Quadratic Deviations from Ohm's Law in *n*-Type InSb

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Measurements of the resistivity of *n*-type InSb of various carrier concentrations have been made as a function of electric field strength at low temperatures. The electric fields were kept small enough so that only a slight heating of the electron distribution occurred and the electron mobility μ satisfied the relation

$$\mu=\mu_0(1+\beta F^2),$$

where μ_0 is the ohmic mobility and F the electric field.

Analysis of our experimental values of β yield information about the processes by which electrons lose momentum and energy and about the influence of carrier degeneracy and electron-electron scattering upon these processes. For example, at 4.2°K, piezoelectric scattering is responsible for most of the energy loss, while near 77°K polar optical scattering, enhanced by strong electron-electron scattering, is responsible for energy loss, at least in zero magnetic field.

A novel means of changing the sign of the deviation from Ohm's law near 77°K was observed by applying a strong magnetic field.

I. INTRODUCTION

THE mobility of charge carriers in a semiconductor becomes dependent on the strength of the applied electric field at moderate field strengths.[1,2] This phenomenon is due to the charge carrier distribution no longer being in thermal equilibrium with the lattice. To a certain approximation at least, the carriers can be thought of as being in an energy distribution having a temperature higher than that of the lattice, and thus have been called "hot" carriers.

Hot carrier effects are observable in semiconductors because the high mobility of the charge carriers results in high power gain per carrier from a given electric field, while the relatively small number of carriers (compared to that in a metal, for example) allows the total power input to remain small.

We shall be concerned with small rises in carrier temperature T_e such that $(T_e-T)/T<1$ where T is the lattice temperature. For such "warm" carriers the mobility is given by[1]

$$\mu=\mu_0(1+\beta F^2), \tag{1.1}$$

where μ_0 is the Ohm's law mobility, F is the electric field strength, and β is a quantity which depends on the energy gain and loss rates of the carriers. In the relaxation time approximation[2]

$$|\beta|\sim\mu_{L0}\mu_0, \tag{1.2}$$

where μ_{L0} is the lattice scattering mobility associated with the energy loss process for $F\rightarrow 0$ and μ_0 is the Ohm's law mobility as before. The sign of β is positive when the momentum relaxation time τ increases with increasing carrier energy, e.g., ionized impurity scattering, and negative when τ decreases with increasing carrier energy, e.g., for certain types of lattice scattering. The regime where (1.1) holds has been studied in germanium having few enough carriers so that classical statistics are applicable.[2] Since germanium is nonionic, the charge carriers lose energy to the lattice via scattering due to the strain field potentials associated with acoustic or optical vibration modes of the lattice atoms.

We shall be concerned with a slightly ionic semiconductor, InSb. This affords the possibility of studying energy loss via scattering due to the polarization associated with lattice vibrations. Of course, scattering due to the strain field potentials is still operative, but it has been shown that lattice scattering in *n*-InSb may be almost completely via the polar interaction,[3] e.g., above 200°K polar optical mode scattering along with electron-hole scattering determines the mobility. Our measurements were made at temperatures $\leqslant$90°K where the mobility has not been fully explained so that the deviations we observe can be used to determine which type of lattice scattering is important and whether it limits the mobility significantly. Limitation of the mobility by lattice scattering is to be expected only in the purer samples in the temperature region from 50°K to 90°K perhaps.

At lower temperatures (1.2 to 20°K) ionized impurity scattering is expected to be predominant in limiting the mobility for all samples. However, since scattering by impurity ions ordinarily causes insignificant energy loss by the carriers, the lattice vibrations are still responsible for energy loss. At liquid helium temperatures, only acoustic modes of vibration are probably important, but, in an ionic material lacking a center of symmetry like InSb, there is a piezoelectric polarization as well as a deformation potential associated with these modes. Which of these interactions is important in the energy loss process will be deducible from our data.

The effects of carrier distribution degeneracy can also be studied in *n*-InSb. This is because the small

[1] W. Shockley, Bell System Tech. J. **30**, 990 (1951).
[2] J. B. Gunn, *Progress in Semiconductors*, edited by A. F. Gibson (John Wiley & Sons, Inc., New York, 1957), Vol. II, p. 213.
[3] H. Ehrenreich, J. Phys. Chem. Solids **2**, 131 (1957).

electron mass permits Fermi energies greater than the thermal energy to be achieved for quite small carrier concentrations. We have studied the effects of degeneracy by making measurements at liquid helium temperatures on samples having various carrier concentrations.

A novel way to change the sign of β in n-InSb suggested itself because of the possibility of quantizing electron motion by a magnetic field and thereby, under appropriate conditions, changing the energy dependence of the relaxation time from a decreasing to an increasing function of energy. We looked for and found that the sign of β could be changed by application of a strong magnetic field.[4]

For a scattering mechanism causing momentum loss which is not describable by a relaxation time at electron energies of importance, the above remarks have to be recast in terms of scattering rates. In addition, if the momentum loss mechanism has a steep maximum at some energy much greater than thermal energy, kT, strong electron-electron scattering may completely alter the otherwise expected value of β. Such a case involving polar optical scattering has been studied theoretically by Stratton.[5] Some of our experimental results require such an explanation incorporating strong $e-e$ scattering.

II. THEORY

The theory of mobility variation due to slight heating of carriers above the lattice temperature has been worked out in detail for a number of cases.[5,6] The principles involved in any case are the same. Some forms for the distributions of the carriers in energy and momentum must be calculated or assumed. The parameters characterizing these distributions are determined by equating the rates at which momentum and energy are gained by the carriers from the electric field to the respective rates at which they are lost due to scattering. When the carrier distribution functions have been thus determined, the dependence of the mobility on electric field strength can, at least in principle, be calculated.

A. Momentum Loss by Ionized Impurity Scattering

For interpreting some of our data it is useful to extend theory to cover the case of impurity scattering determining the momentum relaxation time, piezoelectric scattering[7] being responsible for energy loss, and the carrier distribution having arbitrary statistical degeneracy. Before doing so we shall summarize some theoretical results of Greene for a similar case[8,9] since they can be used in our extension.

Greene assumed that the electron energy distribution function is given by

$$f=1/(e^{(E-\zeta')/kT_e}+1), \tag{2.1}$$

where E is the electron energy, T_e is the electron temperature and ζ' is the Fermi energy for carriers heated by the applied electric field, and that the high temperature or classical approximation, holds for the phonon distribution, i.e.,

$$N_q=1/(e^{\hbar qs/kT}-1)\rightarrow kT/\hbar qs-\tfrac{1}{2}, \tag{2.2}$$

where q is the phonon wave number and s is the velocity of sound. Under these conditions he found that when the momentum relaxation time is given by

$$\tau\sim E^z, \tag{2.3}$$

where E is electron energy and z is a numerical constant, the mobility is given by[9]

$$\mu=\mu_0(1+\delta)^{z+\frac{3}{2}}F_{z+\frac{1}{2}}(\zeta'/kT_e)/F_{z+\frac{1}{2}}(\zeta/kT), \tag{2.4}$$

where

$$\delta\equiv(T_e-T)/T,\quad F_p(\eta)=\int_0^\infty \frac{x^p dx}{e^{x-\eta}+1},$$

and ζ is the Fermi energy for carriers when the electric field is equal to zero. For a constant concentration of charge carriers Greene found that for $\delta\ll 1$ Eq. (2.4) becomes,

