

Beta-Gamma Directional Correlation in  $\text{Rb}^{86}$  and  $\text{Eu}^{152}\dagger$ HELMUT J. FISCHBECK\* AND ROGER G. WILKINSON  
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Measurements are reported on the energy dependence of the beta-gamma angular correlation coefficient in the special first forbidden cases of  $\text{Rb}^{86}$  and  $\text{Eu}^{152}$ . Energy selection of the beta rays was accomplished with a magnetic beta-ray spectrometer having good transmission. It is shown that  $\text{Rb}^{86}$  cannot be analyzed in terms of the so-called modified  $B_{ij}$  approximation. The  $\text{Eu}^{152}$  decay, on the other hand, can be treated quite satisfactorily in this approximation. It is concluded in this case that a strong selection-rule effect is operative to suppress certain of the matrix elements relative to  $B_{ij}$ . The nuclear parameter  $Y$  (Kotani's notation) is found to be  $0.69 \pm 0.06$ . This agrees well with the value which results from the measured spectrum shape factor.

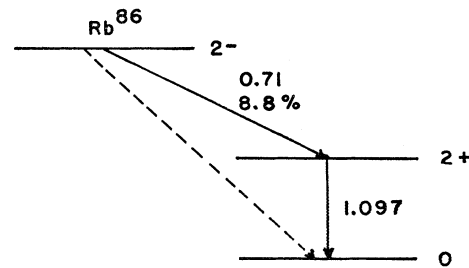
## I. INTRODUCTION

THE theory of the beta-decay process has been greatly clarified by the discovery that only the  $V$  and  $A$  interactions characterize the process, with  $C_A = C_{A'}$  and  $C_V = C_{V'}$ .<sup>1</sup> With the interaction Hamiltonian thus uniquely established, it has been possible to formulate the theory of the various observables associated with beta decay in a tractable form, suitable for comparison with experimental results. In particular, theoretical expressions for the spectrum shape correction factor and for beta-gamma directional correlations, with and without beta- or gamma-ray polarization, have been developed by Morita and Morita,<sup>2</sup> and by Kotani and Ross.<sup>3-6</sup> In view of this, it is of current interest to study the energy dependence of the beta-gamma angular anisotropy especially for first forbidden cases.

In principle, careful measurements should yield a quantitative determination of the relative contributions of the matrix elements which govern the transition. However, in spite of the great simplification which now prevails in the theory, there are in general six matrix elements involved in the first forbidden nonunique transitions and their determination requires information from several of the six or more observables.<sup>7</sup> It has been suggested<sup>6,8</sup> that special cases may exist among the first forbidden nonunique decays for which the  $B_{ij}$  parameter dominates to the extent that the transition resembles a unique transition. One then expects a large  $ft$  value, a nonstatistical shape, and a large beta-gamma angular anisotropy. Kotani<sup>6</sup> has pointed out that such

a situation may arise from a selection rule effect which serves to suppress certain matrix elements relative to  $B_{ij}$ . The so-called "modified  $B_{ij}$  approximation" of the theory, introduced by Kotani<sup>6</sup> and Matumoto *et al.*<sup>8</sup> to describe certain of these cases, considerably simplifies the interpretation of experimental results, once it is established that the approximation applies. In this instance, simple combinations of nuclear parameters may be determined from only two of the observables, for example, from the spectrum shape correction factor and the beta-gamma correlation coefficient.

The present paper is concerned with beta-gamma correlation measurements, without polarization detection, in two nuclei which have been suggested as possible examples for which the modified  $B_{ij}$  approximation may be valid,<sup>6,8</sup>  $\text{Rb}^{86}$  and  $\text{Eu}^{152}$ . The well-known decay of  $\text{Rb}^{86}$  involves two beta-groups, as shown in Fig. 1. The inner group,  $W_0 = 0.717$  Mev, has a rather high  $ft$  value,  $\log ft = 7.9$ , for a spin sequence  $2^-(\beta)2^+$  and is followed by a 1.079-Mev gamma ray. There is some evidence for a nonstatistical spectrum shape. Robinson and Langer<sup>9</sup> have reported a small deviation from an allowed shape, especially at low energies, although this may be due to other effects. The beta-gamma angular correlation as a function of the beta-ray energy has been studied by Stevenson and Deutsch<sup>10</sup> and Klewer.<sup>11</sup> While the results in both cases indicate a rather large anisotropy, they lack sufficient accuracy for detailed analysis. Morita and Morita<sup>2</sup> have attempted to analyze

FIG. 1. Decay scheme of  $\text{Rb}^{86}$ .

