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Endothermic Deuteron Stripping Reactions. I. Stripping Near Threshold*†

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Wilkinson's explanation of the good agreement between theory and experiment for low energy, low Q value, (d,p) and (d,n) stripping reactions is restated from the point of view of the dispersion relations derivation of the Butler stripping formula. From this point of view, the kinematical conditions which should lead to the least distortion of the stripping pattern appear in a simple way. As a consequence of Wilkinson's arguments, it is proposed that the stripping mechanism should contribute significantly to endothermic (d,p) or (d,n) reactions near threshold. Some (d,n) threshold work is discussed, and it is concluded that this work gives some evidence that the stripping mechanism is important near threshold. The information on the reaction mechanism near threshold which can be obtained from γ-ray angular distribution measurements in $(d,p\gamma)$ or $(d,n\gamma)$ reactions is discussed and a general form for the angular distribution function for either plane-wave or distorted-wave stripping is developed. The $B^{11}(d,n)C^{12}$ (15.1-Mev level) reaction is considered in some detail and it is concluded that the experimental evidence is consistent with this reaction proceeding via the stripping mechanism near threshold.

1. INTRODUCTION

IT has been pointed out by Wilkinson[1] that for deuteron energies less than about 2 Mev, (d,p) or (d,n) reactions of low Q values (± 1 Mev or so) show strikingly good stripping patterns, while high Q values often lead to bad stripping patterns—even for high deuteron energies. Here a good stripping pattern is defined as one for which the stripped nucleon has a distribution which is well represented for all angles by the Butler formula with a suitable choice of the effective stripping radius. A large body of experimental data on (d,p) and (d,n) angular distributions support Wilkinson's observation, the latest work being that of Sellschop[2] on the $Li^7(d,p)Li^8$ (ground-state) and $C^{12}(d,p)C^{13}$ (3.09-Mev level) reactions for deuteron energies between 0.5 and 2.5 Mev. Wilkinson gave a simple explanation of this phenomenon. His main argument follows from a consideration of the separation of the stripped nucleon and the target nucleus during the stripping process. If the deuteron energy is low and the Q value is high, then the nucleon that is stripped must obtain most of its high linear momentum from the deuteron ground-state wave function. In order for this to occur, the separation of proton and neutron must be small at the instant of stripping. For low Q values, however, the correspondingly low linear momentum of the stripped nucleon can easily be obtained even for quite large separations of neutron and proton, so that the stripped nucleon need not approach close to the target nucleus for stripping to occur. For low Q values, then, the distortion of the stripping pattern due to nuclear interactions involving the stripped nucleon should be considerably less than in the high Q-value case. For low deuteron energies, the best stripping patterns are expected when the deuteron energy in the center of mass is approximately equal to $-2Q$—that is, when the linear momentum of the stripped nucleon is about half that of the incident deuteron.

In addition to the above consideration, when the Q value is low, the binding energy of the captured nucleon in the residual nucleus is also low (greater than the Q value by the binding energy of the deuteron). In this case the captured nucleon wave function in the final state has a long relaxation length outside the nucleus, and stripping can take place when the captured nucleon is anywhere in this long exponential tail, with the stripped nucleon a correspondingly further distance away from the target nucleus. As the Q value increases, the exponential tail shortens and in order to be captured a nucleon must approach closer to the nuclear surface at the same time as the neutron-proton separation is diminishing.

In order to obtain good stripping patterns, Coulomb distortion as well as nuclear distortion must be small. In this connection, Butler and Hittmair[3] have shown that when the major contribution to stripping comes from partial waves with high angular momenta, the angular distribution should be changed very little by Coulomb effects and, as was pointed out by Wilkinson, the forward peaking of the low Q-value angular distributions are proof in themselves that deuteron partial

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[1] D. H. Wilkinson, Phil. Mag. 3, 1185 (1958).

[2] J. P. F. Sellschop, Phys. Rev. Letters 3, 346 (1959); Phys. Rev. 119, 251 (1960).

[3] S. T. Butler in collaboration with O. H. Hittmair, *Nuclear Stripping Reactions* (Horwitz Publications Inc., Sydney, 1957).

waves of high momenta are indeed involved.[4] Moreover, the main effect of the Coulomb field on the stripping pattern will be to broaden the maxima and shift them toward larger angles (this effect can be largely compensated for by choosing a smaller effective stripping radius). The effect of the Coulomb field should show up more strongly in another way; namely, in an increasingly greater reduction of the cross section as the deuteron energy decreases. For this reason, reduced widths extracted by means of the Butler formula will become smaller as the deuteron energy decreases, even if the stripping pattern is quite good.

It is recalled that Oppenheimer and Phillips first proposed[5] the deuteron stripping mechanism to explain experiments showing that the yield of radioactivity due to (d,p) reactions for low-energy deuterons did not follow the Gamow barrier penetration function. They pointed out that the low binding energy and large average separation of the nucleons in the deuteron allows the neutron to approach the nuclear surface and be captured, while the proton is comparatively far away. This, together with the easy polarizability of the deuteron by the Coulomb field, leads to appreciably larger yields of (d,p) reactions below the Coulomb barrier than would be predicted if both the nucleons had to reach the nuclear surface to form a compound nucleus. Later work by Bethe[6] on the Oppenheimer-Phillips process indicated that the action of the Coulomb barrier on the incident deuterons did not favor the (d,p) stripping reaction over the (d,n) reaction for light $(Z<20)$ elements as much as had been previously indicated; but that the (d,p) stripping reaction was favored relative to compound nucleus formations for the additional reason that the compound nucleus when formed would prefer to decay by neutron emission. An extension of the latter argument is that for a low Q value (d,n) or (d,p) reaction there will be other exit channels which involve either neutrons or protons of considerable more energy. Thus, for either the (d,p) or (d,n) reaction, decay of the compound nucleus by channels other than the one corresponding to the low Q-value reaction in question will be energetically favored because the Coulomb and/or centrifugal barrier will be less.

