Uniqueness Property of the Twofold Vacuum Expectation Value*

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It is shown under general assumptions that if the one-body Green's function equals its free-field value the theory is that of a free field.

WE would like to point out a simple and remarkable generalization of previously known theorems[1,2] regarding the vacuum expectation values of relativistic field theory operators. Namely, if the vacuum expectation value of the simple two-point function coincides with that for a free field at equal times, then all vacuum expectation values coincide with the corresponding free-field values.[3] The proof is essentially elementary. Consider the states,

$$\Psi=\int d^4x[(-\Box^2+m^2)f(x)]\phi(x)|0\rangle \tag{1}$$

where $f(x)$ is an arbitrary space-time function with reasonable properties such that surface terms vanish upon integration by parts. Then Ψ has zero norm since

$$(-\Box^2+m^2)\langle 0|\phi(x)\phi(y)|0\rangle=0. \tag{2}$$

(By assumption, $\langle 0|\phi\phi|0\rangle$ coincides with the free-field function at equal times, and therefore, by analyticity, at arbitrary times.) Consequently $\Psi=0$ if the metric is positive definite. Define

$$j(x)=(-\Box^2+m^2)\phi(x). \tag{3}$$

We see that from $\Psi=0$ if follows that

$$j(x)|0\rangle=0. \tag{4}$$

Accordingly, it is true that

$$\langle 0|j(x)\phi(y_1)\cdots\phi(y_n)|0\rangle=0. \tag{5}$$

It follows that

$$\langle 0|\phi(y_1)\cdots j(x)\cdots\phi(y_n)|0\rangle=0 \tag{6}$$

since both matrix elements are boundary values of the same analytic function of complex variables.[4]

We see immediately that:

(1) The operators satisfy the free-field equations.

(2) Canonical commutation relations imply all the matrix elements are equivalent to the free field ones.

(3) The weaker assumption of the asymptotic condition also yields the same result. (This follows directly from the Yang-Feldman equation.[5])

Although the result is therefore more general, we will assume the canonical commutation rules for simplicity.

It is clear that the same result follows with the assumption that any single vacuum expectation value with an even number of operators coincides with the corresponding free field expectation value. For by the same method we can then construct a null state and hence show that with a single $\phi(x)$ replaced by $j(x)$ we get a zero operator. It will then follow that all higher vacuum expectation values coincide with the free field functions. Finally, by reduction using commutators at equal times, all matrix elements must equal the free ones.

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[1] R. Haag, Kgl. Danske Videnskab. Selskab, Math.-fys. Medd. 29, No. 12 (1955).

[2] O. W. Greenberg, Phys. Rev. **115**, 706 (1959).

[3] We have been informed by R. Jost that the same result has been obtained independently by both R. Jost and B. Schroer.

[4] D. W. Hall and A. S. Wightman, Kgl. Danske Videnskab. Selskab, Math.-fys. Medd. **31**, No. 5 (1957).

[5] D. Feldman and C. N. Yang, Phys. Rev. **79**, 972 (1950).