Some Recent Experimental Tests of the "Clock Paradox"*

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Recent experiments by Pound and Rebka on the temperature dependence of the Mössbauer effect in Fe^{57}, and by Hay, Schiffer, Cranshaw, and Egelstaff using an Fe^{57} absorber on a rotating drum are shown to provide the first direct experimental verification of the time-keeping properties of accelerated clocks such as occur in the classic "clock paradox" of relativity. In the experiment by Pound and Rebka, the thermal vibrations of the lattice impart rms velocities of about $10^{-6}c$, and nearly continuous, randomly-oriented accelerations of the order of $10^{16}g$ to both the source and the absorber nuclei. In the experiment by Hay *et al.* the acceleration of the absorber was $6\times10^4 g$. The photon provides continuous communication of time data between the two nuclei for the duration of the "journey" (the emission time of the quantum). In each case the observed fractional frequency shift $\Delta f/f_0$ which occurs between the source and the absorber is found to be $-v_s^2/2c^2+v_a^2/2c^2$, where v_s and v_a are the rms velocities of the source and the absorber nuclei, respectively. These results are in quantitative agreement with the generally accepted calculations for the "clock paradox", in which two clocks pursue independent paths (at least one of which involves accelerations) in a common inertial frame, but are compared at two or more points where they coincide in space and time. The temperature-dependent experiments also demonstrate that accelerations of the order of $10^{16}g$, arising from lattice vibrations, produce no intrinsic frequency shift in Fe^{57} nuclei to an accuracy exceeding 1 part in 10^{13}.

I. INTRODUCTION

IN his original paper on special relativity, Einstein[1] predicted that a clock which departed from a given point in an inertial frame S, made a trip with a time-varying velocity $v(t)$, and returned to the starting point, would there be found to indicate an elapsed time s which is retarded compared to an identical clock (reading time t) which remained at the starting point, according to

$$s=\int_0^t \{1-[v(t)]^2/c^2\}^{\frac{1}{2}}dt. \qquad (1)$$

The traveling clock runs slow compared to the rest clock. Thus, (1) implies an intrinsic asymmetry between the two clocks, and this is associated with the fact that the rest clock remains in the inertial frame S, but the frame in which the traveling clock is at rest is not inertial, since it experiences accelerations.

The purpose of this paper is to point out that recent experiments have provided the first direct experimental verification of the prediction (1). They show that the clock which experiences the accelerations is unambiguously retarded, compared to the rest clock, and also, in accordance with (1), that the amount of retardation is determined by the average value of v^2 during the trip (for $v\ll c$). Before describing the experiments, however, we shall briefly point out from the experimental or operational point of view why these new tests of relativity are significant.

This problem, which might be described as the transverse Doppler effect for accelerating systems, but which is also known as the "clock paradox," is unusual in that it forms a bridge between the special theory and the general theory. When analyzed in the inertial frame S or any other inertial frame, the result (1) follows directly from the special theory,[2–4] and in this sense the new experimental result might be regarded as being trivial, particularly, since time-dilation for clocks in uniform translation is an established fact.[5–7] When, however, this problem is analyzed in a reference frame attached to the traveling clock—and no theory of relative motion can be regarded as being completely satisfactory unless it can describe the same experiment from the point of view of either of the two bodies which have the relative motion—the problem is far from simple. Einstein,[8] Pauli,[2] Tolman,[9] Møller,[3,4] and McVittie,[10] for example, all resort to the principle of equivalence and general relativity in order to explain from the standpoint of the traveling clock the same result which is predicted by special relativity.[11] Thus, the experimental verification of (1) raises from a new standpoint, the fundamental question: Why are inertial frames privileged above all other reference frames? For, the two clocks are calculated to behave differently for precisely the reason that inertial frames *are* unique.

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1 A. Einstein, Ann. Physik **17**, 891 (1905).

2 W. Pauli, *Theory of Relativity*, translated by G. Feld (Pergamon Press,New York, 1958), pp. 13, 72.

3 C. Møller, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **20**, No. 19 (1943).

4 C. Møller, *The Theory of Relativity* (Oxford University Press, New York, 1952), p. 49.

5 H. E. Ives and C. R. Stilwell, J. Opt. Soc. Am. **28**, 215 (1938); and **31**, 369 (1941).

6 B. Rossi and D. B. Hall, Phys. Rev. **59**, 223 (1941).

7 R. Durbin, H. H. Loar, and W. W. Havens, Phys. Rev. **88**, 179 (1952).

8 A. Einstein, Naturwissenshaften **6**, 697 (1918).

9 R. C. Tolman, *Relativity, Thermodynamics, and Cosmology* (Oxford University Press, New York, 1934), p. 192.

10 G. C. McVittie, Astron. J. **63**, 448 (1958).

11 This position has not been held universally, however. For example, G. Builder [Australian J. Phys. **10**, 246 (1957)] holds that an adequate analysis is possible without resort to general relativity.

