Spin-Orbit Correlations in $\mu-e$ and e^--e^- Scattering

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The imaginary part of the fourth order matrix element for the electromagnetic scattering of two fermions gives rise to a small dependence of the cross section on the quantity $\mathbf{s}\cdot\mathbf{p}_1\times\mathbf{k}_1$, where $\mathbf{s}$ and $\mathbf{p}_1$ are the spin and momenta of one of the initial particles, and $\mathbf{k}_1$ the momentum of the same particle after scattering. The resulting angular asymmetry can probably be measured. The effect does not occur in the lowest order of scattering.

I

THE spin-momentum correlations in nucleon-nucleon scattering (the double scattering experiment), or in Mott scattering, are well known. No such effects seem to have been studied, however, either theoretically or experimentally, in the scattering of electrons, μ mesons, or positrons by atomic electrons. We have calculated the influence of the polarization of one of the initial particles on the total differential cross section for $\mu-e$ and e^--e^- scattering.[1]

The lowest order of scattering does not give any spin-momentum correlation, as will be shown. A fourth order calculation is therefore necessary, but only the imaginary part of the fourth order matrix element contributes to the lowest nonvanishing order of the spin dependence. In fourth order pure $\mu-e$ and e^--e^- scattering, only the diagrams of Fig. 1 have imaginary parts. We calculate the spin dependence of the interference of these diagrams with the second order scattering diagrams. The contribution to the cross section is of order e^6, while the main part of the cross section is of order e^4.

In the calculation of the imaginary part of the fourth order diagrams of Fig. 1, the intermediate fermion lines represent real particles. Hence only angular integrals are involved, and no ultraviolet divergences appear. That the result must also be free of infrared divergences, follows from the following argument. In the calculation of the sixth order scattering cross section, diagrams of the type of Fig. 2 serve to eliminate the infrared divergences. However, these diagrams have no imaginary parts, and hence do not contribute to the spin dependence of the cross section.

The influence of the field of the nucleus has not been considered quantitatively, and our results do not therefore apply directly to the case of scattering of an unpolarized beam by magnetized iron. For the scattering of a polarized beam by hydrogen, however, the effect of the Coulomb field is negligible. The argument for neglecting the rescattering by the nucleus, such as in the diagram of Fig. 3, is as follows. The matrix element is of order e^4, and the contribution to the cross section will therefore be of order e^8, except for interference with the lowest order scattering diagram. Clearly such interference takes place only for vanishing momentum transfer to the nucleus. In this case, however, the scattering by the nucleus is completely spin independent.

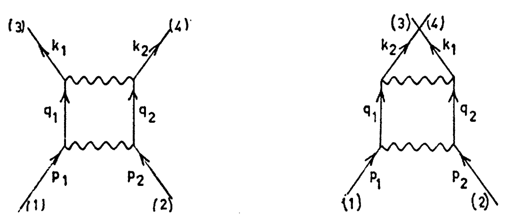

FIG. 1. For distinguishable particles the graph 1(a) is the only one that contributes to the spin-orbit force. For identical particles graph 1(b) must be taken into account as well. Notation: $p_1=(iE_1,\mathbf{p}_1)$, $p_2=(iE_2,-\mathbf{p}_1)$. The other two pairs of momenta have the same energies, but space parts $\mathbf{q}_1$, $-\mathbf{q}_1$ and $\mathbf{k}_1$, $-\mathbf{k}_1$, respectively. The particle whose spin is analyzed is that with momentum p_1. For identical particles we write $E_1=E_2=E$.

II

We shall sketch the calculation for the case of distinguishable particles, using unitarity to calculate the imaginary part of the fourth order scattering

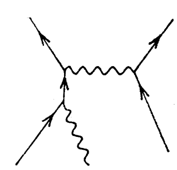

FIG. 2. Graphs of this kind are important for the sixth order cross section, as they help eliminate the infrared divergences. The corresponding amplitudes are real, however, and do not contribute to the spin-orbit correlations.

