Ground States of Odd-Odd Nuclei*

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The spectrum of odd-odd nuclei is investigated with a general interaction of the type $V_0+\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2V_1$. It is shown that an inspection of the properties of some Racah coefficients is enough to determine the ground-state spin for particle-particle or hole-hole configurations in most cases. The results agree qualitatively with empirical evidence summarized by Brennan and Bernstein if V_0 and V_1 are both attractive; this agreement is insensitive to the range and other details of either V_0 or V_1.

A RECENT analysis of the spins of odd-odd nuclei[1] has revealed an outstanding regularity. For an odd-odd nucleus with p protons in the j_1 orbit and n neutrons in the j_2 orbit, provided $(2j_1+1-2p)(2j_2+1-2n)\geqslant 0$, Brennan and Bernstein find that the ground-state spin J_0 of the configuration $[(j_1^p(J_p)j_2^n(J_n)]$ is given by

$$J_0=|J_p-J_n| \quad \text{if} \quad j_1=l_1\pm\tfrac{1}{2},\quad j_2=l_2\mp\tfrac{1}{2};$$
$$J_0=J_p+J_n \quad \text{or} \quad |J_p-J_n| \quad \text{if} \quad j_1=l_1\pm\tfrac{1}{2},\quad j_2=l_2\pm\tfrac{1}{2}.$$

Here J_p and J_n are the total angular momenta of the protons and the neutrons separately. They are taken from neighboring odd-even nuclei and are assumed to be good quantum numbers for the ground-state wave function of the odd-odd nucleus. There are only a few cases of particle-hole configurations [i.e., $(2j_1+1-2p)\times(2j_2+1-2n)<0$] and there does not seem to be a clear rule about the ground-state spin in these cases.

The simplicity of these modified Nordheim rules[2] and their success in reproducing the experimental data suggests that their validity is due to general features of the residual proton-neutron interaction rather than to its details. Furthermore, the odd-group model, which asserts that the ground-state configuration has well defined values of J_p and J_n, is only approximately valid.[3] The modified Nordheim rule can therefore hold only if the arguments for its validity are not too sensitive to small corrections to the odd-group model.

A possible explanation of the modified Nordheim rules is obtained if we consider the general proton-neutron interaction:

$$V_{pn}=V_0(r_1,r_2,\cos\omega_{12})+\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2V_1(r_1,r_2,\cos\omega_{12}). \quad (1)$$

The special case in which V_0 and V_1 are both proportional to $\delta(|\mathbf{r}_1-\mathbf{r}_2|)$ was studied earlier by Schwartz and others.[4] Here we do not restrict V_0 and V_1 except in requiring their rotational invariance and that they represent attractive forces. The latter requirement is made more specific by assuming that all the Slater integrals of both V_0 and V_1 are negative:

$$F_k^{(n)}=\frac{2k+1}{2}\int R^2{}_{n_1l_1}(r_1)R^2{}_{n_2l_2}(r_2)\times\left[\int_{-1}^{1}V_n(r_1,r_2,\cos\omega_{12})P_k(\cos\omega_{12})d(\cos\omega_{12})\right]\times dr_1dr_2\leqslant 0,\quad n=0,1. \quad (2)$$

We consider first the case of a single proton in j_1 and a single neutron in j_2. The splitting of the different levels of the configuration (j_1j_2) resulting from the interaction (1) can then be written as

$$\begin{aligned}\langle j_1j_2J|V_0+\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2V_1|j_1j_2J\rangle&=W(J)-S(J),\\ W(J)&=\langle j_1j_2J|V_0+V_1|j_1j_2J\rangle,\\ S(J)&=2(2j_1+1)(2j_2+1)\begin{Bmatrix}l_1&l_2&J\\ j_2&j_1&\tfrac{1}{2}\end{Bmatrix}^2\langle l_1l_2J|V_1|l_1l_2J\rangle.\end{aligned} \quad (3)$$