$$\mu=\mu_0\left\{1+\delta\left[(z+\tfrac{3}{2})-3(z+\tfrac{1}{2})\frac{F_{\frac{1}{2}}F_{z-\frac{1}{2}}}{F_{-\frac{1}{2}}F_{z+\frac{1}{2}}}\right]\right\}, \tag{2.5}$$

where the F_p's are functions of ζ/kT. For energy loss due to acoustic deformation potential scattering

$$\delta_{\mathrm{Ac}}=\frac{\mu_I(0)\mu_{\mathrm{Ac}}{}^{e-e}(0)}{3s^2}F^2, \tag{2.6}$$

where s is the velocity of longitudinal sound waves, F is the electric field strength, $\mu_I(0)$ is the ohmic impurity scattering mobility, and $\mu_{\mathrm{Ac}}{}^{e-e}(0)$ is the ohmic acoustic scattering mobility when electron-electron scattering is much stronger than the acoustic scattering. More precisely for strong $e-e$ scattering we have by following Keyes[10]

$$\mu_{\mathrm{Ac}}{}^{e-e}(0)=\frac{e}{m}\frac{\int E^{\frac{3}{2}}(\partial f_0/\partial E)dE}{\int E^{\frac{3}{2}}\tau_{\mathrm{Ac}}{}^{-1}(\partial f_0/\partial E)dE}, \tag{2.7}$$

[4] R. J. Sladek and F. S. Black, Jr., Bull. Am. Phys. Soc. **3**, 378 (1958).
[5] R. Stratton, Proc. Roy. Soc. (London) **A246**, 406 (1958).
[6] M. S. Sodha, Phys. Rev. **108**, 1375 (1957) and Phys. Rev. **107**, 1266 (1957).
[7] H. J. G. Meijer and D. Polder, Physica **19**, 255 (1953).
[8] R. F. Greene, J. Electronics and Control **3**, 387 (1957).
[9] R. F. Greene (private communication).
[10] R. W. Keyes, J. Phys. Chem. Solids **6**, 1 (1958).

where e is the electronic charge, m is the effective electron mass, τ_{Ac} is the momentum relaxation time for acoustic scattering, and $f_0=1/[e^{(E-\zeta)/kT}+1]$.

The reason for expressing δ in Eq. (2.6) and later in Eq. (2.12) in terms of a lattice scattering mobility calculated for strong electron-electron scattering is two-fold. First, in calculating the total energy loss rate, it is $1/\tau$ (times a function of energy) which must be averaged over the electron distribution [see Eq. (2.11) for example], and, second in our samples at low temperatures the mean free path for electron-electron scattering, l_{ee}, calculated from theory turns out to be much smaller than the mean-free path for scattering by phonons. (See Sec. IV.) The theoretical expression for l_{ee} is[11]

$$l_{ee}=\frac{1}{4\pi n\lambda_D^2}\left(\frac{\zeta}{kT}\right)^2\pi\left[\left(\frac{\pi a_0^*}{2a_e}\right)^2+\left(\frac{a_0^*}{4\lambda_D}\right)^2\right], \quad (2.8)$$

where a_0^*=effective Bohr radius, $a_e=(3/4\pi n)^{\frac{1}{3}}$, n is the electron concentration, and λ_D is the Debye screening length. Equation (2.8) is for a statistically degenerate electron distribution ($\zeta\gg kT$) in which case

$$\lambda_D^2=\left(\frac{\pi}{3}\right)^{\frac{1}{3}}\hbar^2K/4e^2mn^{\frac{1}{3}}=1.35\times10^{-9}\frac{Kn^{-\frac{1}{3}}}{m/m_0}\text{ cm}^2, \quad (2.9)$$

where K is the dielectric constant (=16 for InSb) and m is the effective electron mass ($=0.013m_0$ for InSb near the band edge). The right hand equality in Eq. (2.9) holds when the electron concentration is in cc^{-1}.

An average mean-free path for phonon scattering, l_L, is obtainable from the lattice scattering mobility μ_L deduced from our measured deviations from Ohm's law by means of the relation

$$l_L=\frac{(2m\zeta)^{\frac{1}{2}}}{e}\mu_L$$
$$=3.57\times10^{-11}(\zeta/k)^{\frac{1}{2}}(\mu_L)_{\text{cm}^2\text{ v}^{-1}\text{ s}^{-1}}\text{ cm}. \quad (2.10)$$

For energy loss via piezoelectric scattering we modified Greene's treatment[8] (which is for energy loss via deformation potential scattering). The resultant energy loss rate via piezoelectric scattering is

$$-\left(\frac{dE}{dt}\right)_{PE}=\frac{2ms'^2}{kT\tau_{PE}}\left\{-kT+\frac{\hbar^2|k|^2}{2m}\left(1-\frac{2T}{T_e}f\right)\right\}\times[1-f], \quad (2.11)$$

where k is the electron wave vector, τ_{PE} is the momentum relaxation time for piezoelectric scattering,[7] and f is given by Eq. (2.1). The quantity s' is the velocity of sound weighted by the polarization and averaged over crystallographic directions.[7]

[11] H. Jones, *Encyclopedia of Physics*, edited by S. Flügge (Springer-Verlag, Berlin, Germany, 1956), Vol. 19, Part I, pp. 227–9.

Averaging the right-hand side of (2.11) over the electron distribution and equating the result to the power gained from the electric field we obtain

$$\delta_{PE}=\frac{1}{3}\frac{\mu_I(0)\mu_{PE}{}^{e-e}(0)}{s'^2}F^2, \quad (2.12)$$

where $\mu_{PE}{}^{e-e}(0)$ is the ohmic piezoelectric scattering mobility when electron-electron scattering is much stronger than piezoelectric scattering or, more precisely, is defined by a relation similar to Eq. (2.7) with τ_{PE} replacing τ_{Ac}.

For energy loss to phonons via either the strain or polarization fields associated with acoustic modes when $\zeta\gg kT$ the mobility given by Eq. (2.5) reduces to

$$\mu_I=\mu_I(0)\left[1+C\left(\frac{kT}{\zeta}\right)^2\frac{\mu_I(0)\mu_L{}^{e-e}(0)}{s^2}F^2\right], \quad (2.13)$$

where C is a constant which depends on the value of z, $\mu_L{}^{e-e}(0)$ stands for either $\mu_{Ac}{}^{e-e}(0)$ or $\mu_{PE}{}^{e-e}(0)$. In the first case s^2 stands for the square of the velocity of longitudinal sound waves and in the second case for s'^2. For $z=1$ and electron-electron scattering much stronger that lattice scattering $C=5\pi^2/18$.

From Eq. (2.13) it can be seen that the coefficient of F^2, i.e., β, has an explicit inverse dependence on the Fermi energy, ζ. Thus a sample with more carriers and hence higher Fermi energy, should exhibit a smaller relative deviation from Ohm's law for a given electric field. In addition β also depends on ζ through $\mu_L{}^{e-e}(0)$, because, in contrast to the case of classical statistics, $\mu_L{}^{e-e}(0)$ depends on the Fermi energy, as we shall see presently.

