Elastic Scattering of Protons by He^3 and of Neutrons by H^3

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The differential scattering cross sections for the elastic scattering of protons by He^3 and of neutrons by H^3 were calculated at the center-of-mass energies 7.31, 6.45, 3.72, and 10.72 Mev, respectively, on the basis of nuclear shell model with the spin-orbit interaction term. The calculations were made with a square well potential and the effect of changing the shape of the well illustrated. The calculated cross sections with the square well potential were found to be in better agreement in the backward direction with the measured cross sections than those with shaped potential well. The polarization of the scattered particles has been calculated so that the double scattering experiments will test the theory.

INTRODUCTION

SWAN[1] has theoretically investigated the scattering of neutrons by H^3 and of protons by He^3 in the energy range 2.5–14 Mev using Wheeler's resonating group structure method. Experiments on the elastic scattering of protons by He^3 have been done by Lovberg,[2] Sweetman,[3] Famularo *et al.*,[4] and of neutrons by H^3 by Coon *et al.*[5] The work of Swan is extensive, but the agreement of the calculated cross sections with the experimentally observed values is not good.

In view of the importance of the p-He^3 and n-H^3, we decided to make a phenomenological approach to the problem. We have considered He^3 and H^3 as single bodies with definite radii. A formula for the nuclear radius of the form

$$R=(0.70+1.26A^{\frac{1}{3}})\times 10^{-13}\text{ cm}$$

was used throughout the numerical computations, following a suggestion by Weisskopf.[6] Then one has only a two-body scattering problem to deal with, instead of the more complicated one-body–three-body scattering problem that has been considered by Swan. We have assumed that when the incident nucleon enters the target nucleus, it behaves as if it is in an average potential well, as has been done by Feshbach, Porter, and Weisskopf.[7] We have taken the potential depth to be real since the possibility of a reaction in p-He^3 and n-H^3 scattering is small. Further, we have considered a strong spin-orbit coupling which is indicated by the success of the j-j coupling model of Mayer[8] and Haxel, Jensen, and Suess.[9]

THEORY

Let $\mathbf{k}_0$ and $\mathbf{k}$ be the direction of incidence and that of scattering of a particle beam. If the vector $\mathbf{n}$ be the normal to the plane in which scattering occurs, defined by

$$\mathbf{k}\times\mathbf{k}_0=\mathbf{n}k^2\sin\theta, \tag{1}$$

then the differential scattering cross section of the incident beam of polarization $\mathbf{P}_i$ can be written as[10]

$$\sigma(\theta)=(AA^*+BB^*)\left\{1+\frac{A^*B+B^*A}{AA^*+BB^*}\mathbf{P}_i\cdot\mathbf{n}\right\}, \tag{2}$$

where

$$A(\theta)=k^{-1}\sum_{l=0}^{\infty}\{(l+1)\exp(i\delta_l^+)\sin\delta_l^+ + l\exp(i\delta_l^-)\sin\delta_l^-\}P_l(\cos\theta),$$

$$B(\theta)=k^{-1}i\sum_{l=0}^{\infty}\{\exp(i\delta_l^+)\sin\delta_l^+ - \exp(i\delta_l^-)\sin\delta_l^-\}P_l'(\cos\theta)\sin\theta,$$

δ_l^+ being the total phase shift for $j=l+\frac{1}{2}$, and δ_l^- that for $j=l-\frac{1}{2}$, $P_l(\cos\theta)$ the Legendre polynomial of order l, and $P_l'(\cos\theta)$ its derivative with respect to its argument.

