$B^{10}(d,p)B^{11}$ Reaction and the Configurations of B^{11}†

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A study of the states of B^{11} has been made by high-resolution measurements of angular distributions and relative intensities of proton groups from $B^{10}(d,p)B^{11}$. The use of Butler analysis and calculated reduced widths leads to the following spin and parity assignments for excited states: 2.14 Mev, $\frac{1}{2}-$; 4.46 Mev, $\frac{5}{2}-$; 5.03 Mev, $\frac{3}{2}-$; 6.76 Mev, $\frac{7}{2}-$; 8.57 Mev, $(\frac{1}{2}+)$; 8.92 Mev, $\frac{5}{2}+$; 9.19 Mev, $\frac{7}{2}+$; 9.28 Mev, $\frac{5}{2}+$. The states of B^{11} exhibit a subdivision into three classes. The first five levels from ground state to 6.76 Mev, all of odd parity, correspond closely to the theoretical predictions of Kurath. The next four states of 6.81, 7.30, 7.99, and 8.57 Mev presumably result from highly mixed configurations which is evidenced by their weak stripping intensities. The third group consists of three single-particle states of even parity at 8.92, 9.19, and 9.28 Mev. The latter two states form a *jj*-double level arising from the configuration $p^6 2s$.

I. INTRODUCTION

THE relatively simple structure of nuclei with unfilled $1p$-shell configurations has been to a large extent responsible for the considerable experimental and theoretical activity in the He^4 to O^{16} region. Significant gaps, however, remain in the experimental assignments, making a valid test of theory difficult. A case in point is the nucleus B^{11} for which the spins and parities for the most part have remained uncertain. In an effort to clarify the situation, an investigation of the states of B^{11} has been undertaken using the $B^{10}(d,p)B^{11}$ reaction. The results are of particular interest inasmuch as they are contradictory to some earlier assignments.[1,2] A preliminary account[3,4] of part of the work has been given earlier.

The well-established experimental energy level scheme[4] for B^{11} is shown in Fig. 1 up to an excitation of 10 Mev. The low-lying states of B^{11} are believed to arise mainly from excitations within the $1p$ shell, that is, with configurations comprised of $p_{\frac{1}{2}}$ and $p_{\frac{3}{2}}$ nucleons. On this assumption the intermediate-coupling shell model[5,6] predicts the ordering of lowest states to be $\frac{3}{2}-$, $\frac{1}{2}-$, $\frac{7}{2}-$, $\frac{5}{2}-$, and $\frac{3}{2}-$, with some freedom of ordering depending upon a particular choice of intermediate-coupling parameter $\zeta=a/K$ and the radial integral parameter L/K. The theoretical level scheme proposed by Kurath[6] for $\zeta=6.0$, $L/K=6.8$, and $K=0.92$ Mev is also shown in Fig. 1 adjacent to the experimental levels. The stripping reaction is especially well suited to the selection and study of these odd-parity ($l_n=1$) states.

Experimentally the angular distributions[7–10] of protons from $B^{10}(d,p)B^{11}$ show neutron capture with

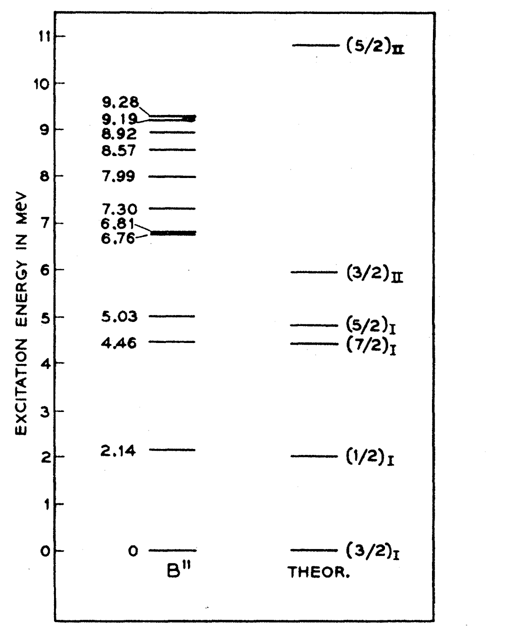

FIG. 1. Energy level schemes for B^{11}. The experimental scheme on the left shows the known states of B^{11} up to excitation energy near 9 Mev (see references 2 and 4). On the right the theoretical scheme calculated by Kurath (reference 6) gives the low-lying states of B^{11} arising from excitations in the $1p$ shell. Here the choice $\zeta=6.0$, $L/K=6.8$, and $K=0.92$ Mev was used. The theoretical states are labeled by the J values in parentheses with Roman numeral subscripts denoting the order of occurrence for levels of a given J value.

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1 G. A. Jones and D. H. Wilkinson, Phys. Rev. **88**, 423 (1952). The original assignments have recently been revised with the aid of stripping data. See G. A. Jones, C. M. P. Johnson, and D. H. Wilkinson, Phil. Mag. **4**, 796 (1959).
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7 N. T. S. Evans and W. C. Parkinson, Proc. Phys. Soc. (London) **A67**, 684 (1954).
8 S. A. Cox and R. M. Williamson, Phys. Rev. **105**, 1799 (1957).
9 B. Zeidman and J. M. Fowler, Phys. Rev. **112**, 2020 (1958).
10 M. Croissiaux, thesis, University of Strasbourg, 1959 (unpublished).

$l_n=1$ for the ground state B_0^{11} and for the first four excited states: $B^{11}{}_{2.14\text{ Mev}}$, $B^{11}{}_{4.46\text{ Mev}}$, $B^{11}{}_{5.03\text{ Mev}}$, $B^{11}{}_{6.76-6.81\text{ Mev}}$ (doublet). Two points, however, require further clarification. First, the anomalous angular distribution for $B^{11}{}_{2.14\text{ Mev}}$ leaves the $l_n=1$ interpretation somewhat in doubt. Although the angular distribution is not inconsistent with the proposed spin-flip mechanism[11] with $l_n=1$ capture leading to a $J=\frac{1}{2}-$ state, this level warrants careful examination to exclude the possibility of an unresolved doublet or contributions from target impurities. Second, it is not certain which member (or whether possibly both) of the $B^{11}{}_{6.76-6.81\text{ Mev}}$ doublet is formed with $l_n=1$. High-resolution measurements were made to help settle these questions. The assignment of character J^π to these odd-parity states was facilitated by the comparison of intensity measurements of outgoing proton groups with reduced widths calculated from the intermediate-coupling shell-model wave functions.

Since B^{11} has a large neutron binding energy, $E_n\sim 11.5$ Mev, favorable for an unambiguous interpretation of (d,p) stripping data even for levels of high excitation, the present investigation was extended to the region above 7 Mev. Except for work[12] at low deuteron energies, angular distributions have not been reported for the 6.81-, 7.30-, 7.99-, and 8.57-Mev states of B^{11}. Measurements at higher deuteron energy are essential in determining the character of these levels.

Particular interest was centered on the three closely spaced levels of B^{11} in the region of 9-Mev excitation which have not previously been resolved in (d,p) angular distribution measurements. Because considerable data[4] exist involving α-γ and γ-γ angular correlations and γ-ray intensity measurements, a determination of parities and assignment of spins to the initial states of the cascades is of immediate use in the analyzing of the structure of the lower levels. The results obtained suggest that $B^{11}{}_{8.92\text{ Mev}}$, $B^{11}{}_{9.19\text{ Mev}}$, and $B^{11}{}_{9.28\text{ Mev}}$ are formed with single-particle $2s$- and $1d$-shell configurations. Calculations are given along with supporting experimental evidence that the 90-kev doublet $B^{11}{}_{9.19-9.28\text{ Mev}}$ is formed by the coupling of a single $2s$ nucleon to an essentially unperturbed B^{10} core $(J=3+)$ to form states of $J=\frac{7}{2}+$ and $J=\frac{5}{2}+$, respectively.

II. EXPERIMENTAL PROCEDURES AND RESULTS

The bombarding deuterons from the Michigan cyclotron had an energy of 7.78 ± 0.01 Mev. Magnetic focusing of the deuteron beam resulted in an energy spread across the 2-mm wide target of less than 12 kev with a beam intensity of ≈ 1 μa. A double-focusing magnetic spectrometer was used for analysis of the reaction products. The details of the instrumentation have been given elsewhere.[13] The reaction products were recorded on Kodak NTB 100-micron nuclear emulsion plates at the image plane of the analyzer magnet. The plates were scanned by microscope in swaths 0.5 mm wide.

The targets were prepared by evaporating enriched boron ($\approx$95% B^{10}) from a carbon crucible onto gold leaf. This crucible was made by carving a hollow in a spectroscopically pure $\frac{1}{4}$-in. graphite rod.[14] In the evaporator the rod was clamped at each end by water-cooled copper jaws and heated to $\approx$2400°C by a current of 200 amperes. As the evaporation progressed, the target thickness could be estimated by watching the formation of Newton's rings on the plate supporting the target. Targets made in this manner were found to be extraordinarily stable; no deterioration was evidenced even after a hundred hours of bombardment with a 1-microampere beam. The thinnest targets, employed when the highest resolution was required, had a thickness of $\approx$0.2 mg/cm² corresponding to a deuteron energy loss of 15 kev at 10 Mev. The over-all resolution of the system with these targets was better than 20 kev. Somewhat thicker targets, $\approx$0.4 mg/cm², were used when intensity was of primary consideration.

Positive identification of the $B^{10}(d,p)B^{11}$ peaks was made by observing the kinematic change of energy of the proton groups as a function of scattering angle and by reference to the ubiquitous carbon and oxygen groups. In talking angular distributions the magnetic field of the analyzer was changed for different scattering angles to keep the peak under study at the same position in the image plane. This ruled out errors arising from uncertainty in solid angle corrections.

Spectra

The proton spectrum for $B^{10}(d,p)B^{11}$ taken at $\theta_{\text{lab}}=20°$ with 30-kev resolution is shown in Fig. 2. Proton groups from all known states of B^{11} below 9.5-Mev excitation were found; no evidence of new levels having an intensity greater than 5% of the ground-state group was uncovered. The 10.32-Mev level of B^{11} observed earlier by the $B^{10}(d,p)B^{11}$ reaction was not seen in the present spectrum due to interference at low angles with the extremely strong groups from $C^{12}(d,p)C^{13*}$.

