Parity-Nonconserving Internucleon Potentials. II. Effects in Electromagnetic Transitions*

R. J. BLIN-STOYLE†
Department of Physics and Laboratory for Nuclear Science, Massachusetts Institute of Technology, Cambridge, Massachusetts

(Received May 9, 1960)

A detailed investigation is made of the way in which parity-nonconserving (PNC) internucleon potentials lead to parity impurities in nuclear states and hence to pseudoscalar asymmetries in the emission of gamma radiation. Explicit expressions are obtained for the angular distribution of unpolarized radiation (a) in emission from nuclei polarized by non-nuclear methods, (b) in β-γ angular correlations, (c) in polarized thermal neutron capture radiation and also for the magnitude of the circular polarization of radiation from an arbitrarily oriented nuclear system. The magnitudes of these effects are then estimated for the case of a transition between low-lying nuclear states and also for a ground-state transition following neutron capture. Finally a critique of the γ-ray transitions so far used in experimental investigations of PNC effects is given. It is concluded that many transitions in particularly simple nuclei are insensitive to PNC effects and that at present all that can be stated with any confidence is that $\mathfrak{F}\lesssim 10^{-4}-10^{-5}$.

I. INTRODUCTION

IN an earlier paper[1] a discussion is given of the form a parity-nonconserving (PNC) internucleon potential might be expected to take because of invariance requirements. Further, the form of the specific PNC potentials arising in lowest order from the conserved self-interacting current description of weak interactions is deduced. A velocity-dependent and a static potential are obtained having the following forms:

$$\mathcal{V}_{\text{static}}=\frac{-Gf^2}{2\pi\hbar c}\left[\frac{1}{r^4}+\frac{2}{\mu r^5}+\frac{1}{\mu^2 r^6}\right]e^{-2\mu r}\mathbf{r}\cdot(\boldsymbol{\sigma}^{(1)}\times\boldsymbol{\sigma}^{(2)})\times[\boldsymbol{\tau}^{(1)}\cdot\boldsymbol{\tau}^{(2)}-\tau_z^{(1)}\tau_z^{(2)}], \quad (1)$$

$$\mathcal{V}_{\text{vel. dep.}}=-(G/4mc)(\boldsymbol{\sigma}^{(1)}-\boldsymbol{\sigma}^{(2)})\cdot\mathbf{p}_{12}\delta(\mathbf{r})\times[\boldsymbol{\tau}^{(1)}\cdot\boldsymbol{\tau}^{(2)}-\tau_z^{(1)}\tau_z^{(2)}], \quad (2)$$

where μ, $\mathbf{r}$, $\mathbf{p}$, $\boldsymbol{\sigma}$, and $\boldsymbol{\tau}$ have their usual meanings and G and f are, respectively, the weak four-fermion and pion-nucleon pseudovector coupling constants.

Wilkinson[2] has classified those experiments in low-energy nuclear physics suitable for the detection of parity-nonconserving effects resulting from potentials of this type, and it would seem from his analysis that the apparently most sensitive are those involving the measurement of asymmetries, circular polarizations, etc., in gamma transitions between nuclear states. Such experiments are the most promising because, in general, they are sensitive to the *amplitudes* of parity impurities in nuclear states. The object of the present paper is to investigate in more detail the way in which PNC potentials will lead to such parity impurities and how they will manifest themselves in electromagnetic transitions. In Secs. II–IV the geometrical problem of angular distributions, etc., is discussed, in Sec. V the magnitude of the parity nonconserving effects is investigated and in Sec. VI the suitability of the electromagnetic transitions so far used in experiments is commented on.

II. ASYMMETRY EFFECTS IN GAMMA-TRANSITIONS. GENERAL RESULTS

Consider a gamma transition between the nuclear substates $|j'm'\rangle$ and $|jm\rangle$, both of which may contain components having opposite parity to the predominant parity of the state. The appropriate matrix element[3] for the transition can be written

$$\langle j'm'|\mathcal{H}(\mathbf{A})|jm\rangle=\pi\sum_{LMP}(2L+1)^{\frac{1}{2}}f(P)D_{MP}^{(L)}(\mathbf{k})\times[\langle j'm'|\mathcal{H}(A_L^M(m))|jm\rangle+(-P)\langle j'm'|\mathcal{H}(A_L^M(e))|jm\rangle], \quad (3)$$

where $f(P)$ ($P=+1$ or -1 corresponding to left or right circular polarization) determines the polarization of the photon emitted in the direction $\mathbf{k}$ and $\mathcal{H}(A_L^M(m))$ and $\mathcal{H}(A_L^M(e))$ are the usual magnetic and electric multipole operators but with additional phase factors[4] i^L and i^{L+1}, respectively.[3]

In such a transition interference effects will arise between multipoles of opposite parity because of parity impurities in the states $|jm\rangle$ and $|j'm'\rangle$ and will manifest themselves as pseudoscalar quantities in the angular distribution of the emitted radiation. The angular distribution $I_{m,m-M}$ of the radiation emitted in the transition $|jm\rangle\rightarrow|j'\,m-M\rangle$ taking into account the contribution from all multipoles and for arbitrary polarization of the radiation is obtained by taking the square modulus of the matrix element (3). The calcula-

* This work is supported in part by funds provided by the U. S. Atomic Energy Commission, the Office of Naval Research and the Air Force Office of Scientific Research.
† On Sabbatical leave from the Clarendon Laboratory, Oxford, England.
[1] R. J. Blin-Stoyle, Phys. Rev. **118**, 1605 (1960); referred to as I.
[2] D. H. Wilkinson, Phys. Rev. **109**, 1603 (1958).
[3] L. C. Biedenharn and M. E. Rose, Revs. Modern Phys. **25**, 729 (1953).
[4] Such a choice of phase factors ensures that the nuclear reduced matrix elements are relative real. See reference 3.

tion is straightforward using the techniques of reference 3 and the following expression is obtained:

$$I_{m,m-M}=|\langle j'm-M|\mathcal{H}(\mathbf{A})|jm\rangle|^2$$
$$=\pi^2\sum_{LL'}[(2L+1)(2L'+1)]^{\frac{1}{2}}C(j'Lj;m-MM)$$
$$\times C(j'L'j;m-MM)F_{LL'}{}^M(\mathbf{k}),\quad(4)$$

where the C's are Clebsch-Gordan coefficients, and

$$\begin{aligned}F_{LL'}{}^M(\mathbf{k})=(-)^{M+1}\textstyle\sum_\nu\{&C(LL'\nu;-11)C(LL'\nu;-MM)\\&\times P_\nu(\cos\theta)[G_{LL'}{}^\nu(m_L{}^*m_{L'}+e_L{}^*e_{L'})\\&+H_{LL'}{}^\nu(e_L{}^*m_{L'}+m_L{}^*e_{L'})]\\&+C(LL'\nu;11)C(LL'\nu;-MM)\\&\times[(\nu-2)!/(\nu+2)!]^{\frac{1}{2}}P_\nu{}^{(2)}(\cos\theta)\\&\times[I_{LL'}{}^\nu(m_L{}^*m_{L'}-e_L{}^*e_{L'})\\&+J_{LL'}{}^\nu(e_L{}^*m_{L'}-m_L{}^*e_{L'})]\}.\end{aligned}\quad(5)$$