The above equations for the variation of mobility with electric field strength were derived from energy loss relations which neglected the effect of screening of the lattice scattering potential by the mobile electrons even though an appreciable effect due to screening is expected for carrier concentrations large enough so that $\zeta\gtrsim kT$. The reason for this neglect is that great complication would attend the inclusion of screening in the energy loss equations. However, the essential features of the screening can, we believe, be reproduced if the lattice scattering mobilities in the above relations are replaced by mobilities in which the effects of screening have been included so we shall consider these mobilities next.

Since even the transport integrals for the pertinent lattice scattering mobilities cannot, in general, be expressed in a simple analytic form, when screening effects are included, we shall first give expressions for the mobilities neglecting screening and then derive correction factors due to the latter which are exact only for the case of complete degeneracy.

By adapting deformation potential theory for the mobility[12] to the case of arbitrary degeneracy and

[12] W. Schockley, *Electrons and Holes in Semiconductors* (D. Van Nostrand Company, Inc., Princeton, New Jersey, 1950), p. 278.

strong $e-e$ scattering, we obtain when the classical approximation holds for phonons, i.e., when Eq. (2.3) holds,

$$\mu_{\mathrm{Ac}}{}^{e-e}(0)=\frac{3e}{4\sqrt{2}}\frac{\rho s^2}{E_1{}^2}\frac{\pi\hbar^4}{m^{\frac{5}{2}}}\frac{1}{(kT)^{\frac{3}{2}}}\frac{F_{\frac{1}{2}}(\zeta/kT)}{F_1(\zeta/kT)}, \quad (2.14)$$

where ρ is the density and E_1 the deformation potential constant. For n-InSb with $\rho=5.8$ g/cc, $E_1=7.2$ ev, $s=3.7\times10^5$ cm/sec, and $m=0.013m_0$, we obtain

$$\mu_{\mathrm{Ac}}{}^{e-e}(0)=2.60\times10^{10}T^{-\frac{3}{2}}F_{\frac{1}{2}}(\zeta/kT)/F_1(\zeta/kT)\ \mathrm{cm}^2\ \mathrm{v}^{-1}\ s^{-1}. \quad (2.15)$$

By using the relaxation time for piezoelectric scattering given by Meijer and Polder,[7] we obtain the theoretical piezoelectric scattering mobility for the case of a classical phonon distribution,

$$\mu_{\mathrm{PE}}{}^{e-e}(0)=\frac{3\sqrt{2}}{16\pi}\frac{\hbar^2K^2}{m^{\frac{3}{2}}ee_{14}{}^2}\frac{1}{(kT)^{\frac{1}{2}}}\frac{F_{\frac{1}{2}}(\zeta/kT)}{F_0(\zeta/kT)}\times\left[\frac{16}{13(c_{11}+c_{12}+4c_{44}+16\pi e_{14}{}^2/K)}+\frac{6}{13(c_{44}+4\pi e_{14}{}^2/K)}\right]^{-1}, \quad (2.16)$$

where K is the dielectric constant, e_{14} is the piezoelectric constant, and the c_{ij}'s are elastic constants. For n-InSb $c_{11}=6.66\times10^{11}$, $c_{12}=3.35\times10^{11}$, and $c_{44}=3.14\times10^{11}$ dynes/cm^2. Thus

$$\mu_{\mathrm{PE}}{}^{e-e}(0)=\frac{1.80\times10^{17}}{e_{14}{}^2}\frac{1}{T^{\frac{1}{2}}}\frac{F_{\frac{1}{2}}(\zeta/kT)}{F_0(\zeta/kT)}\ \mathrm{cm}^2\ \mathrm{v}^{-1}\ s^{-1}, \quad (2.17)$$

where e_{14} is in (dynes)$^{\frac{1}{2}}$/cm. Unfortunately the value of e_{14} has not been measured in InSb. It is possible to calculate a value for e_{14} by means of the relation[13]

$$e_{14}{}^2=\frac{K_s-K}{4\pi}(c_{12}-c_{44})c_{44}/c_{12}, \quad (2.18)$$

where the K_s is the static and K the optical dielectric constant (for InSb, $K_s=17.5$ and $K=16$) and the c_{ij}'s are elastic constants (given above for InSb). For InSb Eq. (2.17) yields a value for e_{14} of 5.2×10^4 (dynes)$^{\frac{1}{2}}$/cm.

Now we shall consider the effect of screening on the acoustic and piezoelectric scattering mobilities. Since our samples are moderately degenerate we shall obtain a correction factor to apply to each of the mobilities given by Eqs. (2.15) and (2.17) by finding an expression for the screened momentum relaxation time for each type of lattice scattering, dividing it by the respective unscreened relaxation time, and evaluating the resultant ratios at the Fermi energy. The way the screened relaxation times were obtained was to include a factor of $1/[1+\lambda_D{}^2q^2]$ in the q'th Fourier component of the matrix element of the scattering potential which is then squared and integrated over the phonon distribution (q is the phonon wave number). The degenerate form was used for λ_D. The screened momentum relaxation times are given by

$$\left(\frac{1}{\tau_{\mathrm{Ac}}}\right)_s=\left(\frac{1}{\tau_{\mathrm{Ac}}}\right)\frac{3}{8}\left(\frac{E_s}{E}\right)^{\frac{1}{2}}\times[2(E/E_s)^{\frac{1}{2}}-\arctan 2(E/E_s)^{\frac{1}{2}}], \quad (2.19)$$

and

$$\left(\frac{1}{\tau_{\mathrm{PE}}}\right)_s=\left(\frac{1}{\tau_{\mathrm{PE}}}\right)\frac{E_s}{4E}\ln(4E/E_s+1), \quad (2.20)$$

where E_s is the screening energy$=\hbar^2/2m\lambda_D{}^2$ with λ_D being the Debye screening length.

B. Polar Optical Scattering

Since polar optical mode scattering has been found to be very important in limiting the mobility in n-InSb above 200°K,[3] this mechanism may be at least partly responsible for momentum and energy loss by the electron distribution in our samples at temperatures in the vicinity of 77°K. Thus we shall quote the theoretical results of Stratton[5] for the case of polar optical mode scattering being responsible for both momentum and energy loss. His results are for nondegenerate statistics with either weak or strong electron-electron scattering. For lattice temperatures much smaller than the optical phonon temperature, θ, he found that, when electron-electron scattering is weak, there are no quadratic deviations from Ohm's law. This is because the momentum relaxation time due to polar optical scattering τ_{po}, is independent of energy at energies small compared to the optical phonon energy, $k\theta$, and moderate heating of the carriers doesn't alter τ_{po}. Hence the mobility, which depends on a weighted average of τ_{po} over the electron distribution, is independent of electric field strength.