In Fig. 3 enlarged portions of the spectrum of Fig. 2 reveal the weak proton groups from $B^{11}{}_{2.14\text{ Mev}}$ and the 6.81-Mev member of the $B^{11}{}_{6.76-6.81\text{ Mev}}$ doublet. In Fig. 3(a) nothing is seen to suggest another B^{11} state near $B^{11}{}_{2.14\text{ Mev}}$. Furthermore, no interference from target impurities is evidenced, except for the well-separated weak groups identified by their residual nucleus as $S^{33}{}_0$, $C^{14}{}_0$, and $Si^{29}{}_0$, arising from known

[11] J. C. Hensel and W. C. Parkinson, Phys. Rev. **110**, 128 (1958), and references therein.
[12] B. Sjögren, Arkiv Fysik **11**, 383 (1957).
[13] D. R. Bach, W. T. Childs, R. W. Hockney, P. V. C. Hough, and W. C. Parkinson, Rev. Sci. Instr. **27**, 516 (1956).
[14] This material was obtained from the United Carbon Products Company, Inc., Bay City, Michigan.

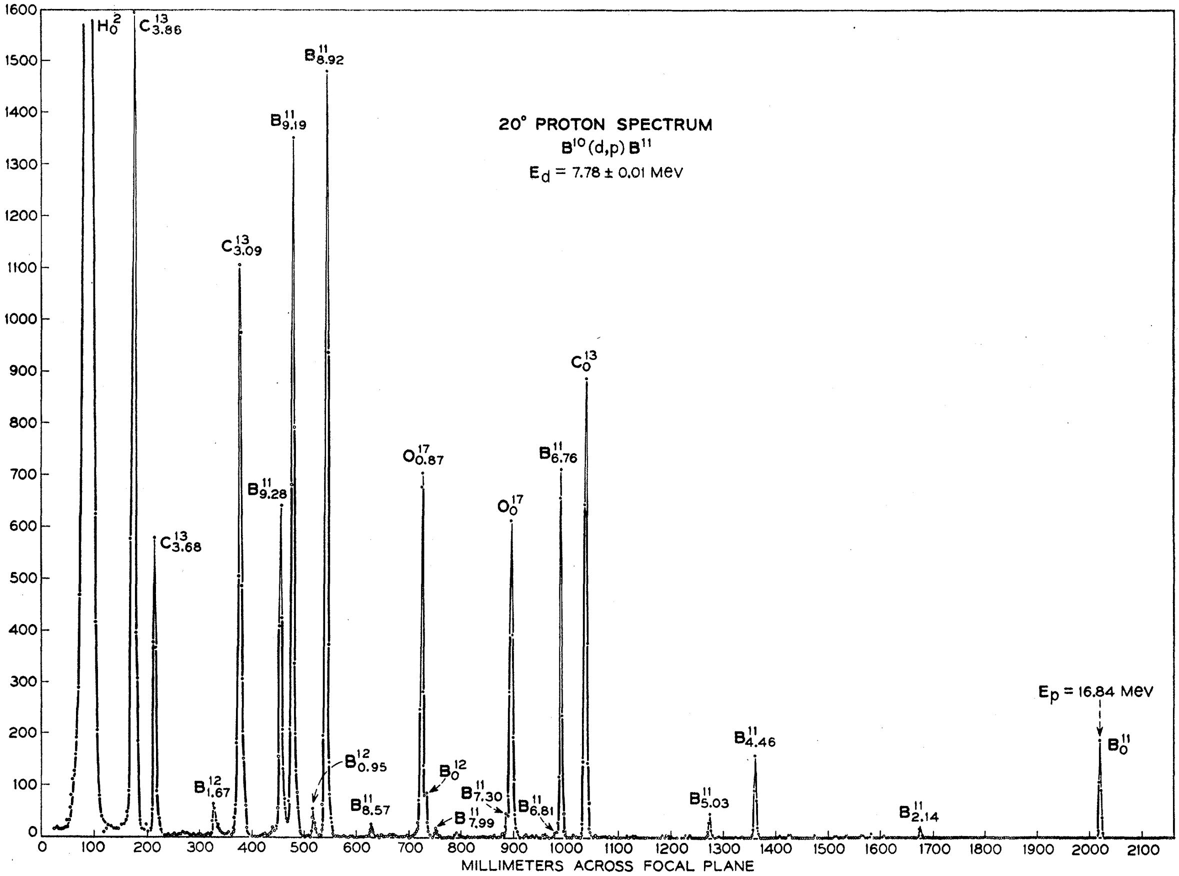

FIG. 2. Proton spectrum for $B^{10}(d,p)B^{11}$ taken at a reaction angle of 20° and recorded on Kodak NTB 100-μ track plates. The groups are identified by the residual nucleus in the (d,p) reaction. The "thick" B^{10} targets limited the resolution to 30 kev. The intensities of the peaks have been corrected for variation of solid angle across the image plane of the analyzer magnet.

impurities in the enriched B^{11} and contaminants from the evaporator.

Figure 4 shows sample spectra for the three closely spaced states of high excitation at 8.92, 9.19, and 9.28 Mev. The spectra at 1.6°, as well as most other data at scattering angles below 15°, were obtained with the aid of a velocity filter[15] of crossed electric and magnetic fields to eliminate the usual interference from reactions caused by beam deuterons striking the walls of the analyzer vacuum chamber.

Angular Distributions

Angular distributions have been taken for protons associated with levels in B^{11} at excitations of 0, 2.14, 6.76, 7.99, 8.92, 9.19, and 9.28 Mev. Calculation of Butler curves to fit the data was greatly facilitated by tables prepared by Lubitz.[16]

The angular distribution for $B^{11}{}_0$ shown in Fig. 5 was measured with a resolution of 35 kev. The data, taken for reference, clearly indicate neutron capture with $l_n=1$ with a stripping radius $r_0=5.1\times10^{-13}$ cm, in good agreement with earlier work.

The weak proton group from $B^{11}{}_{2.14\text{ Mev}}$ required a thick target again limiting the resolution to 35 kev. The angular distribution in Fig. 6 cannot be fitted with any combination of Butler curves having the same parity. Except for the abrupt rise in intensity at forward angles (below $\theta_{\text{c.m.}}=15°$) the measurements agree well with other work.[7,9] Measurements at forward angles with the beam energy increased from $E_d=7.78$ Mev to $E_d=7.92$ Mev gave the same angular distribution. This rules against the possibility that the rise might be due to sharp resonances in the excitation function leading to rapid variations in the angular distribution with energy.

The angular distributions for the 4.46- and 5.03-Mev levels have been measured earlier by Evans and Parkinson,[7] but the 6.76–6.81 Mev doublet remained unresolved. This doublet was effectively separated in the present work with thin targets permitting 20-kev resolution. The 6.76-Mev member of the 6.76–6.81 Mev

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[15] L. H. Th. Rietjens, O. M. Bilaniuk, and W. C. Parkinson, Rev. Sci. Instr. 29, 768 (1958).

[16] C. R. Lubitz, "Numerical Table of Butler-Born Approximation Stripping Cross Sections," H. M. Randall Laboratory of Physics, University of Michigan, 1957 (unpublished).

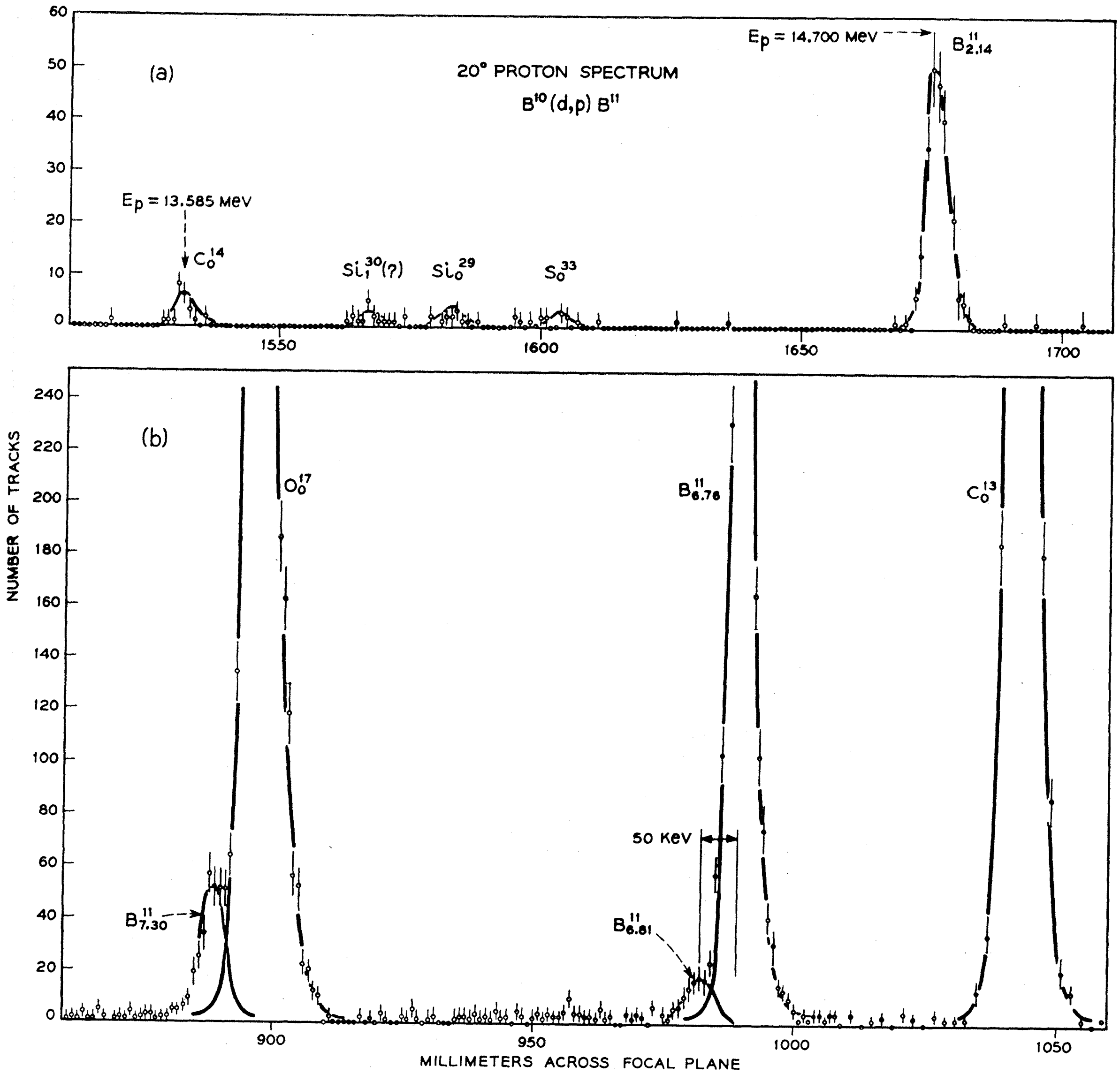

FIG. 3. Enlarged 20° proton spectrum for $B^{10}(d,p)B^{11}$ showing (a) the region near the weak proton group from $B^{11}{}_{2.4\ \mathrm{Mev}}$ and (b) the region near the doublet $B^{11}{}_{6.76\ \mathrm{Mev}}-B^{11}{}_{6.81\ \mathrm{Mev}}$. The small peaks in (a) indicate very weak groups from traces of known impurities in the target.

