Measurement of Thermal Fluctuations in Radiation*

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The thermal fluctuations in radiation are described by the Einstein-Fowler relation $\langle(\Delta E)^2\rangle = kT^2(\partial\langle E\rangle/\partial T)$. Two terms contribute to this fluctuation, a photon shot noise term and an expression predicted by wave theory. In the present experiment the second term has been measured.

1. INTRODUCTION

IN 1911 Einstein[1] derived the equation

$$\langle(\Delta E)^2\rangle = KT^2\partial\langle E\rangle/\partial T \tag{1}$$

expressing the statistical relationship between the energy fluctuation and the specific heat of radiation in thermal equilibrium with its surroundings. For black-body radiation this can be written as

$$\langle(\Delta E)^2\rangle = h\nu\langle E\rangle\left[1+\frac{1}{\exp(h\nu/kT)-1}\right], \tag{2}$$

where T is the absolute temperature, h is Planck's constant, k is Boltzmann's constant, ν is the spectral frequency of the radiation, and brackets $\langle\ \rangle$ denote ensemble average. The fluctuation in the number of photons in the ensemble is obtained by dividing expression (2) by $h^2\nu^2$:

$$\langle(\Delta N)^2\rangle = \langle N\rangle\left[1+\frac{1}{\exp(h\nu/kT)-1}\right]. \tag{3}$$

This can be rewritten, as

$$\langle(\Delta N)^2\rangle = \langle N\rangle[1+\langle N\rangle/g], \tag{4}$$

where g is the number of phase cells in the volume of phase space occupied by the photons. The term $\langle N\rangle^2/g$ has been measured by Brown and Twiss,[2] although they interpret it on a wave-theoretical basis. The measurements reported here differ from those of Brown and Twiss, of Rebka and Pound,[3] and of Brannen, Ferguson, and Wehlau[4] mainly in technical detail.

2. THE EXPERIMENT

Light from an incandescent tungsten filament S (Fig. 1) passes through an infrared transmitting filter F and is incident on a half-silvered mirror (HSM). The reflected light falls on one lead sulfide detector, D_1, while the transmitted light is incident on another one, D_2. The output signals of the two detectors are electronically correlated (Fig. 2). Purcell[5] has shown that the cross correlation $\langle\Delta N_1\Delta N_2\rangle$ between the number of photons incident on two detectors, D_1, and D_2, is equal to one-fourth of the fluctuation term $\langle N\rangle^2/g$ of Eq. (4). Thus, by electronically cross-correlating the output of the two detectors, one can determine the fluctuation in the radiation from source S.

The advantage of the correlation technique can be understood by considering two detectors receiving signals s_1, s_2 and having independent noises n_1 and n_2. When the two detector output signals are electronically multiplied, one obtains the product

$$(s_1+n_1)(s_2+n_2) = s_1s_2+s_2n_1+s_1n_2+n_1n_2. \tag{5}$$

If s_1 and s_2 have identical frequencies, the product term s_1s_2 is always positive or always negative, depending on the phase relation of the two signals. The other terms are positive or negative in some random sequence. Let $s_1=s_2\equiv s$. On integrating expression (5) over a long period of time t, the random terms average to zero, and the integral is dominated by the term s^2. If the system has a response time τ, the correlator output signal $\mathcal{S}$ will be s^2t/τ. The noise $\mathfrak{N}$ in the correlator out-

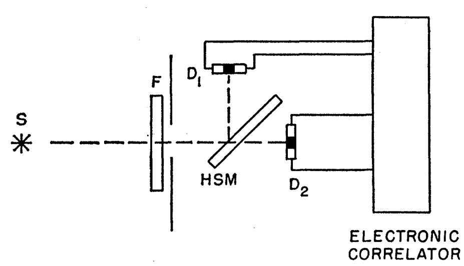

FIG. 1. Schematical diagram of a representative correlating system. Light from the source S passes through an optical filter F and is divided into two beams by a half-silvered mirror HSM before impinging on detectors D_1 or D_2. The detector signals are amplified and correlated in the correlating electronics.