$W(J)$ has the characteristic features of a spectrum resulting from a pure Wigner force. By examining the contributions of each multipole-multipole interaction separately, it can be shown that under the conditions (2), the lowest state of $W(J)$ is always that with $J=|j_1-j_2|$, and the state next to it is almost always that with $J=j_1+j_2$. (Exceptions may occur for $j_1=j_2\geqslant\frac{5}{2}$ and a dominant quadrupole-quadrupole interaction, in which case $J=|j_1-j_2|+1$ is slightly lower than $J=j_1+j_2$; this, however, can be shown not to affect our considerations appreciably.)

From the observed doublet splittings in nuclei for which $j_1=\frac{1}{2}$ or $j_2=\frac{1}{2}$ it can be inferred that V_1 is small compared to V_0 ($V_1\approx 0.1V_0$). The same conclusion is arrived at also in a number of other, independent, ways.[1,4] The contribution of $S(J)$, the singlet part of $\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2V_1$, is therefore small compared to that of $W(J)$. However, it turns out that $S(J)$ actually does determine the order of levels near the ground state because

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$S(J=|j_1-j_2|)-S(J=j_1+j_2)$ is *not* small compared with $W(J=|j_1-j_2|)-W(J=j_1+j_2)$.

Using the explicit formula[5] for the Racah coefficient in $S(J)$, we find that

$$\begin{Bmatrix} l_1 & l_2 & J \\ j_2 & j_1 & \frac{1}{2} \end{Bmatrix}^2 = \frac{1}{\epsilon_1\epsilon_2}\begin{cases} [J(J+1)-(j_2-j_1)(j_2-j_1\pm1)] & \text{for } j_1=l_1\pm\frac{1}{2} \quad j_2=l_2\mp\frac{1}{2} \\ [(j_1+j_2)(j_1+j_2+1)-J(J+1)] & \text{for } j_1=l_1+\frac{1}{2} \quad j_2=l_2+\frac{1}{2}, \\ [(l_1+l_2)(l_1+l_2+1)-J(J+1)] & \text{for } j_1=l_1-\frac{1}{2} \quad j_2=l_2-\frac{1}{2} \end{cases} \qquad (4)$$

where $\epsilon=(j+l+\frac{1}{2})(j+l+\frac{3}{2})$. The effects of $S(J)$, which always *reduces* the binding of the state considered, *increase* therefore with J if $j_1=l_1\pm\frac{1}{2}$, $j_2=l_2\mp\frac{1}{2}$, whereas they *decrease* with J if $j_1=l_1\pm\frac{1}{2}$, $j_2=l_2\pm\frac{1}{2}$. Hence, if we consider $W(J)-S(J)$, the state with $J=|j_1-j_2|$ remains the ground state in the former case (Nordheim's strong rule), whereas it gets closer to $J=j_1+j_2$, and may even cross it, in the latter case. Thus, for $j_1=l_1\pm\frac{1}{2}$, $j_2=l_2\pm\frac{1}{2}$, either $J=|j_1-j_2|$ or $J=j_1+j_2$ may be the ground state, the two levels being at any rate close to each other.

If there are more protons and neutrons in the orbits j_1 and j_2, and if both proton and neutron configurations have seniority $v=1$ ($J_p=j_1$, $J_n=j_2$), the situation remains the same[4] as long as $(2j_1+1-2p)(2j_2+1-2p) \geqslant 0$. For particle-hole configurations, however, V_0 becomes effectively a *repulsive* interaction, the order of the levels is inverted, and nothing definite can be said about the ground-state spin.