doublet was found to be approximately 50 times more intense at 20° than the 6.81-Mev member. Because of the proximity of the much stronger group, the angular distribution of the 6.81-Mev level could not be obtained, but its interference in the angular distribution of the 6.76-Mev state was ruled out.

The angular distribution for $B^{11}{}_{6.76\ \mathrm{Mev}}$ in Fig. 7 indicates neutron capture with $l_n=1$, in agreement with previous measurements because of the extreme weakness of the 6.81-Mev group. The experimental points were fitted by a Butler curve with $r_0=5.1\times10^{-13}$ cm—the same r_0 as used for the ground state. The unusually good fit of the Butler curves and the high intensity of the proton group suggest a single-particle configuration for the $B^{11}{}_{6.76\ \mathrm{Mev}}$ final state.

The angular distribution for $B^{11}{}_{7.30\ \mathrm{Mev}}$ could not be reliably measured at scattering angles $\theta=0-30°$ due to interference from a very strong group from $O^{16}(d,p)O^{17}{}_0$ resulting from oxygen contamination in the target.

The angular distribution for the proton groups corresponding to the 7.99-Mev and 8.57-Mev levels of B^{11}, shown in Fig. 8 and Fig. 9, respectively, suggest that the latter results from an $l_n=2$ neutron capture ($r_0=5.1\times10^{-13}$ cm) without $l_n=0$ admixture. No assignment of parity can be made for the 7.99-Mev state from the stripping data since the angular distribution is nearly isotropic.

The angular distribution for the 8.92-Mev state of B^{11} in Fig. 10 indicates a mixture of $l_n=0$ and $l_n=2$. The theoretical Butler curves give a reasonable fit to the data for $r_0=6.7\times10^{-13}$ cm, somewhat larger than the radius used for the low-lying states. A somewhat improved fit, incidentally, can be achieved by using different stripping radii for the $l_n=0$ and $l_n=2$ components; a smaller value of r_0 for $l_n=0$ works best.

Angular distributions for the 9.19- and 9.28-Mev doublet, shown in Fig. 11 and Fig. 12, respectively, both suggest $l_n=0$ capture with $r_0=6.0\times10^{-13}$ cm; any attempt to fit the data with $l_n=1$ leads to radii which are far too large.

Relative Intensities

The intensities of the first nine proton groups from $B^{10}(d,p)B^{11}$, measured relative to the ground-state group, are presented in Table I. The scattering angle 20° corresponds approximately to the peaks of the $l_n=1$ angular distributions for the low-lying states. In

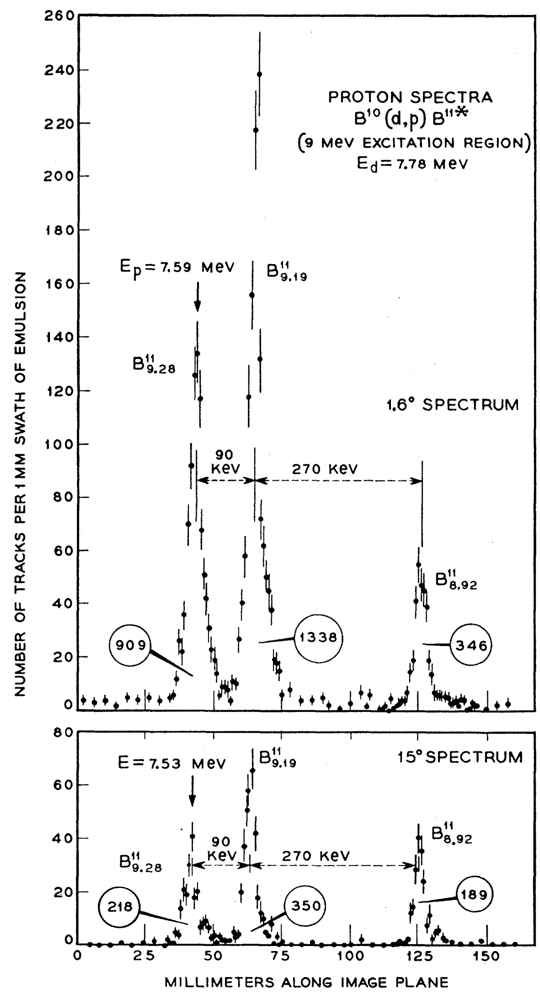

FIG. 4. Proton spectra for $B^{10}(d,p)B^{11}$ at the 9-Mev excitation region showing the three intense proton groups from $B^{11}{}_{8.92\text{ Mev}}$, $B^{11}{}_{9.19\text{ Mev}}$, and $B^{11}{}_{9.28\text{ Mev}}$. The forward-angle spectrum at 1.6° was taken with the aid of a velocity filter of crossed magnetic and electric fields to prevent the deuteron beam from entering the magnetic particle analyzer.

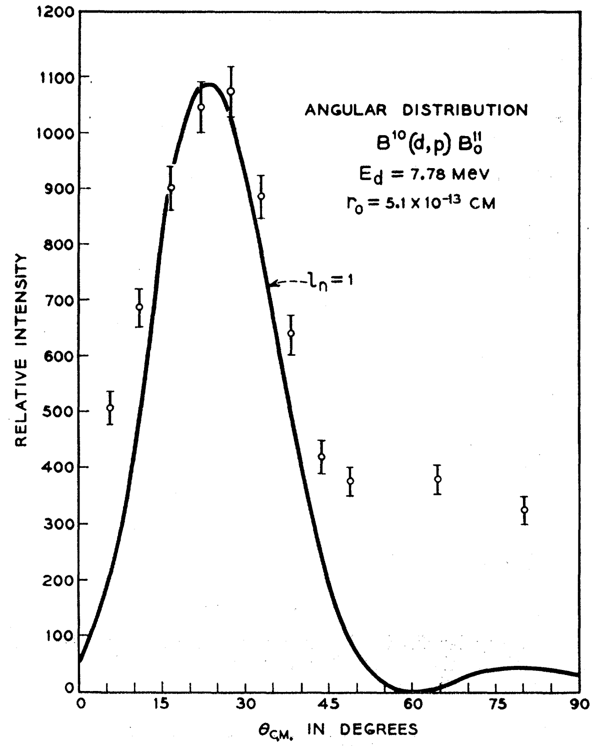

FIG. 5. Angular distribution of protons from $B^{10}(d,p)B^{11}{}_0$. The experimental data are fitted with a Butler curve with $l_n=1$, $r_0=5.1\times10^{-13}$ cm.

addition, the relative intensities of the 9-Mev triplet were measured at $\theta_{\text{lab}}=5°$—as close to the $l_n=0$ peaks at $\theta=0°$ as could be reached reliably.

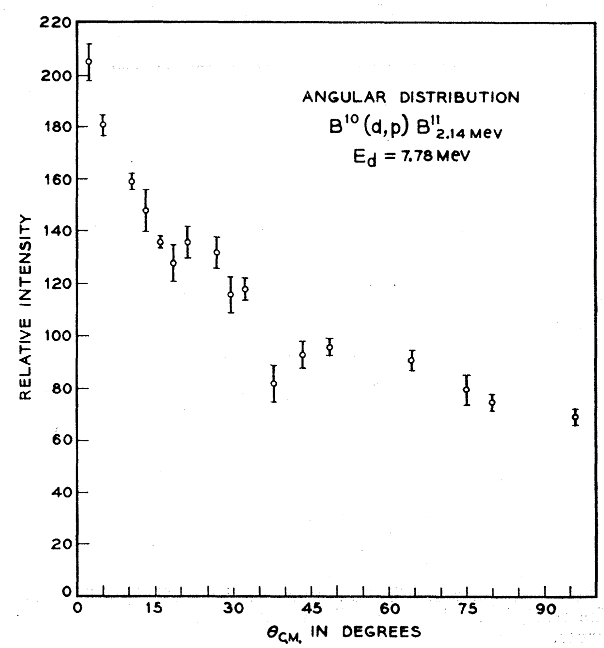

FIG. 6. Angular distribution of protons from $B^{10}(d,p)B^{11}{}_{2.14\text{ Mev}}$. No combination of Butler curves gives a satisfactory fit to this angular distribution.

