar{\Xi}^++p\rightarrow\bar{\Xi}^++p \tag{5}$$

at the same energy. For predominant S- and P-wave interactions, the center-of-mass angular distribution will be of the form $G(x)=\alpha+\beta x+\gamma x^2$.

In the special case of a single dominant interacting state, further information on the nature of this state, to supplement that obtained from reaction (2), might be obtained by comparing the observed angular distribution in the elastic scattering with the distributions predicted for certain definite transitions. These are listed in Table II for the incident states involved in Table I.

TABLE II. Some possible transitions and angular distributions for the reaction $\bar{\Xi}^++p\rightarrow\bar{\Xi}^++p$.

Initial state	Final state	$G(x)$ = center-of-mass angular distribution
${}^1S_0$	${}^1S_0$	1
${}^3P_0$	${}^3P_0$	1
${}^3P_2$	${}^3P_2$	$21x^2+13$
${}^1D_2$	${}^1D_2$	$9x^4-6x^2+1$
${}^3D_2$	${}^3D_2$	$-27x^4+30x^2+1$

III. CONCLUSIONS

The above analysis indicates that if a sufficient number of low-energy $\bar{\Xi}^+$ can be observed to annihilate in hydrogen, a study of the frequency of occurrence of reaction (2) and of the angular distribution of the final K^+ pair can provide a relatively clean determination of the relative cascade-nucleon parity. A test of the identity of the K^+ mesons is provided by the predicted absence of odd powers of $\cos\theta$ in the center-of-mass angular distribution.