† Supported by the Joint Program of the Office of Naval Research and the U. S. Atomic Energy Commission.

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<sup>1</sup> For a survey of the literature pertaining to this point, see E. J. Konopinski, Annual Review of Nuclear Science (Annual Reviews, Inc., Palo Alto, California, 1959), Vol. 9, p. 99.

<sup>2</sup> M. Morita and R. S. Morita, Phys. Rev. **109**, 2048 (1958).

<sup>3</sup> T. Kotani and M. Ross, Phys. Rev. Letters **1**, 140 (1958).

<sup>4</sup> T. Kotani and M. Ross, Progr. Theoret. Phys. (Kyoto) **20**, 643 (1958).

<sup>5</sup> T. Kotani and M. Ross, Phys. Rev. **113**, 622 (1958).

<sup>6</sup> T. Kotani, Phys. Rev. **114**, 795 (1958).

<sup>7</sup> A list of the various observables is given in the Appendix of reference 6.

<sup>8</sup> Z. Matumoto, M. Morita, and M. Yamada, Bull. Kobayasi Phys. Inst. Research **5**, 210 (1955).

<sup>9</sup> R. L. Robinson and L. M. Langer, Phys. Rev. **112**, 481 (1958).

<sup>10</sup> D. T. Stevenson and M. Deutsch, Phys. Rev. **83**, 1202 (1951).

<sup>11</sup> H. Klewer, Z. angew. Physik **11**, 81 (1959).

Deutsch's data in terms of the modified  $B_{ij}$  approximation, but because of the large experimental errors, the outcome is uncertain.

Of interest in the decay of 13-year Eu<sup>152</sup>, a portion of which is shown in Fig. 2, is the outer beta group with end-point energy 1.483 Mev and the subsequent 0.344-Mev gamma ray to the ground state of Gd<sup>152</sup>. This beta-transition is characterized by an unusually large  $ft$  value,  $\log ft=11.7$ , and a nonstatistical, nonunique spectrum shape.<sup>12</sup> The spin-parity sequence is very likely  $3^-(\beta)2^+(\gamma)0^+$ , since the ground-state spin of Eu<sup>152</sup> has been found to be  $J_0=3$ .<sup>13</sup> The results of Dulaney, Braden, and Wyly<sup>14</sup> and Bhattacharjee and Mitra<sup>15</sup> on the beta-gamma angular correlation of this cascade, indicate that the anisotropy is large and might be interpreted within the framework of the modified  $B_{ij}$  approximation. However, their results when analyzed in this approximation, are somewhat at variance with Langer's measured shape factor.

In our measurements we have reduced the complicating effects which are often present in scintillation measurements by the use of magnetic analysis of the beta-ray energies. Sufficient data were gathered in most cases to achieve at least 5% accuracy. It will be shown that the beta-gamma angular correlation in the Eu<sup>152</sup> case is quite adequately described by the modified  $B_{ij}$  approximation, while in the case of Rb<sup>86</sup> this is not possible.

II. THE MODIFIED  $B_{ij}$  APPROXIMATION

It will be convenient to summarize the aspects of the theory of beta-gamma angular correlation which will be of use for comparison with experimental data. In Kotani's<sup>6</sup> notation, the six well-known first forbidden matrix elements for ranks  $\lambda=0, 1, 2$  may be incorporated for our purposes into two new parameters,  $V$  and  $Y$ , as follows:

$$\begin{aligned} \eta V &= C_A \int i\gamma_5 + \xi \eta w & \text{for } \lambda=0, \\ \eta Y &= -C_V \int i\alpha - \xi \eta(u+x) & \text{for } \lambda=1, \\ \eta &= C_A \int B_{ij} & \text{for } \lambda=2, \end{aligned} \tag{1}$$

where

$$\eta w = C_A \int \boldsymbol{\sigma} \cdot \mathbf{r}, \quad \eta u = C_A \int i\boldsymbol{\sigma} \times \mathbf{r}, \quad \eta x = -C_V \int \mathbf{r}. \tag{2}$$

<sup>12</sup> L. M. Langer, D. R. Smith, and M. P. Klein, Bull. Am. Phys. Soc. 4, 426 (1959).