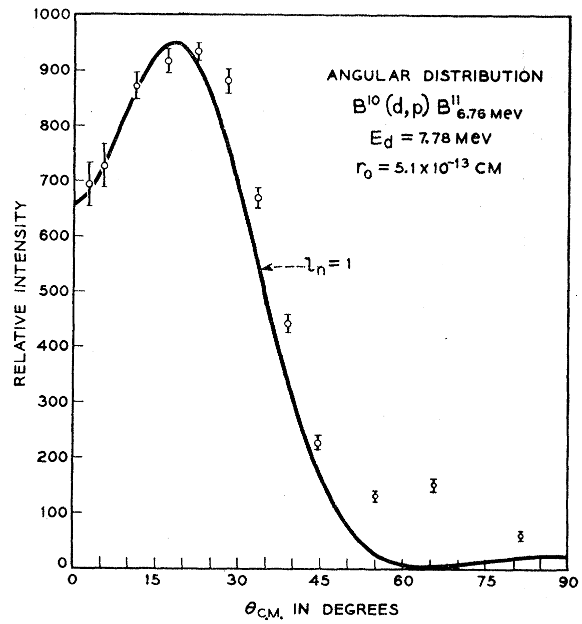

FIG. 7. Angular distribution of protons from $B^{10}(d,p)B^{11}{}_{6.76\text{ Mev}}$. The experimental data are fitted with a Butler curve having $l_n=1$ and $r_0=5.1\times10^{-13}$ cm.

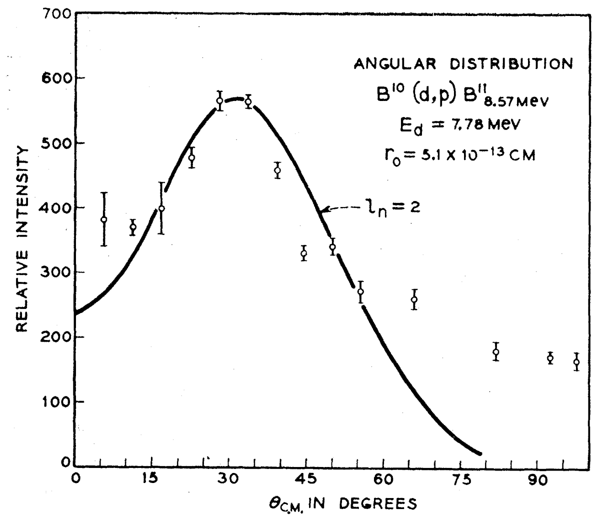

FIG. 9. Angular distribution of protons from $B^{10}(d,p)B^{11}{}_{8.57\text{ Mev}}$. The data are fitted by a Butler curve for $l_n=2$ and $r_0=5.1\times10^{-13}$ cm. There is no evidence for an $l_n=0$ admixture.

III. LOW-LYING STATES OF ODD PARITY

Reduced Widths

In their early stripping experiments, Holt and Marsham[17] employed the neutron capture probability Λ_{l_n} as an aid in determining spin assignments via the $(2J+1)$ statistical factor contained in the Λ_{l_n}, the results, under favorable conditions, being largely independent of the details of the Butler theory. The weakness of this method lies in the fact that Λ_{l_n} contains also the reduced width which is sensitive to the initial-state and final-state nuclear wave function. Recently it has been stressed by French and co-workers[18] that important information about the structure of the nuclear states and angular momentum coupling

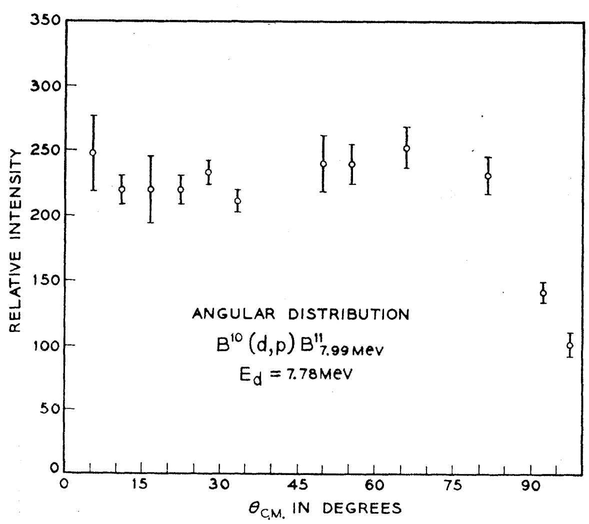

FIG. 8. Angular distribution of protons from $B^{10}(d,p)B^{11}{}_{7.99\text{ Mev}}$. The nearly isotropic angular distribution rules out a fit by Butler curves.

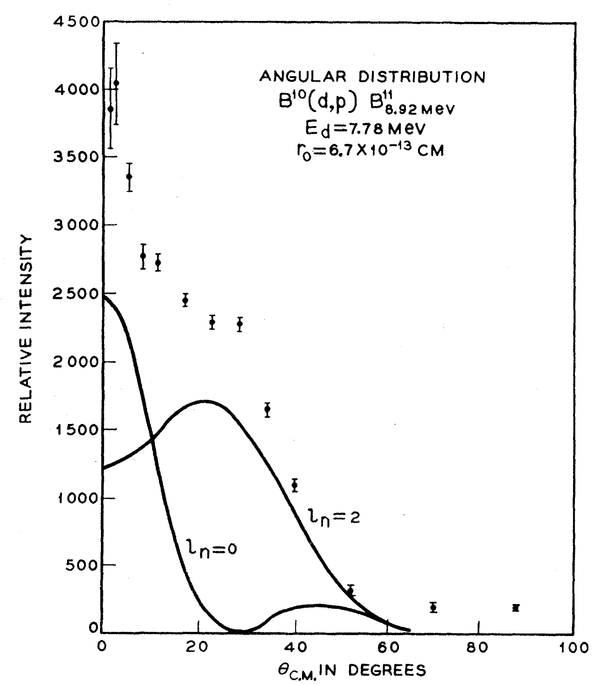

FIG. 10. Angular distribution of protons from $B^{10}(d,p)B^{11}{}_{8.92\text{ Mev}}$. The data are fitted by Butler curves for $l_n=0$ and $l_n=2$ with $r_0=6.7\times10^{-13}$ cm.

[17] J. R. Holt and T. N. Marsham, Proc. Phys. Soc. (London) **A66**, 1063 (1953).

[18] J. B. French and B. J. Raz, Phys. Rev. **104**, 1411 (1956), and references therein.

schemes can be deduced from the reduced widths by comparing the experimental stripping intensities with reduced width derived from calculated nuclear wave functions. Although exceptional cases[19] have been successfully handled, the best results are usually expected when relative intensities are compared between states of the same nucleus formed by the same value of l_n. Using the data in Table I, this procedure has been applied to the low-lying odd-parity states of B^{11} to supply additional information necessary for the complete classification of their characters as well as to provide insight into their structures.

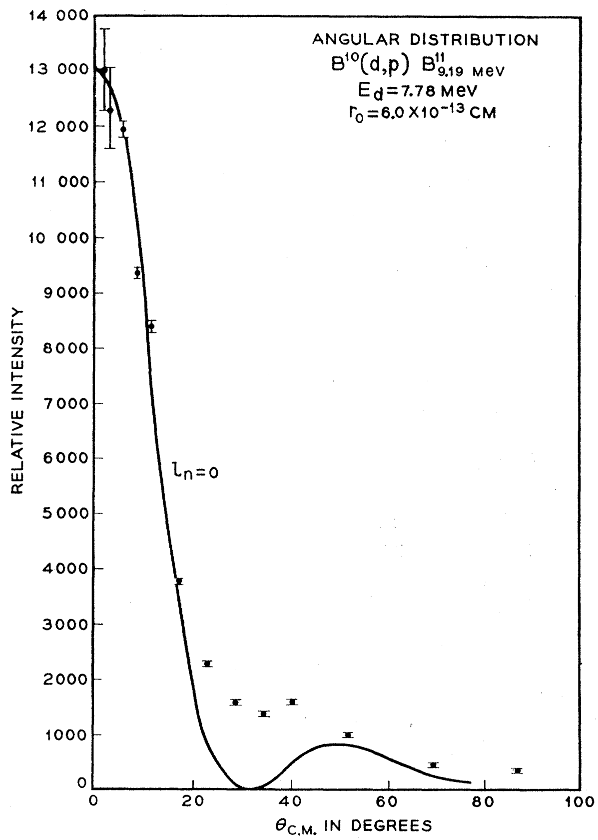

FIG. 11. Angular distribution of protons from $B^{11}(d,p)B^{11}{}_{9.19\ \mathrm{Mev}}$. The data are fitted by a Butler curve for $l_n=0$ and $r_0=6.0\times10^{-13}$ cm. An attempt to fit the data assuming $l_n=1$ would require an unreasonably large value of r_0.

Using the notation of French and Raz,[18] we can write the differential cross section

$$\sigma(\theta)=[(2J+1)/(2J_0+1)]S_1\Phi_1(\theta), \qquad (1)$$

for neutron capture, via deuteron stripping, into an initial nuclear state having spin J_0 and leading to a final nuclear state with spin J. The sum over l_n which usually appears in (1) reduces in the present case to one term with $l_n=1$. The $\Phi_1(\theta)$ is the intrinsic Butler single-particle cross section for neutron capture with $l_n=1$. All information on the nuclear structure is contained in the reduced width[20] S_1.

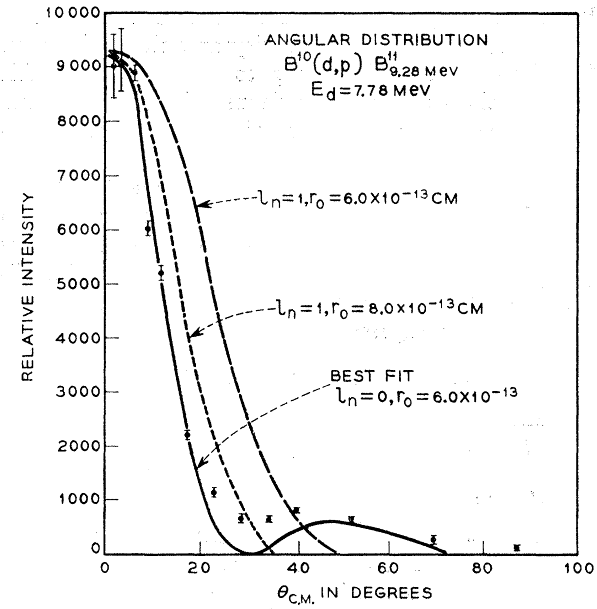

FIG. 12. Angular distribution of proton from $B^{10}(d,p)B^{11}{}_{9.28\ \mathrm{Mev}}$. The data are fitted best by a Butler curve for $l_n=0$ and $r_0=6.0\times10^{-13}$ cm. As shown by the dashed curves, an attempt to fit the data assuming $l_n=1$ would require unreasonably large r_0 values.

