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Photoneutron Disintegration Below the Giant Resonance: Beryllium-9 and Carbon-13

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The photoneutron cross sections of Be^9 and C^{13} have been calculated. A single particle model is used to describe the incident bound and final continuum distorted states. Using reasonable potential well parameters, the calculated $Be(\gamma,n)$ cross section near threshold is in agreement with experiment. Cross sections for transitions to both the $s_{\frac{1}{2}}$ and $d_{\frac{3}{2}}$ final states in C^{12} are calculated for γ-ray energies below 10 Mev. The calculated cross section is three times as large as the measured value at 6.4 Mev. The thermal neutron capture cross section in C^{12} was calculated and the error persisted in magnitude and sign. Origins of the inaccuracies in the model are discussed. Angular distribution and polarization formulas are presented.

I. INTRODUCTION

THE understanding of nuclear reactions has been complicated by the appearance of many nuclear models. The particular model used often changes with the nuclear reaction studied, the nature of the reactants, and even the angular range of the final state particles. The situation as regards photonuclear reactions is especially confused because of additional complications introduced by the experiments themselves. For example the γ-ray spectrum in s-state neutron capture of a complex nucleus reflects only indirectly and weakly the primary capture process. The observed spectrum is, in fact, often an inseparable mixture of direct and cascade transitions. On the other hand, photonuclear experiments, if carefully planned, can be of extreme value in the study of the nucleus. The primary advantages of γ-ray experiments are as follows:

1. The γ-ray probe interacts only weakly with the nuclear system and therefore the disturbance of the nucleus under investigation is at a minimum. This is similar to the weak electromagnetic interaction of charged particles in Coulomb excitation or disintegration.
2. The theoretical analysis is often simplified because only a few partial waves are needed in the theoretical calculation.

From a purely theoretical viewpoint, the experiments that are most likely to furnish valuable information about nuclear structure are both elastic and inelastic scattering of γ rays. The energy of the incident and final state γ rays should be measured as accurately as possible. Elastic scattering of γ rays has been studied by Fuller and Hayward.[1] Most of these measurements are in the giant resonance region. However, there are some data at lower energies.

The (γ,n) nuclear reaction cross-section analysis differs from the (γ,γ) in that strong neutron final state distortions can modify the cross section appreciably. The extraction of ground-state nuclear knowledge becomes more difficult. On the other hand, the sensitivity of the calculation to final state neutron wave function distortion permits one to draw conclusions about the distorting potential well. Also, information about the convergence of multiple scattering expansions of nuclear reaction calculations can be secured.

The photoneutron reactions studied here are limited to energies within a few Mev of threshold where only a few neutron partial waves are needed in the calculation. The target nuclei considered are Be^9 and C^{13}. These nuclei have $J=\frac{3}{2}(-)$ and $\frac{1}{2}(-)$ states, respectively. Therefore, if the final state nucleus is in its ground state and if $E1$ transitions are assumed, the neutron in the final continuum state is either an s or a d wave.[2] Be^9 is an especially interesting target nucleus because final neutron state distortion effects are large. The reasons for the anticipated importance of final state distortion in Be are the following:

1. The neutron-nucleus scattering length changes sign near Be^8.
2. An analysis of the Be^9-p inelastic scattering cross section suggests that the scattering length in Be^8-n system is large.[3]
3. The energy required to remove the last neutron in Be^9 is only 1.667 Mev. A weakly bound neutron enhances distortion effects since it emphasizes the importance of the asymptotic continuum distorted state wave function.

* Operated for the U. S. Atomic Energy Commission by the General Electric Company.
† Operated for the U. S. Atomic Energy Commission by Union Carbide Corporation.

[1] E. G. Fuller and E. Hayward, Phys. Rev. **101**, 692 (1956).

[2] s or d final neutron states are allowed in $E1$ transitions when the target and final nuclear states have different parities and when their angular momenta do not differ by more than $\frac{3}{2}$. Be^9 and C^{13} are considered here because experimental results are available for these nuclei.

[3] D. W. Miller, Phys. Rev. **109**, 1669 (1958).

The C^{13} target nucleus is of interest for still other reasons. Unlike the Be^9 reaction, the $C^{12}+n$ final state elastic scattering phase shifts for energies below 2 Mev are relatively well known. In addition, the most weakly bound state in C^{13} may be considered to be a $p_{\frac{1}{2}}$ state. This particular state has the same spin and parity as the entire nucleus. This suggests use of a single particle model to describe the (γ,n) reaction. For final neutron energies above 2 Mev in the $C^{12}-n$ system compound nucleus resonances are present. Their contribution to the (γ,n) cross section can also be investigated.

In this report the (γ,n) reaction cross section is calculated in the direct interaction model. The target nucleus is assumed to be in a single particle state whose spin, parity, and binding energy are the same as the actual nucleus. The radial dependence is found by assuming a Saxon well with spin orbit coupling. The final neutron continuum wave function is distorted by a similar real potential well. The well parameters are constrained to give the elastic scattering of the final system when the elastic scattering is known.

Similar direct interaction calculations of nuclear photodisintegration cross sections have been reported in the literature. The first successful direct interaction model calculation of the (γ,n) cross section in Be^9 was reported by one of the authors.[4] The s-wave contribution of the predicted cross sections differs only slightly from the predictions reported here. However, the potential well required to distort the final s-state neutron was unrealistic. The potential well was only 3 Mev deep and it is therefore in conflict with reasonable estimates of shell and optical model parameters. The potential well abruptly dropped to zero at a large nuclear radius of 5f.

Since it has been found that the calculated photonuclear cross section depends sensitively on the surface behavior of the continuum potential well, the square well approximation is not a good one. The Be^9 photodisintegration calculation was also repeated by Sexl[5] and Bergmann[6] with similar approximations and results. Fujii[7] has extended the application of the direct interaction model to the calculation of the sub-giant resonance photo effect in C^{13}, N^{14}, and F^{19}. Reasonable volume spin orbit and central potentials were assumed. However, the potentials were assumed to be real square wells in the energy region where they should be complex. Improved distorting potential wells could have been found by requiring that the final state contain the phase shifts that describe elastic scattering.

A model similar to the Austern[8] model for (n,p) reactions was introduced by De Sabbata.[9] In this direct interaction model the final state distortion is completely neglected and the integrations are performed from only the nuclear surface to infinity. Simplicity is the primary virtue of this model, which cannot hope to describe the photonuclear effect at low energies where distortions are, in general, important.

In an early calculation of the direct dipole emission of protons by atomic nuclei, Courant[10] generated bound and distorted continuum final states with the same square well. Assumption of this equal potential well simplified the calculation and qualitatively explained the presence of the large number of photo-protons but the calculation should be repeated using a velocity dependent complex well with a diffuse nuclear surface.