As described by Wilkinson, the conditions which are responsible for the Oppenheimer-Phillips process will also lead to less distortion of the stripping pattern in low Q-value (d,p) and (d,n) reactions both by reducing nuclear distortion and by inhibiting the competing compound nucleus process. In a sense then, Wilkinson's explanation of the goodness of low Q-value stripping patterns bridges the gap between the Oppenheimer-Phillips process, for which the Coulomb barrier is quite important, and the seemingly quite different

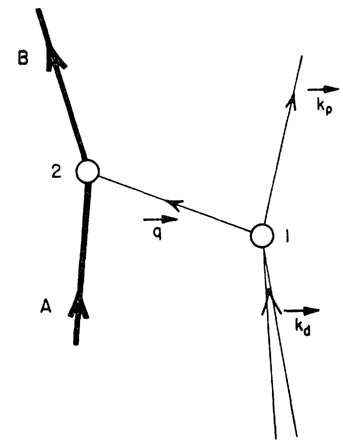

FIG. 1. Lowest order Feynman graph for the reaction $A(d,p)B$ with incoming deuteron wave number $\mathbf{k}_d$, outgoing proton $\mathbf{k}_p$, and momentum transfer $\mathbf{q}$ (from reference 8).

Butler stripping for which the Coulomb barrier can be neglected without appreciable error.

Wilkinson's main argument for the excellent agreement of many low Q-value stripping patterns with the Butler theory can be restated from the point of view of the dispersion relations derivation[7,8] of the Butler stripping cross section, giving some additional insight into the stripping mechanism. From the Feynman graph corresponding to the lowest order Born approximation for the stripping process (Fig. 1) the energy denominator for the captured nucleon is $1/(q^2+k_c^2)$ where ik_c is the wave number of the captured nucleon in its final state and q is the momentum with which the captured nucleon approaches the nuclear surface. There is then a simple pole in the unphysical region $q^2=-k_c^2$. In terms of the dispersion relations approach, the Butler formula is valid when this energy denominator is small compared to energy denominators of other poles and cuts or, in the case that there are other poles a comparable distance away, when the residues of these other poles are small compared to the stripping residue; therefore, saying a (d,p) or (d,n) reaction is close to the stripping pole is generally equivalent to saying the stripping cross section is large compared to that of other processes.

Assuming an infinitely heavy nucleus, the energy dependence of the denominator of the stripping pole can be written as

$$D=(\hbar^2/2m)(q^2+k_c^2)=3E+2Q+\epsilon-2(2E^2+2EQ)^{\frac{1}{2}}\cos\theta, \quad (1.1)$$

where all energies are in Mev, E is the deuteron energy in the center-of-mass system (the deuteron energy in the laboratory frame will be designated by E_d), θ is the center-of-mass angle of the stripped nucleon, and $\epsilon(=2.226$ Mev) is the deuteron binding energy. For $\theta=0°$, D has its smallest value at $E=-2Q$, while for positive Q values, D has a broad minimum at $E=Q$.

[4] The reason for the predominance of high partial waves in low Q-value stripping is discussed later in this section.
[5] J. R. Oppenheimer and M. Phillips, Phys. Rev. 48, 500 (1935).
[6] H. A. Bethe, Phys. Rev. 53, 39 (1938).
[7] R. D. Amado and R. Blankenbecler (to be published).
[8] R. D. Amado, Phys. Rev. Letters 2, 399 (1959).

The minimum in D at $E=-2Q$ is equal to ϵ, while at $E=Q$ it is $\epsilon+Q$; therefore, for $\theta=0°$ the stripping reaction is closest to the pole at $E\sim-2Q$.

The uncertainty principle can be used to estimate the dependence on deuteron energy of the proton-neutron separation at the instance of stripping. During the stripping process the captured nucleon must change its mean square momentum by $\Delta K^2=q^2+k_c^2$, and this virtual process must take place in a distance ΔR such that

$$\Delta R\Delta K\sim 1, \tag{1.2}$$

which gives

$$\Delta R\sim 4.6\times 10^{-13}D^{-\frac{1}{2}}\text{ cm}, \tag{1.3}$$

where D is given by (1.1) in Mev. We interpret ΔR as roughly equal to the distance from the nuclear surface to the mean position of the captured nucleon, or in other words, as the deuteron radius at the instant of stripping. Thus, Eq. (1.3) gives an estimate (within a factor of about 2) of the distance of the stripped nucleon from the nuclear surface at the instant of stripping as a function of the energy denominator D. In agreement with Wilkinson then, we find that for $E\sim-2Q$ the stripping process is, *on the average*, largest compared to other processes, while at the same time the stripped nucleon has its largest distance from the nuclear surface. In applying the uncertainty principle to obtain the estimate of Eq. (1.3) we have implicitly made the adiabatic assumption that the internal motion of the deuteron is much faster than the motion of the center of mass of the deuteron relative to the target nucleus; thus Eq. (1.3) should be most valid for low deuteron energies, and therefore in the region $E\sim-2Q$, with $Q\sim-1$ Mev.

