Collective Quadrupole Effects in Light Nuclei

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The recent measurements of the lifetimes of the first excited states of the mirror nuclei O^{17} and F^{17} have raised new interest in the various theoretical interpretations of these lifetimes. In this work the weak-coupling collective model of Bohr and Mottelson is applied to these $E2$ transitions and to the similar $E2$ transitions that have been measured in N^{16}, F^{19}, and Ne^{19}. If harmonic oscillator radial wave functions are used in evaluating the radial integrals in the theory, the predictions match the experimental results for the $E2$ transition probabilities in N^{16}, O^{17}, F^{17}, and F^{19} and the quadrupole moment of O^{17}. The theoretical prediction is an order of magnitude smaller than the experimental result for transition probability of the first excited state of Ne^{19}.

I. INTRODUCTION

THE recent measurements[1] of the lifetimes of the first excited states of the mirror nuclei O^{17} and F^{17} have raised new interest in the various theoretical interpretations of these lifetimes. The earliest measurement[2] of the lifetime in O^{17} clearly indicated that a single-particle picture was inadequate and collective effects were necessary for a sufficient explanation. Since that measurement, much theoretical interest has been centered on the particular mechanism that accounts for these collective effects. Three different approaches have been used in this field.

1. The inclusion of first-order corrections to the shell-model picture by Blin-Stoyle,[3] Amado,[4] de-Shalit,[5] and others, and most recently by Barton[6,7] has been a very interesting approach that has been able to explain the qualitative features of this collective enhancement within the framework of the shell model.

2. Fallieros and Farrell[8] have recently presented an alternative approach in terms of core polarization due to virtual creation and annihilation of nucleon-hole pairs in the nucleus. This approach also reproduces the qualitative features of the collective enhancement in a more elegant formalism.

3. The third approach has been based on the weak-coupling collective model of Bohr and Mottelson.[9] This approach has also been explored by a large number of authors[5] and seems also to be able to give qualitative explanation of the phenonema. In this paper a detailed review of the formalism of this third approach is presented and compared with the most recent experimental data on N^{16}, O^{17}, F^{17}, F^{19} and Ne^{19}. This comparison shows quantitative agreement with the data. The relative values of the parameters needed for this agreement are also the values obtained when harmonic oscillator radial wave functions are used to evaluate these parameters. The approach is somewhat analogous to the approach of, for instance, de-Shalit[5] using a shell-model description. The main differences are that the collective formalism gives an additional effective charge to each *nucleon* rather than to each neutron and this effective charge is proportional to Z. This dependence on Z appears necessary to give a quantitative fit to the data and comes naturally from the weak-coupling collective model. This type of dependence is not usually present in the shell-model framework but the work of Elliott[10] and of Kurath[11] clearly suggests that the present approach must certainly be equivalent to some shell-model picture. With this in mind, this paper presents a detailed derivation of the weak-coupling approach so that all the assumptions will clearly stand out, in the hope that this will aid in the development of the equivalence between this approach and a more fundamental shell-model approach.

II. EVALUATION OF THE QUADRUPOLE OPERATOR

The basic operator that is involved in both electric quadrupole γ-ray emission and electric quadrupole moments is the electric quadrupole operator $M(2\mu)$ defined as

$$M(2\mu)=\sum_{i=1}^{A} e_i r_i^2 Y_2^\mu(\theta_i,\varphi_i), \tag{1}$$

where e_i is the charge on the ith nucleon; Y_2^μ is the normalized spherical harmonic; $r_i\theta_i\varphi_i$ are the spherical coordinates that refer to the ith nucleon. The transition probability $T(E2)$ for $E2$ γ-ray emission is defined[9] as

$$T(E2)=\frac{4\pi}{75\hbar}\left(\frac{E_\gamma}{\hbar c}\right)^5 B(E2), \tag{2}$$

[1] J. V. Kane, R. E. Pixley, R. B. Schwartz, and A. Schwarzschild, preceding paper [Phys. Rev. **120**, 162 (1960)].
[2] J. Thurion and V. L. Telegdi, Phys. Rev. **92**, 1253 (1953).
[3] R. J. Blin-Stoyle, Proc. Phys. Soc. (London) **A66**, 1158 (1953).
[4] R. D. Amado and R. J. Blin-Stoyle, Proc. Phys. Soc. (London) **A70**, 532 (1957); and R. D. Amado, Phys. Rev. **108**, 1462 (1957).
[5] A. de-Shalit, Phys. Rev. **113**, 547 (1959). This paper also has a list of references to much of this work.
[6] G. Barton, Nuclear Phys. **11**, 466 (1959).
[7] G. Barton, D. N. Brink, and L. M. Delves, Nuclear Phys. **14**, 256 (1959).
[8] S. Fallieros and R. A. Ferrell, Phys. Rev. **116**, 660 (1959).
[9] A. Bohr and B. R. Mottelson, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **27**, No. 16 (1953).
[10] J. P. Elliott, Proc. Roy. Soc. (London) **A245**, 128, 562 (1958).
[11] D. Kurath and L. Picman, Nuclear Phys. **10**, 313 (1959); and D. Kurath, Nuclear Phys. (to be published).