When electron-electron scattering is strong, Stratton found that quadratic deviations from Ohm's law do occur even when $T\ll\theta$. He gives the following expression for the mobility for $T<\theta$ when electron-electron scattering is strong enough to determine the momentum and energy distributions of the electrons and polar optical scattering determines the momentum and energy loss rates of the electron distribution,

$$\mu_{\mathrm{po}}{}^{e-e}=\mu_{\mathrm{po}}{}^{e-e}(0)[1+A(\gamma_0)F^2/F_0{}^2], \quad (2.21)$$

provided $F\ll F_0/\sqrt{A}$, where

$$F_0=(1/e)(K^{-1}-K_s{}^{-1})me^2k\theta/\hbar^2,$$

$$A(\gamma_0)=\frac{3\pi[(1+\gamma_0)K_1(\gamma_0/2)-3\gamma_0K_0(\gamma_0/2)]}{8N_0{}^2\gamma_0{}^3\exp(\gamma_0)K_0(\gamma_0/2)[K_1(\gamma_0/2)]^2},$$

[13] W. G. Cady, *Piezoelectricity* (McGraw-Hill Book Company, Inc., New York, 1946), p. 743.

and $\gamma_0=T/\theta$, T=lattice temperature, $N_0=[e^{\theta/T}-1]^{-1}$ and the K_i's are Bessel functions. Stratton gives a plot of $A(\gamma_0)$. When $T/\theta\rightarrow 0$, Eq. (2.21) reduces to

$$\mu_{\mathrm{po}}{}^{e-e}=\mu_{\mathrm{po}}{}^{e-e}(0)\left[1-\frac{2\theta}{3T}\frac{[\mu_{\mathrm{po}}{}^{e-e}(0)]^2}{k\theta/m}F^2\right]. \quad (2.22)$$

Note that β, the coefficient of F^2, is negative in contrast to the case when ionized impurity scattering dominates the momentum loss process.

In order for electron-electron scattering to be strong enough for Eq. (2.21) and hence Eq. (2.22) to be valid, Stratton finds that the electron concentration must be greater than

$$n_E\simeq(eF_0k\theta/2\pi e^{*4})(T_e/\theta)^{\frac{1}{2}}\exp(-\theta/T_e), \quad (2.23)$$

where e^* is the effective charge of the lattice ions, or at least greater than

$$n_p=n_E(T_e/\theta)[\exp(\theta/T-\theta/T_e)]^{-1}. \quad (2.24)$$

Equations (2.23) and (2.24) give the electron concentrations above which energy or momentum, respectively, is lost to other electrons faster than it is lost directly to the lattice via polar optical scattering.

C. Quantum Limit

When the motion of electrons in a nondegenerate distribution is quantized by a magnetic field, B, strong enough so that $\hbar\omega>kT$, where $\omega=eB/mc$, the so-called quantum limit has been reached. In this regime the momentum relaxation times for certain lattice scattering mechanisms have an energy dependence opposite to that in zero magnetic field.[14] When this is true, according to a relaxation time model at least, quadratic deviations from Ohm's law should be of opposite sign in the quantum limit compared to the zero magnetic field case.

The only quantitative theoretical result for the variation of electron mobility with electric field strength in the quantum limit has been worked out by Yafet for the case of acoustic lattice scattering causing momentum and energy losses.[15] Although this type of scattering is probably not the important one in n-InSb near 77°K, where we have quantum limit data, we shall quote the theoretical expression for the mobility because it will be useful in interpreting our data. It is

$$\mu_{\mathrm{Ac}}{}^B=\mu_{\mathrm{Ac}}{}^B(0)\left[1+\tfrac{1}{2}\frac{[\mu_{\mathrm{Ac}}{}^B(0)]^2}{s^2}\frac{kT}{\hbar\omega}F^2\right], \quad (2.25)$$

where the B superscripts indicate the presence of a strong longitudinal magnetic field. Note that in Eq. (2.25) the coefficient of F^2, $\beta_{\mathrm{Ac}}{}^B$, is positive whereas in zero magnetic β_{Ac} would be negative

$$\{\beta_{\mathrm{Ac}}\sim-[\mu_{\mathrm{Ac}}(0)]^2/s^2\}.$$

The $kT/\hbar\omega$ factor in Eq. (2.25) arises because quantization forbids electronic transitions involving phonons having less than a certain minimum momentum.

[14] P. N. Argyres and E. N. Adams, Phys. Rev. **104**, 900 (1956) and E. N. Adams and T. D. Holstein, J. Phys. Chem. Solids **10**, 254 (1959).

[15] H. Yafet (private communication).

TABLE I. Sample characteristics.

Sample	Source	Electron concentration (cc^{-1})	Hall mobility at 77°K ($cm^2\ v^{-1}\ s^{-1}$)	Hall mobility at 4.18°K ($cm^2\ v^{-1}\ s^{-1}$)
3.3–13	National Bureau of Standards	3.3×10^{13}	112 000	8790
3.7–13	Westinghouse Research Laboratories	3.7×10^{13}	111 000	···
2.8–14	Ohio Semiconductor	2.8×10^{14}	522 000	···
3.0–14	Westinghouse Research Laboratories	3.0×10^{14}	201 000	37 600
6.5–14	Ohio Semiconductor	6.5×10^{14}	279 000	···
9.4–14	Westinghouse Materials Engineering	9.4×10^{14}	82 000	18 700
3.9–15	Chicago Midway Laboratories	3.9×10^{15}	130 000	54 500
8.7–15	Westinghouse Materials Engineering	8.7×10^{15}	146 000	110 000

III. EXPERIMENTAL DETAILS

A. Specimens

The specimens were all n-type InSb cut, lapped, and etched to about $10\times2.5\times1$ mm in size. Each had only a few large grains. Two current and four potential leads of No. 36 or No. 40 copper wire were attached with InSb solder.

Some pertinent properties of the specimens are given in Table I which also identifies their source. Note that the number identifying a sample is an abbreviation for the carrier concentration. To minimize extraneous effects, e.g., sample inhomogeneity, only samples in which measurements of Hall effect and resistivity on different sets of leads agreed well were used and in which there was no anomalous rise of Hall coefficient with magnetic field strength, such a rise having been correlated with sample inhomogeneity by Bate.[16]

B. Experimental Technique

Measurements were made by means of a dc potentiometer system with the samples immersed in a bath of liquid helium, hydrogen, nitrogen or oxygen. Temperatures were determined from the vapor pressure of the baths. Magnetic fields were provided by an auto-

[16] R. T. Bate, R. K. Willardson, and A. C. Beer, Bull. Am. Phys. Soc. **5**, 152 (1960).

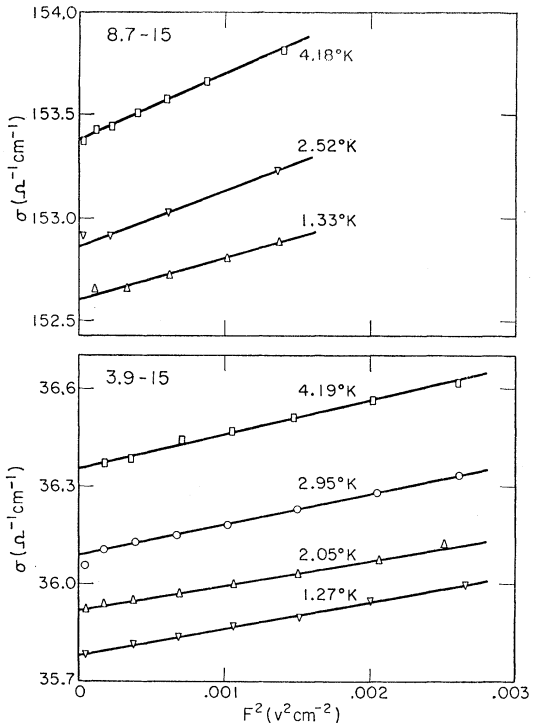

FIG. 1. Variation of electrical conductivity, σ, with electric field, F, for n-InSb samples having 8.7×10^{15} and 3.9×10^{15} electrons/cc. σ is plotted versus F^2 for various liquid helium temperatures.

matically controlled A.D. Little Electromagnet and measured with a rotating coil fluxmeter.