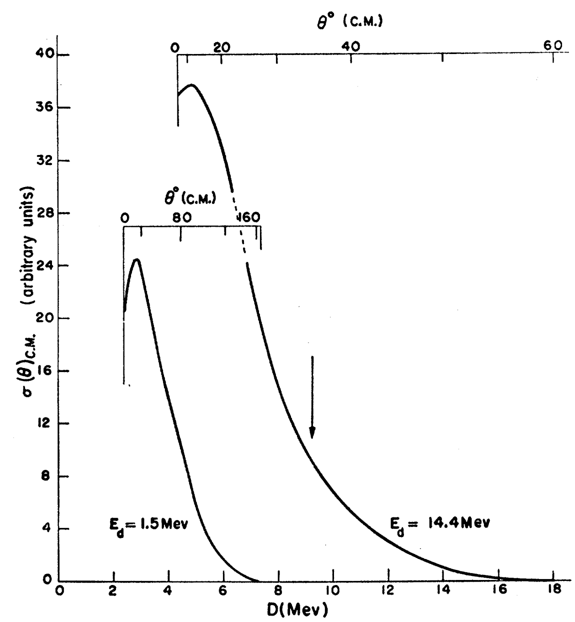

FIG. 2. Stripping patterns for the $Li^7(d,p)Li^8$ (ground-state) reaction ($Q=-0.192$ Mev). The relative differential cross section obtained from Butler theory is plotted against thedistance D in Mev from the stripping pole (see text). The two curves are identified by the deuteron energy in the laboratory frame of reference. The scales at the top of the curves give the corresponding center-of-mass angles of the outgoing proton. For the $E_d=14.4$-Mev curve, $D=58.2$ Mev at $\theta=180°$. The observed 1.5-Mev stripping pattern (reference 2) was found to fit the Butler theory for all angles, while the 14.4-Mev stripping pattern (reference 9) was observed to deviate from the Butler theory for value of D greater than $\sim$9 Mev (to the right of the vertical arrow).

As was pointed out by Butler and Hittmair,[3] a criterion for the validity of the simple stripping formula is in brief that the contribution to a (d,p) or (d,n) reaction be predominantly from high partial waves. As is clear from Eq. (1.3), the cross section at the stripping pole ($D=0$) is made up from infinitely high partial waves; and at the pole, Butler theory is exact except for the Coulomb correction to the cross section.[8] Near the pole the contribution to the cross section is predominantly from high partial waves, so that an alternative criterion for the validity of the simple stripping formula is, as stated by Amado,[8] that the Butler theory is valid when it is "riding the shoulder" of the pole. Butler and Hittmair[3] have shown that when the major contribution to stripping comes from high partial waves, the angular distribution should be changed very little by Coulomb effects, and in this case, have shown that there is a simple angle-independent multiplicative factor which corrects the stripping cross section for Coulomb effects. Thus from this point of view, it is clear why low Q-value angular distributions are distorted very little by Coulomb effects even for low deuteron energies.

It is well known that there is less agreement between the Butler theory and the experimental (d,p) or (d,n) angular distributions as the scattering angle, θ, increases. From the present point of view this is so because the distance from the pole increases with θ—i.e., D increases with θ as is clear from Eq. (1.1). A rather dramatic illustration of the fact that the validity of the simple Butler theory is dependent on the distance from the stripping pole is provided by a comparison of angular distributions obtained for the $Li^7(d,p)Li^8$ (ground-state) reaction at deuteron energies of 1.5 Mev[2] and 14.4 Mev.[9] In Fig. 2, the theoretical Butler angular distributions for this reaction at these two energies are shown plotted against the distance from the stripping pole in Mev. The $Li^7(d,p)Li^8$ ground-state reaction has a Q value of -0.192 Mev and a reduced width close to the single-particle value—which means an inherently large stripping cross section (or residue at the pole). Thus this reaction is one for which Wilkinson would predict a good stripping pattern for low deuteron energies as long as the deuteron energy were not appreciably influenced by compound nucleus formation. In actual fact, Sellschop[2] found that at $E_d=1.5$ Mev the angular distribution could be fitted by the Butler theory for *all* angles. In contrast to this, Levine *et al.*[9]

[9] S. H. Levine, R. S. Bender, and J. N. McGruer, Phys. Rev. **97**, 1249 (1955).

obtained an angular distribution at $E_d = 14.4$ Mev which deviated from the Butler formula at about 35°—that is, at a distance from the pole ($D \cong 9$ Mev) beyond that reached by the 1.5-Mev distribution at $\theta = 180°$. Other experiments on the $\mathrm{Li}^7(d,p)\mathrm{Li}^8$ (gound-state) reaction with deuteron energies of 1.9 Mev[2] and 8 Mev[10] also indicate that the angular distributions deviate from the Butler formula at $D \cong 9$ Mev. This fact can be interpreted as meaning that for this reaction other processes cannot be neglected for distances from the stripping pole greater than ~ 9 Mev, partly because of the decreased stripping cross section and partly because the proton is approaching close enough to the nuclear surface for $D \gtrsim 9$ Mev for nuclear and Coulomb distortions to become noticeable [see Eq. (1.3)]. An examination of other stripping reactions shows that the value of D at which the angular distribution deviates from the Butler formula varies considerably from reaction to reaction. Thus, as is certainly expected, other variables besides those emphasized in the present simplified discussion play an important part in determining the angular dependence of the stripping cross section; however, for those reactions which were examined, the value of D at which an angular distribution deviates from the Butler formula was not noticeably dependent on the deuteron energy.

The remainder of this paper will be concerned with the relative importance of stripping and compound nucleus formation near threshold ($E = -Q$) in endothermic (d,p) or (d,n) reactions. Because of the large Coulomb and nuclear interactions which could accompany very small values of the outgoing linear momentum, it would be surprising if good stripping patterns were obtained for endothermic stripping reactions near threshold. However, it is expected that the stripping contribution (no matter how distorted) to endothermic (d,n) or (d,p) reactions at threshold competes with the compound nucleus contribution to a much greater extent than had been popularly supposed prior to Wilkinson's appraisal of stripping at low energies. This is because for $E = -Q$ the energy denominator of the stripping pole is $D = \epsilon - Q$ which increases to $D = \epsilon$ for $E = -2Q$. Therefore, for Q values of ~ -1 Mev the reaction proceeds quite close to the stripping pole from threshold to deuteron energies several Mev above threshold.