Put another way, noninertial frames may be identified experimentally not only by the deflections of accelerometers, but also by the distinctive behavior of clocks. Thus, as was first pointed by Ives,[12] out of many clocks undergoing various relative motions, the one which runs the fastest has the lowest rms velocity with respect to an inertial frame.

Also, this experiment provides a new way of measuring speed. Given that the rest clock is in an inertial frame (as indicated by the null reading of attached accelerometers, for example), one can measure the rms velocity of the traveling clock (in units of the speed of light) with respect to the specified inertial frame (over the duration of the trip) merely by reading the two clock faces at each of the two spatial coincidences. This determination of average speed is novel from the operational point of view. It involves no distance measurements relating to the path of the traveling clock, no kinetic energy measurements, and no mass measurements.

In addition, the operations employed in the measurement of the relativistic time difference in the "clock paradox" experiment are, in principle, completely independent of the propagation properties of light. For example, one can simply photograph the two clock faces at each of the two instants of spatial coincidence. By contrast, all time dilatation experiments between clocks in uniform relative translation necessarily involve the propagation properties of light signals, since in this latter case the clocks can be in spatial coincidence only once, and the use of light signals in some form is unavoidable (in at least one of the inertial frames) in order to locate the terminus of the (nonproper) time interval. If there is any lesson we have learned from the theory of relativity, it is that we must be critical whenever seemingly familiar quantities are measured by new or different operations.

It is the completely unambiguous nature of the result in the "clock paradox" experiment which is, perhaps, its most unique feature. Here for the first time, one is comparing a proper time interval in one inertial frame to what might be described as the sum of proper time intervals which were collected by the traveling clock in several different inertial frames. The result is completely unambiguous: One particular clock certainly runs fast, and the other certainly runs slow. By contrast, in experiments involving uniform translation (where one is comparing a proper time interval in one inertial frame with a nonproper time interval in another inertial frame) the clock rates (as determined by the prescribed operational procedures) are ambiguous, that is, the observers in each frame measure the *other* clock to be running slow.

The new experiments are also significant because there is a small minority, notably Dingle[13] and Cullwick,[14] who have remained unconvinced by the theoretical arguments based on the conventional interpretation of the theory of relativity.[15–20] Dingle and Cullwick might be characterized as being "pure relativists." They hold that in determining relativistic effects it is only relative motion between two physical objects which has meaning and what the rest of the universe is doing does not make any difference. They accept the usual time dilatation as observed between clocks in uniform translation, but predict that in the case of the "clock paradox," there will be no relative retardation. The new experiments, however, demonstrate the contrary. Thus, whatever its theoretical difficulties may be, "pure relativism" is untenable experimentally. The experimental results, however, are in full agreement with the conventional theory which assigns a unique status to inertial frames, and which, as Ives[12] first noted and Builder[21] discusses at length, denies the possibility of predicting the time difference of the two clocks solely from the knowledge of their *relative* motion.

From the conceptual standpoint, the problem of what happens physically to the traveling clock during its trip is not simple. The returning clock has been permanently altered by its trip (it has suffered a change in phase). The effect is uniquely associated with the fact that acceleration has occurred, but it is quantitatively related not to the acceleration, but to the average speed. Supposing, for convenience, that the acceleration takes place in a very small interval and that the clock is unchanged by the acceleration process *per se*, it is clear that essentially all of the phase difference is accumulated [as (1) implies] during the constant-velocity regions of the path. Since this effect is observable without dependence either on the propagation properties of light, or upon any measurement operations using meter sticks, it cannot be dismissed as being an "apparent effect" having to do somehow with the processes of determining what happens at distant points. One is led therefore to the conclusion that clocks having a velocity in an inertial frame are literally slowed down *by the speed itself*. It is this very deduction which makes the generally accepted prediction regarding the "clock paradox" unacceptable to Dingle,[22] but which has led both Ives[12] and Builder[21] to consider interpretations of special relativity in which an ether plays an important role, at least from the philosophical point of view.