<sup>13</sup> M. Abraham, R. W. Kedzie, and C. D. Jeffries, Phys. Rev. 108, 58 (1957).

<sup>14</sup> H. Dulaney, C. H. Braden, and L. D. Wyly, Phys. Rev. 117, 1092 (1960).

<sup>15</sup> S. K. Bhattacharjee and S. K. Mitra, Nuovo cimento 16, 175 (1960).

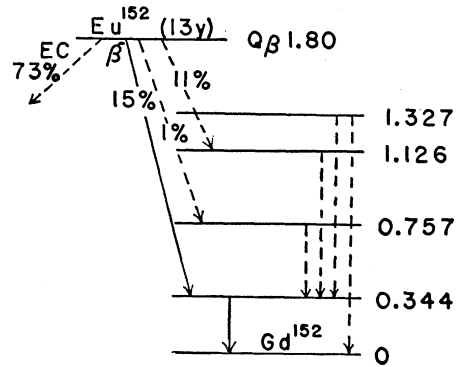


FIG. 2. Partial decay scheme of Eu<sup>152</sup>. The solid lines denote the transitions of interest in this work.

In these expressions  $\xi$  is defined by  $\xi=\alpha Z/2\rho$ , where  $\rho$  is the nuclear radius in units of the electron Compton wavelength and  $\alpha$  is the fine-structure constant. The beta-gamma angular distribution has the form

$$N(W,\vartheta) = 1 + \epsilon P_2(\cos\vartheta), \tag{3}$$

where  $\epsilon$  is the angular correlation coefficient and  $\vartheta$  is the angle between the directions of emission of the beta and gamma rays. In the same approximation in which (1) and (2) are valid, namely when finite nuclear size effects are ignored, Kotani<sup>6</sup> has shown that the energy dependence of the directional correlation coefficient is given by

$$\epsilon = (p^2/W)(R_3 + eW)[k/C(W)]. \tag{4}$$

Here  $R_3$ ,  $k$ , and  $e$  are independent of the energy  $W$  in the point-charge approximation and are functions of the matrix elements (1) and (2). Expressions for these quantities and for the spectrum shape correction factor,  $C(W)$ , valid to order  $(\alpha Z)^2$ , are given in the Appendix of reference 6.

In the modified  $B_{ij}$  approximation it is assumed that the matrix elements  $x$ ,  $u$ , and  $w$  may be neglected unless multiplied by the factor  $\xi \approx 10$ . This approximation is therefore characterized by

$$x = u = w = 0, \quad Y \neq 0, \quad V \neq 0, \quad \text{but } |V| \text{ and } |Y| < \xi. \tag{5}$$

In this instance, the nuclear size effect can be readily included and the energy-dependent Coulomb factors  $\lambda_i$ , which otherwise are unity, are retained. A small modification of the definitions (1) of  $Y$  and  $V$  is also made.<sup>16</sup> In the modified  $B_{ij}$  approximation we have the following expressions for the angular correlation coefficient and the shape factor:

For  $3(\beta)2(\gamma)0$  transitions,

$$\epsilon = - (p^2/W)[(\lambda_2 Y)/7 + (\lambda_1 W)/42][C(W)]^{-1}, \tag{6}$$

$$C(W) = (1/12)(q^2 + \lambda_1 p^2) + Y^2. \tag{7}$$

<sup>16</sup> See reference 6, p. 805.

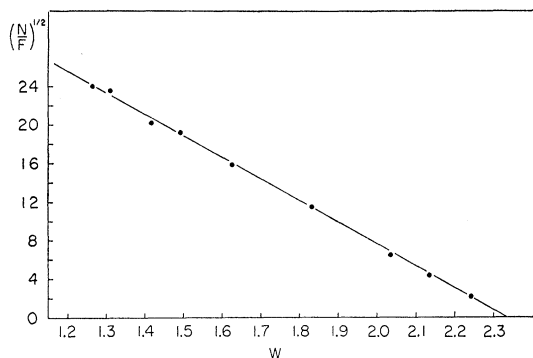


FIG. 3. Fermi plot of the  $\text{Rb}^{86}$  isotropic inner beta spectrum.  $N$  is the isotropic beta-gamma coincidence rate normalized to the gamma-ray singles. The extrapolated end point,  $W_0=2.34$ , is in good agreement with the accepted value  $W_0=2.40$  ( $m_0c^2$ ).