IV. RESULTS AND DISCUSSION

A. Low-Temperature Region

The dependence of the electrical conductivity of n-InSb samples of various carrier concentrations on electric field strength is given for liquid helium and

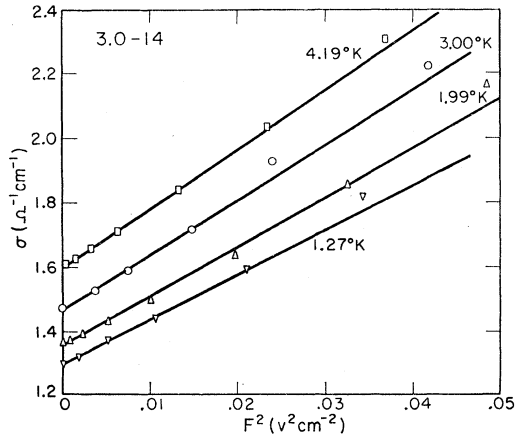

FIG. 2. Variation of electrical conductivity, σ, with electric field, F, for n-InSb having 3.0×10^{14} electrons/cc. σ is plotted versus F^2 for various liquid helium temperatures.

liquid hydrogen temperatures in Figs. 1 to 4. The conductivity σ, for a fixed lattice temperature has been plotted versus the square of the electric field strength, F. Straight lines of positive slope resulted at low electric fields except for sample 3.3–13 at 4.2°K. (Note that sample 3.3–13 has the smallest carrier concentration which may make the electron temperature approximation a poor one.)

The linear dependence of σ on F^2 indicates variation of the mobility due to moderate heating of the carriers by the electric field as predicted by Eq. (1.1). That σ increases with field strength is to be expected when ionized impurity scattering determines the momentum loss rate of the electrons. Other evidence for the latter is provided by the fact that the magnitude and temperature dependence of the observed Ohmic mobility at the temperature in question are explainable using available theory for ionized impurity scattering.[17]

The deviation of σ versus F^2 from linearity at the higher electric fields reached in samples 3.0–14 and

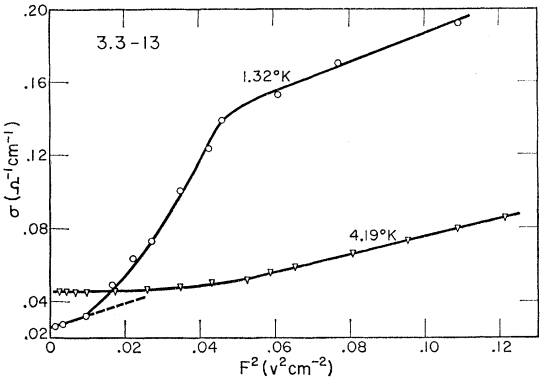

FIG. 3. Variation of electrical conductivity, σ, with electric field, F, for n-InSb having 3.3×10^{13} electrons/cc. σ is plotted versus F^2 for 1.32°K and 4.19°K.

3.3–13 are due to terms in the conductivity dependent on higher powers of the electric field. Such terms become effective when $\delta=(T_e-T)/T$ is not small compared to one. Using Eq. (2.5) and the observed values of β (given by the slopes of σ versus F^2) we note that for sample 3.0–14 at 4.2°K $\delta\approx1$ when $F^2\approx0.027$ v^2 cm^{-2}. We shall not discuss nonquadratic deviations from Ohm's law. (Putley has observed such deviations in n-InSb at liquid helium temperatures and attempted to fit calculated curves to the experimental σ versus F^2 curves with some limited success.)[18]

The smaller rate of increase of σ at the highest fields in sample 3.3–13 at 1.32°K and sample 3.0–14 at liquid hydrogen temperatures suggests that an additional energy loss mechanism is becoming important. A rough estimate indicates that this mechanism may

[17] See for example, F. J. Blatt, *Solid State Physics*, edited by F. Seitz and D. Turnbull (Academic Press, Inc., New York, 1957), Vol. 4, p. 343ff.

[18] E. H. Putley, Proc. Phys. Soc. (London) **A73**, 280 (1959).

be polar optical mode scattering in the case of the liquid hydrogen temperature data. In our analysis we shall be concerned only with deviations in σ which are proportional to F^2 as $F \rightarrow 0$.

A summary of the liquid helium and liquid hydrogen results for various samples is presented in Fig. 4 where $\beta = \Delta\mu/\mu_0 F^2$ is plotted versus lattice temperature. The parameter β was determined from the slope of the linear part of the σ versus F^2 curve for the sample in question. Values of β are also given for an additional sample besides those for which data were given in Figs. 1 to 4. From Fig. 5 it can be seen that β is smaller the higher the carrier concentration. This suggests the presence of the effect contained in Eqs. (2.5) and (2.13) due to the carrier distribution being more degenerate in samples having more carriers.

To test this hypothesis we have plotted β/μ_0 versus carrier concentration in Fig. 6. The carrier concentrations were obtained from the Hall coefficient and are listed in Table I. The Ohm's law mobility, μ_0, is calcu-

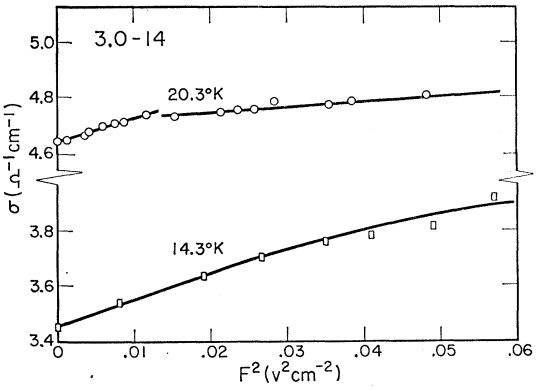

FIG. 4. Variation of electrical conductivity, σ, with electric field, F, for n-InSb having 3.0×10^{14} electrons/cc. σ is plotted versus F^2 for two liquid hydrogen temperatures.

lated from the measured conductivity and the number of carriers. Data for a lattice temperature of 4.18°K are used rather than lower temperature data because the classical approximation for the phonon distribution, i.e., Eq. (2.3), applies at 4.2° but not at 1.3°K, and it is for this approximation that the relations given in Sec. II were obtained. At 4.18°K the Fermi energy of all samples having more than 3×10^{14} carriers/cc is large enough so that the simpler Eq. (2.13) rather than Eq. (2.4) can be used to interpret our results. From Fig. 6 we see that our experimental β/μ_0 varies roughly like $1/n$. The behavior predicted by Eq. (2.13) is $\beta/\mu_0 \sim \mu_L/\zeta^2$, and since $\zeta \sim n^{\frac{1}{3}}$, if μ_L were independent of n, this would give $\beta/\mu_0 \sim 1/n^{\frac{2}{3}}$. The somewhat low value of β/μ_0 for the 3.0–14 sample may be due to incomplete statistical degeneracy. At any rate the $1/\zeta^2$ factor accounts for the general dependence of the experimental β/μ_0 on n.