Here

$$\begin{aligned}G_{LL'}{}^\nu&=|f(1)|^2+(-)^{L+L'-\nu}|f(-1)|^2,\\H_{LL'}{}^\nu&=|f(1)|^2-(-)^{L+L'-\nu}|f(-1)|^2,\\I_{LL'}{}^\nu&=f^*(-1)f(1)+(-)^{L+L'-\nu}f^*(1)f(-1),\\J_{LL'}{}^\nu&=f^*(-1)f(1)-(-)^{L+L'-\nu}f^*(1)f(-1),\end{aligned}\quad(6)$$

and m_L and e_L are the reduced matrix elements

$$m_L=\langle j'\|\mathcal{H}(A_L(m))\|j\rangle;\quad e_L=\langle j'\|\mathcal{H}(A_L(e))\|j\rangle.\quad(7)$$

To proceed further it is necessary to specify both the orientation of the initial nuclear system, that is, the distribution of the nuclear substates $|jm\rangle$, and also the polarization of the emitted radiation. We consider two cases which are likely to be most useful from the point of view of detection of parity nonconservation.

III. ANGULAR DISTRIBUTION OF UNPOLARIZED RADIATION FROM AN ORIENTED NUCLEAR SYSTEM

Suppose that the initial nuclear state has been polarized by some mechanism and that the probability[5] of finding the nucleus in the substate $|jm\rangle$ is $p(m)$. The resulting angular distribution of the emitted radiation is then given by

$$W=\sum_{mM}p(m)I_{m,m-M}.\quad(8)$$

Now[6] for a plane wave with electric field polarized at an angle χ to the direction defined by the intersection of the plane normal to k and the plane of k and the axis of quantization:

$$f(P)=(1/\sqrt{2})\exp(-iP\chi).$$

Thus,

$$\begin{aligned}G_{LL'}{}^\nu&=\delta_{L+L'+\nu,\text{even}},\\H_{LL'}{}^\nu&=\delta_{L+L'+\nu,\text{odd}},\\I_{LL'}{}^\nu&=\cos2\chi\delta_{L+L'+\nu,\text{even}}-i\sin2\chi\delta_{L+L'+\nu,\text{odd}},\\J_{LL'}{}^\nu&=\cos2\chi\delta_{L+L'+\nu,\text{odd}}-i\sin2\chi\delta_{L+L'+\nu,\text{even}}.\end{aligned}\quad(9)$$

However, if polarization insensitive detectors are used for the detection of the radiation, then an average has to be taken over χ, giving

$$(I_{LL'}{}^\nu)_{\text{average}}=(J_{LL'}{}^\nu)_{\text{average}}=0.$$

The resulting angular distribution can then be simplified to the following form:

$$W(\theta)=\pi^2\sum_{LL'\nu}B_\nu(j)F_\nu(LL'j'j)$$
$$\times[\delta_{L+L'+\nu,\text{even}}(m_L{}^*m_{L'}+e_L{}^*e_{L'})$$
$$+\delta_{L+L'+\nu,\text{odd}}(m_L{}^*e_{L'}+e_L{}^*m_{L'})]P_\nu(\cos\theta),\quad(10)$$

where $B_\nu(j)$ is given by

$$B_\nu(j)=\textstyle\sum_m(2\nu+1)^{\frac{1}{2}}C(j\nu j;m0)p(m),\quad(11)$$

and is a parameter introduced by various authors[7,8] to describe the nature of the orientation of a nucleus. Its properties in some specific cases will be discussed a little later. The factor $F_\nu(LL'j'j)$ is defined by

$$F_\nu(LL'j'j)=(-)^{j'+3j-1}[(2j+1)(2L+1)(2L'+1)]^{\frac{1}{2}}$$
$$\times C(LL'\nu;1-1)W(LL'jj;\nu j').\quad(12)$$

It has been used frequently in the discussion of angular distributions and has been tabulated by Biedenharn and Rose[3,9] and Alder *et al.*[10]

Inspection of the expression (10) shows that the terms indicating breakdown of parity conservation, namely those with ν odd, can occur in two ways. Firstly there can be interference between multipoles of like character (i.e., electric or magnetic) but whose orders differ by an odd number (e.g., $E1$-$E2$ interference). Secondly there can be interference between multipoles of unlike character but which have the same multipolarity (e.g., $E1$-$M1$) or multipolarity differing by an even number (e.g., $E2$-$M4$). In practice the most likely possibility is interference between electric and magnetic multipoles of the same order; however, it is possible that in some exceptional cases, the other types of interference may be important. In any case, it is clear that a breakdown in parity conservation will manifest itself in a forward backward asymmetry of the emitted radiation relative

[5] The assumed noncoherence of the substates m is valid for all the cases considered in this paper. See reference 3 for a discussion of this point.

[6] M. E. Rose, *Multipole Fields* (John Wiley & Sons, Inc., New York, 1955).

[7] S. R. DeGroot and H. A. Tolhoek, Beta- and Gamma-Ray Spectroscopy, edited by K. Siegbahn (Interscience Publishers Inc., New York, 1955).

[8] T. P. Gray and G. R. Satchler, Proc. Phys. Soc. (London) **A68**, 349 (1955).

[9] Note that in the notation of reference 3 $F_\nu(Lj'j)=F_\nu(LLj'j)$ of the present paper and reference 10.

[10] K. Alder, B. Stech, and A. Winther, Phys. Rev. **107**, 728 (1957).

to the orientation axis of the initial nuclei. Such an asymmetry depends not only on a breakdown of parity conservation, but also on the nonvanishing of the $B_\nu(j)$ with ν odd, and for the effect to be large this requires that these terms be as large as possible. Three types of method for producing large $B_\nu(j)$ (ν odd) are discussed in the following sections.

A. The Orientation Parameter $B_\nu(j)$

The following explicit forms for the $B_\nu(j)$ are to be noticed

$$\begin{aligned}
B_0(j)&=1,\\
B_1(j)&=\sqrt{3}\frac{\sum_m mp(m)}{[j(j+1)]^{\frac{1}{2}}},\\
B_2(j)&=(5)^{\frac{1}{2}}\frac{\sum_m[m^2-\frac{1}{3}j(j+1)]p(m)}{[j(j+1)(2j-1)(2j+3)]^{\frac{1}{2}}}.
\end{aligned}\tag{13}$$

Apart from a normalization factor, $B_1(j)$ represents the *polarization* and $B_2(j)$ represents the *alinement* of the initial nuclear system. At the present time it seems unlikely that higher values of $B_2(j)$ would be important and in all probability the main forward-backward asymmetry is expected to arise from the term $B_1(j)$.