For  $2(\beta)2(\gamma)0$  transitions,

$$\epsilon = (p^2/W)[(\lambda_2 V)(1/56)^{1/2} - (\lambda_2 V)(1/21)^{1/2} - (\lambda_1/112)W][C(W)]^{-1}, \quad (8)$$

$$C(W) = (1/12)(q^2 + \lambda_1 p^2) + V^2 + Y^2. \quad (9)$$

In these expressions  $q^2 = (W_0 - W)^2$ ,  $p^2 = W^2 - 1$ . The factors  $\lambda_1$  and  $\lambda_2$  have been tabulated by Kotani and Ross.<sup>5</sup>

It may be noted that experimental results for both  $C(W)$  and  $\epsilon(W)$  are necessary before information concerning  $Y$  and  $V$  can be extracted from these formulas, since the validity of the approximation must first be established.

### III. APPARATUS

A  $180^\circ$ -shaped magnetic field beta-ray spectrometer was used in these experiments to define the beta energy. The spectrometer has been modified to permit beta-gamma coincidence measurements at two positions of the gamma detector. The field shape and central baffle setting are such as to focus median-plane electrons which leave the source  $15^\circ$  either side of the mean path (10-cm radius). Since no electrons are accepted which leave the source at angles less than  $90^\circ$  to the source-detector axis, measurements are restricted to the mean angles  $75^\circ$  and  $165^\circ$  between the beta and gamma ray. The gamma rays emitted at these angles pass out of the vacuum chamber through 5-mil thick aluminum ports before detection. The transmission of the instrument is set at approximately 5% by the use of a one-cm exit slit. At this transmission the measured momentum resolution is 5%. The effective solid angle at the beta detector is then 0.005 steradian.

The radiation detectors were RCA 6342-A photomultipliers with a 1-inch  $\times$  1-inch NaI(Tl) crystal for the gamma rays and a  $\frac{1}{4}$ -inch thick plastic scintillator for the beta rays. The usual precautions were employed to minimize gain shifts due to the magnetic field. The use of light pipes and concentric high- and low-level magnetic shields essentially eliminated this effect. The

shift of the  $\text{Cs}^{137}$  photopeak recorded in the gamma-ray counter was less than one percent when the counter was moved from the  $75^\circ$  to  $165^\circ$  position, with the magnetic field set for focusing 1.4-Mev electrons. The contribution of scattered gamma quanta was minimized by surrounding the gamma detector with appropriate cylindrical lead shields. When necessary, aluminum absorbers were used to prevent beta rays from striking the gamma detector. Gamma-counter solid angles were 0.010 steradian for the  $\text{Rb}^{86}$  study and 0.004 steradian for  $\text{Eu}^{152}$ .

The electronic equipment consisted of a modification of the slow-fast coincidence circuit described by Johansson.<sup>17</sup> A trigger time correction circuit makes it possible to trigger fast coincidence pulse generators with the relatively slow pulses from conventional non-overload linear amplifiers without appreciable coincidence loss. The pulse generators are gated by differential discriminators which select the desired energy range. Resolving times of  $2\tau=20$ –40 millimicroseconds have been used without measurable coincidence loss. The singles counts were recorded concurrently.

### IV. MEASUREMENTS

The observable in these experiments is not  $\epsilon(W)$ , but rather the differential beta-gamma coincidence counting rate at the  $165^\circ$  and  $75^\circ$  positions of the gamma counter, relative to the rate at the  $75^\circ$  position. In all cases this anisotropy was determined with the beta differential discriminator window set to take in substantially the entire pulse-height distribution due to beta rays of 5% energy spread. The gamma discriminator was set generally to accept the full-energy peak. In the  $\text{Rb}^{86}$  case, it proved to be quite safe to include a small portion of the Compton distribution. In general, true-to-chance ratios were of the order of 10:1, but in the  $\text{Rb}^{86}$  experiment, the presence of the strong outer

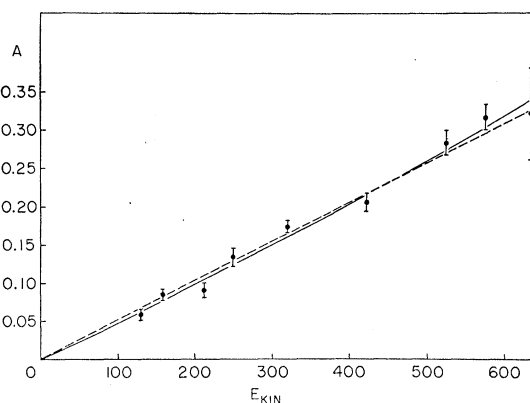


FIG. 4. Energy dependence of the anisotropy  $A$  for  $\text{Rb}^{86}$ . The solid curve was calculated from the theoretical expression for  $A$ , with  $ek/\lambda_2=0.021$  and  $Rk/\lambda_2=0.0774$ . For comparison a straight line has been included (dashed curve).