The polarization of the scattered beam is given by

$$\mathbf{P}=\frac{AA^*\mathbf{P}_i+(AB^*+BA^*)\mathbf{n}+i(A^*B-B^*A)\mathbf{P}_i\times\mathbf{n}+BB^*(2\mathbf{P}_i\cdot\mathbf{nn}-\mathbf{P}_i)}{AA^*+BB^*+(A^*B+B^*A)\mathbf{P}_i\cdot\mathbf{n}}. \tag{3}$$

If the incident beam be unpolarized, this last equation reduces to

$$P=\frac{AB^*+BA^*}{AA^*+BB^*}\mathbf{n}=P(\theta)\mathbf{n}. \tag{4}$$

Thus, in this case, the resulting polarization is directed along the normal to the scattering plane and is de-

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[1] P. Swan, Proc. Phys. Soc. (London) **A66**, 740 (1953).
[2] R. H. Lovberg, Phys. Rev. **103**, 1393 (1956).
[3] D. R. Sweetman, Phil. Mag. **46**, 358 (1955).
[4] K. F. Famularo, R. J. S. Brown, H. D. Holmgren, and T. F. Straton, Phys. Rev. **93**, 928 (A) (1954).
[5] J. H. Coon, C. K. Bockelman, and H. H. Barschall, Phys. Rev. **81**, 33 (1951).
[6] V. Weisskopf (private communication).
[7] H. Feshbach, C. E. Porter and, V. F. Weisskopf, Phys. Rev. **90**, 166 (1953); **96**, 448 (1954).
[8] M. G. Mayer, Phys. Rev. **78**, 16, 22 (1950).
[9] O. Haxel, J. H. D. Jensen, and H. E. Suess, Phys. Rev. **75**, 1766 (1949).
[10] J. V. Lepore, Phys. Rev. **79**, 134 (1950).

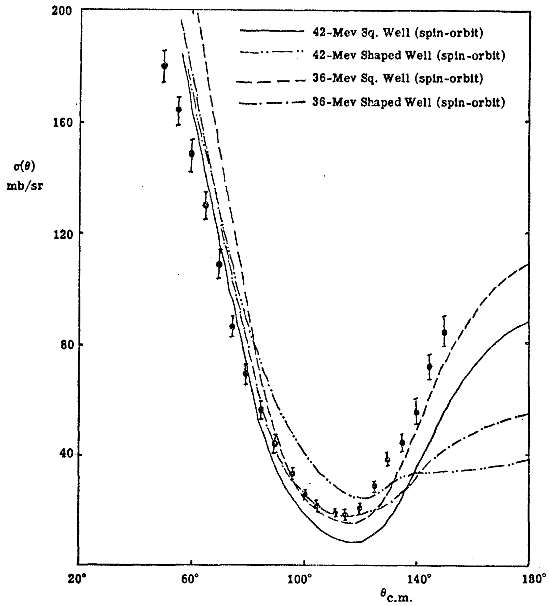

FIG. 1. p-He^3 scattering, $E_p=9.75$ Mev. The experimental results are of Lovberg.[2]

pendent upon the interference between the two parts of the scattered wave.

We have taken the spin-orbit interaction term to be[11]

$$H'(r)=\left(\frac{\hbar^2}{2\mu Mc^2}\right)\left(\frac{1}{r}\frac{dV}{dr}\right)_{\text{av}}\mathbf{l}\cdot\mathbf{S}, \tag{5}$$

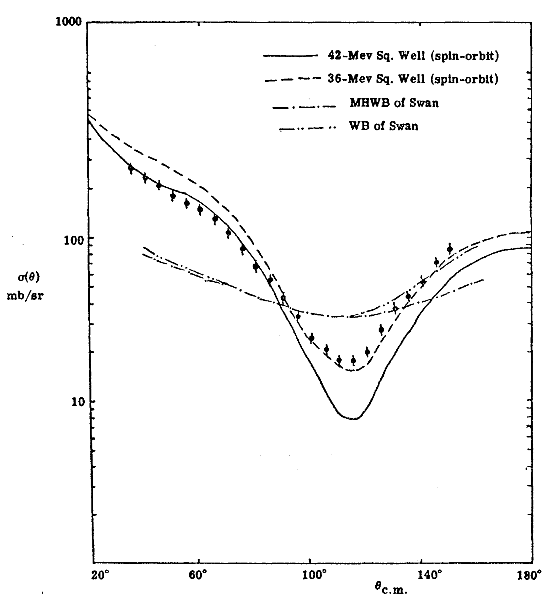

FIG. 2. p-He^3 scattering, $E_p=9.75$ Mev. Comparison of our results with those of Swan.[1]

[11] D. R. Inglis, Revs. Modern Phys. **25**, 390 (1953).

where μ is the mass of the π meson and M that of the nucleon.