The main purpose of this work is to find means of assessing the relative importance of stripping and compound nucleus formation near threshold without recourse to detailed and laborious theoretical calculation. In the next section, the presently available experimental evidence relating to the relative contributions of these two reaction mechanisms is discussed. It is concluded that the (d,n) work of the Rice Institute group[11–15] gives some evidence that the stripping mechanism is important near threshold; however, this evidence is not conclusive. In Sec. 3 the information on the reaction mechanism which can be obtained from γ-ray angular distribution measurements in $(d,p\gamma)$ or $(d,n\gamma)$ reactions is discussed, and as an example the $\mathrm{B}^{11}(d,n)\mathrm{C}^{12}$ (15.1-Mev level) reaction is considered in some detail. In the paper following as Part II, an application of this method to the $\mathrm{C}^{12}(d,p)\mathrm{C}^{13}$ reaction is described.

2. ENDOTHERMIC (d,n) REACTIONS NEAR THRESHOLD

The most obvious way to determine the relative contributions of stripping and compound nucleus formation in (d,p) or (d,n) reactions is from a study of the angular distributions of the outgoing nucleons. The difficulty of doing this for moderate incoming and outgoing energies has been recently re-emphasized.[16] The main complications which deter such an analysis are interference between stripping and compound nucleus formation and the initial and final state interactions, both nuclear and Coulomb. For an endothermic (d,p) or (d,n) reaction near threshold, the situation is even worse and it becomes practically impossible to distinguish compound nucleus formation and stripping by means of the angular distributions. This is the case for the same reasons which apply at moderate deuteron energies, and for the additional reason that both mechanisms (in the absence of interference) will give rise to isotropic, or nearly isotropic, distributions when the wave number k_f of the outgoing nucleon is small compared to the wave number of the deuteron. The stripping distribution is expected to be relatively flat because the differential cross section becomes insensitive to θ for $k_f/k_d \ll 1$; and the compound nucleus cross section will most likely be isotropic because s-wave nucleon emission is greatly favored for $k_f/k_d \ll 1$.[17] In any case, it would be experimentally difficult to determine angular distributions very close to threshold because of the low energy of the outgoing nucleus.

A second possible way of assessing the relative contributions of stripping and compound nucleus formation near threshold is to study the behavior of

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[10] J. R. Holt and T. N. Marsham, Proc. Phys. Soc. (London) **A66**, 1032 (1953).

[11] T. W. Bonner and C. F. Cook, Phys. Rev. **96**, 122 (1954).

[12] J. B. Marion, R. M. Brugger, and T. W. Bonner, Phys. Rev. **100**, 46 (1955).

[13] J. B. Marion, T. W. Bonner, and C. F. Cook, Phys. Rev. **100**, 847 (1955).

[14] M. V. Harlow, J. B. Marion, R. A. Chapman, and T. W. Bonner, Phys. Rev. **101**, 214 (1956).

[15] J. C. Slattery, R. A. Chapman, and T. W. Bonner, Phys. Rev. **108**, 809 (1957).

[16] A. Elwyn, J. V. Kane, S. Ofer, and D. H. Wilkinson, Phys. Rev. **116**, 1490 (1959).

[17] If the compound nuclear process with $l=0$ nucleon emission were forbidden by some selection rule so that $l \geq 1$ nucleon emission were predominant, then, of course, the nuclear angular distribution could deviate significantly from isotropy. However, the compound nucleus cross section would probably be too small in this case to allow a meaningful measurement of the angular distribution.

the total cross section. Consider the deuteron energy region within 100 kev or so of threshold. Here the (d,p) reaction is suppressed by the Coulomb barrier regardless of the reaction mechanism and, in fact, at the present time no endothermic (d,p) reactions have been studied within 200 kev of threshold. It would appear that studies of endothermic deuteron-induced reactions right at threshold must, for experimental reasons, be confined to (d,n) reactions. For (d,n) thresholds occurring at deuteron energies of 1 Mev or so, the barrier transmission for the incoming deuteron or captured proton is essentially constant from threshold to several hundred kev above threshold; thus, the excitation function of the (d,n) reaction near threshold will be, to a good approximation, a function of the outgoing neutron only. In the energy interval in which all magnitudes can be considered constant except those which vanish at threshold, the cross section will then be proportional to $(E-E_0)^{\frac{1}{2}}$ for s-wave neutron emission following compound nucleus formation or for a direct interaction mechanism. Here $E_0=-Q$ is the center-of-mass energy at threshold and

$$k_n \propto (E-E_0)^{\frac{1}{2}}.$$

Quite a few (d,n) thresholds have been observed, mostly by Bonner and his collaborators[11–15]; however, the main purpose of these investigations was to obtain accurate reaction Q values, and not much information on the excitation functions close to threshold has been obtained.