<sup>17</sup> B. Johansson, *Nuclear Instr.* **1**, 274 (1957).

group lowered this ratio to about 1:1 near the inner beta-ray end point.

The average measured anisotropy at each beta-ray energy was derived from 15 cycles with total coincidences ranging from 15 000 to 20 000 counts. Coincidence rates, corrected for accidentals, were normalized to the singles rates in the usual way. To facilitate comparison with other workers, the standard anisotropy,  $A = [N(180) - N(90)]/N(90)$ , corrected for finite solid angle, was extracted from the data. The quantity  $\epsilon(W)$  was then obtained from  $\epsilon = 2A/(3+A)$ . The error assignment is based on a statistical analysis of the fluctuations in the data instead of the customary estimation based on the total number of coincidences. Probable errors are quoted.

To insure reliability of the data, considerable care was taken to minimize drifts and counting losses in the electronics, and to eliminate asymmetries due to geometry, scattering, etc. The Fermi plot of the Rb<sup>86</sup> beta-gamma coincidence rate normalized to the gamma-ray singles, shown in Fig. 3, provides a severe test of the constancy of the counting efficiency and gain in the beta channel, as well as the constancy of the coincidence efficiency. The data shown in Fig. 3 represent a span in time of over 1200 hours. Over this same time interval the average fluctuation per week in the gamma-ray singles was about one percent, and the coincidence resolving time drift was less than one percent per week. Further evidence for stability in the gamma channel is afforded by the measured half-life of 460 hours as determined from the gamma-ray singles. This is to be compared with the prevailing value of 450 hours.<sup>18</sup>

The intrinsic asymmetry in the gamma-ray singles rates and the coincidence rates at the 75° and 165° positions was essentially negligible. Without the cylindrical lead shield around the NaI crystal the singles rates differed by 2% when the spectrometer was adjusted to focus 1-Mev electrons. With the shield in

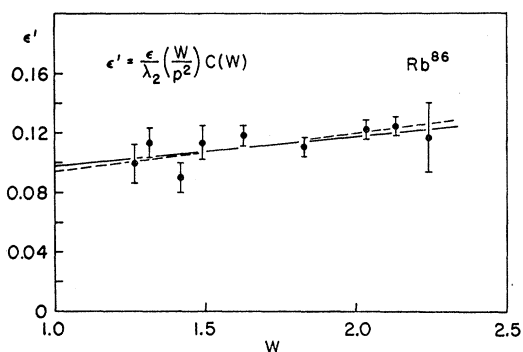


FIG. 5. A plot of the "reduced" correlation coefficient  $\epsilon'$  vs energy ( $m_0c^2$  units). The solid straight line corresponds to  $ek/\lambda_2 = 0.021$  and  $R_3k/\lambda_2 = 0.077$ , and the shape factor of Langer and Robinson. The dashed curve corresponds to the best straight line through the data for  $C(W) = 1$ .

<sup>18</sup> D. Strominger, J. M. Hollander, and G. T. Seaborg, *Revs. Modern Phys.* **30**, 585 (1958).

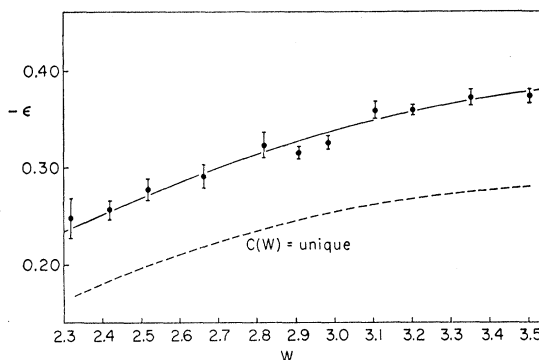


FIG. 6. Energy dependence of the correlation coefficient in the Eu<sup>152</sup> case. The solid curve is a theoretical one with  $Y = 0.69$  or  $0.23$ .

place the asymmetry was reduced to 0.25%. The residual anisotropy is probably due to the difference in the magnetic field in the two positions. For the highest energy encountered in these experiments the effect was no more than 0.5%. The possibility of a built-in coincidence anisotropy was checked by measuring the beta-gamma correlation in the allowed Na<sup>22</sup> decay. A value of  $\epsilon = -0.003 \pm 0.001$ , based on a total of 80 000 coincidences, was obtained for the anisotropy between positrons at 0.39 Mev and the 1.276-Mev gamma ray. This is in agreement with a measurement by Steffen<sup>19</sup> and suggests that the intrinsic asymmetry of the coincidence spectrometer is less than 0.3%.