Several other successful direct interaction calculations of the nucleon capture cross sections using square well potentials have been reported by several authors.[11–15]

The primary difference between the direct interaction (γ,n) calculations reported here and in previous calculations is in the form of the distorting potential. A surface instead of a volume spin orbit potential is used. Instead of a square well potential, a diffuse edge Saxon potential well is assumed. In fact, the introduction of a diffuse edge to the nucleus enables us to calculate the s-wave distribution of the (γ,n) cross section in agreement with the experimental results. Instead of a very small value of 3 Mev used by Guth,[4] a reasonable value of 32 Mev was used to distort the final state neutron wave function. The nuclear radius is 3f, consistent with electron scattering experiments, and the diffuseness parameter is 1.2f. The large diffuseness parameter reflects the fact that Be^8 is a weakly bound structure.

The calculation of the $C^{13}(\gamma,n)$ cross section depends sensitively on the diffuseness of the nuclear surface. Unlike Be, however, the potential well parameters are determined by the constraint that the final state distorting potential gives the component of the elastic scattering phase shifts that vary slowly with energy. The central potential was chosen to give the experimentally[16] determined energy dependence of the s-wave phase shifts. The central potential that predicts the s-wave phase shifts is not unique since potential well parameters sets with different diffusenesses are possible. This ambiguity is partially removed by requiring that the $d_{\frac{3}{2}}$ phase shifts also agree with experiment. The magnitude of the spin orbit coupling is set by requiring that the $d_{\frac{3}{2}}$ wave is resonant at 3.37 Mev in the center-of-mass system. The width of the resonance depends

[4] E. Guth, Phys. Rev. **55**, 511 (1939). E. Guth and C. J. Mullin, Phys. Rev. **74**, 832 (1948). E. Guth and C. J. Mullin, Phys. Rev. **76**, 234 (1949).

[5] Th. Sexl, Acta Phys. Austriaca **3**, 277 (1949).

[6] O. Bergmann, Acta Phys. Austriaca **4**, 338 (1951).

[7] S. Fujii, Progr. Theoret. Phys. (Kyoto) **21**, 511 (1959).

[8] N. Austern, S. T. Butler, and H. McManus, Phys. Rev. **92**, 350 (1953).

[9] V. De Sabbata and A. Tomasini, Nuovo cimento **13**, 1268 (1959).

[10] E. Courant, Phys. Rev. **82**, 703 (1951).

[11] D. H. Wilkinson, Phil. Mag. **45**, 259 (1954).

[12] A. M. Lane, Nuclear Phys. **11**, 625 (1959). A. M. Lane and J. E. Lynn, Nuclear Phys. **11**, 646 (1959).

[13] A. V. Shutko and D. F. Zautskii, J. Exptl. Theoret. Phys. (U.S.S.R.) **29**, 866 (1955) [translation: Soviet Phys.-JETP **2**, 769 (1956)].

[14] L. K. Peker, J. Exptl. Theoret. Phys. (U.S.S.R.) **29**, 865 (1955) [translation: Soviet Phys.-JETP **2**, 753 (1956)].

[15] T. J. Krieger, N. C. Francis and R. M. Rockmore, Bull. Am. Phys. Soc. **4**, 272 (1959).

[16] J. E. Wills, J. K. Bair, H. O. Cohn and H. B. Willard, Phys. Rev. **109**, 891 (1958).

primarily on the diffuseness of the nuclear surface. A diffuseness parameter of 0.4f yields the best $d_{\frac{3}{2}}$ wave phase shifts. With the best elastic scattering parameters the (γ,n) cross sections at 6.4 Mev γ-ray energy is three times the experimental values of Edge.[17]

Another encouraging result is the predicted cross section of the inverse reaction in carbon. Using the best elastic scattering potential well parameters, the thermal neutron (n,γ) capture cross section for the ground-state transition was calculated. The predicted cross section is also three times the experimental results.

In Sec. II the angular distribution, polarization, and total photoneutron cross sections are obtained. The Be^9 and C^{13} results are discussed in Sec. III. Section IV includes a general discussion and conclusions. In Appendix A, the formal basis of the single particle model is discussed briefly.

II. GENERAL THEORY

We consider the interaction between the incoming photon and the single neutron lying outside of the core of A nucleons. Above threshold, the neutron is able to overcome the binding energy and emerge as an outgoing wave distorted by the potential of the residual nucleus. We have considered dipole interactions, i.e., where $|J_i-J_f|\leq 1$ and there is a change in parity. Thus the electric dipole operator may be written as

$$\mathbf{r}\cdot\boldsymbol{\varepsilon}=\frac{4\pi}{3}r\sum_{\lambda} Y_1^{\lambda}(\mathbf{r})Y_1^{\lambda *}(\boldsymbol{\varepsilon}). \tag{2.1}$$

The shell model potential used throughout was

$$V(r)=V_R(r)+V_{SO}(r),$$

where

$$V_R(r)=\frac{V_0}{1+e^{(r-R)/a}},\ \text{a Saxon potential,}^{18}$$
$$V_{SO}(r)=-\frac{V_{SO}}{V_0}\left(\frac{\hbar}{\mu_0 c}\right)^2\frac{1}{r}\frac{dV_R(r)}{dr}\boldsymbol{\sigma}\cdot\mathbf{L}, \tag{2.2}$$

and $\hbar/\mu_0 c$ is the π-meson Compton wavelength. Appropriate parameters were determined on the basis of experimental evidence.

The differential cross section for this interaction is given by[19]

$$\sigma_d(\gamma,n)=\frac{kp\mu e'^2}{2\pi\hbar^2 c}|\langle f|\mathbf{r}\cdot\boldsymbol{\varepsilon}|i\rangle|^2, \tag{2.3}$$

where k is the energy of the incoming photon, p is the momentum of the outgoing neutron in the center-of-mass system, μ is the reduced mass of the system, and e' is the effective change $[e'=-(eZ/A)]$.