where E_γ is the energy of the emitted γ ray, c is the velocity of light, and

$$B(E2)=\frac{1}{2I_i+1}\sum_{M_iM_f}|\langle I_i|M(2\mu)|I_f\rangle|^2$$
$$=\frac{1}{2I_i+1}|\langle I_i\|M(2)\|I_f\rangle|^2$$
$$=\left|\frac{\langle I_i|M(20)|I_f\rangle}{C(I_f2I_i;M_f0)}\right|^2, \quad (3)$$

where I_i is the spin of the initial state. I_f is the spin of the final state, and $\langle I_i\|M(2)\|I_f\rangle$ indicates a reduced matrix element due to Racah.[12] The mean life of the state due to $E2$ γ-ray emission τ, is $1/T(E2)$.

The quadrupole moment Q may be defined also in terms of $M(2\mu)$ as follows:

$$Q=(16\pi/5)^{\frac{1}{2}}\langle I_f|M(20)|I_f\rangle. \quad (4)$$

These definitions may be applied to collective behavior of the nucleus as a whole. In this situation $M(20)$ becomes an integration over the nuclear volume rather than a sum over individual nucleons and this modified $M(20)$ will be indicated by $M^1(20)$:

$$M^1(20)=\int\rho(\mathbf{r})r^2Y_2^0(\theta\varphi)r^2drd\Omega, \quad (5)$$

where $\rho(\mathbf{r})$ is the charge density inside the nucleus. For a uniform spherical charge distribution, $\rho(\mathbf{r})$ is equal to $3Ze/4\pi R_0^3$. This should be a suitable first approximation to $\rho(\mathbf{r})$. With this assumption, the radial integral may now be performed:

$$M^1(20)=\frac{3Ze}{4\pi R_0^3}\int\frac{R^5}{5}Y_2^0(\theta\varphi)d\Omega. \quad (6)$$

The nuclear surface is described by the vector $\mathbf{R}$ and, following Bohr and Mottelson,[9] for small departures from spherical shape,

$$R(\theta\varphi)=R_0[1+\textstyle\sum_\nu\alpha_\nu Y_2^\nu(\theta\varphi)]. \quad (7)$$

Now to lowest order,

$$M^1(20)=(3Ze/4\pi)R_0^2\alpha_0. \quad (8)$$

In the weak-coupling formalism of Bohr and Mottelson the α_μ's may be defined in terms of creation and destruction operators, b_μ^* and b_μ, for phonons, the quanta of surface oscillation:

$$\alpha_\mu=\left(\frac{\hbar\omega}{2C}\right)^{\frac{1}{2}}[b_\mu+(-1)^\mu b_{-\mu}^*], \quad (9)$$

where $\hbar\omega$ is the energy associated with each phonon and C is the surface-deformation parameter in the surface potential energy. Thus

$$M^1(20)=\frac{3Ze}{4\pi}R_0^2\left(\frac{\hbar\omega}{2C}\right)^{\frac{1}{2}}[b_0+b_0^*]. \quad (10)$$

The interaction Hamiltonian for the coupling of the nuclear surface to the particles is

$$H_{\text{int}}=-\textstyle\sum_i k(r_i)\sum_\mu\alpha_\mu Y_2^\mu(\theta_i\varphi_i). \quad (11)$$

Note that this interaction does not involve the charge distribution so that it will not mix states of different isotopic spin. Both Barton[6,7] and Fallieros and Ferrell[8] discuss the amount of isotopic spin mixing occurring in these transitions.

If only states of no phonons or one phonon are considered, the matrix elements for $M(20)$ may be easily evaluated using perturbation theory to determine the size of the coefficients for the various parts of the wave function.

The basis wave functions used are $|\mathbf{J}\mathbf{R}P(\mathbf{I})\rangle$ in which the nucleons are coupled to give a spin of $\mathbf{J}$; P phonons of spin 2 are coupled to give a spin of $\mathbf{R}$; and then $\mathbf{J}$ and $\mathbf{R}$ are coupled to give a total spin of $\mathbf{I}$. The nucleon part of the wave functions is antisymmetric under interchange of nucleons, and the phonon part is symmetric under interchange of phonons.