Using our experimental values of β along with Eqs. (2.5) and (2.6) or (2.12) we deduced μ_L, the lattice

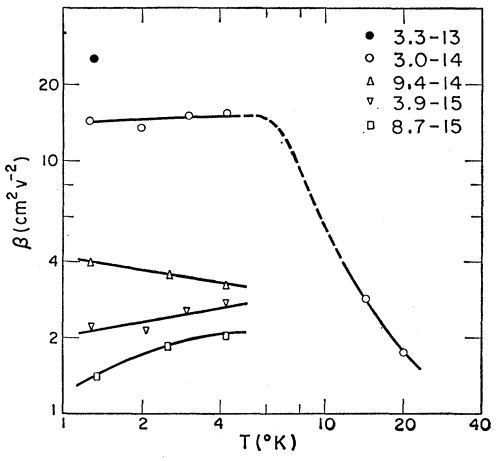

FIG. 5. Dependence of β on temperature for samples of n-InSb with various electron concentrations. $\beta \equiv [\sigma(F)-\sigma(0)]/\sigma(0)F^2$ when σ versus F^2 is linear.

scattering mobility connected with the energy loss mechanism, for each of our samples at liquid helium temperatures and also for sample 3.0–14 at liquid hydrogen temperatures. The resultant values of μ_L at 4.18°K are plotted versus carrier concentration in Fig. 7. From these μ_L values we calculated the value of the associated average mean free path l_L by means of Eq. (2.10). These mean-free path values are listed in Table II and are quite large (of the order of $\sim$1 cm). Indeed the mean-free path for energy loss may be even longer (as discussed by Greene).[8] Both l_L and the associated m.f.p. for energy loss can be longer than the sample dimensions because the electron suffers many changes of direction between phonon scatterings due to the very short m.f.p. for scattering by ionized impurities, l_I (see Table II). Specifically the net rms distance

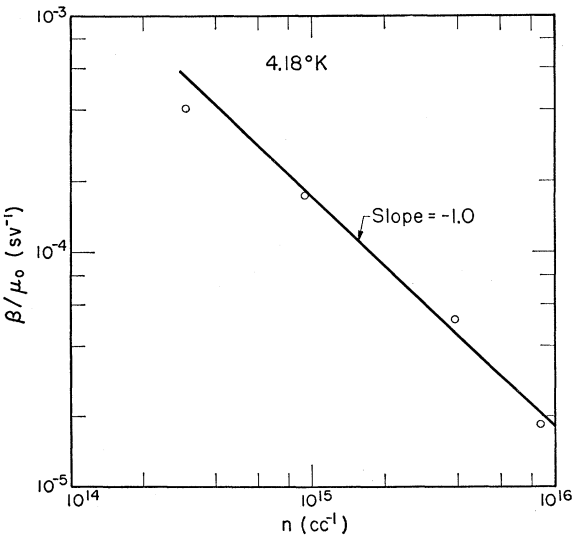

FIG. 6. Dependence of the experimental β/μ_0 at 4.2°K on electron concentration. μ_0 is the Ohm's law mobility. For comparison a straight line of slope $= -1$ has been included.

TABLE II. Characteristic energies and lengths for degenerate samples $T=4.18°K$.

Sample	ζ/kT	l_{ee} (10^{-3} cm)	l_L (10^{-3} cm)	l_I (10^{-6} cm)	$(l_I l_L)^{\frac{1}{2}}$ (10^{-3} cm)	λ_D (10^{-6} cm)	E_s/k (°K)	ζ/E_s
3.0–14	3.2	0.46	56	4.6	0.51	5.0	13	1.0
9.4–14	7.4	2.4	105	3.7	0.62	4.1	20.3	1.5
3.9–15	19.4	18.5	313	17.5	2.34	3.24	32.5	2.5
8.7–15	33	49.3	425	46.3	4.43	2.85	42.1	3.3

traversed between scatterings by phonons is $[(l_L/l_I)]^{\frac{1}{2}}l_I = (l_L l_I)^{\frac{1}{2}}$, since l_L/l_I gives the average number of scatterings by ionized impurities between scatterings by the phonons. Table II lists the values of $(l_L/l_I)^{\frac{1}{2}}$ for the various samples. It is satisfying to note that for each sample $(l_L l_I)^{\frac{1}{2}}$ is much smaller than the sample dimensions. Since scatterings by impurities are so much more frequent than scattering by phonons, the question arises as to whether a small but ordinarily neglected energy loss due to ionized impurities may be important. Greene has estimated the energy loss via scattering by ionized impurities.[9] It turns out to be small compared to the energy loss to acoustic phonons under the present conditions.

To determine the type of lattice scattering responsible for energy loss, we have plotted in Fig. 7 the theoretical mobilities for screened acoustic and screened piezoelectric scattering along with the lattice mobility deduced from our experimental values of β. The theoretical mobilities were calculated by applying a screening factor correction given by Eq. (2.19) or (2.20), in which we set $E=\zeta$, to the mobility given by Eq. (2.15) or (2.17), respectively.

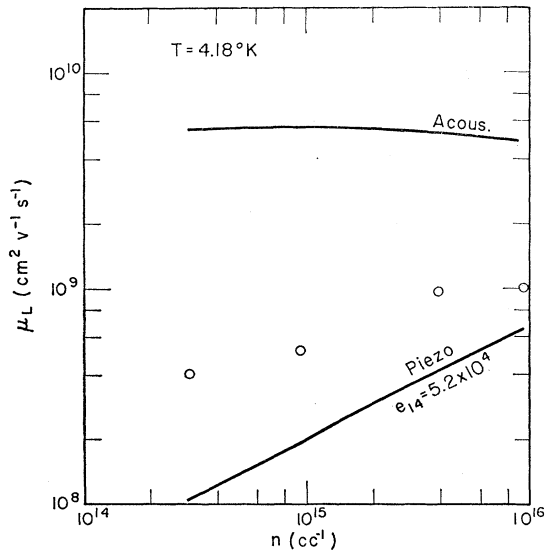

FIG. 7. Lattice scattering mobility at 4.18°K as a function of electron concentration. The circles give the values of μ_L deduced from the experimental values of β. The curves are calculated from theory for piezoelectric and acoustic scattering assuming strong electron-electron scattering and include a correction for screening of the lattice potential by the electrons.

Note that μ_L deduced from the experimental β varies with carrier concentration in about the same way as the "piezo" curve and is closer to it than to the "acous" curve. To obtain more quantitative agreement between μ_L from β and the theoretical piezoelectric scattering mobility would require e_{14} be smaller than the theoretical value by about a factor of two. Such a smaller value of e_{14} is quite reasonable, since it is known that in ZnS the measured value of e_{14} is a factor of about five smaller than the theoretically calculated one.[13] With a value of e_{14} of $\frac{1}{2}$ the theoretical one all the energy loss at 4.2°K can be accounted for by piezoelectric scattering alone.

Next we deduced the temperature dependence of μ_L from our experimental values of β by using Eqs. (2.5) and (2.6) or (2.12). The results are presented in Fig. 8. From this figure we can see that at liquid helium temperatures the deduced μ_L varies about like $T^{-1.7}$ for three of the samples and about like T^{-2} for sample 9.4–14. The reason for the different temperature dependence of μ_L in sample 9.4–14 is not known, although it may be noted from Table I, that this sample has a lower ohmic mobility than would fit the general mobility versus n pattern.