The case in which the proton and neutron configurations do not have seniority $v=1$ is slightly more complicated. Consider first V_C. We have, using the ordinary Slater expansion,[5]

$$\langle j_1^p(J_p)j_2^n(J_n)J|\textstyle\sum V_0(p,n)|j_1^p(J_p)j_2^n(J_n)J\rangle$$
$$=\textstyle\sum_k(-1)^{J_p+J_n+J}\begin{Bmatrix} J_p & J_n & J \\ J_n & J_p & k \end{Bmatrix}$$
$$\times(j_1^pJ_p\|\textstyle\sum_p C_{(p)}{}^{(k)}\|j_1^pJ_p)$$
$$\times(j_2^nJ_n\|\textstyle\sum_n C_{(n)}{}^{(k)}\|j_2^nJ_n)F_k{}^{(0)}, \qquad (5)$$

where $C_\kappa{}^{(k)}(p)=[4\pi/(2k+1)]^{\frac{1}{2}}Y_{k\kappa}(\theta_p,\varphi_p)$. Using fractional parentage coefficients we can write the reduced matrix elements in (5) in the form

$$(j_1^pJ_p\|\textstyle\sum_p C^{(k)}(p)\|j_1^pJ_p)$$
$$=p\cdot(2J_p+1)(j_1\|C^{(k)}\|j_1)f_k{}^{(p)}, \qquad (6)$$

where

$$f_k{}^{(p)}=\sum_{J_p'}(-1)^{J_p'+j_1+J_p+k}\begin{Bmatrix} j_1 & J_p & J_p' \\ J_p & j_1 & k \end{Bmatrix}$$
$$\times|(j_1^{p-1}(J_p')j_1J_p|\}j_1^pJ_p)|^2. \qquad (7)$$

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TABLE I. Values of f_k, Eq. (7), for $p=3$ and $J_p=j_1-1$.

j_1 \ k	0	2	4	6	1	3	5	7
5/2	0.204	0.000			0.045	−0.095		
7/2	0.144	0.051	−0.033		0.036	−0.013	−0.055	
9/2	0.112	0.054	−0.002	−0.026	0.030	0.003	−0.017	−0.038

The summation in (7) can be carried out, in cases of interest, either by using tabulated values of the fractional parentage coefficients[6] or the explicit formula derived for the fractional parentage coefficients for some simple cases.[7] The results are given in Table I. As seen from this table the correction factor f_k decreases with increasing k. Since the low multipoles are probably the dominant ones in V_0, we see from Eq. (6) and Table I that, at least for the interesting cases $p=3$ and $J_p=j_1-1$ (or similarly for n), the modified Slater integrals $F_k'=f_k{}^{(p)}f_k{}^{(n)}F_k$, still represent an essentially attractive interaction. Thus also in this case $J=|J_p-J_n|$ will be the spin of the lowest state and $J=J_p+J_n$ that of the next one as long as we confine ourselves to V_0.

The effects of $\sum \boldsymbol{\sigma}_p\cdot\boldsymbol{\sigma}_nV_1(p,n)$ can be analyzed in a similar way. We obtain[4]

$$\langle j_1^p(J_p)j_2^n(J_n)J|\sum_{p,n}\boldsymbol{\sigma}_p\cdot\boldsymbol{\sigma}_nV_1(p,n)|j_1^p(J_p)j_2^n(J_n)J\rangle$$
$$=\sum_{k,k'}(-1)^{J_p+J_n+J}\begin{Bmatrix} J_p & J_n & J \\ J_n & J_p & k' \end{Bmatrix}$$
$$\times(j_1\|T_1{}^{(1k)k'}\|j_1)(j_2\|T_2{}^{(1k)k'}\|j_2)$$
$$\times f_{k'}{}^{(p)}f_{k'}{}^{(n)}F_k{}^{(1)}p(2J_p+1)n(2J_n+1), \qquad (8)$$

where[4] $T^{(1k)k'}=[\boldsymbol{\sigma}\times C^{(k)}]^{(k')}$ is the irreducible tensor of order k' constructed from $\boldsymbol{\sigma}$ and $C^{(k)}$. The product $(j_1\|T_1{}^{(1k)k'}\|j_1)(j_2\|T_2{}^{(1k)k'}\|j_2)$ vanishes for even values of k'. For odd values of k' its sign is $(-1)^{(j_1-j_2)+(l_1-l_2)}$. On the other hand, we have

$$\begin{Bmatrix} J_p & J_n & J_p+J_n \\ J_n & J_p & k' \end{Bmatrix}\geqslant 0,$$

$$(-1)^{2J_n+k'}\begin{Bmatrix} J_p & J_n & |J_p-J_n| \\ J_n & J_p & k' \end{Bmatrix}\geqslant 0.$$