States in p-shell nuclei are known not to be consistent with either extreme jj or LS coupling. Consequently, the calculation of S_1 leading to states of B^{11} was carried out using B^{10} and B^{11} intermediate-coupling wave functions of Kurath.[21] Briefly, they may be described as follows. The wave functions Ψ_A for states of B^{11}

TABLE I. Relative intensities of proton groups from the reaction $B^{10}(d,p)B^{11}$. All values are normalized with respect to the ground-state group at $\theta_{\mathrm{lab}}=20°$. The uncertainty in intensities corresponding to the 8.92-, 9.19-, and 9.28-Mev states is less than 2% relative to each other.

B^{11} excitation energy (Mev)	Rel. intensity $\theta_{\mathrm{lab}}=5°$	Rel. intensity $\theta_{\mathrm{lab}}=20°$
0		1.00
2.14		0.12±0.01
4.46		0.88±0.04
5.03		0.22±0.02
6.76		4.86±0.20
6.81		0.11±0.03
7.30		0.24±0.04
7.99		0.09±0.04
8.57		0.17±0.02
8.92	14.8±0.6	10.1 ±0.4
9.19	48 ±3	9.2 ±0.4
9.28	35.5±2	4.8 ±0.2

[19] O. M. Bilaniuk and P. V. C. Hough, Phys. Rev. 108, 305 (1957), and reference 18.

[20] Strictly speaking, S_1 is the reduced width in units of the single-particle reduced width θ_0^2. To avoid confusion which sometimes arises on this point, we have chosen to follow the convention of French and Raz (reference 18) and simply call S_1 the reduced width.

[21] D. Kurath (private communication). See also reference 6.

having total angular momentum J and isotropic spin T are constructed from antisymmetric basis wave functions ψ_α, each belonging to one of the possible configurations of $n=7$ p nucleons. The ψ_α's are themselves linear combinations of products of single-particle shell-model wave functions (a sum of Slater determinants) for a harmonic oscillator well. The state wave functions Ψ_A are, therefore, a linear combination of the jj-coupling wave functions ψ_α, the coefficients $K_\alpha(n)$ emerging from the diagonalization of a Hamiltonian consisting of spin-orbit and exchange potentials.

Thus we can express Ψ_A as

$$\Psi_A(J,T)=\textstyle\sum_\alpha K_\alpha(7)\psi_\alpha(J,T). \quad (2\text{-a})$$

Similarly, for 6 nucleons we can write the B^{10} ground-state wave function

$$\Psi_B(J_0=3,\ T_0=0)=\textstyle\sum_\beta K_\beta(6)\psi_\beta(J_0=3,\ T_0=0). \quad (2\text{-b})$$

The diagonalization was carried out by Kurath for a variety of values of the radial integral parameter L/K and intermediate-coupling parameter $\zeta=a/K$.

In jj-coupling notation the reduced width S_{l_n} is written[22] as a sum of terms $\theta_{j_nl_n}{}^2$,

$$S_{l_n}=n\sum_{j_n}\theta_{j_nl_n}{}^2, \quad (3\text{-a})$$

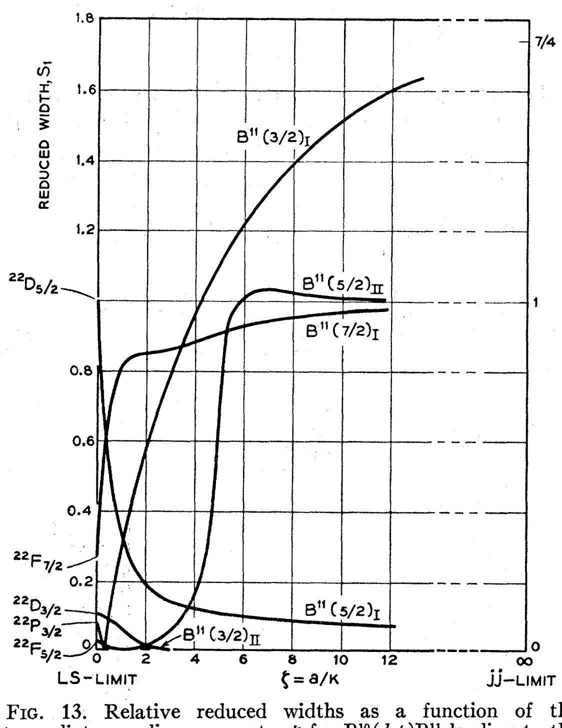

FIG. 13. Relative reduced widths as a function of the intermediate-coupling parameter ζ for $B^{10}(d,p)B^{11}$ leading to the low-lying odd-parity states of B^{11}. The radial integral parameter was chosen as $L/K=6.8$. The curves are identified by the J values of the state using the notation of Fig. 1 and also by the LS supermultiplets in the LS limit at the left. The values of the reduced widths in the jj limit are given at the right ($\zeta=\infty$).

[22] This equation differs slightly from that given by French and Raz (reference 18). See Bilaniuk and Hough (reference 19).

which becomes

$$S_1=7(\theta_{\frac{1}{2},1}{}^2+\theta_{\frac{3}{2},1}{}^2), \quad (3\text{-b})$$

where $j_n=l_n\pm\frac{1}{2}$ is the total angular momentum of the captured neutron. The amplitudes $\theta_{j_nl_n}$ contain the diagonalization parameters $K(n)$:

$$\theta_{j_nl_n}=\textstyle\sum_{\alpha\beta}(-1)^{J_0-J+j_n}K_\alpha(7)K_\beta(6)\times\langle\psi_\beta(J_0,T_0)\times j_n(7)]\!]\psi_\alpha(J,T)\rangle. \quad (4)$$

The factors $\langle\ ]\!]\ \rangle$ are coefficients of fractional parentage (cfp) given by the overlap

$$\langle\psi_\beta\times j_n(7)]\!]\psi_\alpha\rangle=\int\psi_\alpha(1\cdots7)[\psi_\beta(1\cdots6)\times\Phi_{j_n}(7)]d1\cdots d7. \quad (5)$$

Here, $\Phi_{j_n}(7)$ is the wave function of the seventh nucleon. The $\times$ denotes vector coupling. The jj cfp's were computed in this way directly from the (J_0,T_0) and (J,T) basis wave functions.

Using Kurath's coefficients K,[21] the reduced widths S_1 were evaluated from Eqs. (3) and (4), and the results have been plotted as a function of the intermediate-coupling parameter ζ in Fig. 13. The curves are identified by the LS supermultiplets in the $\zeta=0$ limit and also by the J values and Kurath's level order (see Fig. 1). Since the wave functions reduce to a single LS term for $\zeta=0$, the S_1 were checked at $\zeta=0$ using the appropriate LS-coupling analog[23] to Eq. (4) and the LS cfp's of Jahn and van Weiringen.[24] The lowest $J=\frac{1}{2}$ state, $B^{11}(\frac{1}{2})_{\rm I}$, is not included in Fig. 13 because its reduced width for stripping is zero. The second $J=\frac{5}{2}$ state, $B^{11}(\frac{5}{2})_{\rm II}$, is expected to lie very high, ~10–11 Mev, and is not dealt with in the present work.

Spin Assignments

With respect to the ground state, the relative intensities of proton groups from the (d,p) reaction are, according to Eq. (1),

$$R=\frac{\sigma(\theta)}{\sigma_{\rm gnd}(\theta)}=\frac{(2J+1)}{(2J_{\rm gnd}+1)}\frac{S_1\Phi_1(\theta)}{S_{\rm gnd}\Phi_{\rm gnd}(\theta)}. \quad (6)$$

We can compare the calculated S_1 directly with the experimental intensities in Table I by extracting the ratio of single-particle reduced widths Φ_1 from (6). A basis for this is provided by the Butler-Born cross sections computed by Lubitz.[16] The measured relative intensities with kinematic factors (evaluated at 20°) divided out are given in the third column of Table II. For conciseness we have collapsed the statistical factor into $\bar{S}$, $(2J+1)S_1=\bar{S}_1$. It may be seen that the ratio $S_1/S_{\rm gnd}=0.86$ for $B^{11}{}_{6.76\ \rm Mev}$ could be consistent only

[23] T. Auerbach and J. B. French, Phys. Rev. **98**, 1276 (1955), Eq. (5).

[24] H. A. Jahn and H. Van Wieringen, Proc. Roy. Soc. (London) **209**, 502 (1951).

with the state $B^{11}(\frac{7}{2})_{I}$ of Fig. 13. Assuming this assignment, the intermediate coupling parameter is fixed at $\zeta=4.0$ for a best fit. The calculated ratios for the $B^{11}(\frac{5}{2})_{I}$ and $B^{11}(\frac{3}{2})_{II}$ excited states are, therefore, 0.12 and 0.00, respectively, which match best $B^{11}{}_{4.46\text{ Mev}}$ and $B^{11}{}_{5.03\text{ Mev}}$, respectively. For these latter states the agreement can be improved by subtracting from the measured cross sections the contribution from scattering processes other than stripping. From the angular distributions[7] it appears that these can represent as much as 40% of the total cross section at the $l_n=1$ peak. The relatively small contribution of these effects to the angular distributions of $B^{11}{}_0$ and $B^{11}{}_{6.76\text{ Mev}}$ was the main reason for choosing the coupling parameter $\zeta=4.0$ on basis of relative intensities of these latter states alone.

A nonstripping mechanism presumably involving a spin flip[11] for the outgoing proton must account for the entire cross section for $B^{11}{}_{2.14\text{ Mev}}$, since selection rules forbid stripping to this $J=\frac{1}{2}$ state. These effects are reflected in the anomalous appearance of the angular distribution in Fig. 6.

TABLE II. Comparison of calculated and experimental relative reduced widths. The experimental relative intensities with kinematic factors extracted $\bar{S}/\bar{S}_{\text{gnd}}$ in column 3, in conjunction with the assumed J values of column 4, lead to the experimental reduced widths relative to the ground state. The theoretical values for $\zeta=4.0$ are taken from Fig. 13.