The initial (bound) wave function is taken to be

$$\Psi_i=\sum_{m_i\sigma_i} C(l_i\tfrac{1}{2}j_i;\, m_i\sigma_i\mu_i)Y_{l_i}^{m_i}(\mathbf{r})\frac{u_i(r)}{r}\chi_{\frac{1}{2}}^{\sigma_i}, \tag{2.4}$$

where $u_i(r)/r$ is the solution to the single particle Schrödinger equation. The adjoint final wave function, a distorted plane wave, is

$$\Psi_f^{\dagger}=4\pi\sum_{l_fm_fm_f'j_f\mu_f\sigma_f'}(-i)^{l_f}C(l_f\tfrac{1}{2}j_f;\, m_f'\sigma_f'\mu_f)$$
$$\times C(l_f\tfrac{1}{2}j_f;\, m_f\sigma_f\mu_f)[Y_{l_f}^{m_f'}(\mathbf{r})]^{\dagger}Y_{l_f}^{m_f}(\mathbf{p})$$
$$\times\left[\frac{\eta_{l_f}{}^{j_f}u_{l_fj_f}{}^{+}-u_{l_fj_f}{}^{-}}{2ipr}\right](\chi_{\frac{1}{2}}^{\sigma_f'})^{\dagger}, \tag{2.5}$$

where $u_{lj}^{\pm}$ are the outgoing and incoming parts of the radial wave function and η_l^j represents the distortion produced by the residual nucleus and is related to the phase shift δ_l^j by $\eta_l^j=\exp(2i\delta_l^j)$. The radial wave functions, u_{lj}, are independent of j when the distorting potential vanishes. Inside of the well, however, the potential and therefore, u, are j dependent. In the following equations the j dependence of u will be suppressed. The expressions for the differential and total cross sections are

$$\sigma_d=\frac{ke'^2\mu}{36p\hbar^2c}\sum_{l_fl_f'j_fj_f'\epsilon L}\frac{i^{l_f-l_f'}(-)^{j_i-\frac{1}{2}}\,\mathrm{R.I.}_{l_f}{}^{j_f}\,\mathrm{R.I.}_{l_f'}{}^{j_f'*}}{(2j_i+1)[(2j_f+1)(2j_f'+1)]^{\frac{1}{2}}}$$
$$\times P_L(\mathbf{p},\boldsymbol{\varepsilon})Z(l_fj_fl_ij_i;\tfrac{1}{2}1)Z(l_f'j_f'l_ij_i;\tfrac{1}{2}1)$$
$$+Z(l_fj_fl_f'j_f';\tfrac{1}{2}L)Z(1j_f1j_f';j_iL); \tag{2.6}$$

$$\sigma_T=\frac{2\pi ke'^2\mu}{9p\hbar^2c}\sum_{l_fj_f}\frac{|\mathrm{R.I.}_{l_f}{}^{j_f}|^2}{2j_i+1}Z^2(l_fj_fl_ij_i;\tfrac{1}{2}1); \tag{2.7}$$

where

$$\mathrm{R.I.}_l{}^j=\int_0^\infty ru_i(r)(\eta_l{}^ju_l{}^+(r)-u_l{}^-(r))dr$$
$$=2i\exp(i\delta_l{}^j)\ \mathrm{Im}\exp(i\delta_l{}^j)\int_0^\infty ru_iu_l{}^+dr, \tag{2.8}$$

$P_L(\mathbf{p},\boldsymbol{\varepsilon})$ is the Lth order Legendre polynominal of the angle between the neutron direction, $\mathbf{p}$, and the γ-ray polarization, $\boldsymbol{\varepsilon}$, and Z $(abcd;ef)$ is the coefficient introduced by Blatt and Biederharn.[20] The polarization of the outgoing neutron is proportional to the trace of the expectation value of the Pauli spin matrix.[21] The

[17] R. Edge (private communication).
[18] R. Woods and D. S. Saxon, Phys. Rev. **95**, 577 (1954).
[19] R. G. Sachs, *Nuclear Theory* (Addison-Wesley Publishing Company, Inc., Reading, Massachusetts, 1953), p. 238.
[20] J. M. Blatt and L. C. Biederharn, Phys. Rev. **82**, 123 (1951). L. C. Biederharn, Oak Ridge National Laboratory Report ORNL-1098, 1952 (unpublished).
[21] W. Czyz and J. Sawicki, Nuovo cimento **3**, 864 (1956).

formula for the polarization is

$$P=(\sigma_d)^{-1}\frac{k\mu e'^2}{\hbar^2 cp}\frac{\sqrt{5}}{48}$$
$$\times\sum_{l_f l_f' j_f j_f'} i(-)^{j_f-j_i+l_f}\,\mathrm{R.I.}_{l_f}{}^{j_f}\,\mathrm{R.I.}_{l_f'}{}^{j_f'*}$$
$$\times[(2l_f+1)(2l_f'+1)]^{\frac{1}{2}}C(l_f l_f'2;000)\sin 2\theta$$
$$\times X(l_f'l_f2;j_f'j_f2;\tfrac{1}{2}\tfrac{1}{2}1)Z(1j_fl_i\tfrac{1}{2};j_il_f)$$
$$\times Z(1j_f'l_i\tfrac{1}{2};j_il_f')Z(1j_f'1j_f;j_i2), \quad (2.9)$$

where θ is the angle between photon and neutron directions, the z axis is taken to be in the $k\times p$ direction, and X is the $9-j$ coefficient.[22]

The calculations of the cross sections can be greatly simplified if the radial integral is evaluated approximately. When the integration is limited to the region where the distorting potential vanishes, the integration is elementary. The results for the transitions of primary interest will be presented here. The target is assumed to be in a p state and the continuum nucleon is in either an s or a d state. Then the approximate radial integral becomes

$$\mathrm{R.I.}_0{}^{\frac{1}{2}}=\frac{2Ni\exp(i\delta_0{}^{\frac{1}{2}})}{\alpha(\alpha^2+q^2)^2}\{q\cos(qR+\delta_0{}^{\frac{1}{2}})$$
$$\times[\alpha^3R+3\alpha^2+q^2\alpha R+q^2]$$
$$+\alpha\sin(qR+\delta_0{}^{\frac{1}{2}})[2\alpha^2+\alpha^3R+q^2\alpha R]\}, \quad (2.10)$$

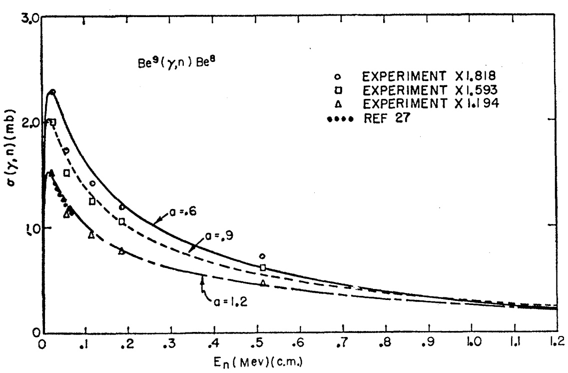


FIG. 1. The s-wave photoneutron cross section for Be^9 plotted as a function of the energy in the center-of-mass system, ($E_n=k-$B.E.), for values of the diffuseness parameter of 0.6, 0.9, and 1.2f. The experimental points at $E_n=0.023$ Mev and 0.183 Mev are those of Gibbons *et al.*,[25] the value at $E_n=0.51$ that of Hamermesh and Kimball[27] and the other enclosed values those of John *et al.*[26] The experimental points of Miller *et al.*[28] are obtained by using a bremsstrahlung spectrum and are normalized to the $a=1.2$f curve.