Assuming that dV/dr exists only at the surface of the nucleus and that a nucleon has uniform probability to be anywhere within the nucleus, we get

$$\left(\frac{1}{r}\frac{dV}{dr}\right)_{\text{av}}=\frac{1}{r}\frac{\Delta V}{\Delta r}\cdot\frac{4\pi r^2\Delta r}{\frac{4}{3}\pi r^3}=\frac{3\Delta V}{r^2}. \tag{6}$$

We have taken ΔV equal to the drop of potential from 90% of its maximum value to 10% of its value, i.e., $\Delta V=0.8V$ where V is the potential to which a nucleon is subjected as it moves from outside to inside of the nucleus.

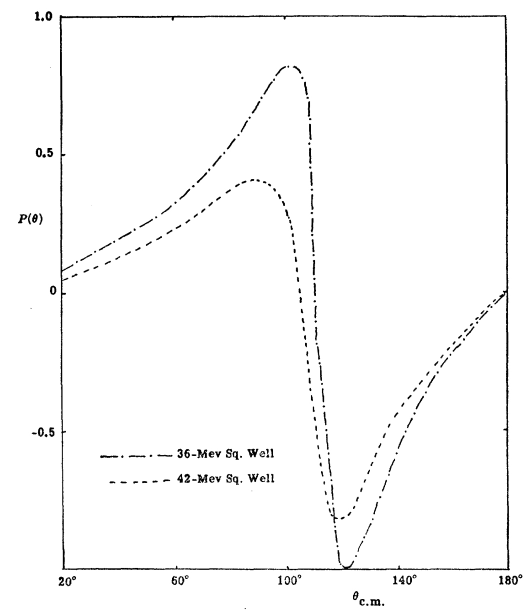

FIG. 3. Polarization of protons scattered by He^3. $E_p=9.75$ Mev.

The spin-orbit interaction energy now comes out to be

$$(1.2\beta V/R^2)(\mathbf{l}\cdot\mathbf{S}), \tag{7}$$

where $\beta=\hbar^2/2\mu Mc^2$, and R is the radius of the nucleus.

If now D is the depth of the square well nuclear potential, then the effective potential depth for a nucleon, when spin-orbit interaction is taken into account, is

$$D+(1.2\beta V/R^2)(\mathbf{l}\cdot\mathbf{S}), \tag{8}$$

where $V=D+Z'e^2/R$, and $Z'e^2/R$ is the height of the Coulomb barrier for the proton.

If we assume that D is the depth of the square well which is the best approximation to the diffuse surface potential well, $\{-V/[1+\exp(r-R)/d]\}$, in the least-square sense, then it can be shown that

$$V=V_0\{1+\epsilon d/R\}, \tag{9}$$

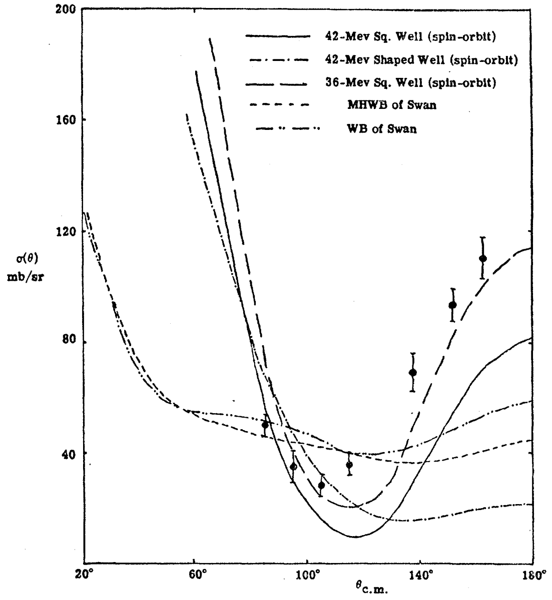

FIG. 4. p-He^3 scattering, $E_{\text{c.m.}}=6.45$ Mev. Comparison of our angular distribution with those of Swan. The experimental results are of Sweetman (private communication).