We now consider three ways of polarizing the initial nuclear state, all of which have been used experimentally.

B. Polarization by Non-Nuclear Methods

Two groups of methods for orienting nuclei have been proposed which depend essentially on the interaction of the nuclear electromagnetic moments with electromagnetic fields. On the one hand there are the methods which depend on the separation in energy of the nuclear magnetic substates and then ensure that these are unequally populated by reducing the temperature of the system so far that the low-lying states become preferentially populated, i.e., the $p(m)$ are not equal to one another. On the other hand, there are the optical and microwave methods which depend on the atomic absorption and emission of radiation; in these preferential nuclear magnetic state populations may be obtained by emission of atomic radiation through different channels from those by which it was absorbed. These methods are reviewed and the orientation parameters quoted by Blin-Stoyle and Grace[11] and will not be further considered here.

C. β-γ Angular Correlations

It is now well established that in beta decay, parity is not conserved and it therefore follows that the final nuclear state following a beta transition will be highly polarized in the direction of the emitted beta ray. Thus, if parity is not conserved in the succeeding γ emission, a forward-backward asymmetry is to be expected in the β-γ angular correlation.[12]

We now restrict ourselves to the case of an allowed β decay $j_\beta\rightarrow j$, where j_β is the spin of the β-emitting state and j that of the daughter (γ emitting) nucleus. In such a decay, the angular momentum restrictions are such that only $B_0(j)$ and $B_1(j)$ are nonvanishing. Now $B_0(j)=1$ and $B_1(j)$ can easily be deduced from the expression for the angular distribution of β particles from a polarized nucleus by applying time reversal arguments. A discussion of the relation between the asymmetry factor in such a process and the polarization of the nucleus following the time reversed process is given for the general case by Satchler.[13] Taking the expression for the asymmetry in β decay from a polarized nucleus from the work of Jackson *et al.*[14] and comparing with Satchler's[13] results, it follows that in the allowed β decay $j_\beta\rightarrow j$ with j_β *unpolarized*, the polarization of j along the axis of emission of the β particles is

$$P=-A\frac{j+1}{3j}\frac{p_e}{E_e},\tag{14}$$

where p_e and E_e are the momentum and energy of the emitted β particles and A is given by the two equations,

$$\begin{aligned}
A\xi=|M_{\mathrm{GT}}|^2\lambda_{j_\beta j}\bigg[&\pm2\,\mathrm{Re}(C_TC_T'^*-C_AC_A'^*)\\
&+\frac{\alpha Zm}{p_e}2\,\mathrm{Im}(C_TC_A'^*+C_T'C_A^*)\bigg]\\
&+\delta_{j_\beta j}M_{\mathrm{F}}M_{\mathrm{GT}}\left(\frac{j}{j+1}\right)^{\frac{1}{2}}\bigg[2\,\mathrm{Re}(C_SC_T'^*+C_S'C_T^*\\
&-C_VC_A'^*-C_V'C_A^*)\pm\frac{\alpha Zm}{p_e}2\,\mathrm{Im}(C_SC_A'^*\\
&+C_S'C_A^*-C_VC_T'^*-C_V'C_T^*)\bigg],
\end{aligned}\tag{15}$$

$$\begin{aligned}
\xi=&|M_{\mathrm{F}}|^2[|C_S|^2+|C_V|^2+|C_S'|^2+|C_V'|^2]\\
&+|M_{\mathrm{GT}}|^2[|C_T|^2+|C_A|^2+|C_T'|^2+|C_A'|^2],
\end{aligned}\tag{16}$$

with

$$\begin{aligned}
\lambda_{j_\beta j}&=1 &&\text{for}\quad j_\beta=j-1,\\
&=1/(j+1) &&\text{for}\quad j_\beta=j,\\
&=-j/(j+1) &&\text{for}\quad j_\beta=j+1.
\end{aligned}$$

The remaining symbols in (15) refer to the nature of the

[11] R. J. Blin-Stoyle and M. A. Grace, *Handbuch der Physik*, edited by S. Flügge (Springer-Verlag, Berlin, 1957), Vol. XLII.

[12] F. Boehm and U. Hauser (to be published).

[13] G. R. Satchler, Nuclear Phys. 8, 65 (1958).

[14] J. D. Jackson, S. B. Treiman, and H. W. Wyld, Jr., Nuclear Phys. 4, 206 (1957).

β-decay interaction and have their usual significance and the $\pm$ signs correspond to electron or positron emission.

Now the polarization P is defined by

$$P=\frac{1}{j}\sum mp(m), \tag{17}$$

so that, comparing (13), (14), and (17)

$$B_1=-A\left(\frac{j+1}{3j}\right)^{\frac{1}{2}}\frac{p_e}{E_e}. \tag{18}$$

There is considerable evidence at the present time that in β decay $C_S=C_T=C_S'=C_T'=0$, $C_A=C_A'=\text{real}$, $C_V=C_V'=\text{real}$. We therefore specialize the preceding results to this case and also approximate $Z=0$. Thus

$$B_1(j)=\frac{p_e}{E_e}\left(\frac{j+1}{3j}\right)^{\frac{1}{2}} \times\left[\pm\lambda_{j\beta j}+2\delta_{j\beta j}\gamma\left(\frac{j}{j+1}\right)^{\frac{1}{2}}\right]\Big/[1+\gamma^2], \tag{19}$$

where $\gamma=C_VM_F/C_AM_{GT}$.

Now in the most likely case of practical interest the interference term indicative of parity nonconservation in the angular distribution of the following γ radiation will be between electric and magnetic multipoles of the same order L (say). Thus, remembering $\nu=0$, 1 only, (10) can be written

$$W(\theta)=\pi^2\{(|m_L|^2+|e_L|^2) +B_1(j)F_1(LLj'j)(m_L^*e_L+e_L^*m_L)\cos\theta\}, \tag{20}$$

$$\propto(1+\alpha\cos\theta), \tag{21}$$

where, using (18) and (19) the asymmetry factor α is given by

$$\alpha=\frac{2p_e}{E_e}\left\{\left[\pm\lambda_{j\beta j}+2\delta_{j\beta j}\gamma\left(\frac{j}{j+1}\right)^{\frac{1}{2}}\right]\Big/[1+\gamma^2]\right\} \times F_1(LLj'j)\frac{m_L^*e_L}{|m_L|^2+|e_L|^2}. \tag{22}$$

Here, the fact, referred to earlier, has been used that m_L and e_L are relatively real. It is now a straightforward matter to use formula (22) for any particular β-γ angular correlation.[15]

D. Capture of Polarized Thermal Neutrons

Haas *et al.*[16] have suggested that parity-nonconserving effects might show up as a forward-backward asymmetry in the angular distribution of γ rays emitted by a compound nuclear state formed by the capture of polarized S-wave neutrons. Since neutrons have spin $\frac{1}{2}$ it follows that in describing the orientation properties of the state resulting from the capture process, only $B_0(j)$ and $B_1(j)$ are nonvanishing.