The wave functions for the initial state and final state may be described in perturbation theory by

$$\psi_i=|\mathbf{J}_i00\mathbf{I}_i\rangle+\sum_{J'}\frac{\langle\mathbf{J}'21\mathbf{I}_i|H_{\text{int}}|\mathbf{J}_i00\mathbf{I}_i\rangle}{E_{J_i}-(E_{J'}+\hbar\omega)}|\mathbf{J}'21\mathbf{I}_i\rangle, \quad (12)$$

$$\psi_f=|\mathbf{J}_f00\mathbf{I}_f\rangle+\sum_{J''}\frac{\langle\mathbf{J}''21\mathbf{I}_f|H_{\text{int}}|\mathbf{J}_f00\mathbf{I}_f\rangle}{E_{J_f}-(E_{J''}+\hbar\omega)}|\mathbf{J}''21\mathbf{I}_f\rangle. \quad (13)$$

The matrix elements for H_{int} are easily evaluated by the general formulas given previously by the author.[13] The operator $M^1(20)$ is diagonal in $\mathbf{J}$, and thus very few terms in the summation over $\mathbf{J}'$ and $\mathbf{J}''$ actually contribute to $M^1(20)$. Specifically

$$\langle\psi_i|M^1(20)|\psi_f\rangle$$
$$=\frac{3Ze}{4\pi}R_0^2\left(\frac{\hbar\omega}{2C}\right)^{\frac{1}{2}}$$
$$\times\left\{\langle\mathbf{J}_i00\mathbf{I}_i|b_0|\mathbf{J}_i21\mathbf{I}_f\rangle\frac{\langle\mathbf{J}_i21\mathbf{I}_f|H_{\text{int}}|\mathbf{J}_f00\mathbf{I}_f\rangle}{(E_{J_f}-E_{J_i})-\hbar\omega}\right.$$
$$\left.+\langle\mathbf{J}_f21\mathbf{I}_i|b_0^*|\mathbf{J}_f00\mathbf{I}_f\rangle\frac{\langle\mathbf{J}_f21I_i|H_{\text{int}}|J_i00I_i\rangle}{(E_{J_i}-E_{J_f})-\hbar\omega}\right\}. \quad (14)$$

Note that the second term in the bracket is equivalent to the first term with $\mathbf{J}_i\mathbf{I}_i\mathbf{J}_f\mathbf{I}_f\rightleftarrows\mathbf{J}_f\mathbf{I}_f\mathbf{J}_i\mathbf{I}_i$ so that it is necessary to evaluate only one of these terms explicitly.

[12] G. Racah, Phys. Rev. 62, 438 (1942).

[13] B. James Raz, Phys. Rev. 114, 1116 (1959).

By using the techniques and general formulas described in reference 13, $\langle|b_0|\rangle$ and $\langle|H_{\rm int}|\rangle$ may be easily evaluated.

If $E_{J_i}-E_{J_f}$ is set equal to ΔE, the final result is

$$\langle\psi_i|M^1(20)|\psi_f\rangle = \frac{3Ze}{4\pi}\frac{R_0^2}{C}\frac{(\hbar\omega)^2}{(\hbar\omega)^2-(\Delta E)^2} \times\langle J_i|\textstyle\sum_n Y_2^0(n)|J_f\rangle\langle J_i|\sum_n k(r_n)|J_f\rangle. \quad (15)$$

This in turn shows that the operator $M^1(20)$ is equal to

$$\frac{3ZeR_0^2}{4\pi}\sum_n \frac{k(r_n)}{C}\frac{(\hbar\omega)^2}{(\hbar\omega)^2-(\Delta E_n)^2}Y_2^0(\theta_n\varphi_n), \quad (16A)$$

or in general

$$M^1(2\mu)=\frac{3ZeR_0^2}{4\pi}\sum_n \frac{k(r_n)}{C} \times\frac{(\hbar\omega)^2}{(\hbar\omega)^2-(\Delta E_n)^2}Y_2^\mu(\theta_n\varphi_n). \quad (16B)$$

This is only an illustration of the derivation of Formula VII 12 in Bohr and Mottelson[9] but it is useful to display the assumptions explicitly.

The operator $M(2\mu)$ may now be considered as composed of two parts

$$M^0(2\mu)=\sum_{i=1}^N e_i r_i^2 Y_2^\mu(\theta_i\varphi_i), \quad (17A)$$

and

$$M^1(2\mu)=\frac{3Ze}{4\pi C}R_0^2\sum_{i=1}^N k(r_i)\frac{(\hbar\omega)^2}{(\hbar\omega)^2-(\Delta E_i)^2}Y_2^\mu(\theta_i\varphi_i), \quad (17B)$$

where the summation is only over nucleons outside the closed shell in both cases, and where $M(2\mu)=M^0(2\mu)+M^1(2\mu)$. The first term represents the single-particle matrix elements and the second term represents the collective influence due to the interaction of the extra-core nucleons with the collective modes of the nucleus. Since the angular dependence of both M^0 and M^1 is the same their angular matrix elements will be identical and the only difference will come in the different dependence on radial integrals and on Z and R_0.