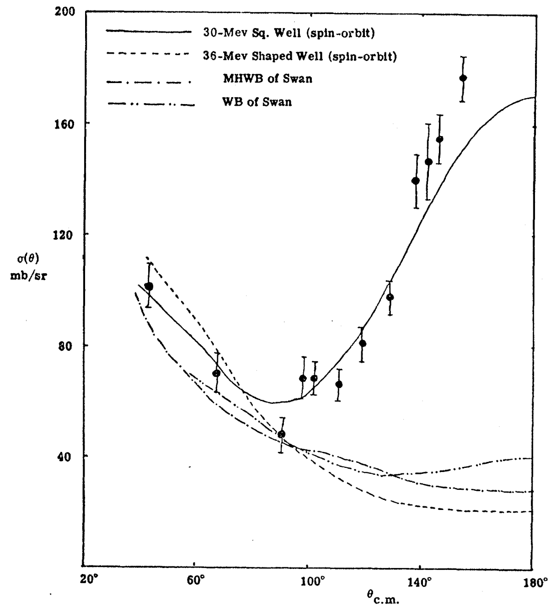

FIG. 6. p-He^3 scattering, $E_{\text{c.m.}}=3.72$ Mev. The experimental results are of Sweetman.[3]

where $V_0=D+Z'e^2/R$ and $\epsilon=1-\ln 2$. Solving the radial wave equation both for the square well and the diffuse surface potential it can be shown that the logarithmic derivative for the diffuse surface potential is

$$f_{\text{sh}}=\pm\left\{f_{\text{sq}}^2-\frac{2m}{\hbar^2}V_0\epsilon dR\right\}^{\frac{1}{2}}, \tag{10}$$

where f_{sq} is the logarithmic derivative for square well.

The positive sign is to be taken when f_{sh} is real and the negative sign when f_{sh} is imaginary.

When the spin-orbit interaction is taken into account, we have, instead of (10), the expression

$$f_{\text{sh}}=\pm\left\{f_{\text{sq}}-\frac{2m}{\hbar^2}V_0\epsilon dR\left(1+\frac{1.2\beta}{R^2}\mathbf{l}\cdot\mathbf{S}\right)\right\}^{\frac{1}{2}}. \tag{11}$$

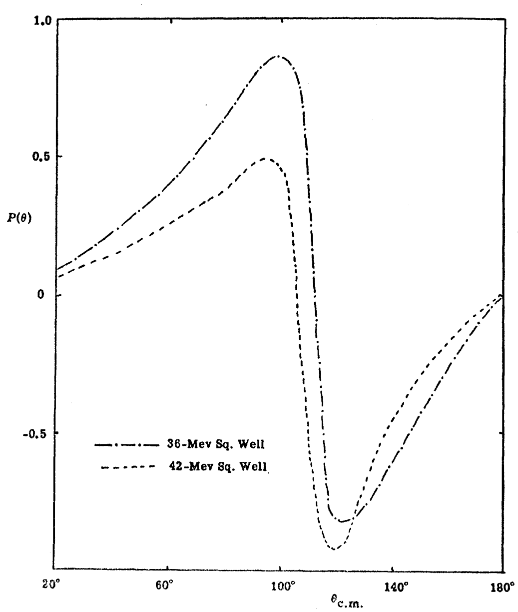

FIG. 5. Polarization of protons scattered by He^3. $E_{\text{c.m.}}=6.45$ Mev.