Let the spin of the initial nuclear state (before capture) be j_i and let the spin of the compound (γ-emitting) state be j. If $\epsilon_\mu(\mu=\pm\frac{1}{2})$ is the probability of a neutron having a z component μ, the orientation weighing factor $p(m)$ for the compound state j is given by

$$p(m)=\frac{2}{2j+1}\sum_\mu \epsilon_\mu|C(j_i\tfrac{1}{2}j;m-\mu\mu)|^2. \tag{23}$$

Inserting this expression into Eq. (11), $B_0(j)$ and $B_1(j)$ can then be calculated. We obtain

$$B_0(j)=1,$$

$$B_1(j)=\frac{j_i(j_i+1)-j(j+1)-\frac{3}{4}}{[3j(j+1)]^{\frac{1}{2}}}P_n, \tag{24}$$

where P_n is the polarization of the captured neutrons, given by

$$P_n=(\epsilon_{\frac{1}{2}}-\epsilon_{-\frac{1}{2}})/(\epsilon_{\frac{1}{2}}+\epsilon_{-\frac{1}{2}}). \tag{25}$$

Substitution of $B_0(j)$ and $B_1(j)$ into (10) at once gives the angular distribution of the emitted γ rays in the transition $j\rightarrow j'$ relative to the polarization direction of the neutrons. For the special case of interference between two multipoles m_L and e_L of the same order, the expression for the angular distribution simplifies to

$$W(\theta)=\pi^2\left\{(|m_L|^2+|e_L|^2) +P_n\frac{j_i(j_i+1)-j(j+1)-\frac{3}{4}}{[3j(j+1)]^{\frac{1}{2}}}F_1(LLj'j) \times(m_L^*e_L+e_L^*m_L)\cos\theta\right\}, \tag{26}$$

$$\propto(1+\alpha\cos\theta), \tag{27}$$

where the asymmetry factor α is given by

$$\alpha=2P_n\frac{j_i(j_i+1)-j(j+1)-\frac{3}{4}}{[3j(j+1)]^{\frac{1}{2}}} \times F_1(LLj'j)\frac{m_L^*e_L}{|m_L|^2+|e_L|^2}, \tag{28}$$

α has been quoted for the particular case $j_i=0$, $j=\frac{1}{2}$, $j'=\frac{1}{2}$, $L=1$ by Haas *et al.*[16]

---

[15] A formula essentially identical with this has also been obtained by L. Kruger [Z. Physik **157**, 369 (1959)].

[16] R. Haas, L. B. Leipuner, and R. K. Adair, Phys. Rev. **116**, 1221 (1959).

IV. EMISSION OF CIRCULARLY POLARIZED RADIATION

If detectors sensitive to circular polarization are used for the detection of emitted γ rays, then, even if the initial nuclear system is unpolarized, there will be a resultant circular polarization if parity is not conserved. This corresponds, for example, to the longitudinal polarization of electrons observed in the β decay of unpolarized nuclei. For generality we consider the case of an arbitrarily oriented nuclear system and then specialize to the case of unpolarized nuclei. The former case might, in fact, be of interest since an orientation may be produced in forming a suitable γ-emitting state.[2]

The distribution of left circularly polarized radiation $W_L(\theta)$ from an oriented nucleus is given by taking $f(1)=1$ and $f(-1)=0$ in (4) and (5), multiplying by $p(m)$, and summing over m and M. Similarly the distribution of right circularly polarized radiation $W_R(\theta)$ is given by following the same procedure but with $f(1)=0$ and $f(-1)=1$. Using the methods already described, the following expression is obtained for the degree δ of circular polarization:

$$\delta=\frac{W_L(\theta)-W_R(\theta)}{W_L(\theta)+W_R(\theta)}$$

$$=\frac{\sum_{LL'\nu} B_\nu(j)F_\nu(LL'j'j)[(m_L{}^*m_{L'}+e_L{}^*e_{L'})\delta_{L+L'+\nu,\mathrm{odd}}+(m_L{}^*e_{L'}+e_L{}^*m_{L'})\delta_{L+L'+\nu,\mathrm{even}}]P_\nu(\cos\theta)}{\sum_{LL'\nu} B_\nu(j)F_\nu(LL'j'j)[(m_L{}^*m_{L'}+e_L{}^*e_{L'})\delta_{L+L'+\nu,\mathrm{even}}+(m_L{}^*e_{L'}+e_L{}^*m_{L'})\delta_{L+L'+\nu,\mathrm{odd}}]P_\nu(\cos\theta)}. \quad (29)$$

In the particularly simple case that the initial nucleus is unpolarized, the degree of circular polarization reduces to

$$\delta=2\sum_L m_L{}^*e_L/\sum_L(|m_L|^2+|e_L|^2). \quad (30)$$

In this case an effect is only obtained if there is interference between electric and magnetic multipoles of the same order.

V. EXPECTED MAGNITUDE OF THE EFFECTS

All the effects discussed in the preceding sections depend for their observation on the nonvanishing of interference terms between multipoles having opposite parity. We now have to make some estimate of the magnitude of these interference terms.

Let the nuclear wave function for a state i be written

$$\Psi_i=\psi_i+\eta\varphi_i, \quad (31)$$

where ψ_i and φ_i have opposite parity but the same angular momentum properties and η, the amplitude of admixture of φ_i, is taken to be very small. In general η will be of the order $\mathfrak{F}$, where $\mathfrak{F}$ is qualitatively defined to be the ratio of the strengths of the parity-nonconserving and parity-conserving internucleon potentials.[1,17] ψ_i is referred to as the *regular* part and φ_i as the *irregular* part of the wave function.

Now ψ_i is one of the complete set of nuclear states for the nucleus under consideration and is an eigenfunction of a Hamiltonian H which includes the strong (assumed parity conserving) interactions and commutes with the parity operator. Thus

$$H\psi_i=E_i\psi_i. \quad (32)$$

The irregular part of the wave function φ_i can then be computed using perturbation theory and treating $\mathcal{V}$, the PNC internucleon potential, as a perturbation. Thus

$$\Psi_i=\psi_i+\sum_{j\neq i}\frac{\langle j|\mathcal{V}|i\rangle}{\Delta_{ij}}\psi_j, \quad (33)$$

where $\Delta_{ij}=E_i-E_j$.