The standard techniques for evaluating $M^0(2\mu)$ give answers immediately for $M^1(2\mu)$ also. The coefficient

$$e_{\rm eff}=\frac{3ZeR_0^2}{4\pi C}\frac{\langle|k(r_i)|\rangle}{\langle|r_i^2|\rangle}\frac{(\hbar\omega)^2}{(\hbar\omega)^2-(\Delta E_i)^2} \quad (17C)$$

serves the role of an additional effective charge given to each extra core nucleon.

The relative signs of $\langle|k(r)|\rangle$ and $\langle|r^2|\rangle$ determine if $M^1(20)$ enhances a proton transition or decreases it. It will be shown that $\langle|k(r)|\rangle$ and $\langle|r^2|\rangle$ must have the same sign in order to agree with the lifetime of the γ-ray transition in F^{17}. Since $k(r)$ is usually considered to be nonzero only near the nuclear surface and since the effect of r^2 is also mainly near the nuclear surface, their signs are expected to be the same, in general, and are shown to be the same for harmonic oscillator radial wave functions.

III. COMPARISON WITH EXPERIMENTAL RESULTS

There are now several lifetime measurements of the $E2$ transitions[1,14,15] between the $2s_{\frac{1}{2}}$ and the $1d_{\frac{5}{2}}$ single-particle states in the region of $A=16$. These transitions occur in N^{16}, O^{17}, and F^{17}. O^{17} and F^{17} are both nuclei with only one nucleon outside the closed 16 shell. In N^{16} there are four low-lying levels that are almost pure $p_{\frac{1}{2}}$ proton hole coupled to either a $1d_{\frac{5}{2}}$ neutron or a $2s_{\frac{1}{2}}$ neutron. Shell-model calculations of Elliott and Flowers[16] on N^{16} indicate that the ground state may be described by 96% pure $(p_{\frac{1}{2}}^{-1},d_{\frac{5}{2}})_2$ and the first excited state by pure $(p_{\frac{1}{2}}^{-1},s_{\frac{1}{2}})_0$. Since $M(2\mu)$ is a one-particle tensor operator of rank two, the $p_{\frac{1}{2}}$-to-$p_{\frac{1}{2}}$ proton transition does not contribute to $M(2\mu)$. Therefore, aside from a factor due to the angular momentum coupling in N^{16}, the $0^-\rightarrow 2^-$ transition is equivalent to the $\frac{1}{2}^+\rightarrow\frac{5}{2}^+$ transitions in F^{17} and O^{17}.

In N^{16} and O^{17}, $M^0(2\mu)=0$ since these are neutron transitions while in F^{17}, both $M^0(2\mu)$ and $M^1(2\mu)$ contribute and their relative phases are important.

In this region of the periodic table ΔE_i is much smaller than $\hbar\omega$ so that the factor $(\hbar\omega)^2/[(\hbar\omega)^2-(\Delta E_i)^2]$ in equation 16B is set equal to 1. For these calculations, we also assume $\langle 2s|k(r)|1d\rangle/C$ are equal for the three transitions. With these assumptions,

$$\left[\frac{B(E2)}{a^2Z^2R_0^4}\right]_{N^{16}}=\left[\frac{B(E2)}{a^2Z^2R_0^4}\right]_{O^{17}}, \quad (18)$$

and

$$\left[\frac{B(E2)}{(3ZR_0^2a/4\pi)^2}\right]_{O^{17}} =\left[\frac{B(E2)}{[(3ZR_0^2a/4\pi)+\langle 2s|r^2|1d\rangle]^2}\right]_{F^{17}}, \quad (19)$$

where

$$a=\frac{\langle 2s|k(r)|1d\rangle(\hbar\omega)^2}{C[(\hbar\omega)^2-(\Delta E_i)^2]}.$$

Now the mean life τ is related to $B(E2)$ by $\tau\propto 1/E^5B(E2)$, so that the above relationships coupled with the assumptions that $R_0\propto A^{\frac{1}{3}}$, $a_{O^{17}}=a_{F^{17}}=a_{N^{16}}$, and

[14] W. Zimmermann, Phys. Rev. **114**, 867 (1959).
[15] J. Freeman and R. C. Hanna, Nuclear Phys. 4, 599 (1957).
[16] J. P. Elliott and B. H. Flowers, Proc. Roy. Soc. (London) **A242**, 57 (1957). See also R. A. Ferrell, *Proceedings of the University of Pittsburgh Conference on Nuclear Structure, June 6–8, 1957*, edited by S. Meshkov (University of Pittsburgh and Office of Ordnance Research, U. S. Army, 1957); and reference 8.