The temperature dependences of μ_L deduced from β are greater than expected from theory using the classical approximation for the phonon distribution which predicts a $1/T$ variation for either acoustic or piezoelectric scattering when the electron distribution is statistically degenerate. A possible explanation of the strong temperature dependence of μ_L is that since Eq. (2.3) fails to hold somewhere below 4.2°K in the samples of interest (see Table III) there are fewer phonons available to scatter the electrons and hence a higher μ_L than otherwise. Although the mobilities and energy loss rates could be calculated for the case when Eq. (2.3) fails, we have not done so because it would require a prohibitive amount of numerical integrations.

TABLE III. Temperature at which maximum phonon energy[a] equals thermal energy.

Sample	$T=\hbar q_{\max} s/k$
3.0–14	1.2°K
9.4–14	1.7
3.9–15	2.8
8.7–15	3.6

[a] Calculated from $\hbar s q_{\max} \approx 2(2ms^2\zeta)^{\frac{1}{2}}$.

B. Vicinity of 77°K

The variation of the electrical conductivity of a number of samples at 77°K with electric field is presented in Fig. 9. The ratio of the conductivity to that at zero electric field is plotted versus the square of the electric field. For each sample the data fall along a straight line. The slope of the line depends on the sample in question. For sample 3.3–13 which has the lowest mobility the slope is positive while for the other samples with higher mobilities the slopes are negative, being more negative the higher the mobility.

The linear dependence of the conductivities on the square of the electric field are interpretable by means of

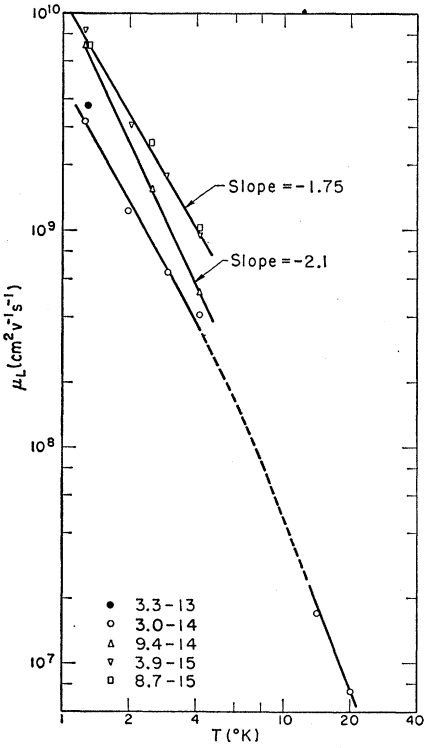

FIG. 8. Temperature dependence of the lattice scattering mobility, μ_L, deduced from experimental values of β. Slopes of straight lines drawn through some of the data are indicated.

Eq. (1.1) as variation of the mobility due to small amounts of carrier heating by the electric field. The effects in the various samples can be interpreted qualitatively as follows.

The positive deviation from Ohm's law for sample 3.3–13 is due to the mobility in this sample being determined by ionized impurity scattering. Evidence for the latter is provided by the low value of the mobility (112 000 $cm^2\ v^{-1}\ s^{-1}$) compared to that of other samples and the fact that the mobility of this sample is an increasing function of temperature around 77°K. (See Fig. 12.)

The different amounts of negative deviations from Ohm's law for most of the samples are due to lattice

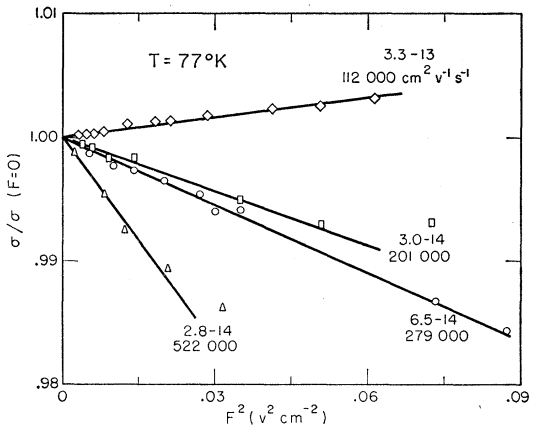

FIG. 9. The relative conductivity versus the square of the electric field for a number of n-InSb samples at 77°K.

scattering being important to different degrees (with impurity scattering accounting for the remainder of the momentum loss). Evidence for the importance of lattice scattering is provided by the fact that in these samples the mobility is a decreasing function of temperature around 77°K (e.g., see Fig. 11).

As yet it is not known what types of lattice scattering are important in n-InSb around 77°K. However the value of the mobility calculated when scattering is due either to the deformation potential or to the piezoelectric polarization associated with acoustic modes is much higher than observed in the purest material. Thus we shall consider polar optical scattering which has been shown to predominate in n-InSb above 200°K.[3] Calculation shows that polar optical mode scattering also gives much too high a value for the mobility at 77°K to be important when electron-electron scattering is negligible. However, as noted in Sec. II, the effectiveness of polar optical mode scattering in limiting the mobility is greatly enhanced when there is strong electron-electron scattering. To see whether we should expect strong electron-electron scattering in our samples we have used Eqs. (2.23) and (2.24) to calculate the critical electron concentrations for which momentum and energy transfers to other electrons equals that to polar optical modes. These critical concentrations are given for a number of temperatures in Table IV. The samples we measured in this temperature range had electron concentrations between 3.3×10^{13} and 6.5×10^{14} cc^{-1}. Thus at 77°K n_p at least

TABLE IV. Minimum electron concentrations needed for electron-electron scattering to determine energy distribution, n_E, and momentum distribution, n_p, when polar optical scattering is operative.

T	n_E	n_p
90°K	14.6×10^{14} cc^{-1}	4.54×10^{14} cc^{-1}
77	7.76	2.06
73	6.2	1.56
65	3.62	0.81

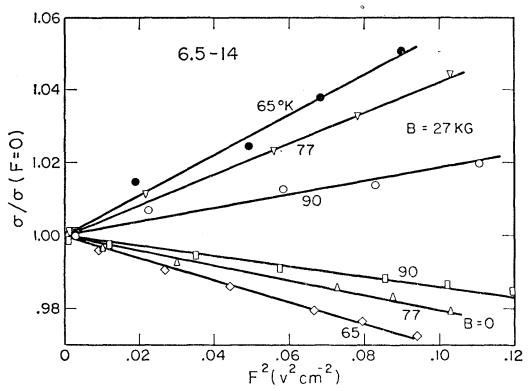

FIG. 10. Comparison of the variation of the relative conductivity of n-InSb, sample 6.5–14, in a strong longitudinal magnetic field and in zero magnetic field at temperatures between 65°K and 90°K.

falls within this range of concentrations indicating that $e-e$ scattering may be quite important.

With the inclusion of strong electron-electron scattering, the negative deviations from Ohm's law which we observe at 77°K can be explained qualitatively as due to polar optical mode lattice scattering. (For weak $e-e$ scattering there should be no quadratic deviations from Ohm's law due to polar optical scattering when $T\ll\theta$.)[5] The higher the mobility the more negative the deviation is, in general accord with the theoretical Eq. (2.21) or (2.22). For a quantitative comparison with the theory we note that for sample 2.8–14 with the highest mobility and thus with the most nearly pure lattice scattering, β at 77°K has a value of -0.53 v^{-2} cm^2 while Eq. (2.21) predicts a value of -0.0087 v^{-2} cm^2. Thus $\beta_{\text{exp}}\approx 60\beta_{\text{theor}}$ and the energy loss rate in our sample is almost two order of magnitude smaller than predicted by theory. This large discrepancy raises the question as to whether the theory is at all applicable. If it is, the discrepancy may be due to electron-electron interaction being relatively less effective in enhancing the energy loss than the momentum loss to polar optical modes in our samples. Perhaps this might occur because the electron concentrations in our samples lie between the critical concentrations at which electron-electron scattering determines the energy and momentum distributions of the electrons.