In order to proceed further it is now necessary to be more specific. There are essentially two cases to consider according as the electromagnetic transition is between low-lying nuclear states, as is usually the case in emission from oriented nuclei and in a β-γ sequence, or between a state of relatively high energy (≈ 8 Mev) and a low-lying state as in radiative neutron capture. In the former case it is a fair approximation to treat the ψ_i as essentially shell-model states whereas in the latter the capture state will be highly complicated. We consider these two cases separately. Further we restrict our considerations to the most usual case, namely that in which the regular part of the γ transition goes via the magnetic multipole m_L and the irregular part goes via the electric multipole e_L. In this case an enhancement effect is to be expected because of the fact that normally the strength of an electric multipole transition is greater than that of a magnetic multipole transition of the same order.[2]

A. Electromagnetic Transition Between Low-Lying States

Let the transition be between the nuclear states Ψ_1 and Ψ_2 which have relative parity $(-)^{L+1}$. By (33)

$$\begin{aligned}\Psi_1&=\psi_1+\sum_{i\neq 1}\frac{\langle i|\mathcal{V}|1\rangle}{\Delta_{1i}}\psi_i,\\ \Psi_2&=\psi_2+\sum_{j\neq 2}\frac{\langle j|\mathcal{V}|2\rangle}{\Delta_{2j}}\psi_j.\end{aligned} \quad (34)$$

[17] T. D. Lee and C. N. Yang, Phys. Rev. **104**, 254 (1956).

The matrix element of the magnetic multipole operator between these two states is

$$\langle\Psi_2|m_L|\Psi_1\rangle=\langle 2|m_L|1\rangle, \tag{35}$$

and that of the electric multipole operator is

$$\langle\Psi_2|e_L|\Psi_1\rangle=\sum_{i\neq 1}\frac{\langle 2|e_L|i\rangle\langle i|\mathcal{V}|1\rangle}{\Delta_{1i}}+\sum_{j\neq 2}\frac{\langle 2|\mathcal{V}|j\rangle\langle j|e_L|1\rangle}{\Delta_{2j}}. \tag{36}$$

Now diagonal matrix elements of $\mathcal{V}$ between states ψ_i which have a well-defined parity are zero. This means that the i and j summations in (36) can be extended over all i and j. Further, since e_L is a sum of single-particle operators, $|i\rangle$ differs by at most one-particle excitation from $|2\rangle$ and similarly $|j\rangle$ from $|1\rangle$. Also if an oscillator description is used for the shell model states, then selection rules require that this particle is excited through L-oscillator levels. Thus $\Delta_{1i}=\Delta_{2j}=-L\hbar\omega$, where $\hbar\omega$ is the oscillator spacing. This means that closure can now be performed in the two summations to give

$$\langle\Psi_2|e_L|\Psi_1\rangle\approx-\frac{\langle 2|e_L\mathcal{V}+\mathcal{V}e_L|1\rangle}{L\hbar\omega}. \tag{37}$$

This result is exact for an oscillator model and might be expected to be approximately true in general.

Equation (37) is a formula which can be used as a basis for calculations in any specific case. We now consider, as an example, the particular case common to many experiments so far performed in which $L=1$ and we have $M1$-$E1$ interference in light nuclei.

B. $M1$-$E1$ Interference in Light Nuclei

Consider the situation of a regular magnetic dipole transition and an irregular electric dipole transition between nuclear states that are adequately described in terms of a configuration of equivalent particles (either in LS or jj-coupling). This will be the approximate situation when considering transitions in light nuclei (e.g., $A\leqslant 16$). Taking $e_1=\frac{1}{2}e\sum_i(1+\tau_z^{(i)})\mathbf{r}_i$, where $\tau_z^{(i)}$ is the z component of the isotopic spin operator for the ith nucleon and writing $\mathcal{V}=\frac{1}{2}\sum_{j\neq k}\mathcal{V}_{jk}$, we have from (37)

$$\langle\Psi_2|e_1|\Psi_1\rangle=\frac{-e}{4\hbar\omega}\langle 2|\sum_{i,j\neq k}\{(1+\tau_z^{(i)})\mathbf{r}_i\mathcal{V}_{jk}+\mathcal{V}_{jk}(1+\tau_z^{(i)})\mathbf{r}_i\}|1\rangle. \tag{38}$$

But in the summation if $i\neq j$ or k, then the above matrix element will vanish since it would reduce to the product of matrix elements of two odd-parity operators ($\mathbf{r}_i$ and $\mathcal{V}_{jk}$) with respect to particle states having the same parity (since $|1\rangle$ and $|2\rangle$ are equivalent particle wave functions). Thus, considering only those terms for which $i=j$ or $i=k$, (38) reduces to

$$\langle\Psi_2|e_1|\Psi_1\rangle=-\frac{e}{2\hbar\omega}\langle 2|\sum_{j\neq k}\{(1+\tau_z^{(j)})\mathbf{r}_j\mathcal{V}_{jk}+\mathcal{V}_{jk}(1+\tau_z^{(j)})\mathbf{r}_j\}|1\rangle. \tag{39}$$

Now take $\mathcal{V}_{jk}$ to have the form (1). The isotopic spin dependence can be written $(\tau_x^{(j)}\tau_x^{(k)}+\tau_y^{(j)}\tau_y^{(k)})$ and $\tau_z^{(j)}$ anticommutes with $\mathcal{V}_{jk}$. The isotopic spin dependence of (39) therefore disappears to give

$$\langle\Psi_2|e_1|\Psi_1\rangle=-(e/2\hbar\omega)\langle 2|\sum_{j\neq k}(r_j+r_k)\mathcal{V}_{jk}|1\rangle. \tag{40}$$

But in LS coupling for a nucleus having $\leqslant 4$ equivalent particles, the low-lying states are spatially symmetric under interchange of two particles, whereas the operator in the above matrix element is spatially antisymmetric. The matrix element therefore vanishes. This would be the situation, for example in the $\frac{1}{2}-\rightarrow\frac{3}{2}-$ transition in Li^7 which has been used in this way.

In an equivalent-particle jj-description, the matrix element reduces to a sum of two-particle matrix elements

$$\langle\Psi_2|e_1|\Psi_1\rangle\sim\sum_{J_1T_1J_2T_2}C_{J_1T_1J_2T_2}\times\langle(j^2)J_2T_2|(\mathbf{r}_1+\mathbf{r}_2)\mathcal{V}_{12}|(j^2)J_1T_1\rangle, \tag{41}$$

where the (real) coefficients $C_{J_1T_1J_2T_2}$ are combinations of fractional parentage coefficients and J and T are the total angular momentum and total isotopic spins of two particles. For reasons of antisymmetry, when $T=0$, $J=1, 3, 5\cdots$ and when $T=1$, $J=0, 2, 4\cdots$. Thus, since $(\mathbf{r}_1+\mathbf{r}_2)\mathcal{V}_{12}$ transforms as $D^{(0)}+D^{(2)}$ in isotopic spin space and $D^{(1)}$ in ordinary space, it follows that only diagonal matrix elements can contribute to the sum (41). The matrix element is therefore real since the operator is Hermitian. However, because of the choice of phase of the multipole operators (see section II) this matrix element must be imaginary (it is real relative to $\langle\Psi_1|m_1|\Psi_2\rangle$ with $m_1\sim i\mu$, where μ is the magnetic moment operator, which is clearly imaginary). Thus this matrix element vanishes.