B^{11} excitation energy (Mev)	Rel. intensity $\theta_{\text{lab}}=20°$	$\bar{S}/\bar{S}_{\text{gnd}}$	J	Expt. S/S_{gnd}	Calc. S/S_{gnd} $\zeta=4.0$
0	1.00	1.00	3/2	1.00	1.00
2.14	0.12±0.01	0.09	1/2	0.18	0
4.46	0.88±0.04	0.46	5/2	0.31	0.12
5.03	0.22±0.02	0.11	3/2	0.11	0.00
6.76	4.86±0.20	1.72	7/2	0.86	0.86

The results are summarized in Table II. The proposed assignments $\frac{3}{2}-$, $\frac{1}{2}-$, $\frac{5}{2}-$, $\frac{3}{2}-$, and $\frac{7}{2}-$ for the first five states, respectively, of B^{11} are in complete agreement with the assignments[25] given by the Chalk River group based on γ-ray decay schemes from the $Be^9(He^3,p)B^{11}$ reaction. In addition, the value of $\zeta=4.0$, determined from the stripping relative widths, is very close to their value $\zeta\approx4.5$ derived from the γ-ray branching ratios. Both of these values provide nearly an optimum fit of the calculated $B^{11}{}_0$ magnetic dipole moment 2.8–2.9 (depending upon L/K) with the experimental value of 2.69.

No attempt was made to improve the agreement of the calculated and experimental reduced widths by varying the intermediate-coupling parameters ζ for B^{10} and B^{11} independently. The trend is well known to be toward stronger jj coupling as the $1p$ shell fills; consequently, one would expect the value of ζ to be slightly smaller for B^{10} than for B^{11}. The small difference, however, is not expected to change the results in any significant way. A second possibility that was not explored was variation of L/K. Although changes of L/K have little effect on the calculated positions of the low-lying states of B^{10} and B^{11}, a choice of L/K other than $L/K=6.8$ might lead to a better fit for the reduced widths. Again the change is not expected to alter the interpretations.

[25] A. J. Ferguson, H. E. Gove, J. A. Kuehner, A. E. Litherland, E. Almqvist, and D. A. Bromley, Phys. Rev. Letters 1, 414 (1958), and A. J. Ferguson (private communication).

TABLE III. Contributions of p-shell configurations to the states of B^{11}. The values given were calculated by Kurath[a] for $\zeta=4.5$ and $L/K=6.8$.

	$(p_{\frac{3}{2}})^7$	$(p_{\frac{3}{2}})^6 p_{\frac{1}{2}}$	$(p_{\frac{3}{2}})^5(p_{\frac{1}{2}})^2$	$(p_{\frac{3}{2}})^4(p_{\frac{1}{2}})^3$	$(p_{\frac{3}{2}})^3(p_{\frac{1}{2}})^4$
$B^{11}(\frac{3}{2})_{I}$	0.545	0.055	0.348	0.043	0.010
$B^{11}(\frac{1}{2})_{I}$	...	0.636	0.221	0.143	0.001
$B^{11}(\frac{5}{2})_{I}$	...	0.628	0.180	0.186	0.005
$B^{11}(\frac{7}{2})_{I}$	...	0.704	0.245	0.048	0.003
$B^{11}(\frac{3}{2})_{II}$	0.015	0.420	0.395	0.128	0.043

[a] See references 6 and 21.

Configurations

Clearly for $\zeta=4.0$ the description of the low-lying B^{11} levels in terms of one single jj-coupling configuration is not possible; instead we list in Table III the contribution from each of the p-shell configurations to a given B^{11} state. Even this actually oversimplifies the situation since a configuration generally gives rise to several (J,T) states with individual amplitudes [Eq. (2)]; for example, $(p_{\frac{3}{2}})^5(p_{\frac{1}{2}})^2$ gives five states with $J=\frac{3}{2}$, $T=\frac{1}{2}$, three states with $J=\frac{1}{2}$, $T=\frac{1}{2}$, etc. Nevertheless, from Table III some general properties of the B^{11} states can be pointed out.

In the limit of extreme jj coupling ($\zeta\rightarrow\infty$), $(p_{\frac{3}{2}})^7$ is the configuration of $B^{11}(\frac{3}{2})_{I}$ while the other four states in question belong to $(p_{\frac{3}{2}})^6 p_{\frac{1}{2}}$. Thus, Table III suggests that jj coupling still dominates in B^{11} for values of the intermediate-coupling parameter as low as $\zeta=4.0$. This fact is also reflected in Fig. 13 where the reduced widths change relatively little for $\zeta>4$ compared to the abrupt changes near the LS limit. Therefore, it is qualitatively correct to assume that jj coupling prevails and, furthermore, that the B^{10} ground state has a configuration $(p_{\frac{3}{2}})^6$.[26]

In Table IV, configurations of the low-lying B^{11} states in the jj extreme along with the parentage of each state are listed. Since the B^{10} configuration is $(p_{\frac{3}{2}})^6$ we see that p-wave stripping from $B^{10}{}_0$ can only go to states having $(p_{\frac{3}{2}})^6$ $(J_0=3, T_0=0)$ as a parent; i.e., a nonvanishing cfp in the first column of Table IV. This yields the following reduced widths [$S_1=n(\text{cfp})^2$] at the jj limit: $S_1=7/4$ for $B^{11}(\frac{3}{2})_{I}$; $S_1=1$ for $B^{11}(\frac{7}{2})_{I}$ and $B^{11}(\frac{5}{2})_{II}$; and $S_1=0$ for $B^{11}(\frac{1}{2})$, $B^{11}(\frac{5}{2})_{I}$, and $B^{11}(\frac{3}{2})_{II}$

[26] The actual contribution of $(p_{\frac{3}{2}})^6$ in the B^{10} ground-state wave function is 0.654 for $\zeta=4.5$.

TABLE IV. Coefficients of fractional parentage for the principal configurations of the low-lying levels of B^{11}. The $(J_0,T_0)=(3,0)$ state of $(p_{3/2})^6$ in column 1 corresponds to the ground state of B^{10}. The states of B^{11} are identified at the right.

	(J,T) \ (J_0,T_0)	$(p_{3/2})^6$ (3,0)	(2,1)	(1,0)	(0,1)	
$(p_{3/2})^7$	$(\frac{3}{2},\frac{1}{2})$	$-(7/28)^{1/2}$	$-(15/28)^{1/2}$	$(3/28)^{1/2}$	$-(3/28)^{1/2}$	$B^{11}(\frac{3}{2})_{I}$
$(p_{3/2})^6p_{1/2}$	$(\frac{7}{2},\frac{1}{2})$	$(1/7)^{1/2}$	0	0	0	$B^{11}(\frac{7}{2})_{I}$
	$(\frac{5}{2},\frac{1}{2})$	$(1/7)^{1/2}$	0	0	0	$B^{11}(\frac{5}{2})_{II}$
	$(\frac{5}{2},\frac{1}{2})$	0	$-(1/7)^{1/2}$	0	0	$B^{11}(\frac{5}{2})_{I}$
	$(\frac{3}{2},\frac{1}{2})$	0	$(1/14)^{1/2}$	$(1/14)^{1/2}$	0	$B^{11}(\frac{3}{2})_{II}$
	$(\frac{1}{2},\frac{1}{2})$	0	0	0	$(1/7)^{1/2}$	$B^{11}(\frac{1}{2})_{I}$

(see also Fig. 13). Of the great number of possible p-shell states only three, namely $B^{11}(\frac{3}{2})_{I}$, $B^{11}(\frac{7}{2})_{I}$, and $B^{11}(\frac{5}{2})_{II}$, have an overlap with $B^{10}{}_0$ in the jj limit. As seen in Fig. 13, this general behavior persists down to $\zeta\approx 4$, hence these three states alone should appear strongly in a (d,p) spectrum. Since $B^{11}(\frac{5}{2})_{II}$ is predicted to lie above 10-Mev excitation and accordingly was not seen in this investigation, only two strong $l=1$ groups are expected. In this way, the large experimental intensities of proton groups from $B^{11}{}_0$ and $B^{11}{}_{6.76\text{ Mev}}$ as well as the relatively low intensities for the other low-lying states are accounted for. The two states $B^{11}(\frac{7}{2})_{I}$ and $B^{11}(\frac{5}{2})_{II}$, with spins $\frac{7}{2}$ and $J=\frac{5}{2}$, respectively, can be crudely pictured as formed by coupling a $p_{1/2}$ neutron to the $J=3$ core of $B^{10}{}_0$. Likewise, the ground state of B^{11} is formed by coupling a $p_{3/2}$ neutron to the same core; however, the Pauli principle allows only one state to be formed this way.

IV. STATES OF MEDIUM EXCITATION

While there is yet insufficient evidence to allow assignments to be made for the 8.57-, 7.99-, 7.30-, and 6.81-Mev levels, it is not inappropriate here to include a few somewhat speculative remarks. The extremely small widths for the (d,p) transition to these states suggest that they arise from highly mixed configurations rather than from single-particle configurations of B^{10} $(J=3)$ core plus one extra neutron. None of them is a likely candidate for a p-shell state since all of the low-excitation members of this class are presumably accounted for, and the next state from the p shell, $B^{11}(\frac{5}{2})_{II}$, is expected considerably higher, at $\approx$10–11 Mev. Since the low-intensity states lie near but just below the single-particle states centered around 9 Mev, their formation may very likely proceed by capture of an s or d neutron accompanied by recoupling and pairing in the p-shell "core." Such a recoupling would involve core states of B^{10} with $J_0=3$, $T_0=0$ going to states having partial configurations of 6 nucleons with $(J_0,T_0)=(2,1)$, (1,0), and (0,1). In intermediate coupling the energy separation for the $J_0=0$, 1, and 3 states is rather small as evidenced by the positions of the first three states of B^{10}, all of which lie within 2 Mev. Indeed, intermediate-coupling calculations[6] show that the $J_0=1$ level of the 6-nucleon core crosses below the $J_0=3$ level for $\zeta<3$. Thus when an s or d neutron is added, it is expected that there will be a considerable admixture of core states of $J_0=0$, 1, and 3 whenever angular momentum coupling permits, and the result is not expected to resemble a single-particle state. The exception, of course, is pure s-wave capture, leading to a $J=\frac{5}{2}$ or $J=\frac{7}{2}$ final state which can have only a $J_0=3$ core.