[22] See M. E. Rose, *Elementary Theory of Angular Momentum* (John Wiley & Sons, Inc., New York, 1957), p. 193 for the definition of the X-coefficient and additional references.

and

$$\mathrm{R.I.}_2{}^{j}=2Ni\exp(i\delta_2{}^{j})\Big\{\sin(qR+\delta_2{}^{j})$$
$$\times\left[\frac{3-\alpha R}{\alpha^2+q^2}-\frac{2\alpha^2}{(\alpha^2+q^2)^2}+\frac{3}{2q^2R}\right]+\frac{\cos(qR+\delta_2{}^{j})}{\alpha^2+q^2}$$
$$\times\left[\frac{2\alpha q}{\alpha^2+q^2}-\left(qR+\frac{3\alpha}{q}+\frac{q}{\alpha}\right)\right]\Big\}, \quad (2.11)$$

where

$$\alpha^2=2\mu E/\hbar^2,$$
$$q^2=2\mu E/\hbar^2.$$

ϵ and E are the binding and final state neutron energy. The factor N is introduced to normalize the initial bound-state wave function:

$$N\simeq\sqrt{\alpha}.$$

The appropriate radial integrals have been evaluated assuming that R is the Saxon nuclear radius for both s and d final states. The numerical results are in good agreement with the more exact evaluations. A reason for the success of the approximate formula is that the

TABLE I. Beryllium photoneutron cross section for $E=0.023$ Mev.

a (f)	V_0 (Mev)	Theoretical (mb)	Experimental (mb)
0.6	−46.40	2.036	
0.9	−38.48	1.784	1.26
1.2	−32.05	1.338	

direct interaction contribution to the (γ,n) cross section comes primarily from regions outside of the nucleus. The nuclear radius can be taken to be a parameter determined by experiment.

III. RESULTS

A. Beryllium

The ground state of Be^9 has spin and parity $\frac{3}{2}-$. The first excited state at 1.75 Mev has $\frac{1}{2}+$.[23] The minimum γ-ray energy to overcome the binding energy of the last neutron in Be^9 is 1.667 Mev; thus the excitation of the neutron from the ground state to the first excited state would proceed through a dipole interaction.

The ground-state wave function was determined by selecting reasonable values for V_{SO}, R, and a, and then trying to find an eigenvalue V_0 so that the energy would equal the binding energy, -1.667 Mev.

These values were $E=-1.667$ Mev, $V_{SO}=-9.0$ Mev, $R=3.0$f, the Saxon radius,[24] $a=0.6$f, the diffuseness parameter, and $V_0=-26.8$ Mev. The choice of the

[23] F. Ajzenberg-Selove and T. Lauritsen, Nuclear Phys. 11, 1 (1959).

[24] R. Hofstadter, Revs. Modern Phys. 28, 214 (1956).

nuclear radius is consistent with electron scattering experiments. The well depth, V_0, for the final state is chosen so that the ratio of total cross sections at energies 0.023 Mev and 0.183 Mev agrees with the experimental ratio.[25]

The total cross section for photoneutron production from Be^9 is computed from Eq. (2.7). It is

$$\sigma_T = 1.043\times10^{-3}[(1.667+E)/\sqrt{E}]|\text{R.I.}_0^{\frac{1}{2}}|^2\ \text{mb}, \quad (3.1)$$

where the radial integral is given in fermis$^{\frac{3}{2}}$ and E, the energy of the neutron, in Mev. Since we have considered only s-wave neutrons, the angular distribution is isotropic and the polarization is zero. Inspection of Eq. (2.9) and the property of the X-coefficient show that polarization occurs only by interference between two outgoing waves of different states.

An energy spectrum for three different values of a, $a=0.6$, 0.9, and 1.2f, was computed using Eq. (3.1) with the radial integral obtained numerically, and the results plotted in Fig. 1. The experimental points[25–28] are shown on the same figure. It is seen that each of the values of a reproduced the shape of the experimental energy spectrum but the cross section for the most diffuse well approaches the exact values[28] most nearly and the magnitude of the cross section varies inversely as the diffuseness parameter. The values of well depth and cross section at $E=0.023$ Mev for each choice of diffuseness parameter are given in Table I.

TABLE II. Carbon photoneutron cross section for $E=1.5$ Mev.

a (f)	V_0 (Mev)	V_{SO} (Mev)	Theoretical (mb)	Experimental (mb)
0.4	−62.5	−7.062	301	
0.6	−59.3	−7.11	418	94
0.8	−56.6	−9.70	582	

The results are relatively insensitive to the diffuseness parameter. This is because the bulk of the contribution to the s wave (γ,n) cross section comes from neutrons outside of the well.

The potential wells that produce the best (γ,n) cross sections have been used to predict the $Be^8(n,n)Be^8$ s-wave phase shifts. These phase shifts are shown in Fig. 2. With these phase shifts, or preferably the potential well parameters reported here, the neutron-Be^9 inelastic scattering cross section near threshold can be calculated. The reduced width for inelastic scattering should be anomalously large at about 1.75 Mev since both final state neutrons can be distorted strongly.

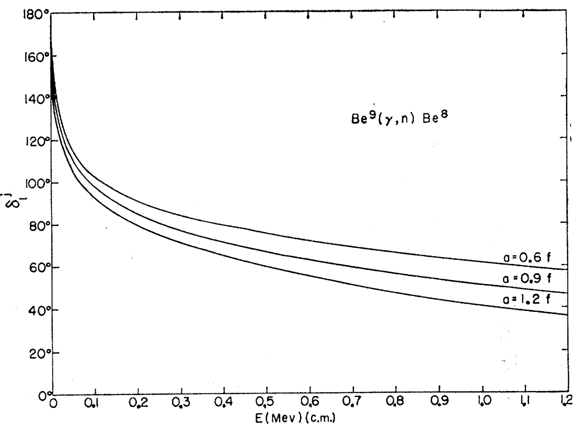


FIG. 2. Calculated s-wave phase shifts for elastic neutron scattering from Be^8 plotted as a function of energy for values of the diffuseness parameter of 0.6, 0.9, and 1.2f.

B. Carbon

The energy levels of C^{13} are given in reference 23. The ground state of C^{13} is $\frac{1}{2}-$. This can be associated with a $p_{\frac{1}{2}}$ neutron in the outer shell with a binding energy of 4.496 Mev. There are few continuum levels whose spins and parities have been determined.