The point of the foregoing discussion is to illustrate that care must be taken in selecting nuclei in which electromagnetic transitions between low-lying nuclear states are to be used to investigate the breakdown of parity conservation. In particular, nuclei which are well described by a simple few-particles LS or jj wave functions are likely to be unsatisfactory in that the irregular multipole matrix element may vanish or be very small for reasons of the type just described thus making the experiment insensitive for the detection of at least certain types of PNC internucleon potential.

We now consider the situation for a neutron-capture gamma-ray transition.

C. Neutron Capture γ-Ray Transition

This case has been discussed to some extent by Haas *et al.*[16] A rather different treatment of the problem is given here.

The situation differs from that holding in the case of a γ transition between low-lying states in that very near the capture state $\psi_{\lambda g}$ is likely to be a state $\psi_{\lambda'\mu}$ having the same angular momentum but opposite parity. The admixture of this state into $\psi_{\lambda g}$ will then be the dominant contribution to the sum in expression (33). Thus, we approximate (33) by

$$\Psi_{\lambda g}=\psi_{\lambda g}+\frac{\langle\lambda'\mu|\mathcal{V}|\lambda g\rangle}{\Delta_{\lambda\lambda'}}\psi_{\lambda'\mu}. \tag{42}$$

The problem then resolves itself into estimating the magnitude of the admixture parameter α, where

$$\alpha=\frac{\langle\lambda'\mu|\mathcal{V}|\lambda g\rangle}{\Delta_{\lambda\lambda'}}=\frac{\mathfrak{M}_{\lambda\lambda'}}{\Delta_{\lambda\lambda'}}. \tag{43}$$

Unfortunately, the situation is too complicated to obtain anything but some average estimate of α. In the following an upper limit is obtained for the average value of α using the same model as Haas *et al.*[16] In this model it is assumed that $\psi_{\lambda g}$, whose corresponding eigenvalue is $E_{\lambda g}$, can be expanded as

$$\psi_{\lambda g}=\sum_i a_i u_{ig}, \tag{44}$$

where the set u_{ig} is approximately equal to the set of N almost degenerate oscillator states belonging to the oscillator level n with energy $n\hbar\omega$ near $E_{\lambda g}$. Thus approximately:

$$\begin{gathered} H_0\psi_{\lambda g}=n\hbar\omega\psi_{\lambda g}, \\ (H_0+V)\psi_{\lambda g}=E_{\lambda g}\psi_{\lambda g}, \end{gathered} \tag{45}$$

where H_0 is the oscillator Hamiltonian and V is a strong parity-conserving internucleon potential representing the difference between the oscillator potential and the true internucleon potential. The opposite-parity state $\psi_{\lambda' u}$ can be described in a similar fashion as belonging to the oscillator level m (of opposite parity to n) with energy $m\hbar\omega$ and degeneracy M.

We now want to obtain some idea of the average values of $\mathfrak{M}_{\lambda\lambda'}$ and $\Delta_{\lambda\lambda'}$. Consider $\sum_{\lambda'}|\mathfrak{M}_{\lambda\lambda'}|^2$. By closure

$$\sum_{\lambda'}|\mathfrak{M}_{\lambda\lambda'}|^2=\langle\lambda g|\mathcal{V}^2|\lambda g\rangle. \tag{46}$$

If $\mathcal{V}$ only had matrix elements between $\psi_{\lambda g}$ and states $\psi_{\lambda' u}$ belonging to the oscillator level m, then

$$\sum_{\lambda'}|\mathfrak{M}_{\lambda\lambda'}|^2=M[\mathfrak{M}_\lambda{}^2]_{\rm av}, \tag{47}$$

where $[\mathfrak{M}_\lambda{}^2]_{\rm av}$ is the mean square average of matrix elements between $\psi_{\lambda g}$ and states of the type $\psi_{\lambda' u}$. However, $\mathcal{V}$ will have matrix elements between $\psi_{\lambda g}$ and other states, so that

$$[\mathfrak{M}_\lambda{}^2]_{\rm av}\leqslant(1/M)\sum_{\lambda'}|\mathfrak{M}_{\lambda\lambda'}|^2. \tag{48}$$

Finally, averaging over the N states $\psi_{\lambda g}$ belonging to the oscillator level n, we have for the mean square average of matrix elements of $\mathcal{V}$ between states of the type $\psi_{\lambda g}$ and $\psi_{\lambda u}$

$$\begin{aligned}[\mathfrak{M}^2]_{\rm av}\leqslant\frac{1}{NM}\sum_{\lambda\lambda'}|\mathfrak{M}_{\lambda\lambda'}|^2=\frac{1}{NM}\sum_\lambda\langle\lambda g|\mathcal{V}^2|\lambda g\rangle \\ \leqslant(1/M)[\langle\lambda g|\mathcal{V}^2|\lambda g\rangle]_{\rm av}.\end{aligned} \tag{49}$$

Now consider the quantity

$$[\langle\lambda g|V^2|\lambda g\rangle]_{\rm av}=(1/N)\sum_\lambda\langle\lambda g|V^2|\lambda g\rangle, \tag{50}$$

where the λ sum is over the N states belonging to the oscillator level n. By Eqs. (45) and (50)

$$\begin{aligned}[\langle\lambda g|V^2|\lambda g\rangle]_{\rm av}&=(1/N)\sum_\lambda\langle\lambda g|(E_{\lambda g}-H_0)^2|\lambda g\rangle \\ &\approx(1/N)\sum_\lambda\langle\lambda g|(E_{\lambda g}-n\hbar\omega)^2|\lambda g\rangle \\ &\approx(1/N)\sum_\lambda(E_{\lambda g}-n\hbar\omega)^2.\end{aligned} \tag{51}$$

But this quantity is the mean square deviation of the energy of the states $\psi_{\lambda g}$ from the oscillator level n and if the level spacing for the states $\psi_{\lambda g}$ is Δ its value must be less than $(N\Delta)^2$ if the states are distributed in the vicinity of n. Conversely

$$[(N\Delta)^2]_{\rm av}\geqslant[\langle\lambda g|V^2|\lambda g\rangle]_{\rm av}. \tag{52}$$

But the mean value of $\Delta_{\lambda\lambda'}$ will also be of the order $([\Delta^2]_{\rm av})^{\frac{1}{2}}$ where by (52)

$$([\Delta^2]_{\rm av})^{\frac{1}{2}}\geqslant(1/N)([\langle\lambda g|V^2|\lambda g\rangle]_{\rm av})^{\frac{1}{2}}. \tag{53}$$