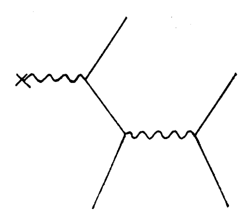

FIG. 3. Rescattering by the nucleus is important for heavy nuclei, but has no bearings on our results in the case of hydrogen.

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[1] Results for Bhabha scattering will soon be reported by C. Fronsdal and B. Jakšic.

amplitude.[2] For definiteness we shall compute the correction to the scattering cross section due to the polarization of one of the initial particles.

The second and fourth order matrix elements are defined as follows:

$$S=1+iM_2+iM_4+\cdots,$$

where M_2 and M_4 are of orders e^2 and e^4, respectively. The "polarization" is defined by

$$P=\frac{\mathrm{tr}[M^\dagger\boldsymbol{\sigma}\cdot\mathbf{s}M]}{\mathrm{tr}[M^\dagger M]}.$$

Here $\boldsymbol{\sigma}$ and $\mathbf{s}$ are the spin matrices and spin direction of one of the particles. The meaning of the matrix multiplication and the relation of P to the cross section will be explained below. It is well known that P vanishes when M is Hermitian or anti-Hermitian. Hence

$$P=2i\frac{\mathrm{tr}[\mathrm{Re}M^\dagger\boldsymbol{\sigma}\cdot\mathbf{s}\,\mathrm{Im}M]}{\mathrm{tr}[M^\dagger M]}.$$

The unitarity of the S matrix, i.e., the condition

$$S^\dagger S=SS^\dagger=1,$$

being an identity in e, gives

$$M_2-M_2^\dagger=0,\quad M_2^\dagger M_2+i(M_4-M_4^\dagger)=0.$$

Hence, to lowest nonvanishing order,

$$P=2i\frac{\mathrm{tr}[M_2^\dagger\boldsymbol{\sigma}\cdot\mathbf{s}\,\mathrm{Im}M_4]}{\mathrm{tr}[M_2^\dagger M_2]},\tag{1}$$

$$\mathrm{Im}M_4=\tfrac{1}{2}M_2^\dagger M_2.\tag{2}$$

In order to evaluate the matrix product in (2), we need M_2 between the initial state and some intermediate state and $M_2^\dagger$ between the intermediate state and the final state. Referring to Fig. 1(a) for the notation:

$$M_2=(2\pi)^{-2}\frac{m_1m_2}{E_1E_2}(p_1-q_1)^{-2}\times\bar\psi(q_1)e\gamma_\mu\psi(p_1)\bar\psi(q_2)e\gamma_\mu\psi(p_2),\tag{3}$$

$$M_2^\dagger=(2\pi)^{-2}\frac{m_1m_2}{E_1E_2}(q_1-k_1)^{-2}\times\bar\psi(k_1)e\gamma_\nu\psi(q_1)\bar\psi(k_2)e\gamma_\nu\psi(q_2).\tag{4}$$

Multiplying (3) by (4), the sum over intermediate spins is effected by means of the substitutions ($j=1, 2$)

$$\psi(q_j)\bar\psi(q_j)\to\frac{-i}{2m_j}(\boldsymbol{q}_j+im_j)\tag{5}$$

[2] An alternative method, by which the imaginary part was extracted from the complete fourth order matrix element, was used for checking. In reference 1 an example of an application of this method is given in detail.

Summing over intermediate momenta means applying the operator

$$\int d^3\mathbf{q}\,\delta(E_{\mathrm{int}}-E_1-E_2)=\frac{pE_1E_2}{E_1+E_2}\int d\Omega.\tag{6}$$

Here E_{int} is the total energy of the intermediate state. The calculation is carried out in the center-of-mass system (for notation see Fig. 1). The angular integral is over the directions of $\mathbf{q}$, with $\mathbf{q}^2=p^2$. For identical particles, (6) is modified by an extra factor of $\frac{1}{2}$ since in that case the intermediate states of momentum $\mathbf{q}$ and $-\mathbf{q}$ are identical. Equations (2) to (6) give

$$\mathrm{Im}M_4=-e^4(2\pi)^{-4}\frac{m_1m_2}{E_1E_2}\frac{p}{8(E_1+E_2)}\times\int\frac{d\Omega}{(q_1-p_1)^2(q_1-k_1)^2}\bar\psi(k_1)\gamma_\mu(\boldsymbol{q}_1+im_1)\times\gamma_\nu\psi(p_1)\bar\psi(k_2)\gamma_\mu(\boldsymbol{q}_2+im_2)\gamma_\nu\psi(p_2).\tag{7}$$