$(\hbar\omega)^2 \gg (\Delta E_i)^2$ lead to

$$[E_\gamma^5 \tau Z^2 R_0^4]_{N^{16}} = [E_\gamma^5 \tau Z^2 R_0^4]_{O^{17}}. \tag{20}$$

Since $E_\gamma(O^{17}) = 871$ kev, $E_\gamma(F^{17}) = 500$ kev, and $E_\gamma(N^{16}) = 119$ kev,[17] this becomes

$$\tau_{N^{16}} = \left(\frac{871}{119}\right)^5 \left(\frac{8}{7}\right)^2 \left(\frac{17}{16}\right)^{\frac{4}{3}} \tau_{O^{17}}. \tag{21}$$

Similarly,

$$\tau_{F^{17}} = \left[\frac{3\times 8R_0^2 a/4\pi}{(3\times 9R_0^2 a/4\pi)+\langle 2s|r^2|1d\rangle}\right]^2 \left(\frac{871}{500}\right)^5 \tau_{O^{17}}, \tag{22}$$

$$\tau_{F^{17}} = \left[\frac{6}{(27/4)+(\pi\langle 2s|r^2|1d\rangle/R_0^2 a)}\right]^2 \left(\frac{871}{500}\right)^5 \tau_{O^{17}}. \tag{22A}$$

Using the recent measurement[1] of $\tau_{O^{17}} = 2.55\times 10^{-10}$ sec$\pm 5\%$ in formula (22) gives $\tau_{N^{16}} = 7.65\times 10^{-6}$ sec $\pm 5\%$. This compares very favorably with Zimmermann's recent result[14] of $\tau_{N^{16}} = 7.83\times 10^{-6}$ sec$\pm 4\%$. (This experimental result is significantly lower than an earlier measurement of Freeman and Hanna[15] of $\tau_{N^{16}} = 9.7\times 10^{-6}$ sec$\pm 7\%$.)

In order to determine $\tau_{F^{17}}$, the ratio $\langle 2s|r^2|1d\rangle/R_0^2 a$ must be known. $(R_0^2 a)^2$ may be determined from $\tau_{O^{17}}$. The evaluation of $\langle 2s|r^2|1d\rangle$ may be performed by assuming radial wave functions for the two wave functions involved.

There exists no completely satisfactory radial dependence for nuclear wave functions. Therefore a different approach is used. A value of $\langle 2s|r^2|1d\rangle$ is found which fits the experimental results and this number is then compared with the predictions for various radial wave functions. The ratio of $\tau_{O^{17}}$ to $\tau_{F^{17}}$ gives the value of $\langle 2s|r^2|1d\rangle/aR_0^2$. The value of $(aR_0^2)^2$ necessary to account for the value of $\tau_{O^{17}}$ is then determined and this value used to determine $|\langle 2s|r^2|1d\rangle|$. The necessary values[1] are

$$\tau_{O^{17}} = 2.55\times 10^{-10}\ \text{sec}\pm 5\%,$$
$$B(E2)_{O^{17}} = 6.45e^2\times 10^{-52}\ \text{cm}^4\pm 5\%,$$
$$|aR_0^2| = 2.60\times 10^{-26}\ \text{cm}^2\pm 3\%,$$
$$\tau_{F^{17}} = 4.45\times 10^{-10}\ \text{sec}\pm 5\%,$$
$$\langle 2s|r^2|1d\rangle/aR_0^2 = 3.67\pm 5\%,$$

and finally

$$|\langle 2s|r^2|1d\rangle| = 9.55\times 10^{-26}\ \text{cm}^2\pm 8\%.$$

Barton[6] has calculated this quantity for the F^{17} transition and obtains values of -9.9 f^2 and -10.0 f^2 for an oscillator well and for a square well with some Coulomb corrections. His most recent result[7] from a detailed machine calculation gives the value of -17.28 f^2.

The results of the present calculations are in excellent agreement with Barton's earlier results. This feature will be discussed later after the values of $\langle |r^2| \rangle$ and $\langle |k(r)| \rangle$ for the quadrupole moment have been investigated.

The sign of $\langle 2s|r^2|1d\rangle$ must be negative due to the opposite signs of the 2s and 1d radial wave function beyond the first node of the 2s function. This same argument would give a negative sign for $\langle 2s|k(r)|1d\rangle$ and thus for a also, in agreement with the observed enhancement due to collective effects in this transition. A comparison of 27/4 and $\pi\langle r^2\rangle/R_0^2 a$ indicates that $\langle |M^0| \rangle / \langle |M^1| \rangle = 1.7$ for F^{17}.

The three $E2$ γ-ray transitions are all in excellent agreement with the relations predicted by the weak-surface-coupling approach using the reasonable assumptions that R_0 varies as $A^{\frac{1}{3}}$ and $\langle 2s|r^2|1d\rangle \sim -10\times 10^{-26}$ cm^2 for F^{17}.