Now write $[\alpha^2]_{\rm av}=[\mathfrak{M}^2]_{\rm av}/[\Delta^2]_{\rm av}$ to give on using (49) and (53)

$$([\alpha^2]_{\rm av})^{\frac{1}{2}}\leqslant\frac{N}{M^{\frac{1}{2}}}\left(\frac{[\langle\lambda g|\mathcal{V}^2|\lambda g\rangle]_{\rm av}}{[\langle\lambda g|V^2|\lambda g\rangle]_{\rm av}}\right)^{\frac{1}{2}}. \tag{54}$$

But $([\langle\lambda g|\mathcal{V}^2|\lambda g\rangle]_{\rm av}/[\langle\lambda g|V^2|\lambda g\rangle]_{\rm av})^{\frac{1}{2}}$ is a very suitable definition of $\mathfrak{F}$ the ratio of the strengths of the PNC and PC potentials. Further $N\approx M$, so that

$$([\alpha^2]_{\rm av})^{\frac{1}{2}}\lesssim\mathfrak{F}N^{\frac{1}{2}}. \tag{55}$$

This upper limit on $([\alpha^2]_{\rm av})^{\frac{1}{2}}$ is to be compared with the result of Haas *et al.*,[16] who obtain essentially [see their equation (2)]

$$([\alpha^2]_{\rm av})^{\frac{1}{2}}=A^{\frac{1}{3}}N^{\frac{1}{2}}\,{}_nH_m'/\hbar\omega, \tag{56}$$

where A is the mass number of the nucleus under consideration, $\hbar\omega$ the oscillator spacing and ${}_nH_m'$ is an effective one-particle matrix element of the PNC potential. This latter quantity is difficult to interpret in view of the fact pointed out in I that, at least in the static approximation, it is impossible to construct a one-particle PNC potential. For $\mathfrak{F}$, Haas *et al.*[16] take the ratio of ${}_nH_m'$ to the depth V_0 of the single particle shell model potential. Thus, their result can be written

$$([\alpha^2]_{\mathrm{av}})^{\frac{1}{2}} = \mathfrak{F}N^{\frac{1}{2}}(A^{\frac{1}{3}}V_0/\hbar\omega), \tag{57}$$

where $(A^{\frac{1}{3}}V_0/\hbar\omega)=40$ for $A=100$, $V_0=40$ Mev, $\hbar\omega=10$ Mev. Now the inequality in (55) is quite a strong one[18] so that comparison of (55) and (57) would imply that $([\alpha^2]_{\mathrm{av}})^{\frac{1}{2}}$ is at least a factor 10^2-10^3 smaller than that taken by Haas *et al.*

VI. CRITIQUE OF γ TRANSITIONS USED IN EXPERIMENTS

In the following, brief remarks are made, in the light of the foregoing discussion, about the γ transitions so far used by experimenters in investigations of parity non-conservation in electromagnetic transitions.

$$\mathbf{B}^{11*}(\tfrac{1}{2}-,\, T=\tfrac{1}{2}) \rightarrow \mathbf{B}^{11}(\tfrac{3}{2}-,\, T=\tfrac{1}{2})\ ^{19}$$

The regular transition here is $M1$ and $M1$-$E1$ interference is to be expected. Since there are 7 equivalent particles in the p shell there is no obvious reason why the interference term should vanish. Furthermore, isotopic spin considerations impose no restrictions and the transition seems to be a useful one to use. Wilkinson[17] obtains $\mathfrak{F}^2 \leqslant 1\times 10^{-7}$.

$$\mathbf{O}^{16*}(1-,\, T=0) \rightarrow \mathbf{O}^{16}(0+,\, T=0)\ ^{19}$$

Here the regular transition is $E1$ but is much inhibited by the $\Delta T=0$ selection rule for electric dipole emission. $M1$-$E1$ interference therefore might be expected to be large. However, with either of the PNC potentials (1) or (2) (even including higher order contributions which will have essentially the same $D^{(0)}+D^{(2)}$ symmetry in isotopic spin space) only parity impurities having $T=0$ or 2 can be admixed. However $\Delta T=2$ is forbidden in an electromagnetic transition and a $\Delta T=0$ $M1$ transition matrix element is inhibited[20] by a factor ≈ 10. Wilkinson's[17] result ($\mathfrak{F}^2 \leqslant 3\times 10^{-8}$) should therefore be modified to $\mathfrak{F}^2 \leqslant 3\times 10^{-6}$ if it is supposed that the PNC potential stems from the self interacting current description of weak interactions.

$$\mathbf{Li}^{7*}(\tfrac{1}{2}-,\, T=\tfrac{1}{2}) \rightarrow \mathbf{Li}^{7}(\tfrac{3}{2}-,\, T=\tfrac{1}{2})\ ^{21}$$

Again the regular transition is $E1$ and we have $M1$-$E1$ interference. However, Li^7 is a nucleus in which an LS or jj description gives a vanishing effect for the potential (1). In intermediate coupling there would be a finite effect, but since it vanishes at the two extremes of coupling it is expected to be small in the intermediate case.

$$\mathbf{K}^{41*}(\tfrac{7}{2}-) \rightarrow \mathbf{K}^{41}(\tfrac{3}{2}+);\ \mathbf{Cs}^{133*}(\tfrac{5}{2}+) \rightarrow \mathbf{Cs}^{133}(\tfrac{7}{2}+);$$
$$\mathbf{Hf}^{177*}(9/2+) \rightarrow \mathbf{Hf}^{177}(\tfrac{7}{2}-)\ ^{12}$$

The regular parts of these transitions, which are between low-lying states, are respectively $M2$, $M1$, and $E1$ and any interference is expected to be with the multipole of the same multipolarity but opposite parity. Each of these transitions follows a β decay and has been used by Boehm and Hauser[12] in searching for a forward-backward asymmetry in the β-γ angular correlation. From the point of view of the investigations of this paper, all that can be said is that the transitions are between complicated many particle states of mixed isotopic spin. Thus, there is no *a priori* reason why any selection rule or principle should vitiate the significance of the experimental results, although an accidental vanishing of any effect cannot be ruled out. After making various theoretical assumptions, Boehm and Hauser[12] came to the conclusion that

$$\mathfrak{F} \leqslant 2.8\times 10^{-3} \cdots 7.5\times 10^{-6}.$$

$$\mathbf{Cd}^{114*}(1+) \rightarrow \mathbf{Cd}^{114}(0+)\ ^{16}$$

This transition, following the capture of polarized thermal neutrons, has been used by Haas *et al.*[16] and an experimental upper limit obtained for the forward-backward asymmetry of the radiation relative to the polarization direction of the neutrons. The regular transition is $M1$ and $M1$-$E1$ interference is expected as in the discussion of section V(c). Using (57), Haas *et al.* obtain $\mathfrak{F} \leqslant 10^{-9}$ and quote $\mathfrak{F} \leqslant 10^{-8}$ after allowing a factor 10 for the uncertainties of their calculation. However, in view of the conclusions of section V(c), probably the most that can be stated with any confidence is that $\mathfrak{F} \leqslant 10^{-5}$, here allowing a factor 100 for the inequalities which appear in the calculations of that section.