Once again the ground-state wave function was computed by selecting reasonable values for V_{SO}, R, and a and finding V_0 such that the binding energy would be an eigenvalue of the appropriate single particle Schrödinger. The values chosen were $E=-4.946$ Mev, $V_{SO}=-9.45$ Mev, $R=2.86$f, $a=0.6$f, and $V_0=-47.08$ Mev. R was found from the formula $R=1.25A^{\frac{1}{3}}$f, and is consistent with electron scattering results. The same value of R was used for the continuum wave function.

The existence of the C^{12} nucleus enables us to partially determine the appropriate continuum parameters in terms of the scattering of neutrons from C^{12}. For incoming neutrons of $E=0.01$ Mev, the cross section for elastic scattering is given in terms of the s-wave phase shift only, $\sigma=4\pi\sin^2\delta_0^{\frac{1}{2}}/q^2$.

The s-wave phase shift is a function of the well-depth and this was adjusted so that the cross sections agree with the thermal experimental value $=4.7b$.[29] The center of the broad d-wave resonance at a neutron energy of 3.37 Mev, corresponding to the 8.33 Mev $\frac{3}{2}+$ level of C^{13}, was used to determine the strength of the spin-orbit part of the potential. The potential parameters used are shown in Table II.

The Saxon well parameters used here are similar to those used to describe the $n-O^{16}$ elastic scattering.[30,31] In the oxygen calculation $V_0=-53.8$ Mev, $V_{SO}=-6.6$ Mev, $a=0.6$f and the value of r_0 is 1.25f where $R=r_0A^{\frac{1}{3}}$.

[25] J. H. Gibbons, R. L. Macklin, J. B. Marion, and H. W. Schmitt, Phys. Rev. **114**, 1319 (1959).
[26] W. John, F. J. Lombard, E. T. Moore, and J. M. Prosser, Bull. Am. Phys. Soc. **5**, 44 (1960).
[27] R. Hamermesh and C. Kimball, Phys. Rev. **90**, 1063 (1953).
[28] W. C. Miller (private communication); R. L. Walter, M. S. Shea, W. C. Miller, Bull. Am. Phys. Soc. **5**, 229 (1960).
[29] *Neutron Cross Sections*, compiled by D. J. Hughes and R. B. Schwartz, Brookhaven National Laboratory Report BNL-325 (Superintendent of Documents, U. S. Government Printing Office, Washington, D. C., 1958), 2nd ed.
[30] J. L. Fowler and H. O. Cohn, Phys. Rev. **109**, 89 (1958).
[31] E. H. Auerbach and N. Francis, Bull. Am. Phys. Soc. 4, 272 (1959).

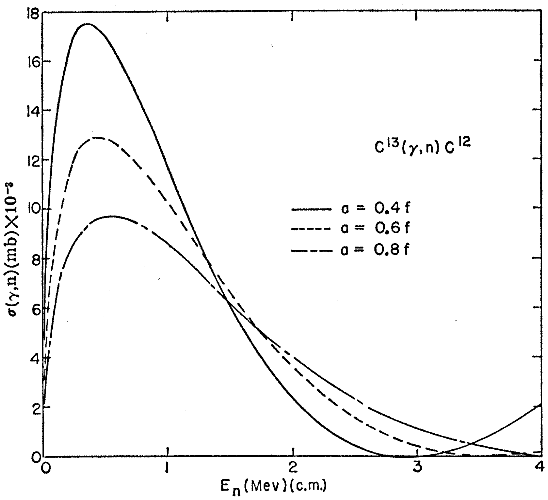


FIG. 3. The calculated s-wave photoneutron cross sections for C^{13} as a function of neutron energy in the center-of-mass system for diffuseness parameters of 0.4, 0.6, and 0.8f.

The total cross section, using Eq. (2.7), is

$$\sigma_T = 1.147\times10^{-3}\frac{4.946+E}{\sqrt{E}} \times[|\mathrm{R.I.}_0^{\frac{1}{2}}|^2+2|\mathrm{R.I.}_2^{\frac{3}{2}}|^2]\mathrm{mb}, \quad (3.2)$$

where E is the energy of the neutron in the center-of-mass system. The differential cross section, using Eq. (2.6), is

$$\sigma_d = 4.562\times10^{-5}\frac{4.946+E}{\sqrt{E}} \times\Bigg\{2|\mathrm{R.I.}_0^{\frac{1}{2}}|^2+|\mathrm{R.I.}_2^{\frac{3}{2}}|^2(2+3\sin^2\theta) -8(3\sin^2\theta-2)\cos(\delta_0^{\frac{1}{2}}-\delta_2^{\frac{3}{2}}) \times\left(\mathrm{Im}\exp(i\delta_0^{\frac{1}{2}})\int_0^\infty ru_iu_0^+dr\right) \times\left(\mathrm{Im}\exp(i\delta_2^{\frac{3}{2}})\int_0^\infty ru_iu_2^+dr\right)\Bigg\}\mathrm{mb}. \quad (3.3)$$

The nuclear matrix elements appearing on the right-hand side of Eq. (3.3) can be determined by using Figs. 3 and 4 since the total cross section can be written as

$$\sigma_T = 1.147\times10^{-3}\frac{4.946+E}{\sqrt{E}} \times4\left[\left(\mathrm{Im}\exp(i\delta_0^{\frac{1}{2}})\int_0^\infty ru_iu_0^+dr\right)^2 +2\left(\mathrm{Im}\exp(i\delta_2^{\frac{3}{2}})\int_0^\infty ru_iu_2^+dr\right)^2\right]\mathrm{mb}. \quad (3.4)$$

The expression for the polarization is computed from Eq. (2.9) and, as has been remarked before, is proportional to the interference between s and d waves:

$$P = -12\sin^2\theta\sin(\delta_0^{\frac{1}{2}}-\delta_2^{\frac{3}{2}}) \times\left(\mathrm{Im}\exp(i\delta_0^{\frac{1}{2}})\int_0^\infty ru_iu_0^+dr\right) \times\left(\mathrm{Im}\exp(i\delta_2^{\frac{3}{2}})\int_0^\infty ru_iu_2^+dr\right) \times\Bigg[2|\mathrm{R.I.}_0^{\frac{1}{2}}|^2+(2+3\sin^2\theta)|\mathrm{R.I.}_2^{\frac{3}{2}}|^2 -8(3\sin^2\theta-2)\cos(\delta_0^{\frac{1}{2}}-\delta_2^{\frac{3}{2}}) \times\left(\mathrm{Im}\exp(i\delta_0^{\frac{1}{2}})\int_0^\infty ru_iu_0^+dr\right) \times\left(\mathrm{Im}\exp(i\delta_2^{\frac{3}{2}})\int_0^\infty ru_iu_2^+dr\right)\Bigg]^{-1}. \quad (3.5)$$

The nuclear matrix elements can again be determined from the total cross section by using Eq. (3.2) or Eq. (3.4).