One example of a well-investigated (d,n) threshold is the $B^{11}(d,n)C^{12}$ (15.1-Mev level) reaction. The threshold was determined to be at $E_d=1627\pm4$ kev using the counter ratio technique[3] and 1633 ± 3 kev from observations on the yield of the 15.1-Mev de-excitation γ rays.[18] In the latter investigation an excitation curve was obtained up to several Mev above threshold, and the closest compound nucleus resonance to threshold was observed at a deuteron energy of 2.180 Mev; therefore, the cross section at threshold is due to stripping[19] and/or contributions from overlapping compound nucleus states. The 15.1-Mev γ-ray yield within 70 kev of threshold was observed to have the $(E-E_0)^{\frac{1}{2}}$ shape expected for s-wave outgoing neutrons following compound nucleus formation or for the stripping mechanism. For higher deuteron energies, the yield increases faster than $(E-E_0)^{\frac{1}{2}}$ in a somewhat linear manner. Because of the large number of adjustable parameters available, it is probable that this yield curve could be fitted assuming either a direct or a compound nucleus reaction mechanism. In any case, nothing conclusive can be said about the (d,n) reaction mechanism near threshold merely by inspection of the energy-dependence of the total cross section.

It is noteworthy that the counter ratio work of the Rice Institute group[11–15] shows in general the most pronounced results (increase of cross section at threshold) for those states which would be expected to be formed by $l_p=0$ protons if stripping were involved. Three of these states at least have, or are expected to have, large proton reduced widths for the target nucleus. These states are the $\frac{1}{2}^+$, N^{13} 2.37-Mev level,[20] the 1^-, N^{14} 5.69-Mev level,[21,22] and the $\frac{1}{2}^+$, F^{17} 0.500-Mev level.[20] The N^{13} and F^{17} states are known to be single-particle $p^n 2s_{\frac{1}{2}}$ states and the N^{14} state probably mostly $p^n 2s_{\frac{1}{2}}$ states.[23,24] The experimental information on these states has several other features in common. For all three the counter ratio increased for a greater energy interval above threshold than is compatible with s-wave emission; p-wave emission could be involved, although the yields seem surprisingly high, while the slow rise above threshold is reminiscent of the original work on exothermic reactions which led to the proposal of the Oppenheimer-Phillips effect. A further similarity is that all three states have been observed[20,21] to have large cross sections for the radiative capture of p-wave protons, and in fact are the only s states in the upper p shell for which such a process has been observed. The involvement of all three states in radiative p-wave proton capture suggests that stripping should take place with a high probability near threshold, because there is a close analogy between radiative capture and stripping. The cross sections for both processes are proportional to the s-wave proton reduced width of the final state for the target and, even more apropos to the present case, both processes are strongly enhanced because of the large extranuclear extension of the final state wave function. For the three states in question this large extension is due to the character of the $p^n 2s$ wave function, as well as to the low binding energy, which, as Wilkinson pointed out, accompanies a low Q value in a (d,p) or (d,n) reaction. We feel that these facts taken together rule rather strongly in favor of the stripping mechanism at threshold for these three states.

For the N^{14} 5.69-Mev state there is an additional argument against compound nucleus formation. The threshold for this state occurs at an unusually low deuteron energy, $E_d=0.422$ Mev. As pointed out by Marion *et al.*,[11,13] compound nucleus formation, if it occurred, would probably proceed through capture of $l=0$ deuterons at such a low energy, and in this case the evidence for p-wave neutron emission would

[18] R. W. Kavanagh and C. A. Barnes, Phys. Rev. **112**, 503 (1958).

[19] We assume throughout this work that exchange stripping is negligible compared to direct stripping.

[20] F. Ajzenberg-Selove and T. Lauritsen, Nuclear Phys. **11**, 1 (1959).

[21] E. K. Warburton, W. T. Pinkston, H. J. Rose, and E. N. Hatch, Bull Am. Phys. Soc. **4**, 219 (1959).

[22] It has not been conclusively proven that the N^{14} 5.69-Mev level has odd-parity; however, the evidence is strong and it is assumed in this work that this state has $J^\pi=1^-$.

[23] E. K. Warburton, H. J. Rose, and E. N. Hatch, Phys. Rev. **114**, 214 (1959).

[24] E. K. Warburton and W. T. Pinkston, Phys. Rev. **118**, 733 (1960).

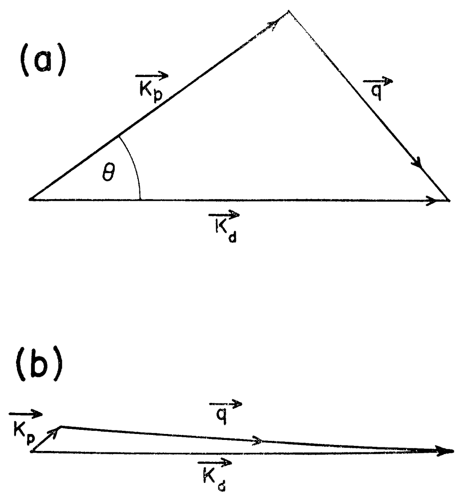

FIG. 3. Linear momentum diagram for an undistorted, plane-wave (d,p) stripping reaction (a) Exothermic reaction at a moderate deuteron energy; (b) Endothermic reaction near threshold.

indicate positive parity for the N^{14} 5.69-Mev level in contradiction with other evidence.[25]

3. ANGULAR DISTRIBUTION OF THE GAMMA RAYS FOLLOWING AN ENDOTHERMIC STRIPPING REACTION

One method which can be used to investigate the interplay of stripping and compound nucleus formation in (d,p) or (d,n) reactions leading to excited states is to study the angular correlation between the outgoing nucleon and the de-excitation γ rays. Examples of such studies are the work of Cox and Williamson,[26] of Martin,[27] and of Bromley *et al.*[28] No measurements have been made, however, of $(d,p\gamma)$ or $(d,n\gamma)$ correlations in the region $-Q\leq E\lesssim -2Q$.