VII. DISCUSSION

The foregoing calculations and the comments on the γ transitions so far used in experiments indicate that the nature of the nucleus selected for an experiment is extremely important. Thus, if it is assumed that the PNC internucleon potential arises from the self-interacting current theory of weak interactions, then because of the isotopic spin character of the current and the charge independent nature of strong interactions, it follows that the PNC potential must always transform as a combination of $D^{(0)}$ and $D^{(2)}$ in isotopic spin space. This means that an experiment which depends on admixing isotopic spin states by a $D^{(1)}$ admixing agent is of no value for the detection of such a potential. This stricture would only apply, of course, to light nuclei where isotopic spin is a good quantum number.

Furthermore, if the isotopic spin dependence is that of the potential (1), then the analysis of section V(b) suggests that it is unwise to chose a γ transition in a nucleus having a one-particle, or particularly simple LS or jj equivalent-particle structure.

---

[18] E.g., for a uniform distribution of levels distributed symmetrically about $n\hbar\omega$ the inequality in (53) alone corresponds to a factor 12.
[19] D. H. Wilkinson, Phys. Rev. **109**, 1610 (1958).
[20] G. Morpurgo, Phys. Rev. **110**, 721 (1958).
[21] D. H. Wilkinson, Phys. Rev. **109**, 1614 (1958).

It is therefore to be expected that γ transitions in rather complicated nuclei have more chance of showing an effect. If one is observed in such a nucleus, well and good. However, if no effect is observed, then because of the very complexity of the nucleus, the negative result is difficult to interpret. As pointed out by Wilkinson,[2] the significance of such negative results only increases in proportion to the number of cases investigated. So far only a few experiments have been carried out and it is probably only safe to say that $\mathfrak{F} \leqslant 10^{-4}-10^{-5}$ with any confidence. This is still two orders of magnitude larger than the value of $\mathfrak{F}$ expected from the potential (1).

ACKNOWLEDGMENTS

Comments on various aspects of this work by Professor H. Feshbach, Professor L. Grodzins, Professor T. D. Lee, and Professor V. F. Weisskopf are gratefully acknowledged.

PHYSICAL REVIEW VOLUME 120, NUMBER 1 OCTOBER 1, 1960

Transition Intensities in the Tl^{208} Beta Decay, the $Bi^{212} \rightarrow Po^{212}$ Decay Scheme, and the Bi^{212} Branching Ratio*

G. SCHUPP, H. DANIEL,† G. W. EAKINS, AND E. N. JENSEN
Institute for Atomic Research and Department of Physics, Iowa State University, Ames, Iowa
(Received April 14, 1960)

Studies were made on the Pb^{212} (ThB) active deposit by means of gamma singles and beta-gamma, gamma-gamma, gamma-alpha, and gamma-gamma-alpha coincidence measurements. The singles and coincidence gamma-ray spectra were recorded on an RCL 256-channel analyzer, and an intermediate-image beta-ray spectrometer was used in the beta-gamma work. Beta intensities of 4.6±0.2, 23.9±0.8, 22.7±0.7, 48.8±2.7, and <0.5% were obtained for the 1.04-, 1.29-, 1.52-, 1.80-, and 2.38-Mev groups, respectively, of the $Tl^{208} \rightarrow Pb^{208}$ decay. Existence of the 1.800-Mev gamma ray in Po^{212} was established and 11.2±0.7% of the $Bi^{212} \rightarrow Po^{212}$ disintegrations were determined to go by way of the 0.727-Mev transition. Relative intensities of 11.1±0.7, 1.7±0.3, 0.66±0.07, 0.16±0.04, 0.99±0.08, 0.49±0.05, 2.8±0.2, and 0.17±0.03 were found for the 0.727-, 0.786-, 0.893-, 0.953-, 1.073-, 1.513-, 1.620-, and 1.800-Mev gamma rays, respectively, in Po^{212}. The ratio of alpha to total disintegrations for the Bi^{212} decay was measured to be 0.3596 ±0.0006.

I. INTRODUCTION

THE nuclei Bi^{212} (ThC) and Tl^{208} (ThC″) are members of the naturally radioactive thorium ($4n$) series. Figure 1 shows the generic relations between the last members of this family. Most of the details of the Tl^{208} decay were already known[1–3] before the beginning of the present work while knowledge of the Bi^{212} beta decay was incomplete.[1,4,5]

Reported values for the intensities of the transitions occurring in the beta decay of these nuclei show considerable disagreement. It was the purpose of this investigation to further clarify the scheme of the beta decay of Bi^{212} and to examine the intensities of the radiations resulting from the decay of Tl^{208}. The level schemes of Po^{212} (ThC′) and Pb^{208} (ThD) are interesting because ${}_{82}Pb_{126}{}^{208}$ has a doubly magic nucleus and ${}_{84}Po_{128}{}^{212}$ has only four nucleons outside the doubly magic core.

The following types of measurements were performed by means of magnetic and scintillation spectrometers: beta- and gamma-ray singles; beta-gamma, gamma-gamma, gamma-alpha, and gamma-gamma-alpha coincidences. The beta-gamma and gamma-gamma coincidences were performed for both decays while the alpha coincidences were used to isolate the Po^{212} gamma rays.

II. EXPERIMENTAL TECHNIQUES

Sources of Pb^{212} (ThB) in equilibrium with its decay products were used in all of the measurements. The magnetic beta-ray spectrometer sources were collected on Al foil of 1.8 mg/cm² and 1-mm diameter whereas those used for the scintillation spectrometers were collected on Pt foil of 50 mg/cm² and 5-mm diameter.

Figure 2 shows the experimental arrangement for the beta-gamma coincidence measurements. The magnetic spectrometer is of the Slätis-Siegbahn type modified[6]

* Contribution No. 871. Work was performed in the Ames Laboratory of the U. S. Atomic Energy Commission.

† On leave from Max Planck Institute for Nuclear Physics, Heidelberg, Germany.

[1] H. Daniel, Ergeb. exakt. Naturw. **32**, 118 (1959). Reviews older work.

[2] E. M. Krisiuk, A. G. Sergeev, G. D. Latyshev, K. I. Il'in, and V. I. Fadeev, J. Exptl. Theoret. Phys. (U.S.S.R.) **33**, 1144 (1958) [translation: Soviet Phys.—JETP **6**, 880 (1958)].

[3] *Nuclear Data Sheets*, edited by C. L. McGinnis (National Academy of Science, Washington, D. C., 1958).

[4] A. G. Sergeev, E. M. Krisiuk, G. D. Latyshev, Yu. N. Trofimov, and A. S. Remmennyi, J. Exptl. Theoret. Phys. (U.S.S.R.) **33**, 1140 (1958) [translation: Soviet Phys.—JETP **6**, 878 (1958)].

[5] J. Burde and B. Rozner, Phys. Rev. **107**, 531 (1957).

[6] To be described in a later publication.