The Rb<sup>86</sup> and Eu<sup>152</sup> sources were prepared from high specific activity samples produced at the Oak Ridge National Laboratory. The sources consisted of a deposit, less than 0.1 mg/cm<sup>2</sup> thick, on thin Zapon or Mylar films. A small amount ( $\sim 4\%$ ) of Eu<sup>154</sup> was present in the Eu<sup>152</sup> sample.

### V. Rb<sup>86</sup> RESULTS

The beta-gamma angular correlation data for Rb<sup>86</sup> are summarized in Table I. The energy dependence of the anisotropy  $A$  is shown in Fig. 4. A positive correlation coefficient  $\epsilon$  is found for this  $2^-(\beta)2^+(\gamma)0^+$  case ranging from 0.04 to 0.20. The form of Eq. (4) indicates that, apart from the  $p^2/W$  factor and  $C(W)$ , the energy dependence of  $\epsilon$  is linear. Accordingly, the "reduced correlation coefficient,"  $\epsilon' = \epsilon C(W)W/p^2$ , is introduced to facilitate comparison of the experimental results with theory. A plot of the experimental values of  $\epsilon'$  versus energy is expected to yield a straight line with slope  $ek$  and intercept  $R_3k$ . A representation of the data in this manner is shown in Fig. 5, with  $\lambda_2$  included in  $\epsilon'$ . This serves to suppress the small energy dependence of  $ek$  and  $R_3k$  when  $\lambda_1$  and  $\lambda_2$  are not set equal to one. It is seen that, within the experimental uncertainties, the data can be represented by a straight line of slope  $ek/\lambda_2 = 0.021$  and intercept  $R_3k/\lambda_2 = 0.077$ , when the shape factor of Robinson and Langer<sup>9</sup> is used. The

<sup>19</sup> R. Steffen, *Phys. Rev. Letters* **3**, 277 (1959).

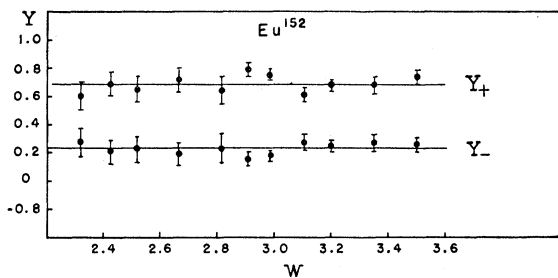


Fig. 7.  $Y_+$  and  $Y_-$  calculated from Eq. (10) in the text with the measured values of  $\epsilon$ .

dashed line in Fig. 5 results when a statistical shape is assumed. In either case a *positive* slope is indicated. It may be seen from Eq. (8) that if the modified  $B_{ij}$  approximation applies, the linear dependence of  $\epsilon'$  on  $W$  should have a *negative* slope,  $ek/\lambda_2 = -\lambda_1/(112\lambda_2) \approx -1/112$ . We conclude that this approximation does not hold for  $\text{Rb}^{86}$  and that small contributions from the matrix elements  $x$ , or  $u$  and  $x$ , are present. A contribution from  $u$  alone would only tend to make  $ek/\lambda_2$  more negative.

TABLE I. Summary of the  $\text{Rb}^{86}$  data.