IV. EXAMINATION OF F^{19} AND Ne^{19}

The mirror nuclei F^{19} and Ne^{19} are both composed of one particle outside a core of 18 nucleons. The ground state is a $\frac{1}{2}^+$ state and there exists a $\frac{5}{2}^+$ state at 198 kev and 241 kev, respectively, in these nuclei. The mean life of the $\frac{5}{2}^+$ level has been measured in both cases. In F^{19} the transition from the $\frac{5}{2}^+$ level to the $\frac{1}{2}^-$ first excited state is very weak[17] and may be ignored in computing the transition probability for the $\frac{5}{2}^+ \rightarrow \frac{1}{2}^+$ transition. The wave functions for both F^{19} and Ne^{19} are expected to be complex mixtures of many different shell-model configurations.[18]

The simple shell-model picture with the addition of weak-coupling collective effects may be tried in this situation to see whether the addition of weak-coupling collective effects can give the effect of configuration mixing for the $\frac{5}{2}^+ \rightarrow \frac{1}{2}^+$ transitions in these nuclei. With this in mind, the assumption is made that these two nuclei are equivalent to F^{17} and O^{17}, respectively, with a pair of equivalent particles coupled to give zero spin added to each to form F^{19} and Ne^{19}. The transitions in both these nuclei are then considered to be $d_{\frac{5}{2}} \rightarrow s_{\frac{1}{2}}$ which may be compared with the $s_{\frac{1}{2}} \rightarrow d_{\frac{5}{2}}$ transitions in F^{17} and O^{17}, respectively.[19] If collective effects are the same in both F^{17} and F^{19}, the general relationships given above give the relationship of $\tau_{F^{19}}$ to $\tau_{F^{17}}$ as

$$\tau_{F^{19}} = \left(\frac{E_\gamma(F^{17})}{E_\gamma(F^{19})}\right)^5 \frac{(2\times\frac{5}{2}+1)}{(2\times\frac{1}{2}+1)} \left(\frac{R_0(F^{17})}{R_0(F^{19})}\right)^4 \tau_{F^{17}}.$$

Using[1,17] $E_\gamma(F^{19}) = 197$ kev, $E_\gamma(F^{17}) = 500$ kev, $\tau_{F^{17}} = 4.45\times 10^{-10}$ sec$\pm 5\%$, and $R_0 \sim A^{\frac{1}{3}}$ gives $\tau_{F^{19}} = 12.2\times 10^{-8}$ sec$\pm 5\%$.

This is to be compared with the experimentally

[17] See F. Ajzenberg-Selove and T. Lauritsen, Nuclear Phys. **11**, 1 (1959).

[18] M. G. Redlich, Phys. Rev. **110**, 468 (1958) and J. P. Elliot and B. H. Flowers, Proc. Roy. Soc. (London) **A229**, 536 (1955).

[19] See also P. Lehmann, A. Lévêque, T. Grjebine, and R. Barloutaud, *Proceedings of the Paris Conference 1959, Nuclear Interactions at Low Energies and the Structure of Nuclei* (Dunod, Paris, 1959), p. 813.

measured value[20] of $12.5\times10^{-8}\pm2\%$. This indicates that the above assumptions are valid for F^{19}. Therefore, the quadrupole moments of the two $\frac{5}{2}^+$ states are expected to be the same. A similar comparison may be made between O^{17} and Ne^{19}:

$$\tau_{Ne^{19}}=\left(\frac{E_\gamma(O^{17})}{E_\gamma(Ne^{19})}\right)^5\frac{(2\times\frac{5}{2}+1)}{(2\times\frac{1}{2}+1)}\frac{8^2}{10^2}\left(\frac{R_0(O^{17})}{R_0(Ne^{19})}\right)^4\tau_{O^{17}}.$$

Using[17] $E_\gamma(Ne^{19})=241$ kev, $E_\gamma(O^{17})=871$ kev, $\tau_{O^{17}}=2.55\times10^{-10}$ sec$\pm5\%$, and $R_0\sim A^{\frac{1}{3}}$ gives $\tau_{Ne^{19}}=26\times10^{-8}$ sec$\pm5\%$.

This does not agree with the experimentally measured value[17] of 1.8×10^{-8} sec$\pm10\%$. The above assumptions therefore work only for F^{19} and not for the mirror nucleus Ne^{19}.

V. EVALUATION OF QUADRUPOLE MOMENTS

The quadrupole moment of the ground state of O^{17} has also been measured[21] and the predictions of the present approach may be compared with this result.