The operation indicated in the numerator of (1) consists of multiplying M_2 by (7), making the substitutions

$$\psi(k_j)\bar\psi(k_j)\to\frac{-i}{2m_j}(\boldsymbol{k}_j+im_j),\tag{8}$$

$$\psi(p_1)\bar\psi(p_1)\to i\gamma_5\boldsymbol{s}\frac{-i}{2m_1}(\boldsymbol{p}_1+im_1),\qquad \psi(p_2)\bar\psi(p_2)\to\frac{-i}{2m_2}(\boldsymbol{p}_2+im_2),\tag{9}$$

and taking the trace. Here s_μ is a four-vector describing the spin of particle 1 in a covariant way.[3] It satisfies

$$s^2=1,\quad s\cdot p_1=0.$$

The result is

$$\mathrm{tr}[M_2^\dagger\boldsymbol{\sigma}\cdot\mathbf{s}\,\mathrm{Im}M_4]=-e^6(2\pi)^{-6}\frac{p}{8E_1^2E_2^2(E_1+E_2)}\times\int\frac{\frac{1}{4}\mathrm{tr}_1\,\frac{1}{4}\mathrm{tr}_2\,d\Omega}{(p_1-k_1)^2(q_1-p_1)^2(q_1-k_1)^2},$$

$$\tfrac{1}{4}\mathrm{tr}_1=\tfrac{1}{4}\mathrm{tr}[i\gamma_5\boldsymbol{s}(\boldsymbol{p}_1+im_1)\times\gamma_\rho(\boldsymbol{k}_1+im_1)\gamma_\mu(\boldsymbol{q}_1+im_1)\gamma_\nu],\tag{10}$$

$$\tfrac{1}{4}\mathrm{tr}_2=\tfrac{1}{4}\mathrm{tr}[(\boldsymbol{p}_2+im_2)\gamma_\rho(\boldsymbol{k}_2+im_2)\times\gamma_\mu(\boldsymbol{q}_2+im_2)\gamma_\nu].$$

The trace calculations are conveniently carried out before the integration. Dropping terms that integrate

[3] See, e.g., C. Fronsdal and H. Überall, Phys. Rev. **111**, 580 (1958), and further references given there.

to zero, we obtain

$$\tfrac{1}{4}\,\mathrm{tr}_1\,\tfrac{1}{4}\,\mathrm{tr}_2=\tfrac{1}{4}\,\mathrm{tr}[i\gamma_5 s k_1 k_2 p_1]\frac{-4im_1}{(E_1+E_2)(p^2+\mathbf{p}_1\cdot\mathbf{k}_1)}\times\bigg\{\frac{E_2}{p^2}(p^2+\mathbf{p}_1\cdot\mathbf{k}_1)(p^2-\mathbf{k}_1\cdot\mathbf{q})(p^2-\mathbf{p}_1\cdot\mathbf{q})-m_2^2(2E_1+E_2)[(p^2-\mathbf{p}_1\cdot\mathbf{q})+(p^2-\mathbf{k}_1\cdot\mathbf{q})-(p^2-\mathbf{p}_1\cdot\mathbf{k}_1)]\bigg\}. \quad (11)$$