It is well known that for the plane wave stripping theory, the $(\mathbf{q},\gamma)$ angular corelation in a $(d,f\gamma)$ reaction is the same as through a resonant (nucleon, γ) capture reaction had taken place in the residual nucleus. In this process $\mathbf{k}_f$ serves the double purpose of defining the direction of $\mathbf{q}$, and of carrying away sufficient energy to make the capture process possible. In general, the angular distribution of the de-excitation γ rays relative to the deuteron beam with particle f unobserved has no such simplicity. At threshold in an endothermic reaction, the angular distribution of the γ rays relative to the deuteron beam, however, is identical to the $(\mathbf{q},\gamma)$ correlation; and near threshold the γ-ray distribution will not be much different from the $(\mathbf{q},\gamma)$ correlation. This distribution occurs because, as illustrated in Fig. 3, the direction of $\mathbf{q}$ is not much different than that of $\mathbf{k}_d$ for $k_f/k_d\ll 1$.

It is much simpler to measure the γ-ray distribution than it is to measure the $(\mathbf{q},\gamma)$ correlation, especially near threshold where the low energy of the outgoing nucleons increases the difficulty of the correlation measurement; therefore, it is of interest to see what information concerning the reaction mechanism can be obtained from γ-ray distributions for endothermic $(d,f\gamma)$ reactions near threshold.

The most serious shortcoming of any attempt to decide the reaction mechanism from measurements of the γ-ray distributions becomes evident when an attempt is made to compare the results to those expected for compound nucleus formation. Except in very special circumstances, the nonzero spins of the deuteron and outgoing nucleon allow such a large choice for the channel spins and for the spin of the compound nucleus states contributing to the cross section that almost any distribution would be compatible with compound nucleus formation. In general then, any information concerning the reaction mechanism near threshold will come from agreement or disagreement of measured γ-ray distributions with the more concrete predictions of stripping. To investigate such distributions, we first consider the dependence on k_f/k_d (hereafter called α) of the γ-ray angular distribution for a $(d,f\gamma)$ reaction involving undistorted plane waves. The effect of distortion on the angular distributions will be considered in the Appendix.

The geometry of the $(d,f\gamma)$ reaction is shown in Fig. 4. The deuteron beam defines the axis of quantization and the γ rays are detected in the xz-plane. For the undistorted plane wave stripping model of Butler, the $(\mathbf{q},\gamma)$ correlation is given by

$$W(\mathbf{q},\mathbf{k}_\gamma;\vartheta')=\sum_{\nu=0}^{\nu_m} a_\nu P_\nu(\cos\vartheta'), \tag{3.1}$$

where ν is even and the a_ν are the coefficients for a resonant (nucleon, γ) capture reaction given, for instance, by Devons and Goldbarb.[29] The angular distribution of the γ rays is obtained by averaging over the intermediate (unobserved) emission of particle f,

$$W(\vartheta)=\int d\Omega_q W(\mathbf{q},\mathbf{k}_\gamma;\vartheta')\sigma_q(\theta_R)\Big/\int d\Omega_q\sigma_q(\theta_R). \tag{3.2}$$

[25] The threshold for this state can be compared to that for the B^{10} 4.77-Mev level which is the only level besides the three discussed here for which the Rice Institute group observed a slow rise above threshold (see reference 11). The threshold for this $J^\pi=2^+$ state occurs at $E_d=0.52$ Mev and indicates a weak cross section at threshold. At this low deuteron energy $l=0$ deuterons are expected to give the predominant contribution to compound nucleus formation and in this case, unlike the situation for the 1^-, N^{14} 5.69-Mev level, p-wave neutron emission would be necessary to form the even-parity B^{10} 4.77-Mev level from the odd-parity Be^9 ground state. Therefore, the B^{10} 4.77-Mev level could be formed from the (d,n) reaction at threshold through compound nucleus formation—as would appear likely because of the very small proton reduced width of this state (reference 20).

[26] S. A. Cox and R. M. Williamson, Phys. Rev. **105**, 1799 (1957).

[27] J. P. Martin, K. S. Quisenberry, and C. A. Low, Jr., Phys. Rev. **120**, 492 (1960).

[28] D. A. Bromley, J. A. Kuehner, and E. Almquist, Bull. Am. Phys. Soc. **5**, 56 (1960).

[29] S. Devons and L. J. B. Goldfarb, *Handbuch der Physik* (Springer-Verlag, Berlin, 1957), Vol. 42, p. 362.

In Eq. (3.2), $\sigma_q(\theta_R)$ is the differential cross section for recoil of the residual nucleus into the solid angle element $d\Omega_q$ at a polar angle θ_R.

The form of $W(\vartheta)$ can be determined from Eqs. (3.1) and (3.2) by means of results developed by Rose[30] to determine the angular resolution correction for γ-ray correlation and distribution measurements. The method is based on the addition theorem which in this case gives

$$P_\nu(\cos\vartheta')=\frac{4\pi}{2\nu+1}\sum_{-\nu}^{\nu} Y_\nu^{m*}(\vartheta,0)Y_\nu^{m}(\theta_R,\phi). \quad (3.3)$$

In the integration over ϕ in Eq. (3.2), only the $m=0$ terms contribute since $\sigma_q(\theta_R)$ is independent of ϕ. Thus

$$W(\vartheta)=\sum_{\nu=0}^{\nu m} a_\nu Q_\nu P_\nu(\cos\vartheta), \quad (3.4)$$

where

$$Q_\nu=\int_0^{\sin^{-1}\alpha} P_\nu(\cos\theta_R)\sigma_q(\theta_R)\sin\theta_R d\theta_R \bigg/ \int_0^{\sin^{-1}\alpha}\sigma_q(\theta_R)\sin\theta_R d\theta_R. \quad (3.5)$$

The upper limit on the integrals corresponds to the maximum value of θ_R which occurs when $\mathbf{k}_f\cdot\mathbf{q}=0$ (see Fig. 3) since α $(=k_f/k_d)$ is fixed for a given reaction. We are interested in γ-ray distributions close enough to threshold so that $P_\nu(\cos\theta_R)$ is always positive, a condition which is insured for $\alpha<0.82$ for $\nu=2$ and $\alpha<0.5$ for $\nu=4$. Then

$$P_\nu((1-\alpha^2)^{\frac{1}{2}})\leq Q_\nu\leq 1, \quad (3.6)$$

and Q_ν is an attenuation coefficient which depends on α only.