The formula for the quadrupole moment involves the diagonal matrix elements of $M(20)$. Therefore the radial matrix elements $\langle 1d|r^2|1d\rangle$ and $\langle 1d|k(r)|1d\rangle$ will come into these evaluations. In O^{17} only the operator $M^1(20)$ contributes to the quadrupole moment. The theoretical value for this quadrupole moment is

$$Q_{O^{17}}=(-24/7)a'R_0^2,$$

where

$$a'=\frac{(\hbar\omega)^2}{(\hbar\omega)^2-(\Delta E)^2}\frac{\langle 1d|k(r)|1d\rangle}{C}.$$

Using the experimental value of $Q=-2.65\times10^{-26}$ cm$^2\pm11\%$, the value $a'R_0^2$ is computed to be $+2.43\times10^{-26}$ cm$^2\pm11\%$. This value coupled with the value of a computed from the lifetime of the first excited state gives:

$$\frac{\langle 1d|k(r)|1d\rangle}{\langle 2s|k(r)|1d\rangle}=-\frac{2.43}{2.60}\pm14\%=-0.94\pm14\%.$$

At this point it is worth while to investigate the theoretical values of these various matrix elements. If harmonic oscillator functions are used, the matrix elements for r^2 are

$$\langle nl|r^2|nl\rangle=[2(n-1)+l+\tfrac{3}{2}]\alpha^{-2};$$

thus

$$\langle 2s|r^2|2s\rangle=\langle 1d|r^2|1d\rangle=\tfrac{7}{2}\alpha^{-2},$$

and

$$\langle 2s|r^2|1d\rangle=-(10)^{\frac{1}{2}}\alpha^{-2},$$

where α is the factor that appears in the exponential $\exp[-\frac{1}{2}(\alpha r)^2]$ of the wave function (see Mayer and Jensen[22]). Note that $\alpha^2=\nu$ in Carlson and Talmi,[23] and $\alpha^2=b^{-2}$ in Barton.[6,7]

Carlson and Talmi[23] have determined a value of $\alpha^2=0.354\times10^{26}$ cm^{-2} for this region of the periodic table, and if this value is used, the matrix elements become $\langle 1d|r^2|1d\rangle=\langle 2s|r^2|2s\rangle=+9.9\times10^{-26}$ cm^2 and $\langle 2s|r^2|1d\rangle=-8.95\times10^{-26}$ cm^2. This last value is in close agreement with the value -9.55×10^{-26} cm$^2\pm8\%$ determined from $\tau_{F^{17}}$ and $\tau_{O^{17}}$.

If the standard assumption is made[9] that $k(r)\propto\delta(r-R_0)$, the relative values of the matrix elements of $k(r)$ may be easily determined by using harmonic oscillator wave functions. This results in

$$\frac{\langle 1d|k(r)|1d\rangle}{\langle 2s|k(r)|1d\rangle}=\left(\frac{2}{5}\right)^{\frac{1}{2}}\frac{(\alpha R_0)^2}{[\frac{3}{2}-(\alpha R_0)^2]}.$$

If R_0 is set equal to $1.30A^{\frac{1}{3}}\times10^{-13}$ cm and α^2 is 0.354×10^{26} cm^{-2}, this ratio is $\langle 1d|k(r)|1d\rangle/\langle 2s|k(r)|1d\rangle=-1.02$, for $A=17$. This is in close agreement with the value $-0.94\pm14\%$ determined from the quadrupole moment and the lifetime of the first excited state of O^{17}. Harmonic oscillator wave functions, then, give reasonable values for the parameters involved.

Within the framework of this approach, values may be predicted for the quadrupole moments of N^{16} and F^{17}. These values are most easily put in the form of ratios, and are

$$Q_{F^{17}}=\left[\frac{(27/4)+(\pi\langle 1d|r^2|1d\rangle/R_0^2a')}{6}\right]Q_{O^{17}},$$

and

$$Q_{N^{16}}=\left[\frac{4\times7}{5\times8}\frac{R_0^2(16)}{R_0^2(17)}\right]Q_{O^{17}}.$$

Using the values $a'R_0^2=+2.43\times10^{-26}$ cm$^2\pm11\%$ determined from $Q_{O^{17}}$, $\langle 1d|r^2|1d\rangle=+9.9\times10^{-26}$ cm^2 from the theoretical calculations for harmonic oscillator wave functions, and the relationship $R_0\propto A^{\frac{1}{3}}$, these ratios become

$$Q_{F^{17}}=3.25Q_{O^{17}},$$
$$Q_{F^{17}}=-8.6\times10^{-26}\text{ cm}^2\pm20\%,$$
$$Q_{N^{16}}=0.67Q_{O^{17}},$$
$$Q_{N^{16}}=-1.8\times10^{-26}\text{ cm}^2\pm11\%.$$

VI. CONCLUSIONS

The basic relationships from the weak-coupling collective model involve the two radial integrals $\langle|k(r)|\rangle$ and $\langle|r^2|\rangle$. These quantities may be calculated from theory or may be determined from experimental results. In this work the radial integrals were determined from experimental results and then compared with the pre-

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TABLE I. A comparison of the theoretical results for relevent radial integrals and the values for these integrals that have been deduced from experiment using the weak-coupling collective model for interpretation.