The total cross section as a function of energy (Eq. 3.2) was obtained. The s- or d-wave contributions are given separately and plotted in Figs. 3 and 4, respectively. Note that the broad d-wave resonance is not centered about a γ-ray energy of 8.33 Mev, the energy at which the $C^{12}(n,n)C^{12}$ cross section is a maximum. The shift of the peak of the (γ,n) cross section with re-

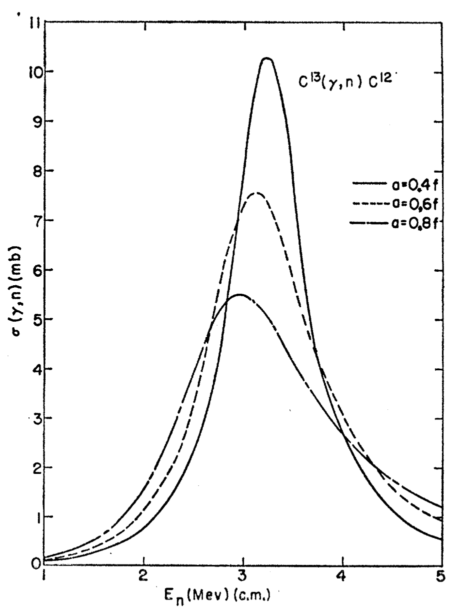


FIG. 4. The calculated d-wave photoneutron cross section for C^{13} as a function of outgoing neutron energy (in the center-of-mass system) for diffuseness parameters of 0.4, 0.6, and 0.8f.

TABLE III. Carbon-12 cross sections for thermal capture.

a (f)	Theoretical (mb)	Experimental (mb)
0.4	7.2	
0.6	4.8	2.50
0.8	3.2	

spect to the elastic peak depends on the width of the resonance. The shift is largest for the broadest resonance. This is a prediction of the direct interaction model that has not as yet been verified experimentally. There is one experimental value of the $\sigma(\gamma,n)$ cross section at a mean γ-ray energy of 6.494.[16] 65% of the γ-ray beam in the experiment was of 6.13-Mev energy, 25% of 6.9-Mev energy and 11% of 7.1-Mev energy. The computed value of the cross section, appropriately weighted for the three different γ-ray energies, is given in Table II. Values are given for each of the diffuseness parameters considered. The reason for this discrepancy is not clear but may be due in part to the fact that the 6.9 Mev γ ray is close to a sharp resonance of $n-C^{12}$ scattering and there may be interference between direct interaction and compound nucleus effects. More accurate γ-ray energies are needed for this problem. It is believed, however, that the factor of three difference between the best theoretical prediction and experiment is primarily due to the importance of corrections to the single particle model when the final state nucleon is in a d-wave state. The large correction terms contain radial integrals which are not quenched because of oscillations in the wave function, and hence are large.

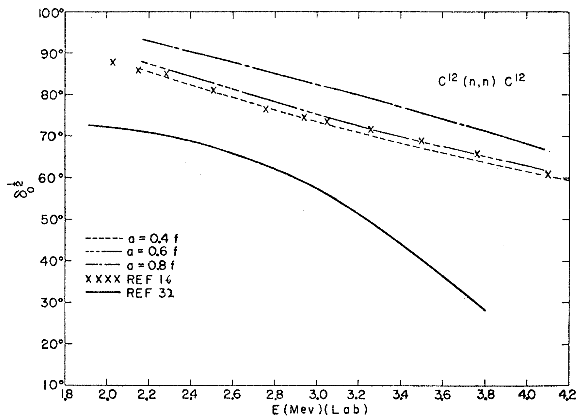
Using the principle of detailed balance, we are also able to compute the cross section for neutron capture. Thus we can write

$$\sigma(n,\gamma)=\sigma(\gamma,n)\frac{k^2}{p^2c^2}\frac{2J_{C^{13}}+1}{2J_{C^{12}}+1}. \tag{3.6}$$

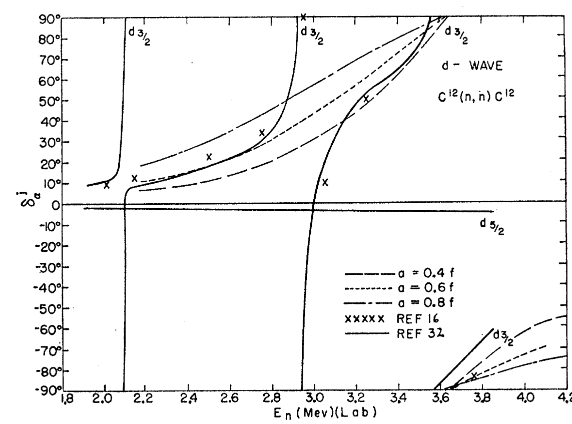
The value of $\sigma(\gamma,n)$ was computed at $E=0.01$ Mev. The cross sections for thermal capture were computed under the assumption that the capture cross section varies inversely as the velocity. These values and the experimental value[29] are given in Table III.

An extrapolation of the results presented in Table III indicates that a diffuseness parameter of 1.0f yields the best thermal neutron capture cross section. This large diffuseness parameter is not quite as large as the 1.2f value of Be^8 but it is considerably larger than the value of $a=0.6$f which is often used in conventional cloudy crystal ball calculations. The diffuseness parameter, $a=1$f, is also inconsistent with the value of $a=0.4$f which is needed to predict the $d_{\frac{3}{2}}$ phase shift energy dependence near the resonance. On the other hand, a is expected to be energy dependent. Because of the energy dependence of the effective interaction of a continuum neutron and the surface nucleons, should increase as the energy increases. Information about the energy dependence of a can be obtained from a careful analysis of $n-C^{12}$ polarization experiments for neutron energies below 4 Mev. In addition, at least the first order corrections to the single particle calculation of the $C^{13}(\gamma,n)C^{12}$ reaction cross section should be performed for the s-wave neutron final state. It is expected that this correction term will be relatively small because of the rapid oscillations of the radial wave functions appearing in the radial integral.

As an additional check on the accuracy of our continuum wave functions, we examined the energy dependence of the continuum phase shifts for various values of total and orbital angular momentum. These were then compared with experimental phase shifts of two different authors.[16,32] These results are plotted in Figs. 5(a) and 5(b). It is seen that there is disagreement



(a)



(b)

FIG. 5. (a) The phase shifts for s-wave elastic scattering for neutrons on C^{12} plotted as a function of neutron energy. The calculated values for three different diffuseness parameters of 0.4, 0.6, and 0.8f are shown as dashed lines. The values determined from experiment are those of Meier *et al.*[32] (the solid line) and Wells *et al.*[16] (b) The phase shifts for d-wave elastic scattering for neutrons on C^{12}. Both experimental values and all calculated values for $d_{\frac{5}{2}}$ scattered waves are given by the line labeled $d_{\frac{5}{2}}$. The $d_{\frac{3}{2}}$ curves bear the same legend as given in the caption for Fig. 5(a).