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[1] A. Einstein, 1st Solvay Congress (1911).

[2] R. Hanbury Brown and R. Q. Twiss, Proc. Roy. Soc. (London) **A242**, 300 (1957); **243**, 291 (1958); **248**, 199 (1958); **248**, 222 (1958).

[3] G. Rebka and R. V. Pound, Nature **180**, 1035 (1957).

[4] E. Brannen, H. S. I. Ferguson, and H. Wehlau, Can. J. Phys. **36**, 871 (1958).

[5] E. M. Purcell, Nature **178**, 1449 (1956).

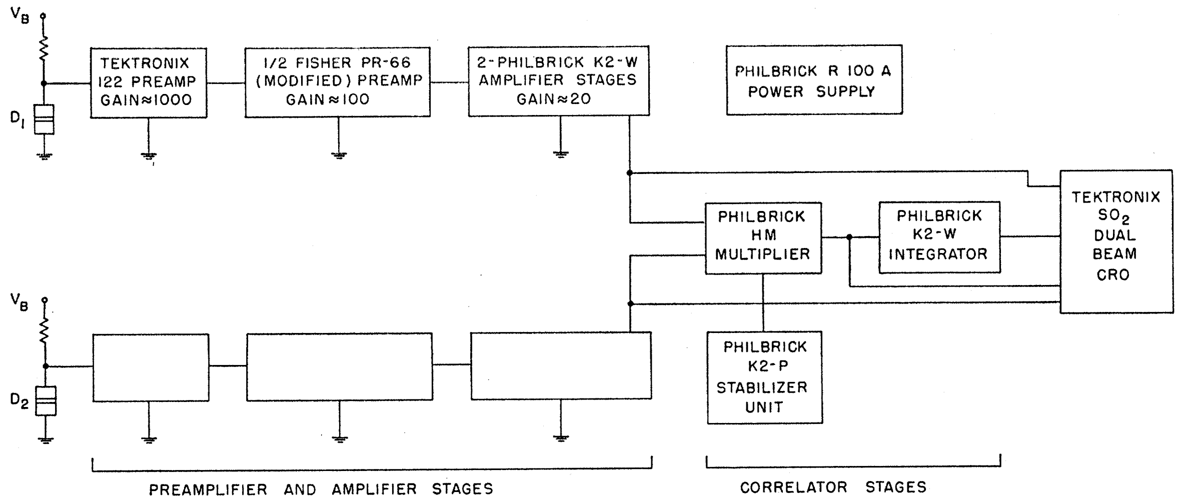

FIG. 2. Block diagram of the electronic system.

put is given by $n_1n_2(t/\tau)^{\frac{1}{2}}$ if $s\ll n_1$, n_2. Let $n_1=n_2\equiv n$, then the correlator output signal-to-noise ratio is

$$\frac{\mathcal{S}}{\mathfrak{N}}=\frac{s^2}{n^2}\left(\frac{t}{\tau}\right)^{\frac{1}{2}}. \tag{6}$$

For a derivation of this equation see Lee, Cheatham, and Wiesner[6] or Goldstein.[7]

In the present experiment the quantity s^2 corresponds to the expected cross correlation $\langle\Delta N_1\Delta N_2\rangle$ appropriately modified by detector response and amplifier terms.

3. THE FLUCTUATION TERM $\langle N\rangle^2/g$ AND THE CROSS CORRELATION IN SIGNALS FROM TWO DETECTORS

Consider a beam of photons emanating from an area A of a blackbody and falling on a detector that subtends a solid angle Ω at A. During a system response time τ, an average number of photons $\langle N\rangle$ is incident on the detector. The volume of phase space, h^3g, occupied by these photons at the source is given in terms of the number of phase cells, $g=2V\Omega\nu_0^2\Delta\nu c^{-3}$, where the factor 2 accounts for the two possible directions of polarization; $V=Ac\tau$ is the volume in extension space occupied by $\langle N\rangle$ photons near the source; ν_0 is the central spectral frequency; and $\Delta\nu$ is the spectral bandwidth of the radiation. The factor $\Omega\nu_0^2\Delta\nu c^{-3}$ is h^{-3} times the extension of the photon beam in momentum space. The fluctuation in the number of photons N incident on the detector in time τ is given by Eq. (4). If the detector has a quantum efficiency ϵ, only a randomly selected fraction ϵ of the incident photons can give rise to a detector signal.