	Theory	Experiment
$\langle 2s\|r^2\|1d\rangle$	-8.95×10^{-26} cm^2 [a]	-9.55×10^{-26} cm$^2\pm8\%$
	-9.9×10^{-26} cm^2 [b]	
	-10.0×10^{-26} cm^2 [c]	
	-17.28×10^{-26} cm^2 [d]	
$\langle 1d\|r^2\|1d\rangle$	$+9.9\times10^{-26}$ cm^2 [a]	no experiment
$\langle 2s\|r^2\|2s\rangle$	$+9.9\times10^{-26}$ cm^2 [a]	no experiment
$\dfrac{\langle 1d\|k(r)\|1d\rangle}{\langle 2s\|k(r)\|1d\rangle}$	-1.02[e]	$-0.94\pm14\%$

[a] Harmonic oscillator wave functions in Sec. V.
[b] Harmonic oscillator wave functions with a slightly larger radius (Barton, reference 6).
[c] Modifed square well in reference 6.
[d] Detailed machine calculation in reference 7.
[e] Harmonic oscillator wave functions as in footnote a, $k(r)\propto\delta(r-R_0)$, and $R_0=1.30A^{\frac{1}{3}}\times10^{-13}$ cm.

dictions from theory. The results are summarized in Table I.

In a few instances the values of these integrals are unnecessary in comparing the ratio of two lifetime measurements. This is true for the ratios of $\tau_{N^{16}}$ to $\tau_{O^{17}}$, $\tau_{F^{19}}$ to $\tau_{F^{17}}$, and $\tau_{Ne^{19}}$ to $\tau_{O^{17}}$. The results for these ratios are given in Table II. The only glaring discrepancy between theory and experiment is in the lifetime of Ne^{19}. The use of this simple theory for either F^{19} or Ne^{19} is not well justified in view of the complex configuration mixing that is most likely present in both these nuclei.[18]

It is gratifying that the theory works for F^{19} but rather surprising that it does not work equally well for both these mirror nuclei.

These results may also be expressed in terms of the size of the effective charge e_{eff} needed to match the experimental results (see Eqs. 17B and 17C). In this terminology the ratio of $e_{\text{eff}}(N^{16})$ to $e_{\text{eff}}(O^{17})$ equals $0.864\pm5\%$ indicating that here e_{eff} is proportional to Z. The ratio of $e+e_{\text{eff}}(F^{19})$ to $e+e_{\text{eff}}(F^{17})$ is $1.02\pm3\%$, while the ratio of $e_{\text{eff}}(Ne^{19})$ to $e_{\text{eff}}(O^{17})$ is $4.75\pm7\%$. This indicates that the last two protons in Ne^{19} do contribute substantially to the transition probability. Professor J. P. Elliot kindly informed the author[24] that when the detailed shell model wave functions[18] are used to calculate the transition probabilities (with e_{eff} set equal to $\frac{1}{2}e$) the results agree with the measured values for both F^{19} and Ne^{19}.

The ratio of $e+e_{\text{eff}}(F^{19})$ to $e_{\text{eff}}(O^{17})$ of $3.05\pm5\%$ may be used to determine the relationship between e and $e_{\text{eff}}(F^{17})$ if some assumptions are made about the relationship between $e_{\text{eff}}(F^{17})$ and $e_{\text{eff}}(O^{17})$.

The treatment in Sec. III assumes that $e_{\text{eff}}(F^{17})$ is set equal to $9/8\ e_{\text{eff}}(O^{17})$. In that case $e_{\text{eff}}(F^{17})$ is equal to $0.56e$. Using this relationship and the measured lifetime of F^{17} gives the value of $\langle 2s|r^2|1d\rangle$ equal to -9.55×10^{-26} cm^2. A more realistic approach would be to set $e_{\text{eff}}(F^{17})$ equal to $e_{\text{eff}}(O^{17})$ since in both cases the last nucleon polarizes the same O^{16} core. In this case $e_{\text{eff}}(F^{17})$ is equal to $0.49e$ and $\langle 2s|r^2|1d\rangle$ is equal to -10.2×10^{-26} cm^2.

TABLE II. A comparison of the theoretical predictions and the experimental results for the ratio of lifetimes of the relevant excited states in nuclei around $A=17$.

	Theory	Experiment
$\tau_{N^{16}}/\tau_{O^{17}}$	3.00×10^4	$3.07\times10^4\pm9\%$
$\tau_{F^{19}}/\tau_{F^{17}}$	2.74×10^2	$2.81\times10^2\pm7\%$
$\tau_{Ne^{19}}/\tau_{O^{17}}$	1.02×10^3	$0.70\times10^2\pm15\%$

The high value for $\langle 2s|r^2|1d\rangle$ obtained by Barton in his most recent work is somewhat surprising and further machine calculations would be quite valuable to check how sensitive this integral is to small changes in the parameters used in the wave function.

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