$E$ (keV)	$W$ ( $m_0c^2$ )	$A$	$\epsilon$
135	1.265	$0.058 \pm 0.008$	$0.038 \pm 0.005$
159	1.311	$0.085 \pm 0.008$	$0.055 \pm 0.005$
212	1.416	$0.092 \pm 0.008$	$0.059 \pm 0.005$
251	1.491	$0.133 \pm 0.013$	$0.085 \pm 0.009$
320	1.625	$0.173 \pm 0.008$	$0.109 \pm 0.005$
423	1.827	$0.206 \pm 0.012$	$0.129 \pm 0.008$
526	2.030	$0.283 \pm 0.015$	$0.172 \pm 0.009$
577	2.130	$0.317 \pm 0.017$	$0.191 \pm 0.010$
634	2.240	$0.321 \pm 0.060$	$0.193 \pm 0.036$

Apparently no strong selection-rule effect is operative in this case to give a large reduction factor for the  $\lambda=1$  matrix elements. Additional experiments and theoretical analysis are necessary to determine the nuclear parameters. In particular, a measurement of the beta-circularly polarized gamma correlation as a function of energy would be useful.

## VI. $\text{Eu}^{152}$ RESULTS

The beta-gamma correlation coefficient in this case was found to be negative and exceptionally large. A summary of the data appears in Table II. It will be noted that the first five values lie in the region of the inner beta-ray group. To correct for the interference of the inner group it is necessary to know the branching ratio of the two groups. Dulaney *et al.*<sup>14</sup> assumed that the outer group has 3.5 times the intensity of the inner group, whereas the decay scheme shown in Fig. 2 suggests a much larger ratio.<sup>18</sup> In the present case the branching ratio was determined directly from the coincidence data by a Fermi analysis of the beta-gamma coincidences, after elimination of the angular depend-

ence. The ratio of the outer to inner beta-ray intensities determined in this way is 2.3. The correction of the lower energy points, with this value of the branching ratio, ranged from 25% at 675 keV to 2% at 929 keV. Since it is necessary to assume that the inner group is isotropic, the assigned error in these cases is generously in excess of the statistical error.

A plot of the correlation coefficient versus energy is shown in Fig. 6. A preliminary analysis incorporating the shape factor measured by Langer *et al.*<sup>12</sup> suggested strongly that Eqs. (6) and (7) apply to this case. In the  $3^-(\beta)2^+(\gamma)0^+$  case an analysis of the data in terms of the modified  $B_{ij}$  approximation can be made without direct recourse to the shape factor. Inspection of expressions (6) and (7) shows that  $Y$  is related only to the observable  $\epsilon$  when  $C(W)$  is eliminated. Therefore in the modified  $B_{ij}$  approximation we have

$$Y = -(\lambda_2/14\mathcal{E}) \pm [(\lambda_2/14\mathcal{E})^2 - (1/12)(q^2 + \lambda_1 p^2) - \lambda_1 W/42\mathcal{E}]^{1/2}, \quad (10)$$

$$\mathcal{E} = \epsilon W/p^2.$$

Clearly, since  $Y$  is independent of energy, the criterion for the validity of the modified  $B_{ij}$  approximation is the constancy of the right-hand side of (10). If expression (10) yields the same two values, within the experimental error, for each of the ten data points of Table II, then it is assured that the approximation holds and the mean value of one of the roots of (10) may be identified as the nuclear parameter  $Y$ . Figure 7 shows that the two values  $Y_+$  and  $Y_-$  (corresponding to the appropriate sign in expression 10) do not depend on  $W$  within the accuracy of the experiment. The average values are

$$Y_+ = 0.69 \pm 0.06, \quad Y_- = 0.23 \pm 0.06.$$

Although these considerations prove conclusively that the  $\text{Eu}^{152}$  data may be analyzed in the framework of the modified  $B_{ij}$  approximation, other data must be at hand to decide whether  $Y_+$  or  $Y_-$  is the nuclear parameter  $Y$ . The measured spectrum shape factor has

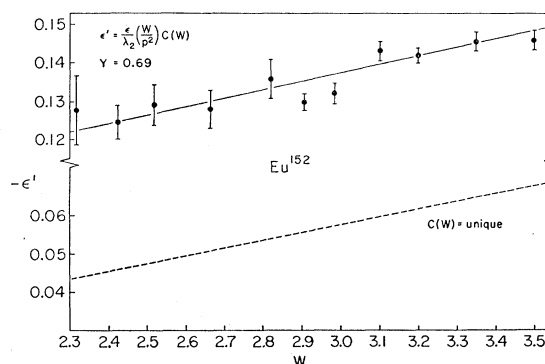


Fig. 8. The experimental values of  $\epsilon'$  compared with the theoretical straight line predicted by the modified  $B_{ij}$  approximation with  $Y=0.69$ . The theoretical slope is  $\lambda_1/42 \approx 0.019$ .