Combining these results, we see that if the dominant contributions to (8) come from low multipoles, for which $f_{k'}\geqslant 0$, then the effect of $\sum \boldsymbol{\sigma}_p\cdot\boldsymbol{\sigma}_nV_1(p,n)$ is to *increase* the separation between $J=|J_p-J_n|$ and $J=J_p+J_n$ if $j_1=l_1\pm\frac{1}{2}$, $j_2=l_2\mp\frac{1}{2}$, and to decrease this separation, or eventually cause the crossing of these levels, if $j_1=l_1\pm\frac{1}{2}$, $j_2=l_2\pm\frac{1}{2}$.

The validity of the modified Nordheim rules as

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formulated by Brennan and Bernstein[1] has thus been demonstrated with relatively few assumptions about the residual proton-neutron interaction in nuclei. More experimental data, especially on the excited states of odd-odd nuclei, are required to deduce more specific information on the residual neutron-proton interaction. With the modified Nordheim rule as formulated by Brennan and Bernstein we can only exclude dominant high-multipole interactions in the residual neutron-proton interaction, if it is of the general form (1). It seems that most of the regularities found so far could be understood with the assumption that the ratio of triplet to singlet parts in the residual interaction is roughly the same as that of the free proton-neutron interactions.[1,8]

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Level Structure of Eu^{153}†

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Gamma rays in Eu^{153} following the decay of Sm^{153} and Gd^{153} have been studied using coincidence and angular correlation methods. Results for the strong transitions are in agreement with the decay scheme given by McCutchen. Measurements were made of spectra of gamma rays in coincidence with the x-ray, 70-kev, and the 97-kev and 103-kev transitions in the decay of Gd^{153}, and with eight energy regions in the decay of Sm^{153}. A number of new, weak transitions were observed in the decay of Sm^{153}, and a consistent decay scheme is proposed. Directional correlation measurements were made on the 70 kev–103 kev cascade from the decay of Sm^{153} and from the decay of Gd^{153}. Possible spin assignments are discussed.

I. INTRODUCTION

THE beta decay of the 47-hour Sm^{153} to Eu^{153} and the electron capture decay of the 225-day Gd^{153} to Eu^{153} have been studied by a number of investigators.[1–25] Figure 1 shows the decay scheme given by McCutchen.[1]

There is general agreement about the levels at 84 kev, 97 kev, 103 kev, 172 kev, and 187 kev. The 70-kev gamma ray has been observed in coincidence with the 103-kev gamma ray in the decay[10] of Sm^{153} and in the decay[1] of Gd^{153}; both transitions[25] are $M1+E2$. The 97-kev gamma ray was first observed by Church and Goldhaber[16] by means of internal conversion measurements on Gd^{153}. This transition[1,21,22] is strongly fed in the decay of Gd^{153}. Recently, Walters *et al.*[15] observed a 97-kev transition with a bent-crystal spectrometer in the Sm^{153} decay with an intensity of less than 5% of the 103-kev gamma ray. The levels at 84 kev and 187 kev, and the corresponding gamma transitions have been observed by Coulomb excitation.[26–28]

The energies of the strong beta components in the Sm^{153} decay have been measured as 803 kev, 698 kev, and 640 kev.[4,8,10,11,13] The 698-kev beta transition has been observed in coincidence with the 103-kev gamma ray, and the 640-kev beta ray has been observed in

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