The variation of the conductivity with electric field strength for sample 6.5–14 at various temperatures is presented in Fig. 10 both for zero magnetic field and for a longitudinal magnetic field of 27 kgauss. In each case σ/σ $(F=0)$ is plotted versus F^2. When no magnetic field is applied, the data for a given temperature fall along a straight line of *negative* slope. However, in the presence of a magnetic field of 27 kgauss, which is sufficient to quantize electron motion, the data for each temperature fall along a straight line of *positive* slope. This change of sign of the deviation from Ohm's law by the magnetic field we interpret as due to the energy dependence of the momentum loss rate in the strong magnetic field case being opposite to that in zero magnetic field.

The observed temperature dependence of β for sample 6.5–14 is plotted in Fig. 11. For zero magnetic field between 65°K and 90°K β remains negative and decreases in absolute value as the temperature as increased. This is expected since in this temperature range the mobility of the sample decreases with increasing temperature. (See Fig. 11.) However the size of the observed temperature dependence is much smaller than predicted by theory as given by Eq. (2.21). The reason for this discrepancy may be the same as that suggested to explain the discrepancy between the observed and theoretical magnitudes of β at 77°K.

To interpret our data in the strong magnetic field case we shall resort to Eq. (2.25) even though the latter is for acoustic (deformation potential) scattering which is probably not the important type of lattice scattering

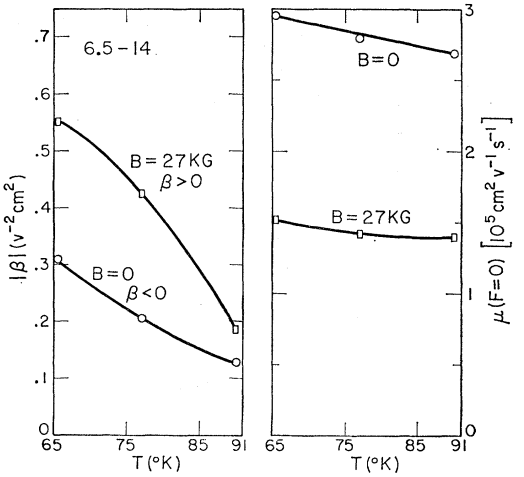

FIG. 11. Temperature dependences of $|\beta|$ and the (Ohm's law) mobility in n-InSb, sample 6.5–14, between 65°K and 90°K in a strong longitudinal magnetic field and in zero magnetic field.

in n-InSb at 77°K. To make Eq. (2.25) more applicable to the case of polar optical mode scattering we replace s^2 by $k\theta/m$. In addition, because of the presence of some impurity scattering we shall replace $[\mu_{Ac}{}^B(0)]^2$ by the product of the observed Ohmic mobility of the sample in question and that of the most pure sample, the latter giving a rough (under) estimate of the lattice scattering mobility. Using Eq. (2.25) in this manner we calculate for sample 6.5–14 at 77°K a value for β of about 0.66×10^{-5} cm^2 v^{-2} which is five orders of magnitude smaller than the observed value of 0.42 cm^2 v^{-2}. Since there is some evidence that piezoelectric scattering may be important at 77°K at least in high purity n-InSb when a strong magnetic field is present,[19] we shall now use Eq. (2.25) without replacing s^2 by $k\theta/m$. (Rather s^2 should be replaced by s'^2 which however is expected to be similar in value to s^2.) Upon doing so, we calculate

[19] R. J. Sladek, J. Phys. Chem. Solids (to be published).

for β a value of 0.21 v^{-2} cm^2 which is only a factor of two smaller than the observed value. It should be noted that, for zero magnetic field, piezoelectric scattering could not be the important type of lattice scattering because, if it were, a negative β would be impossible.

The temperature dependence observed for β in a strong magnetic field in sample 6.5–14 is reasonable in that β decreases as the Ohmic mobility decreases (see Fig. 11) and also presumably the lattice scattering mobility decreases. A more quantitative comparison with theory represented by Eq. (2.25) is ambiguous because of our not knowing very well what mobility values to use in the latter.

For sample 3.3–13 β and the (ohmic) mobility between 65° and 90°K are plotted versus temperature in Fig. 12. Note that both β and the temperature derivative of the mobility, $d\mu/dT$ are positive, while for each of the purer samples, in zero magnetic field, both β and $d\mu/dT$ are negative (e.g., see Fig. 11). Actually the sign of β should be the same as that of

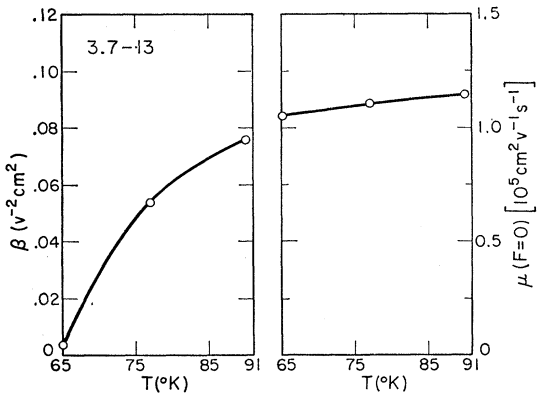

FIG. 12. Temperature dependences of β and the (Ohm's law) mobility in n-InSb, sample 3.3–13, between 65°K and 90°K.

$d\mu/dT_e$ since the signs of both these quantities depend on the process by which momentum is lost from the electron distribution. Thus the observed correlation between the signs of β and $d\mu/dT$ is reasonable although not actually required by the above considerations since $d\mu/dT$ might in principle have a different sign than $d\mu/dT_e$.

For sample 3.3–13 Fig. 12 shows that β increases with increasing temperature between 65°K and 90°K. This is consistent with Eq. (1.2) in that the observed ohmic mobility also increases with temperature in this range. However Eq. (1.2) also states that β should be proportional to the mobility associated with the energy loss process, μ_L. Since the latter presumably decreases as the temperature is increased while μ_0 increases only slightly with increasing temperature, it is not clear that the observed dependence of β on temperature can be accounted for by means of Eq. (1.2).

V. CONCLUSIONS

By measurements of quadratic deviations from Ohm's law in n-InSb at low temperatures, the types of scattering which are important for both momentum and energy loss can be deduced and the effects of carrier degeneracy can be studied.

At liquid helium temperatures impurity scattering limits the mobility, piezoelectric scattering is largely responsible for energy loss by the carriers, and carrier degeneracy effects occur.

At liquid hydrogen temperatures impurity scattering again limits the mobility. Energy loss to polar optical modes may be important after a certain electric field strength is reached.

In the vicinity of 77°K the value of β can be explained qualitatively in terms of the magnitude and temperature dependence of the Ohm's law mobility. Quantitative conclusions from our data in this range are less certain because of the lack or inapplicability of existing theory. Among these conclusions are that (1) polar optical mode scattering enhanced by strong electron-electron scattering is the important type of lattice scattering in zero magnetic field, and (2) that a strong magnetic field changes the sign of deviations from Ohm's law by altering the energy dependence of the momentum loss rate.

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