For simplicity, let a fixed number of photons N be incident on the detector during each interval τ. Then the fluctuation in the number of excited photoelectrons N_e is

$$\langle(\Delta N_e)^2\rangle=\langle N_e^2\rangle-\epsilon^2N^2=\epsilon(1-\epsilon)N, \tag{7}$$

since for a fixed number of incident photons, the photoelectrons have a binomial distribution. On removing the restriction on N and allowing it to fluctuate in accordance with (4), one obtains

$$\langle(\Delta N_e)^2\rangle=\langle N_e^2\rangle-\langle N_e\rangle^2=\langle\epsilon(1-\epsilon)N+\epsilon^2N^2-\epsilon^2\langle N\rangle^2\rangle.$$

Substituting Eq. (4) for quadratic terms in N leads to

$$\langle(\Delta N_e)^2\rangle=\epsilon\langle N\rangle(1+\epsilon\langle N\rangle/g). \tag{8}$$

One should emphasize that this expression for the fluctuation in the number of detector photoelectrons is rigorous only when the detector temperature is absolute zero. A different expression,

$$\langle(\Delta N_e)^2\rangle=\epsilon\langle N\rangle(1+\langle N\rangle/g),$$

was derived by Jones[8] and Fellgett[9] for detectors in thermal equilibrium with the radiant source. Jones, Fellgett, and Twiss[10] showed that these expressions represent extremes of a general equation relating detector temperature to photoelectron fluctuation. In the present experiment, where the source temperature was about 3000°K and the detector temperature was 200°K, expression (8) was an extremely good approximation.

To determine the cross correlation in photocurrents at the two detectors in Fig. 1, one can write

$$\langle(\Delta N_e)^2\rangle=\langle(\Delta N_{e1}+\Delta N_{e2})^2\rangle$$
$$=\langle\Delta N_{e1}^2\rangle+\langle(\Delta N_{e2})^2\rangle+\langle 2\Delta N_{e1}\Delta N_{e2}\rangle. \tag{9}$$

[6] Y. W. Lee, T. P. Cheatham, and J. B. Wiesner, Proc. Inst. Radio Engrs. **38**, 1165 (1950).
[7] S. Goldstein, Proc. Inst. Radio Engrs. **43**, 1663 (1955).
[8] R. C. Jones, in *Advances in Electronics* edited by L. Marton (Academic Press, Inc., New York, 1953), Vol. 5.
[9] P. B. Fellgett, J. Opt. Soc. Am. **39**, 970 (1949).
[10] P. B. Fellgett, R. C. Jones, and R. Q. Twiss, Nature **184**, 967 (1959).

Therefore,

$$\langle 2\Delta N_{e1}\Delta N_{e2}\rangle = \epsilon\langle N\rangle(1+\epsilon\langle N\rangle/g) - 2[\tfrac{1}{2}\epsilon\langle N\rangle(1+\tfrac{1}{2}\epsilon\langle N\rangle/g)], \quad (10)$$

where the first term on the right equals $\langle(\Delta N_e)^2\rangle$, the fluctuation that would be observed if the signals from the two detectors were combined. The term in square brackets is the fluctuation in the output from each of the detectors taken separately. From Eq. (10) we have

$$\langle \Delta N_{e1}\Delta N_{e2}\rangle = \tfrac{1}{4}\epsilon^2\langle N\rangle^2/g. \quad (11)$$

In the optical system sketched in Fig. 1, the source emissivity, p, and filter transmission coefficient, q, also affect the observed photon fluctuations. The fluctuation in a beam of photons after passing through a partially transparent optical filter is given by an expression similar to Eq. (8) with the quantum efficiency ϵ replaced by the transmission coefficient q. One assumes that the filter emits no radiation, so that the filter must be kept at a much lower temperature than the radiating source.