Finally, and aside from considerations of relativity, the experimental observation of clocks under various

[12] H. E. Ives, J. Opt. Soc. Am. **27**, 305 (1937).
[13] H. Dingle, Nature **144**, 888 (1939); Proc. Phys. Soc. (London) **A69**, 925 (1956); Nature **180**, 1275 (1957).
[14] E. G. Cullwick, *Electricity and Magnetism* (Longmans, Green and Company, London, 1957), p. 70.
[15] W. H. McRaea, Nature **167**, 680 (1951).
[16] H. E. Ives, Nature **168**, 246 (1951).
[17] F. S. Crawford, Jr., Nature **179**, 35, 1071 (1957).
[18] C. G. Darwin, Nature **180**, 976 (1957).
[19] G. Builder, Australian J. Phys. **10**, 246 (1957).
[20] E. M. McMillen, Science **126**, 381 (1957).
[21] G. Builder, Australian J. Phys. **11**, 279 (1958).
[22] H. Dingle, Nature **179**, 866 (1957).

conditions of acceleration may be of interest in its own right. For example, changes in the time-keeping properties of a clock (such as a nucleus) under adequately large accelerations might contribute to our understanding of its structure.

In the following sections we shall discuss two recent experiments both of which exploit the recoil-free emission of gamma rays (the Mössbauer effect) in Fe^{57}. The first experiment takes advantage of the differences in nuclear motion associated with the temperature differences between the source and the absorber, and tests the general case of the "clock paradox." The second experiment employs mechanical rotation to produce the necessary acceleration and (partially) tests a special case.

II. THE TEMPERATURE-DEPENDENT MÖSSBAUER EFFECT

Pound and Rebka,[23] working with the 14.4-kev gamma ray which is emitted and absorbed without recoil in Fe^{57}, observed that a difference in temperature ΔT°K between emitter and absorber produced an observable shift Δf in the absorption line as observed by the usual Doppler-shift method. This result was independently predicted by Josephson,[24] who also noted as did Pound and Rebka that it is a consequence of the relativistic time dilation caused by the motion of the nuclei in the crystal lattice due to thermal vibrations. If f_0 is the mean frequency of the emitted gamma ray when the nucleus is at rest in an inertial frame, and f_1 is the *shift* in the frequency of the emitted gamma ray when the nucleus is in a lattice at the temperature T°K, then in the classical limit,

$$f_1/f_0=(1-v^2/c^2)^{\frac{1}{2}}-1\cong -v^2/2c^2 = -3kT/2Mc^2=-2.4\times 10^{-15}T, \quad (2)$$

where v^2 is the mean square velocity of the nucleus of mass M due to the thermal vibrations of the lattice. The shift in the resonant frequency of the absorbing nucleus is also given by (2), so that if the source is at temperature T_s and the absorber is at T_a, then the frequency difference Δf between the source frequency f_s and the absorber frequency f_a (again in the classical limit) is given by

$$\Delta f/f_0=(1-v_s^2/c^2)^{\frac{1}{2}}-(1-v_a^2/c^2)^{\frac{1}{2}} \cong -v_s^2/2c^2+v_a^2/2c^2=-3k\Delta T/2Mc^2 = -2.4\times 10^{-15}\Delta T, \quad (3)$$

where $\Delta T=T_s-T_a$, and $\Delta f=f_s-f_a$.[25] Thus, if the source is at a higher temperature than the absorber, then Δf is negative, that is, the source has a lower frequency than the absorber. If resonance absorption is to occur, the absorber must be given a small velocity away from the source. By contrast, if the source is at a lower temperature than the absorber, then for resonance to occur, it is found necessary that the absorber move toward the source.

Pound and Rebka point out that for the case of iron near room temperature, the numerical coefficient in (3) should be about 0.9 times the classical value. (The Debye temperature is 467°.) Thus, (3) should read $\Delta f/f_0=-2.21\times 10^{-15}\Delta T$. They report an experimental value of $(-2.09\pm 0.24)\times 10^{-15}$/deg K for the constant coefficient, at room temperature. Thus, the second-order Doppler shift gives a satisfactory explanation of the temperature dependence of the mean frequency of the Fe^{57} recoil-free gamma resonance absorption.

III. THE RELATIONSHIP BETWEEN THE TEMPERATURE DEPENDENCE OF THE Fe^{57} RESONANCE AND THE "CLOCK PARADOX"

It is apparent that the Fe^{57} nuclei are playing the role of the clocks of the "clock paradox." Due to their participation in the lattice vibrations, the nuclei have rms velocities (at room temperature) of about 3×10^4 cm/sec—about the speed of a jet airplane. Also, the electromagnetic signals which are sent from the source to the absorber (in the form of photons) are transmitting time data, between the two "space ships," in the same manner that radio signals are used in the analyses of the macroscopic *Gedankenexperiment* (used by Darwin[18] and Builder[19]). (Builder[19] makes a detailed world diagram of the light signals traveling both ways between a "rest clock" and the "traveling clock.") In these analyses, a receiving device at each clock counts the total number of cycles received from the other clock in the time interval between the two points of spatial coincidence. This number is compared to the total number of cycles produced by the local clock during the same two spacetime events. The fact that the Fe^{57} nuclei are undergoing numerous (rather than a few) acceleration processes during the time of the "trip" (that is, during the transmission time for one photon) is a detail. Accelerations are in any case essential, but if they do not produce any intrinsic effects on the clocks, it makes no difference whether they occur frequently or rarely, or are large or small.