This situation is to be contrasted to that for the low-excitation even-parity states of C^{13} for which the C^{12} core states of $J_0=0$ and $J_0=2$ are separated by nearly 4.5 Mev. Inglis[5] has remarked that the state composed of $p^8d_{5/2}$ closely resembles a single-particle state because the $J_0=2$ excitation in the core is almost absent in comparison to the lower lying $J_0=0$.

Fitting into the above picture is the 8.57-Mev state of B^{11} formed with $l_n=2$ capture. The absence of an $l_n=0$ admixture in the observed (d,p) angular distribution for this state suggests an assignment of $J=\frac{1}{2}+$ or $\frac{3}{2}+$ in agreement with the γ-ray intensity measurements. The observed $l_n=2$ and low stripping intensity is consistent with a configuration of p^6d, however, with only a small amount of the $(J_0,T_0)=(3,0)$ component in the p^6 core to give an overlap with the B^{10} ground state.

On the other hand, the absence of the $(J_0,T_0)=(3,0)$ state from the core configuration could be responsible for the isotropic stripping angular distribution observed for the 7.99-Mev level. In this case the (d,p) reaction would have to go by mechanisms other than stripping. Another possibility is that an $s_{1/2}$ neutron is coupled to a rearranged p-shell core to form a final state of $J=\frac{1}{2}+$ or $\frac{3}{2}+$. Since s-wave stripping would then be forbidden the reaction could occur only via a weak d admixture which would easily be obscured by background processes. Favoring the latter interpretation is the fact that a $J=\frac{1}{2}$ classification for the 7.99-Mev level is consistent with the γ-ray branching.

Little can be said for the 7.30- and 6.81-Mev states except that the low (d,p) intensities suggests they might possess some of the features discussed above. Stripping angular distributions for these states could help shed more light on this.

V. SINGLE-PARTICLE STATES OF EVEN PARITY

States of even parity $[(-1)^{A+1}]$ for B^{11} are expected from configurations higher than s^4p^7; in particular, the lowest of the even-parity states can be regarded as belonging to configurations[27] $1s^4p^62s+1s^4p^61d+1s^3p^8$. Lane[28] has proposed that states of this type arise in the p-shell nuclei by weak coupling of a $2s$ or $1d$ nucleon to a definite (J,T) state of a core configuration (in this case s^4p^6). Similarly, even-parity states can also be formed by removal of a $1s$ nucleon from s^4p^8. Whereas in the discussion of p-shell states in Sec. III it was not, strictly speaking, meaningful to consider the coupling of a single p nucleon to a "partial configuration" of p nucleons, we can now specify a definite coupling between the *inequivalent* s or d nucleon and a distinct "core" state. For the first few even-parity levels, the core states are expected to be the ground and lowest excited states of the neighboring nuclei, in particular B^{10}. We propose that the three states of B^{11} near 9 Mev conform with this weak-coupling picture.

jj-Double Level p^6s: $B^{11}{}_{9.19\text{ Mev}}-B^{11}{}_{9.28\text{ Mev}}$

The good fit with theoretical curves of the angular distributions for the 9.19-Mev and 9.28-Mev states of B^{11} shown in Figs. 11 and 12 as well as the high relative intensities of these levels suggest that both result from direct capture of a neutron into the $2s$ shell without appreciable excitation of the B^{10} core. Since the spin of the core is $J_0=3+$, the $j_n=\frac{1}{2}$ total angular momentum of the neutron can couple to the core in two ways resulting in states of $J=\frac{5}{2}+$ or $J=\frac{7}{2}+$. The nuclear two-body forces between the rather weakly bound $2s$ neutron and members of the core can split these otherwise degenerate states into a close doublet—a jj-double level.[5] It should be noted that the spin-orbit potential does not contribute to the splitting since the nucleon in question has s character.

Assuming that the 9.19- and 9.28-Mev levels are formed in the manner outlined above, the intensities should be in the ratio of the statistical factor $(2J+1)$ since the reduced widths are $S_0(2s)=1$ and the single-particle widths $\theta_0^2(2s)$ are essentially equal for both states [the $\theta_0^2(2s)$ should vary very little over 90 kev]. Thus it is expected that the ratio of intensities for the two states would be

$$\sigma(J=\tfrac{5}{2})/\sigma(J=\tfrac{7}{2})=(2\times\tfrac{5}{2}+1)/(2\times\tfrac{7}{2}+1)=0.75.$$

The measured ratio (Table I) is found to be

$$\sigma(9.28)/\sigma(9.19)=0.74\pm0.03,$$

which suggests strongly that the spin and parity of the 9.28-Mev state is $J=\frac{5}{2}+$ and that for the 9.19-Mev state is $J=\frac{7}{2}+$.

These assignments are in disagreement with the early interpretation of (γ,γ) and (α,γ) angular correlation measurements[1,2] but appear to agree with the more recent γ-ray relative intensities.[25,29]

[27] E. C. Halbert and J. B. French, Phys. Rev. **105**, 1563 (1957).
[28] A. M. Lane (unpublished), see reference 27.
[29] P. P. Singh (private communication).

A stringent test of the single-particle nature of these states can be applied using the absolute (d,p) widths. This is especially good for the "pure" $l_n=0$ transitions. Unfortunately, only relative stripping cross sections have been measured for the states of B^{11}. Nevertheless, Macfarlane[30] has estimated the absolute widths on the basis of the present measurements of intensities of these states relative to $B^{11}{}_0$ and the known cross section for the mirror reaction $B^{10}(d,n)C^{11}{}_0$ measured by Maslin, Calvert, and Jaffe[31] at $E_d=9$ Mev. Assuming that the latter cross section is approximately equal to that for $B^{10}(d,p)B^{11}{}_0$, Macfarlane arrives at the absolute widths, granting aforementioned spin assignments,

$$(\theta^2)_{9.19\text{ Mev}}=(\theta^2)_{9.28\text{ Mev}}\approx0.14. \qquad (7)$$

This is consistent with current estimates for the $2s$ single-particle widths,

$$0.11\leq\theta_0^2(2s)\leq0.20, \qquad (8)$$

based on an analysis of $l_n=0$ transitions in the upper $1p$ shell and the $2s$-$1d$ shell. We can conclude from the information at hand that $B^{11}{}_{9.19\text{ Mev}}$ and $B^{11}{}_{9.28\text{ Mev}}$ both fit into a single-particle interpretation. A careful measurement of the cross section for $B^{11}{}_0$ at 7.8 Mev, or better for $B^{11}{}_{9.19\text{ Mev}}$ and $B^{11}{}_{9.28\text{ Mev}}$, would help reduce the 40–50% uncertainty in the estimate [Eq. (7)] for the reduced widths.

In order to compute the splitting of the p^62s double level, the p^6 core function was assumed to consist exclusively of configurations of the B^{10} ground state. This assumption is justified because B^{10} does not possess any low-lying $J=3$ excited states, and because a state of $J=2$ does not appear until an excitation of 3.58 Mev. Although the latter can be expected to contribute slightly to the formation of the p^62s levels, this contribution is assumed to have an insignificant effect on the results of the present calculation.

The B^{10} ground-state wave function, as computed in terms of the ${}^{2T+1,\,2S+1}L$ multiplets by French,[32] was decoupled by normalized Racah coefficients[33] and cfp's[24] into multiplets of a 9-nucleon core plus a p nucleon. The contribution to the total B^{11} binding energies in the $J=\frac{5}{2}+$ and $J=\frac{7}{2}+$ states could thus be evaluated in terms of the central interactions $\langle 1p,2s|H|1p,2s\rangle=E_{ps}$ of the decoupled p particle and the added s particle in their four $(T,S)=(0,0)$; $(0,1)$; $(1,0)$; $(1,1)$ states. The calculation amounted to the

[30] M. J. Macfarlane (private communication). We are indebted to Dr. Macfarlane for allowing us to quote his results here.
[31] E. E. Maslin, J. M. Calvert, and A. A. Jaffe, Proc. Phys. Soc. (London) **A69**, 754 (1956).
[32] J. B. French, University of Pittsburgh Technical Report No. IX, NR022-068, 1958 (unpublished).
[33] M. Simon, J. H. Vander Sluis, and L. C. Biedenharn, University of California Radiation Laboratory Report ORNL-1679, 1954 (unpublished).

evaluation and summation of elements of the form

$$\Big[K(\alpha_1 L_1 S_1 J_1 T_1;\zeta)\bar{X}\begin{pmatrix} L_1 & S_1 & 3 \\ 0 & \frac{1}{2} & \frac{1}{2} \\ L_1 & S & T \end{pmatrix}\langle\Psi_1\times j(6)\|\Psi_2\rangle \times U(L_2 1 L 0; L_1 1)U(S_2\tfrac{1}{2}S\tfrac{1}{2}; S_1 S_{ps}) \times U(\tfrac{1}{2}\tfrac{1}{2}\tfrac{1}{2}\tfrac{1}{2}; 0T_{ps})\Big]^2 E_{ps},$$

where K is the coefficient of the B^{10} wave function, α denotes the space symmetry partition, ζ is the intermediate coupling parameter, and $\bar{X}$ is the $9j$ symbol. Subscripts ps refer to the $p+s$ particle system, subscripts 1 refer to B^{10}, subscripts 2 to the 9-nucleon core, and symbols without subscripts to B^{11}.

Numerical evaluation of the contribution to the total binding energy from the $p+s$ system, with $\zeta=3.8$, yields for $J=\frac{5}{2}$,

$$E(\tfrac{5}{2})=0.0838E_{00}+0.1663E_{01}+0.2515E_{10}+0.4991E_{11}, \quad \text{(9-a)}$$

and for $J=\frac{7}{2}$,

$$E(\tfrac{7}{2})=0.0466E_{00}+0.2036E_{01}+0.1397E_{10}+0.6108E_{11}. \quad \text{(9-b)}$$

Calculations with $\zeta=5.7$ resulted in values very close to the above.