The limited emissivity of a radiant source can be attributed to a change in permittivity, permeability, and electrical conductivity at the source–free space interface. The interface cannot absorb or emit radiant energy; it can only reflect or transmit radiation incident from the interior of the source. Hence, to calculate the fluctuation in a beam of light from a gray source, one can construct a model in which the source is represented by a blackbody surrounded by a partially-reflecting, partially-transparent surface whose transmission coefficient is p. Again, a photon fluctuation expression similar to Eq. (8) can be derived: the efficiency ϵ in (8) is replaced by the parameter p.

If one compounds the source emissivity p, transmission by the medium between source and detector q, and the detector quantum efficiency ϵ, the substitution of the product $qp\epsilon$ for ϵ in Eq. (11) gives a general expression for the cross correlation expected for two detectors illuminated by a source at temperature T:

$$\langle \Delta N_{e1}\Delta N_{e2}\rangle = \frac{\epsilon^2}{4}\langle N\rangle\frac{\langle N\rangle}{g} = \frac{g^2p^2\epsilon^2}{4}g[\exp(h\nu_0/kT)-1]^{-2}. \quad (12)$$

The number of photons incident on the detector, N, already contains the factors p and q if one defines $N\equiv qpN_0$, where N_0 is the number of quanta that would impinge on the two detectors if they were illuminated by a blackbody at the given temperature T, with no photon losses between source and detectors.

4. COMPARISON OF QUANTUM STATISTICAL AND WAVE-THEORETICAL RESULTS

Equation (12) indicates that a knowledge of the source temperature determines the radiation fluctuation only if the parameters p and q are known, and that it is the light intensity at the detectors that actually determines the observed correlation. This was indicated by Kahn,[11] but it also follows immediately from the work of Brown and Twiss[2] and their theory of intensity interferometry. In fact, the apparatus sketched in Fig. 1 is an intensity interferometer. Hence, the result (12) should, in regions of common applicability, be equivalent to the results derived by Brown and Twiss by classical wave-theoretical means. To bring Eq. (12) into a form more suitable for direct comparison, one can rewrite g as

$$g=(a\tau c)(2\omega\nu_0^2\Delta\nu)c^{-3}, \quad (13)$$

where a is the area of the detector and ω is the solid angle subtended by the source at the detector. Hence $\langle N\rangle/g=\langle N\rangle\lambda^2(2\omega a\tau\Delta\nu)^{-1}$ where λ is the wavelength of the radiation $\lambda\equiv c\nu_0^{-1}$. From Eq. (12) we have

$$\langle(\Delta N_{e1})(\Delta N_{e2})\rangle=\epsilon^2\langle N\rangle^2\lambda^2/8\omega a\tau\Delta\nu. \quad (14)$$

Let $n=N/2$ be the number of photons incident on each detector, and let α be the detector voltage response per liberated photoelectron; then

$$s^2=\langle\Delta V_1\Delta V_2\rangle=\epsilon^2\lambda^2\langle N\rangle^2\alpha^2/2\omega a\tau\Delta\nu. \quad (15)$$

If τ is the response time of the electronic system, the electronic frequency bandwidth becomes[12] $\Delta f=1/2\tau$, so that the correlation measured by the electronic equipment becomes

$$\langle C\rangle = MF_1F_2\langle\Delta V_1\Delta V_2\rangle = \beta M\alpha^2\langle N\rangle^2\epsilon^2(F_1F_2)\Delta f(\Delta\nu)^{-1}\lambda^2/\omega a. \quad (16)$$

Here F_1 and F_2 are the gains of the two amplifier channels, and M is a multiplying factor introduced by the electronic multiplier and integrator stages; β is a factor that generalizes the expression for arbitrary polarization; $\beta=1$ for unpolarized light as in Eq. (15), but it becomes 2 for completely polarized light; α is the detector response.