We may estimate the frequency of vibration ν of the lattice from $h\nu=k\Theta$, where Θ is the Debye temperature. We obtain $\nu\cong 10^{13}$ cps. Assuming harmonic oscillation, the maximum acceleration of the nuclei is calculated to be the order of $10^{16}g$, which is very large by macroscopic standards, of course, but very low by atomic or nuclear standards.

Also, the fact that the time signals are transmitted by a single photon rather than by many, as in a radio transmission, is only a matter of detail. The waves representing the photon obey Maxwell's equations, and

[23] R. V. Pound and G. A. Rebka, Jr., Phys. Rev. Letters 4, 274 (1960).

[24] B. D. Josephson, Phys. Rev. Letters 4, 341 (1960).

[25] Although Eq. (2) applies to the general case where the two clocks are both accelerating with respect to a common inertial frame, as Møller[3] points out, this does not change the problem significantly. Builder[21] discusses this case at some length.

are propagated in exact accordance with requirements of special relativity even though they are detected by the usual discrete quantum events. Furthermore, in quantum mechanics, the excitation and de-excitation of states by electromagnetic waves is calculated as if the process were continuous, that is, in a manner essentially identical to that used for macroscopic resonant structures. It is only in the matter of the interpretation and the observation of the calculated state amplitudes that the characteristic discrete quantum effects enter.

The receiving nucleus measures the frequency of the probability waves representing the photons emitted by the source nucleus, using its own characteristic frequency as a standard of comparison. Since the center frequency f_0 is 3.46×10^{18} cps, and since the mean life of the excited state is 1.4×10^{-7} sec, there are about 5×10^{11} complete cycles in the (intense part of the) wave train representing the photon. So sensitive is the receiving nucleus to the frequency of the incoming signal that if the total number of cycles arriving in a given time is say 10^{12}, it will have a large probability of absorbing the quantum, but if the total number of cycles arriving in this interval should fall short of 10^{12} by just *one* cycle (or exceed it by one) the receiving nucleus will have a very low probability of absorbing the quantum. It is of interest to note that if the receiving nucleus is properly Doppler-shifted,[26] so that it is in resonance with the mean frequency of the source, it maintains "phase-lock" with the incoming signals despite the fact that both the transmitting and receiving nuclei have large rms velocites (of the order of 1 part in 10^6 of the velocity of light). This arises from the fact that a complete frequency modulation cycle in the signal (due to the first-order Doppler effect arising from the periodic lattice vibrations) has a duration of only about 10^{-13} sec, during which time about 3×10^5 complete cycles are transmitted (and received). Although the average first-order Doppler frequency shift is $\Delta f=f_0(v/c)=f_0\times10^{-6}$, the velocity never continues long enough in one direction so as to produce a cumulative error in phase of more than about 10^{-1} cycle, or 30°. Under these conditions, therefore, the receiving nucleus may properly be regarded as keeping track of the exact number of cycles arriving in a given time interval—a situation which closely parallels the macroscopic *Gedankenexperiment.*

Since the source and the absorber are physically separated during the entire "journey," and electromagnetic signals are used to connect the two, the actual experiment does not exactly correspond to the *Gedankenexperiment* discussed in the Introduction. The key point, however, is that there is no net change in distance between the source and the absorber during the trip. Thus, no corrections are needed which involve either distance measurements or the propagation properties of light. The operations of time comparison (or, more exactly, frequency comparison) are identical, whether the source and absorber are both at rest in the laboratory, or are moving, and we can assume that any possible effects which might arise from the spatial displacement will not affect the *difference* between the two experimental situations.