Close to threshold the angular distribution of particle f will be isotropic or nearly isotropic. For an isotropic distribution of particle f, the Q_ν can be evaluated from Eq. (3.5), using the identity

$$\sigma_q(\theta_R)\sin\theta_R d\theta_R=\sigma_f(\theta)\sin\theta d\theta,$$

and the geometry of Fig. 3. The result is

$$Q_2=1-\tfrac{3}{8}(1+\alpha^2)+\frac{3}{16\alpha}(1-\alpha^2)^2\ln\frac{1+\alpha}{1-\alpha}, \quad (3.7a)$$

$$Q_4=7/12+(5/12)(1-7\alpha^2)Q_2, \quad (3.7b)$$

for $\sigma_f(\theta)=$constant. Equation (3.7a) differs negligibly from $Q_2=1-\alpha^2$ for $\alpha\leq 0.25$ and by 1.7% for $\alpha=0.50$.

It is expected that the Q_ν will be altered somewhat when the effects of initial and final state interactions are included. The general form of the $(d,f\gamma)$ correlation in the distorted wave case has been given by Huby, Refai, and Satchler.[31] In the Appendix the results of Huby *et al.*[31] are used to obtain the general form of Q_ν for the case in which any of the several selection rules limits ν to 0 and 2, and $l_q=1$ or 2. In general, it is found that any effects of distortion will most probably lead to a decreased Q_2; and, in the region $\alpha<0.25$, any increase in Q_2 that is possible would be negligibly small compared to experimental errors.

As an application of the above considerations, consider the angular distribution of the ground state decay of the C^{12} 15.1-Mev level following the $B^{11}(d,n)C^{12}$ reaction. Since this transition to the 0^+, C^{12} ground state is dipole, the distribution relative to the beam must be of the form $1+A\cos^2\vartheta$ regardless of the reaction mechanism. The anisotropy, A, is given by $I(0^0)/I(90°)-1$. From studies at $E_d=5$ and 8 Mev,[32] the $B^{11}(d,n)C^{12}$ (15.1-Mev level) reaction is known to proceed by capture of $l_o=1$ protons with a proton reduced width within a factor of two of the single particle width so that, from Wilkinson's arguments, it might be expected that this reaction proceeds by $l_p=1$ stripping at threshold and just above threshold as well as at higher deuteron energies.

The anisotropy expected for the 15.1-Mev γ ray following resonant capture of $l_p=1$ protons by the C^{12} 15.1-Mev level is $A=(5-x)/(5+7x)$, where x is the intensity ratio between channel spins 2 and 1.

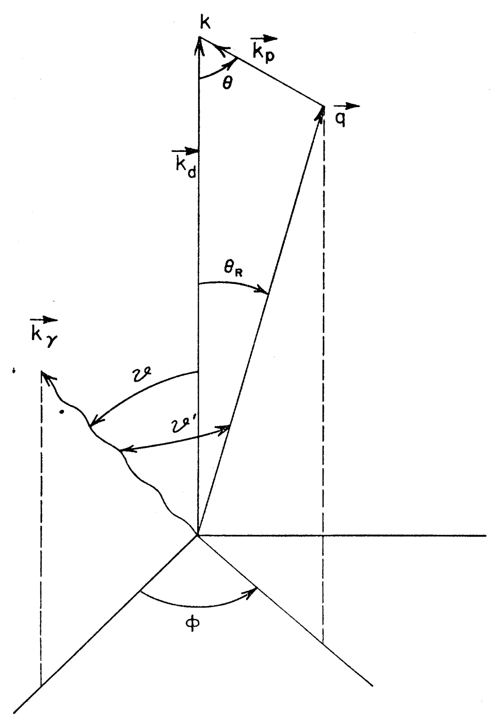

FIG. 4. Geometry for the angular distribution of the de-excitation γ rays in the $(d,p\gamma)$ stripping reaction. All quantities refer to the center-of-mass system. The deuteron beam direction is taken as the z axis. The angle θ is the center-of-mass angle of the outgoing protons. The γ rays are detected in the xz plane.

[30] M. E. Rose, Phys. Rev. 91, 610 (1953).

[31] R. Huby, M. Y. Refai, and G. R. Satchler, Nuclear Phys. 9, 94 (1958).

[32] A. J. Ferguson, H. E. Gove, A. E. Litherland, and R. Batchelor, Bull. Am. Phys. Soc. 5, 45 (1960).

Assuming plane wave stripping theory, Ferguson *et al.*[32] found $A=-0.08\pm0.02$ from $(d,n\gamma)$ angular correlation measurements at $E_d=5$ Mev. This anisotropy corresponds to $x\simeq 12$ and determines the a_ν of (3.1) for this reaction if the plane wave assumption is justified in this case.