The three last terms each gives an infrared divergence. These cancel when a careful integration, using a small photon mass, is carried out. There is no contribution from the longitudual vector mesons thus introduced, because the intermediate states are real states. The result of the integration is

$$\mathrm{tr}[M_2^{\dagger}\boldsymbol{\sigma}\cdot\mathbf{s}\,\mathrm{Im}M_4]=\frac{im_1\pi}{4}e^6(2\pi)^{-6}\frac{\tfrac{1}{4}\,\mathrm{tr}[i\gamma_5 s k_1 k_2 p_1]}{E_1^2E_2^2(E_1+E_2)p^3\sin^2\vartheta}\times\bigg\{\frac{E_2}{E_1+E_2}(1+\cos\vartheta)+\frac{m_2^2}{p^2}\frac{2E_1+E_2}{E_1+E_2}\ln(1-\cos\vartheta)/2\bigg\}, \quad (12)$$

where ϑ is the angle between $\mathbf{p}_1$ and $\mathbf{k}_1$, and everything is expressed in the center-of-mass system. If the spin is transverse, i.e., if in addition to $s^2=1$, $s\cdot p_1=0$ we have $\mathbf{s}\cdot\mathbf{p}_1=0$, then s_μ is purely spacelike and of unit length. Moreover, in this case s_μ is the same in the center-of-mass system as in the rest system of p_1 (which in the case of a polarized target is the laboratory system) or that of p_2. Hence we may write

$$\tfrac{1}{4}\,\mathrm{tr}[i\gamma_5 s k_1 k_2 p_1]=-(E_1+E_2)\mathbf{s}\cdot\mathbf{p}_1\times\mathbf{k}_1, \quad (13)$$

and interpret $\mathbf{s}$ as the rest system spin direction of particle 1.

The denominator in (1) is (for distinguishable particles)

$$\mathrm{tr}[M_2^{\dagger}M_2]=e^4(2\pi)^{-4}\frac{1}{E_1^2E_2^2}\bigg\{\frac{1}{2}+\frac{(E_1+E_2)^2}{2p^2}\frac{1+\cos\vartheta}{(1-\cos\vartheta)^2}+\frac{(m_1m_2/p^2)^2}{(1-\cos\vartheta)^2}\bigg\}. \quad (14)$$

Thus P is given by (1), (12), (13), and (14). If $\mathbf{s}$ is perpendicular to the scattering plane $\mathbf{s}\cdot\mathbf{p}_1\times\mathbf{k}_1=p^2\sin\vartheta$, then

$$P=\frac{1}{137}\frac{m_1}{2p\sin\vartheta}\times\frac{\dfrac{E_2}{E_1+E_2}(1+\cos\vartheta)+\dfrac{m_2^2}{p^2}\dfrac{2E_1+E_2}{E_1+E_2}\ln\dfrac{1-\cos\vartheta}{2}}{\dfrac{1}{2}+\dfrac{(E_1+E_2)^2}{2p^2}\dfrac{1+\cos\vartheta}{(1-\cos\vartheta)^2}+\dfrac{(m_1m_2/p^2)^2}{(1-\cos\vartheta)^2}}. \quad (15)$$

The corresponding formula for identical particles is found to be

$$P=\frac{1}{137}\frac{m}{2p\sin\vartheta}\frac{2\cos\vartheta+3\sin^{-2}(\vartheta/2)(3+m^2/p^2-\cos\vartheta)\ln\cos(\vartheta/2)-3\cos^{-2}(\vartheta/2)(3+m^2/p^2+\cos\vartheta)\ln\sin(\vartheta/2)}{1+4(m^2/p^2+2)^2\sin^{-4}\vartheta+[4-3(m^2/p^2+2)^2]\sin^{-2}\vartheta}. \quad (16)$$

The relation of P to the differential cross section σ is, for complete polarization,[4]

$$\sigma=\sigma_0(1+P).$$

If the spin is with probability η parallel to $\mathbf{s}$, then the degree of polarization is $\xi=2\eta-1$ and the cross section is[4]

$$\sigma=\sigma_0(1+\xi P). \quad (17)$$

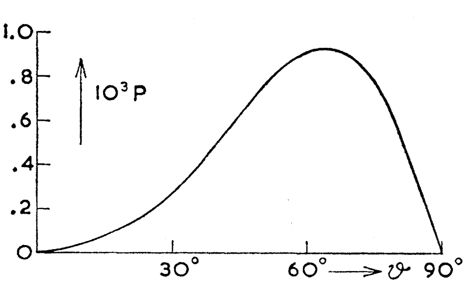

FIG. 4. The fractional asymmetry P for Møller scattering, when the direction of polarization is normal to the scattering plane, as a function of the center-of-mass scattering angle, for a laboratory kinetic energy of 1.0 Mev.