IV. AN EXPERIMENT EMPLOYING MECHANICAL ACCELERATION

Hay, Schiffer, Cranshaw, and Egelstaff[27] placed a Co^{57} source near the center of a rotating wheel, a thin iron absorber in the form of a band around its circumference at a radius of 6.6 cm, and a counter at rest a short distance beyond the absorber. The gamma-ray transmission was observed as a function of the angular velocity of the wheel. The maximum tangential speed was 2.07×10^4 cm/sec, corresponding to a radial acceleration of $6.6\times10^4 g$. The transmission was observed to increase as the velocity of the absorber increased, indicating a shift in the characteristic frequency of the absorber, presumably toward lower frequency, although this point was not tested due to experimental limitations. Since the line shape was known experimentally (for the absorber at rest), the magnitude of the frequency shift due to the second-order, or relativistic, Doppler effect could be measured, and it was found to agree with that predicted by (1), to an accuracy of a few percent.

In this experiment, the absorbing nucleus moves only 2×10^{-8} cm during the quantum transmission time, so that it does not have a chance to make a complete trip.[28] If, however, the quantum transmission time were to be extended over a complete rotation of the wheel (which is possible in principle) the accelerating nucleus would return to its starting point, and the conditions for the "clock paradox" would be met in a literal sense. This is, of course, unnecessary, since, due to the geometrical symmetry, the relative rates of the clocks can be accurately established during a small fraction of a complete revolution. This is a particular case of the "clock paradox" since the acceleration is constant, and is normal to the velocity during the quantum transmission time. Nonetheless, the observed frequency shift is explicable (within experimental error) entirely in terms of the *velocity* of the absorber.

We note that once again that this experiment differs from all measurements of the transverse Doppler effect for uniform translation in that there is no change in the distance separating the source and the moving absorber during the frequency comparison operation. Also, this

[26] Even with $\Delta T=300°$, the velocity needed to produce the first-order Doppler shift which is necessary to correct for the difference in time dilation, is only about 0.01 cm/sec.

[27] J. J. Hay, J. P. Schiffer, T. E. Cranshaw, and P. A. Egelstaff, Phys. Rev. Letters 4, 165 (1960).

[28] Even so, this small motion, if it were in a straight line, would cause a first-order Doppler broadening which is 1000 times as great as the second-order Doppler frequency shift. Thus, the rotational motion is essential for a practical experiment of this type.

distance is identical to that involved in making the corresponding observation in the experiment in which the absorber is at rest.

V. CONCLUSIONS

The experiments of Pound and Rebka[23] demonstrate that the clock with the higher rms velocity invariably runs more slowly than the clock with the lower rms velocity, and, in spite of the very large, randomly-oriented, approximately periodic accelerations of magnitude $10^{16}g$, the observed time differences are explicable (within an experimental error of about 10%) entirely in terms of the *velocities* of the clocks. The maximum second-order velocity-dependent effect occurs when the source and the absorber have the maximum practical temperature difference (one near zero degrees, and one at room temperature), and for this case, $\Delta f/f_0 = v^2/2c^2 = 0.5\times 10^{-12}$. The agreement between the calculated shift and the observed shift in frequency (to within about 10%) implies that any specific acceleration-dependent effect must be at least one order of magnitude smaller, that is, the order of 1 part in 10^{13}, or less.

The rotating wheel experiment of Hay *et al.*[27] demonstrates that clocks which have an acceleration of constant magnitude ($6\times 10^4 g$) directed normal to their velocity in an inertial frame have the predicted second-order Doppler shift.

Although, on a nuclear scale, accelerations of $10^{16}g$ are very small, it is possible that significant distortions of the nuclear structure could still result. Let us make a rough estimate of the magnitude of the deformation of the Fe^{57} nucleus arising from this acceleration. From the "giant" gamma ray resonance at about 15 Mev, one estimates the force constant associated with the relative displacement of the proton and neutron components of the nuclear structure to be 3×10^{23} dynes/cm. Then, noting that the electric forces which cause the acceleration act only on the protons, we find that, under accelerations of about $10^{16}g$, the neutron and proton components of the nucleus should suffer a maximum relative displacement of about 1 part in 10^{13} of the nuclear diameter. Even using the great sensitivity of the Mössbauer resonance, such a small distortion is not likely to produce an observable effect. First, it would have to produce a *relative* shift of the same order of magnitude between the two states which define the resonance. Second, the change in the resonance frequency arising from the acceleration would have to be independent of the direction of the acceleration, for, if it were not, the rapidly varying, cyclical acceleration patterns would have their effects averaged to zero over the emission time of a quantum (in a manner similar to that of the first-order Doppler shift arising from lattice vibrations). We conclude from this rough calculation that the mechanical distortion of the nuclear structure under the accelerations due to the lattice vibrations is very small, but under favorable circumstances an intrinsic acceleration-dependent effect in the resonance frequency might be observable. A detailed analysis based on specific nuclear models is needed for a further evaluation of this possibility. In any case, the experiments to date appear to be adequately explained without recourse to any acceleration-dependent effects.