Kavanagh and Barnes[18] found that the anisotropy relative to the beam of the 15.1-Mev γ ray was in the range $|A|<0.04$ at $E_d=2.5$ Mev decreasing slowly with decreasing energy to -0.13 ± 0.03 at $E_d=1.70$ Mev (70 kev above threshold where $\alpha=0.14$); therefore, the γ-ray anisotropy just above threshold agrees within the errors with the anisotropy obtained at the higher deuteron energies, and thus agrees with the predictions of stripping with no noticeable effects of distortion—i.e., this close to threshold the distortion could only decrease the magnitude of A, since Q_2 differs from unity by at most 3% for $\alpha=0.14$ (see Appendix). At $E_d=2.5$ Mev, where $\alpha=0.40$, the plane wave stripping prediction is $Q_2\geq 0.76$(3.6) which gives $A\leq -0.075$. Thus the results of Kavanagh and Barnes[18] ($|A|<0.04$ at $E_d=2.5$ Mev) indicate that distortion and/or the resonances at $E_d=2.180$ and 3.080 Mev have considerable effect on the $(d,n\gamma)$ distribution between $E_d=1.7$ and 2.5 Mev if in actual fact the reaction is proceeding predominantly by stripping.

It would seem that all the present information on the $B^{11}(d,n)C^{12}$ (15.1-Mev level) reaction is consistent with all or part of the cross section near threshold being due to the stripping mechanism; however, there seems no way to rule out the compound nucleus mechanism on the basis of present information.

The sparsity of information on endothermic (d,p) or (d,n) reactions near threshold makes conclusions concerning the reaction mechanism difficult. More information can be obtained from further measurements of excitation functions and $\gamma-$ray distributions, and such measurements on the $C^{12}(d,p)C^{13}$ (3.86-Mev level) reaction are reported in the paper following as Part II. Measurements of this type are not likely to be conclusive so that more informative (but also more difficult) measurements would be worthwhile. For instance, $(d,n\gamma)$ correlation measurements made just above threshold in the $B^{11}(d,n)C^{12}$ (15.1-Mev level) reaction could conceivably differentiate between the stripping and compound nucleus reaction mechanisms.

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APPENDIX

Huby *et al.*[31] have given the form of the angular correlation function $W(q,\gamma)$ for the case in which the complexity of the correlation is limited by $\nu_m=2$ because of some selection rule or other. By a transformation of coordinate axis, the result of Huby *et al.* is obtained for the geometry of Fig. 4:

$$W(\mathbf{k}_d,\mathbf{q}',\mathbf{k}_\gamma;\vartheta',\vartheta,\phi)=1+a_2\delta P_2(\cos\vartheta')+\tfrac{1}{4}a_2(\beta-\delta)\times[P_2(\cos\vartheta)+\tfrac{1}{2}P_2^2(\cos\vartheta)\cos 2\phi], \quad (1a)$$

where $\mathbf{q}'$—which is the symmetry axis—is in the $\mathbf{k}_d\times\mathbf{q}$ plane (see Fig. 4) and makes an angle ϕ_0 with q, and ϑ' is now the angle between k_γ and q'; a_2 is defined by (3.1), δ and β are defined in terms of parameters given by Huby *et al.*:

$$\delta=4A_2^2/a_2,\quad \beta=-2A_2^0/a_2. \quad (2a)$$

We wish to evaluate the distribution function

$$W(\vartheta)=\int d\Omega_q W(\mathbf{k}_d,\mathbf{q}',\mathbf{k}_\gamma;\vartheta',\vartheta,\phi)\sigma_q(\theta_R)\Big/\int d\Omega_q\sigma_q(\theta_R). \quad (3a)$$

As in Sec. 3, we make use of the addition theorem and the fact that $\sigma_q(\theta_R)$, β, and δ are functions of θ_R only and independent of ϕ. Then[33]

$$W(\vartheta)=1+a_2Q_2P_2(\cos\vartheta) \quad (4a)$$

where Q_2 is defined by

$$Q_2=\int_0^{\sin^{-1}\alpha}\sigma_q(\theta_R)[\delta P_2(\cos[\theta_R-\phi_0])+\tfrac{1}{4}(\beta-\delta)]\times\sin\theta_R d\theta_R\Big/\int_0^{\sin^{-1}\alpha}\sigma_q(\theta_R)\sin\theta_R d\theta_R. \quad (5a)$$

For the plane wave case, $\beta=\delta=1$, $\phi_0=0$, and Eq. (5a) reduces to Eq. (3.5) with $\nu=2$.

Case $l_q=1$. For capture of an $l_n=1$ neutron in a $(d,p\gamma)$ reaction or an $l_p=1$ proton in a $(d,n\gamma)$ reaction, $\beta=1$, $\delta=\lambda$, where λ is defined by Huby *et al.* and is limited by $0\leq\lambda\leq 1$. By means of Eq. (5a) we find $Q_2\leq 1$ and $Q_2-Q_2^p\leq\frac{3}{2}\alpha^2$ where Q_2^p is the attenuation coefficient for plane wave stripping; thus for small enough α, distortion would only be noticeable in a measurement of $W(\vartheta)$ if it caused a decrease in Q_2.

Case $l_q=2$. For this case the results of Huby *et al.* show $2\geq\beta\geq -2$, $(2/\sqrt{3})(1-\frac{1}{4}\beta^2)^{\frac{1}{2}}\geq\delta\geq 0$ and again $Q_2\leq 1$ with $Q_2-Q_2^p\leq\frac{3}{2}\alpha^2$.

For both $l_q=1$ and 2, the results of Satchler and Tobocman[34] lead us to expect that it is more probable that distortion will decrease Q_2 relative to Q_2^p rather than increase it.

[33] This result is given in a more elegant form by G. R. Satchler, Nuclear Phys. **18**, 110 (1960); and by G. R. Satchler and W. Tobocman, Phys. Rev. **118**, 1566 (1960). The latter reference also gives Eq. (3.4).

[34] G. R. Satchler and W. Tobocman, Bull. Am. Phys. Soc. **5**, 30 (1960); and G. R. Satchler and W. Tobocman, Phys. Rev. **118**, 1566 (1960).