[32] R. W. Meier, P. Scherrer, and G. Trumpy, Helv. Phys. Acta 27, 577 (1954).

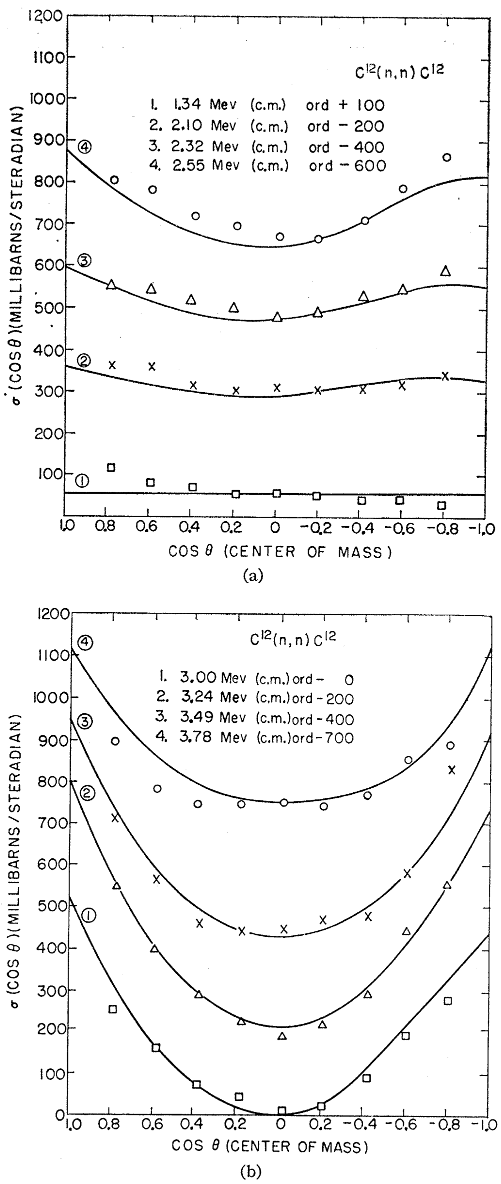


FIG. 6. (a) The angular distribution of neutrons elastically scattered from C^{12}. The solid lines are calculated with a diffuseness parameter of 0.6f. The single points are the experimental values of Wells *et al.*[16] There is a slightly different angular distribution for $a=0.4$f (see text). (b) A continuation of Fig. 6(a) for higher values of neutron energy.

between experiments for *s*- and *d*-wave phase shifts. Our computed phase shifts agree very well with those of Wells *et al.*[16] and reproduce the $d_{\frac{3}{2}}$ single particle resonance level. The disagreement in experimental values seems to be due to an inherent difficulty in producing an unambiguous phase shift analysis of scattering data.

The angular distribution of the elastic scattering of neutrons by C^{12} was computed and compared with the results of Wells *et al.*[16] The results are depicted in Figs. 6(a) and 6(b) where it is seen that reasonable agreement between calculated and experimental results exists. The calculated elastic scattering angular distribution was obtained using $a=0.6$f. A slightly improved angular distribution in the neighborhood of 3.37 Mev was found when $a=0.4$f. However, the angular distributions calculated at neutron energies between 2 and 3 Mev were slightly in closer agreement with experiment when $a=0.6$f. Although the differences mentioned here are slight, an energy dependence of a is suggested. The energy dependence of a is consistent with the conjectured energy dependence needed to reduce the difference between the calculated and measured thermal neutron capture cross section in C^{12} (see Table III). An alternative procedure is to make the potential state dependent.[30,33]

IV. CONCLUSIONS

The general agreement between the experimental values of the calculated thermal (n,γ) cross section in C^{12} and the (γ,n) cross sections in Be^9 is encouraging but it is not considered especially significant since correlations in the final bound and initial continuum states have been neglected. Similarly the fact that the best value of the predicted (γ,n) cross section in C^{13} at 6.4 Mev is a factor of 3 times the experimental result of Edge is not considered to be important. (By assuming that the final d wave is undistorted the (γ,n) prediction is reduced by a factor of 10.) What is considered to be important, however, is the fact that this single particle direct interaction calculation permits a better understanding of the experimental results. The low-energy s wave photoneutron cross section of Be^9 is large near threshold because the distorted s wave has a large amplitude in the all important nuclear surface region. On the other hand the s-wave contribution to the (γ,n) cross section of C^{13} is small because of the oscillations in the continuum wave function results in strong cancellations in the direct interaction results. The d-wave cross section is large even at relatively low energies because its wave function is always positive. All of the calculated cross sections are quite sensitive to the diffuseness parameter, a. Because of this sensitivity the following conclusions may be reached:

1. Square well calculations are not in general to be trusted.
2. The (γ,n) reactions and their inverse may furnish information about the nuclear surface region.
3. When higher order calculations are to be performed, it is desirable to select the best diffuseness parameter preferably from the analysis of elastic scattering angular distributions, polarizations or other experiments.

[33] K. A. Brueckner and D. T. Goldman, Phys. Rev. **116**, 424 (1959). K. A. Brueckner, *Proceedings of the International Conference on the Nuclear Optical Model, Florida State University Studies, No. 32*, edited by A. E. S. Green, C. E. Porter, and D. S. Saxon (The Florida State University, Tallahassee, 1959), p. 145.

The $C^{13}(\gamma,n)C^{12}$ calculations were extended to higher energies. The $d_{\frac{3}{2}}$ contribution decreased as the energy of the neutron increases above 3.37 Mev. However, the s-wave direct interaction cross-section peaks at 15 Mev. The magnitude of the cross section at the peak is quite small and, therefore, its presence cannot completely explain the broad resonance[34] in the $C^{13}(\gamma,n)$ cross section which appears at the base of the giant resonance bump. The magnitude of the direct interaction s-wave cross section may be enhanced considerably by the inclusion of the first correction term to the single particle model. In the correction term, the final state s-wave neutron collides with a $p_{\frac{3}{2}}$ target neutron. After the collision, one nucleon goes to a $p_{\frac{1}{2}}$ bound state while the other goes to a $s_{\frac{1}{2}}$ or $d_{\frac{5}{2}}$ bound state. The energy of this quasi-bound C^{13} state is not very different from the total energy of the system before the collision. The d- or s-state nucleon then goes to a $p_{\frac{3}{2}}$ or $p_{\frac{1}{2}}$ model ground state with the emission of dipole radiation. Even though the radiative transition may involve a one node $2s_{\frac{1}{2}}$ and a zero node $1p_{\frac{1}{2}}$ wave function, the electric dipole radial integral is large because of the importance of the surface region. Of course the $d_{\frac{5}{2}}$ intermediate state transition is expected to be even larger than the transition to the $2s_{\frac{1}{2}}$ intermediate state since the $d_{\frac{5}{2}}$ state has no nodes. An even more plausible explanation of the 14-Mev peak in the C^{13} photodisintegration might be linked with the second order collision amplitude. In this amplitude, two nuclear collisions are required to elevate two C^{12} core nucleons before the emission of the dipole radiation. This amplitude is expected to be resonant and for that reason it may be important even though the even higher order collision corrections are small. The presence of these resonances implies that the 14-Mev broad peak is in fact the net effect of several resonances. These resonances are especially important because the incident s-wave neutron is strongly distorted and can therefore easily excite these not very complex resonances.