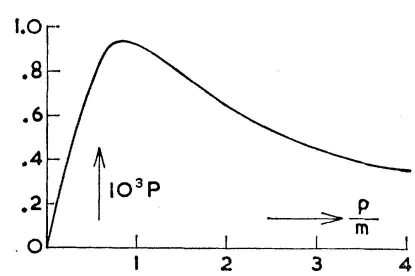

FIG. 5. Same as Fig. 4, showing P versus the laboratory kinetic energy for a center-of-mass scattering angle of 60°.

[4] See, e.g., J. Hamilton, *The Theory of Elementary Particles* (Clarendon Press, Oxford, England, 1959), p. 482.

III

The fractional asymmetry given by (16), for Møller scattering, is plotted in Figs. 4 and 5. An absolute maximum in $P(p,\vartheta)$ occurs near $p/m=1.0$ (i.e., $E_{\text{lab-kin}} \approx 1.0$ Mev), and $\vartheta=60°$. Figure 4 shows the angular dependence of P at the maximal value of p/m and Fig. 5 shows the energy dependence at the maximal value of ϑ.

If (15) is applied to the scattering of μ mesons on a polarized electron target, we obtain numbers of the same order of magnitude as in the case of Møller scattering. In the more realistic case (since hydrogen electrons are not easily polarized) of the scattering of transversally polarized μ mesons on unpolarized electrons, the value of P is more than an order of magnitude smaller.

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Positive Pion Production in p-p Collisions at 420 Mev with Polarized Protons*†

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The asymmetry in the production of positive π mesons in p-p collisions has been studied, using a 420-Mev, 62% polarized proton beam from the Chicago synchrocyclotron, with nuclear emulsions as the pion detector. The asymmetry at 65° in the laboratory for the entire spectrum above 20 Mev in the center-of-mass system is found to be 0.151 ± 0.021, in the direction opposite the elastic scattering that produced the polarized beam. In the region of the spectrum above 40 Mev, results are consistent with those found by other authors for the reaction $p+p\rightarrow\pi^{+}+d$; at lower energies where pions associated with final nucleon p states become predominant, the asymmetry decreases rapidly and may possibly reverse.

I. INTRODUCTION

THE asymmetry of positive pion production in the reaction

$$p+p\rightarrow\pi^{+}+d, \tag{1}$$

with polarized protons, predicted by Marshak and Messiah[1] from the phenomenological theory of Brueckner and Watson,[2] has been studied by a number of authors.[3] Until recently the only results available[4] on the asymmetry of pions from the reaction

$$p+p\rightarrow\pi^{+}+n+p \tag{2}$$

seemed to indicate an opposite asymmetry for this reaction, though recent results by McIlwain *et al.*[5] on about 250 events seem to indicate that this result was spurious. A reversed asymmetry in reaction (2) would be surprising in view of the fact that the angular momentum states that account for reaction (1) are also responsible for the bulk of the spectrum of reaction (2), and this experiment was undertaken to resolve this conflict with the phenomenological theory.

Because of the low cross section for reaction (2), the pion detector must be stable over long periods of time, of high efficiency and solid angle, view the entire spectrum simultaneously, and have a high rejection of spurious background events, with certain identification of positive pions and reasonable energy resolution. Nuclear emulsions insensitive to minimum ionizing particles (Ilford *GB*)[6] possess these features and, in addition, permit internal checks on the beam polarization and the geometric alignment of the apparatus. The emulsions may be area-scanned for pion endings, the pion being identified by its decay and its energy determined from its range. It was felt that results obtained by this technique, though limited in statistical accuracy, might be more convincing than those obtained with the relatively intricate counter telescope that would be required to perform this experiment with counters.

II. APPARATUS AND PROCEDURE

A. The Polarized Proton Beam

The beam was produced by scattering the internal beam of the Chicago synchrocyclotron 13° to the left in a beryllium target, emerged through a magnetic channel, and entered the exit system used with the external proton beam,[7] as shown in Fig. 1. Target

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