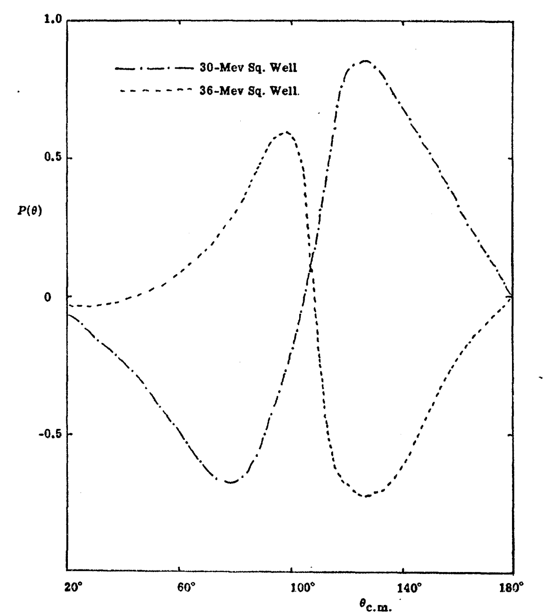

FIG. 7. Polarization of protons scattered by He^3. $E_{\text{c.m.}}=3.72$ Mev.

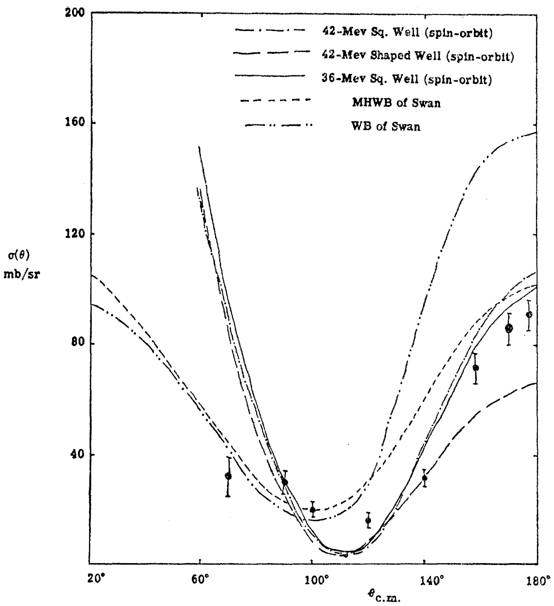

FIG. 8. n-H^3 scattering, $E_n=14.30$ Mev. Comparison of our angular distribution with those of Swan. The experimental results are those of Coon *et al.*[5]

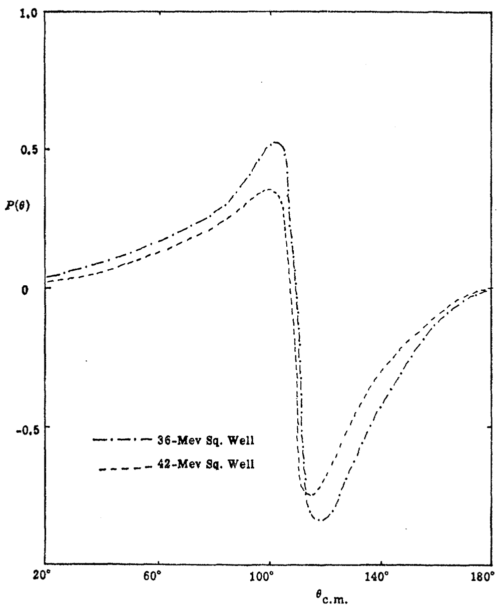

FIG. 9. Polarization of neutrons scattered by H^3. $E_n=14.30$ Mev.

NUMERICAL RESULTS AND GENERAL DISCUSSION

The numerical results are given in Figs. 1–9. Reasonably good agreement with the angular distribution data can be obtained. The polarization of the scattered nucleon calculated on this basis is also shown in the figures. Experiments on the polarization of nucleons in p-He^3 and n-H^3 scattering now can provide a test of the theory.

In these calculations we have disregarded the spin of the target nucleus. There is no compelling *a priori* reason for this assumption. The double scattering experiments should test whether this assumption is physically meaningful.

The effect of using a well of the Saxon shape is also considered. As usual the backward scattering is less pronounced for this case than for the case of the equivalent square well. We see that the data favor the square well.

Further, it seems to us that a single well depth will not fit the experimental data in the energy range considered. For example, we obtained good results with square-well potential of depth 36 Mev for 7.31-Mev and 6.45-Mev p-He^3 scattering, but for p-He^3 scattering at 3.72 Mev better agreement was obtained with a square well depth of 30 Mev. This dependence of the depth of the potential well on the energy has been noted in many other optical model analyses of scattering data.