Simple single particle model direct interaction calculations of the type reported here have been criticized in the literature.[35–37] The argument sometimes stated is that the single particle model does not predict all of the properties of the known ground and excited states of light nuclei. This objection is a valid one. However, the consequences of a single particle model were investigated for the following reasons:

1. To test the sensitivity of the results to the parameters of the model.
2. To calculate as well as possible the leading term in a multiple scattering expansion.
3. To define a good representation in which the correction terms are likely to be as small as possible.

[34] B. C. Cook, Phys. Rev. **106**, 300 (1957).
[35] D. Inglis, Revs. Modern Phys. **25**, 390 (1953).
[36] A. M. Lane and L. Radicatti, Proc. Roy. Soc. (London) **A67**, 167 (1954).
[37] J. B. French, E. C. Halbert, and S. P. Pandya, Phys. Rev. **99**, 1387 (1955).

When the correction terms to the single particle model are calculated, the bound state corrections can be found relatively reliably. However, unless the more accurate wave functions differ drastically from the single particle wave function, the photodisintegration cross section may be insensitive to the modifications. An example of this insensitivity is seen in the Beryllium s-state photoneutron cross section. The result is modified by only 20% after a more refined intermediate coupling calculations of the Be^9 ground state is performed.[38]

The success of this and other direct interaction calculations suggests the following profitable areas of investigation:

1. The direct interaction calculation of all nuclear reactions involving no more than one free nucleon in the initial or final state. This, of course, includes (γ,n), (γ,p), (n,p), and (n,n') reactions and their inverse for light nuclei in the low-energy range. The calculation of the photoneutron cross section in O^{17} is especially important. Here the Born approximation may be adequate for transitions to low-energy p states.
2. The calculation of the correction terms of the model paying particular attention to resonance phenomena.
3. Extended and improved measurements of elastic scattering and polarization of nucleons on light nuclei. Results of this type would sharpen the calculation of distortion effects.
4. Improved measurements of (γ,n), (γ,p) and their inverse reactions emphasizing angular dependence and polarizations to the check the calculations.
5. Extension of the model calculation to more complex nuclei where the resonance level density is not necessarily low. Here the model will predict average cross sections since it is necessary to introduce complex potentials to filter fluctuations and accelerate convergence.
6. Surface nonlocal effects in the single particle wave functions should be considered.

APPENDIX

In this section we consider the formal basis of the model that is used in this report. The corrections to the model are exhibited here but numerical examples of calculations of these correction terms are deferred for a subsequent publication.

Simply stated, the model is a single particle model for the calculation of photodisintegration of atomic nuclei. That is, an incident photon collides with a neutron bound in a single particle state and ejects a neutron into a single particle continuum state. The bound-state neutron is required to have the experimental binding energy. The continuum potential well is adjusted to give

[38] Cecil B. Mast, Notre Dame University thesis, 1956 (unpublished).

the energy dependence of the scattering differential cross sections. Both of these constraints ensure that the asymptotic parts of the single particle wave functions are correct.

This can best be described with the model Hamiltonian

$$H_M = T_0 + V_0 + H_A. \tag{A.1}$$

T_0 and V_0 are the kinetic and potential energies of the neutron in question and H_A is the Hamiltonian of the core consisting of A nucleons:

$$\begin{aligned} T_0 &= p_0^2/2M; \\ V_0 &= V(r_0). \end{aligned} \tag{A.2}$$

The model wave function $\Phi(A+1)$ is generated by the following Schrödinger equation:

$$(E - H_M)\Phi(A+1) = 0. \tag{A.3}$$

For bound and continuum states the energy value E is assumed to be exact. The model wave function $\Phi(A+1)$ is the product of the core wave function of A nucleons in their ground state and a single particle state.

$$\Phi_i = \Psi_0(A)\varphi(r_0), \tag{A.4}$$

and

$$\Phi_f = \Psi_0(A)\chi(r_0). \tag{A.5}$$

The exact solution $\Psi(A+1)$ is related to the model wave function $\Phi(A+1)$ through the model operator M[39,40]:

$$M\Phi(A+1) = \Psi(A+1). \tag{A.6}$$

The operator M satisfies the integral equation

$$M = 1 + \frac{1}{a}WM, \tag{A.7}$$

where

$$W = \sum_{i=1}^{A} v(r_0 - r_i) - \frac{V_0}{A}, \tag{A.8}$$

$$a = E - H_A - T_0 - V_0. \tag{A.9}$$

In the sum over intermediate states, the model wave function in the ground state is included if $E<0$, and only outgoing scattered waves are permitted if $E>0$.

If the correction terms were to be calculated explicitly, a more manifestly convergent expression for M would be introduced. For the present qualitative discussion Eq. (A.7) is adequate.

For bound states, M introduces correlations in the motion of the incident particle and the core particles. These correlations, although non-negligible in their absolute effect, are masked by uncertainties in the continuum wave function.[41] For continuum states, M represents collisions between the incident neutron and the core particles. Of these collisions, the ones of primary importance lead to states in which the core is excited and the continuum particle is de-excited by the same amount. At these energies, the cross section exhibits sharp resonance features.[42] A qualitative calculation of the neutron and radiative widths of these resonances will be presented in a later publication. At present we will limit the calculation to the region between resonances where we shall assume that

$$M = 1.$$

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[39] K. A. Brueckner and C. A. Levinson, Phys. Rev. **97**, 1344 (1955).

[40] R. J. Eden and N. C. Francis, Phys. Rev. **97**, 1366 (1955).

[41] The continuum wave function is of higher energy and for this reason the corrections to the model wave function are more important than for the bound states.

[42] K. A. Brueckner, R. J. Eden, and N. C. Francis, Phys. Rev. **100**, 891 (1955).