TABLE II. Summary of the Eu<sup>152</sup> data.

$E$ (kev)	$W$ ( $m_0c^2$ )	$-A$	$-\epsilon$
674	2.320	0.331±0.027	0.248±0.020
727	2.424	0.340±0.014	0.256±0.010
776	2.519	0.365±0.014	0.277±0.011
850	2.664	0.381±0.015	0.291±0.012
929	2.819	0.418±0.017	0.323±0.013
974	2.906	0.408±0.008	0.315±0.006
1014	3.103	0.457±0.009	0.359±0.007
1125	3.202	0.458±0.006	0.360±0.004
1201	3.350	0.420±0.009	0.372±0.007
1277	3.500	0.473±0.008	0.374±0.006

been reported by Langer *et al.*<sup>12</sup> in the following form:

$$C(W) = (\lambda_1 p^2 + q^2) + 5 \pm 2. \quad (11)$$

This has the form of expression (7), but it is to be noted that this fact alone does not establish the validity of the modified  $B_{ij}$  approximation. However, since the beta-gamma correlation experiment clearly demonstrates that the approximation applies, we may interpret the constant in (11) as  $12Y^2$ . On this basis Langer's experiment yields the value  $Y = \pm 0.65 \pm 0.15$ . We therefore exclude  $Y_-$  and conclude that  $Y = Y_+ = 0.69 \pm 0.06$ . It is instructive to compare our data with the theoretical expression (6) with  $Y$  set equal to 0.69, i.e., with the shape factor  $(1/12)(q^2 + \lambda_1 p^2) + 0.476$ . In Fig. 8 the data are expressed in terms of the reduced correlation coefficient and are in agreement with the theoretical straight line of slope  $\lambda_1/42$ . Because of the rather large errors quoted in the measured shape factor (11) it might be argued that the result is not significantly different from a unique shape ( $\Delta J \neq 2, Y = 0$ ). Bhattacharjee *et al.*,<sup>20</sup> in fact, have reported a unique energy spectrum shape. The dashed line in Fig. 6 and Fig. 8 corresponds to this situation and is definitely not in agreement with our measurements. We can further rule out the possibility of a unique transition  $4^-(\beta)2^+(\gamma)0^+$  since this requires a positive beta-gamma anisotropy.

The foregoing considerations show that the measured energy dependence of  $\epsilon$  is adequately described by the modified  $B_{ij}$  approximation and the matrix element ratio,  $Y = 0.69 \pm 0.06$ , is in good agreement with the spectrum shape result of Langer *et al.*,<sup>12</sup>  $Y = 0.65 \pm 0.15$ .

<sup>20</sup> S. K. Bhattacharjee, T. D. Nainan, S. Raman, and B. Sahai, *Nuovo cimento* 7, 501 (1958).

The nonunique nonstatistical character of the spectrum shape is therefore confirmed. Our value may also be compared with  $Y \approx 1.05$  cited by Bhattacharjee and Mitra<sup>15</sup> and  $Y \approx 0.82$  which follows from the data of Dulaney *et al.*<sup>14</sup> when Kotani's Coulomb factors are used. The somewhat large spread in the values is possibly due to divergent criteria used in applying geometrical corrections and corrections to the interference of the inner beta group, or it may be due to the difference in the methods used for selection of the beta energies. In any case, it appears to be well established that the matrix elements  $x$  and  $u$  are suppressed over  $B_{ij}$  by some selection-rule effect. Even the standard matrix element itself has the very small value  $|C_A \int B_{ij}|^2 = 5.15 \times 10^{-11}$ . The slowness of the transition may be attributed to a large change in the deformation parameter associated with the beta decay of Eu<sup>152</sup> to Gd<sup>152</sup>, since the latter is known to be essentially spherical. Because of this, however, it is not clear to us that the selection-rule effect can be strictly ascribed to  $K$  forbiddenness, as Kotani has suggested.<sup>6</sup> For a more detailed understanding of this decay it would be helpful to have available beta-gamma correlation data in the parallel case of the beta decay of Eu<sup>154</sup> to Gd<sup>154</sup>. Similar anomalies in the  $ft$  value and the shape correction factor exist in this decay but the change in the deformation parameter may be considerably less since Gd<sup>154</sup> is not spherical.<sup>21</sup> Work on this problem is now in progress in this laboratory.

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<sup>21</sup> A. W. Sunyar, *Phys. Rev.* 98, 653 (1955).