The splitting of the double level is equal to the difference of the total binding energies for these two levels:

$$\begin{aligned} E(\tfrac{5}{2})-E(\tfrac{7}{2}) &= 0.0372E_{00}-0.0373E_{01}+0.1118E_{10}-0.1117E_{11} \\ &= 0.0372(E_{00}-E_{01})+0.1118(E_{10}-E_{11}). \quad (10) \end{aligned}$$

A reasonable set[32] of Rosenfeld-type E_{TS} values, -5.0, 4.6, 0.72, -3.0 Mev, gives a splitting of -250 kev, the $J=\frac{5}{2}$ level falling *below* the $J=\frac{7}{2}$ state, contrary to observation. An Inglis-type interaction leads to even greater negative splitting. However, as can be seen from Fig. 10, the 8.92-Mev level contains an appreciable s admixture. The effect of this $J=\frac{5}{2}$ level on the $J=\frac{5}{2}$ member of the $p^6 2s$ doublet is to displace the latter upward, thus mitigating the discrepancy. This question is fully analyzed in another paper.[34]

Single-Particle $p^6 d$ State: $B^{11}_{8.92\text{ Mev}}$

The (d,p) angular distributions for the 8.92-Mev state of B^{11} (Fig. 10) indicates neutron capture with $l_n=0$ and $l_n=2$, again implying even parity for the final state. The s-wave admixture ($\approx 45\%$) restricts the final spin to $\frac{5}{2}+$ or $\frac{7}{2}+$. The level thus appears to be formed by capture of a $d_{\frac{5}{2}}$ neutron which couples with the $J_0=3+$ ground state of the B^{10} core to yield a character of $J=\frac{5}{2}+$ or $\frac{7}{2}+$. Furthermore, the high stripping intensity observed in the formation of $B^{11}_{8.92\text{ Mev}}$ suggests that the core configuration is similar to the ground state of B^{10} and that there is little admixture of states of other J. This apparent purity of the core state is consistent with the "weak coupling" proposal of Lane. Measurements of the absolute (d,p) width, however, are necessary to be certain that the $B^{11}_{8.92\text{ Mev}}$ state has single-particle nature.

An attempt to identify the state $B^{11}_{8.92\text{ Mev}}$ as $B^{11}(\frac{5}{2})_{\text{II}}$ appears to be ruled out since we cannot reconcile the angular distribution with $l_n=1$. Furthermore, from Fig. 13 one sees that the reduced width of $B^{11}(\frac{5}{2})_{\text{II}}$ should always be smaller than the ground-state reduced width. If $B^{11}_{8.92\text{ Mev}}$ has a classification of $J=\frac{5}{2}-$ and is formed by $l_n=1$ capture, then from the intensities in Table I an experimental ratio of reduced widths $S/S_{\text{gnd}}=1.83$ is obtained—a value so large as to cast considerable doubt on such an assumption.

The width of the 8.92-Mev γ transition[25] to the ground state of B^{11} implies an $E1$ transition which rules out a spin of $\frac{7}{2}+$ for the 8.92-Mev state since the ground state of B^{11} has $J=\frac{3}{2}-$. Thus, $J=\frac{5}{2}+$ for $B^{11}_{8.92\text{ Mev}}$ is the most probable assignment. This is in agreement with the γ-ray relative intensities measured from the $Be^9(He^3,p)B^{11}$ reaction[25] of 86:9:5 for the three γ rays from the 8.92-Mev level to the ground state, to the 2.14-Mev and 6.76–6.81 Mev levels (unresolved), and to the 4.46-Mev state. Roughly the ratio 86/5 $\sim(8.92/4.46)^3$ suggests that the 4.46-Mev γ ray also corresponds to an $E1$ transition. Furthermore, an $E1$ transition to the $J=\frac{7}{2}-$ level at 6.76 Mev and possibly a weak $M2$ transition to the 2.14-Mev $J=\frac{1}{2}-$ state could account for the remaining branching of 9%.

VI. CONCLUDING REMARKS

For convenience we have summarized our assignments in Fig. 14 adjacent to the respective energy levels. The levels of B^{11} fall into three definite groups with respect to excitation energy. The lowest consists of the five odd-parity states covering the region 0–6.76 Mev; next there appear four states characterized by weak stripping intensities suggestive of highly mixed configurations and possible even parity; and finally three single-particle states of even parity near 9 Mev.

The close agreement of our assignment with those proposed[25] by the Chalk River Group from γ-ray work has been already mentioned. Further evidence for the character of the B^{11} states is the Be^{11} beta-decay scheme measured by Wilkinson and Alburger.[35] A difficulty arises here, however, because the assignment of the ground state of Be^{11} is not yet firmly established, although the shell model suggests $J=\frac{1}{2}-$. Allowed beta decays are seen to the ground state, 2.14-, 6.76–6.81-, and 7.99-Mev levels in B^{11}. The first two decays are consistent with the $J=\frac{3}{2}-$ and $J=\frac{1}{2}-$ assignments for the respective B^{11} states. The beta decay to the

[34] O. M. Bilaniuk and J. B. French (to be published).

[35] D. H. Wilkinson and D. E. Alburger, Phys. Rev. **113**, 563 (1959).

6.76–6.81 Mev doublet cannot involve the lower, $J=\frac{7}{2}-$ member, so the 6.81-Mev state is probably $\frac{3}{2}-$ or $\frac{1}{2}-$. Finally, a decay to the 7.99-Mev state is reasonable in view of the possible $J=\frac{1}{2}$ assignment. There is an interesting question about the parity of Be^{11} ground state. The log ft values for the beta decay to the ground and first excited states of B^{11} are large enough to be consistent with first-forbidden transitions. Also, the decays to the 6.81- and 7.99-Mev states have lower log ft consistent with allowed transitions. In view of the possibility that these latter states have even parity, we are faced with the unusual situation of even parity for Be^{11}. A final puzzle is the absence of beta decay to the $J=\frac{3}{2}-$, 5.03-Mev state. This conceivably could be due to a nearly vanishing matrix element resulting from the large mixture of configurations found in this state (see Table III).

To the right of the B^{11} energy level scheme in Fig. 14 we have drawn energy levels for C^{11}. The correspondence between these is striking up to 9-Mev excitation. Furthermore, a close parallel can be drawn between the present results from the $B^{10}(d,p)B^{11}$ reaction and the information available for the mirror reaction $B^{10}(d,n)C^{11}$. The (d,n) spectrum[36] at $\theta=0°$ is very similar to the (d,p) spectrum at $\theta=20°$ in Fig. 2. The lowest five states of C^{11} formed with $l_p=1$ are probably in a one-to-one correspondence to the respective first five states of B^{11}. Its very large stripping width strongly suggests that the 6.50-Mev state of C^{11} is the analog to the 6.76-Mev $J=\frac{7}{2}$ state of B^{11}. However, the fairly large cross section leading to the formation of the 6.77-Mev state in C^{11} makes its correspondence with the very weak 6.81-Mev B^{11} state questionable in spite of the failure of a recent search[4] to uncover a new level in this region. The strong single-particle states of B^{11} near 9 Mev probably correspond to the 8.53- and 8.66-Mev states in C^{11} which also have very large (d,n) widths. The analog to the jj-double levels is probably unresolved in the "8.66-Mev state."

Flanking the experimental energy levels in Fig. 14 is a theoretical level scheme for $\zeta=4.0$, $L/K=5.8$, and $K=1.3$ Mev obtained from curves calculated by Kurath.[6] There is clearly a marked correspondence between the low-lying levels of B^{11} and the predicted p-shell states. Although the positions do not match, this must be considered good agreement. Some improvement could be made by going to lower L/K which raises the $J=\frac{7}{2}$ level with respect to the $J=\frac{3}{2}$ and $J=\frac{5}{2}$. An extensive effort to gain an optimum fit was not attempted. No account in the calculation has been made for the presence of small $1f$- and $2p$-shell admixtures nor configurations with 2 nucleons raised

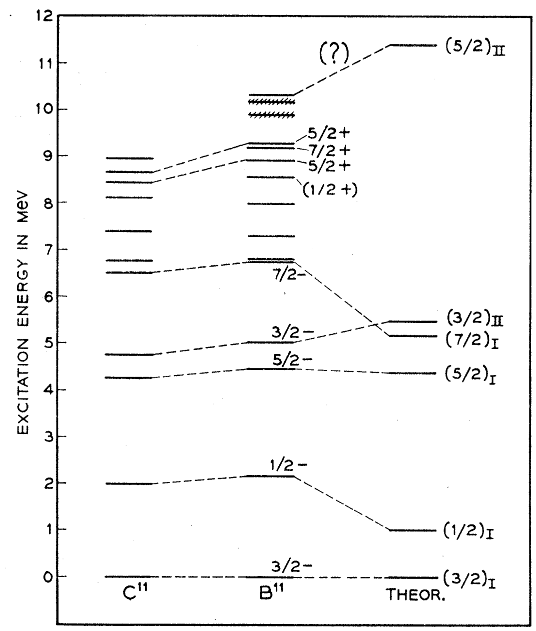

FIG. 14. Energy level schemes for $A=11$, $T_z=\pm\frac{1}{2}$. The proposed spin and parity assignments are indicated for the B^{11} states. A correlation for the odd-parity states of B^{11} is made with the theoretical scheme of Kurath (reference 6) which flanks the experimental levels on the right. The value of intermediate-coupling parameter selected was $\zeta=4.0$ in accord with the experimental results. The value of the radial integral parameter used was $L/K=5.8$ and $K=1.3$ Mev. A possible correspondence with the states of C^{11} is shown at the left.

from the $1s$ core into the p shell. These admixtures would push down the upper states and thus tend to close the large gap between the $J=\frac{1}{2}$ first excited state and the next three states of $J=\frac{5}{2}, \frac{7}{2}, \frac{3}{2}$.

The second $J=\frac{5}{2}$ p-shell state very likely falls in the vicinity of 10 Mev. As mentioned earlier, the (d,p) width to this level should be rather large, and this may correspond to the 10.32-Mev level observed earlier by stripping.[4] The identification of this state would be helpful in testing the model further.

VII. ACKNOWLEDGMENTS

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[36] V. R. Johnson, Phys. Rev. 86, 302 (1952), Fig. 1.