Equation (16) is equivalent to the expressions obtained by Brown and Twiss[2] in the limiting case when $a\omega\gg\lambda^2$. Brown and Twiss replace $\lambda^2/a\omega$ by two Fresnel integrals involving the dimensions of the source and detectors. One of these integrals is a function of $\lambda R/yY$, where y and Y are the detector height and source length (Fig. 3, top). The other integral depends upon $\lambda R/Xx$ and $\lambda R/Xd$, where x and X are the detector and source widths, d is the center-to-center separation of the two detectors as seen from the source, and R is the source-to-detector distance, which is assumed to be large compared to X, x, Y, y, and d. If $\Psi=\pi yY/\lambda R$, and $\Phi=\pi xX/\lambda R$, then the product of the integrals is

[11] F. D. Kahn, Optica Acta (Paris) **5**, 93 (1958).
[12] R. A. Smith, F. E. Jones, and R. P. Chasmar, *The Detection and Measurement of Infrared Radiation* (Clarendon Press, Oxford, England, 1957), p. 244.

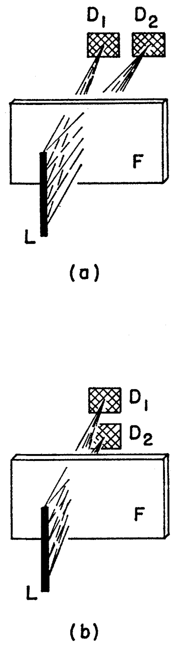

Fig. 3. Light from a long thin filament L passes through an infrared transmitting filter F and is incident on two detectors D_1 and D_2. The detectors can be rotated so that the line joining their centers is either perpendicular or parallel to the length of the filament.

$$\frac{1}{\Psi^2\Phi^2}\int_0^{\Phi}\frac{2\sin^2\phi}{\phi^2}(\Phi-\phi)\cos\left(\frac{2\phi d}{x}\right)d\phi \times\int_0^{\Psi}\frac{2\sin^2\Psi}{\psi^2}(\Psi-\psi)d\psi. \quad (17)$$

5. APPARATUS

Equation (2) states that

$$\langle(\Delta E)^2\rangle = h\nu\langle E\rangle\left[1+\frac{1}{\exp(h\nu/kT)-1}\right], \quad (2)$$

where the first term in the brackets represents the photon shot noise and the second term is the classical fluctuation predicted by electromagnetic wave theory. Both these terms and also the classical to shot noise ratio are monotonically increasing functions of temperature so that the classical fluctuation should be most easily observed at high temperatures. In order to avoid possible source oscillations, arcs and discharges were not used even though they provide very high temperatures. Instead, an incandescent tungsten filament lamp was used. The highest temperatures that could be maintained for several hours were only 3000°K, but the optical properties of tungsten are well established, and the geometry of the straight tungsten filament used permitted an evaluation of the Fresnel integrals in Eq. (17).

In order to determine the most favorable spectral region for an intensity interferometer operating at 3000°K, Fig. 4 has been drawn. It shows the relative values of the classical energy fluctuation per spectral frequency interval, per detector area within the diffraction pattern whose area $a\approx\lambda^2/\omega$, and per time constant τ plotted as a function of wavelength. Figure 5 shows the ratio of classical to shot noise fluctuations plotted as a function of $h\nu/kT$. In the absence of detector and electronic noise, this would represent the signal-to-noise ratio of the fluctuation measurements. On the basis of this curve one would expect to measure the classical fluctuation most conveniently in the radio region. However, this signal-to-noise ratio is not a pertinent quantity if the detector noise is very large, for then the significant expressions are signal power (Fig. 4) and noise equivalent power of the detector. On the basis of these criteria, PbS detectors operating near 3 μ in the infrared were selected for the experiment. Recent improvements in InSb detectors would now make them much more suitable.

In most of the experiments a straight-filament GE 1872 bulb illuminated two Infrared Industries PbS detectors. The spectral bandwidth was defined either by a germanium filter transmission coated for maximum transmission at 2.5 μ or by an interference filter designed to pass only a narrow frequency spike at about 2.65 μ. Two detector arrangements were used:

1. The two detectors were mounted as shown in Fig. 1. A half-silvered mirror transmitted and reflected beams of roughly identical intensity to the two detectors D_1 and D_2.

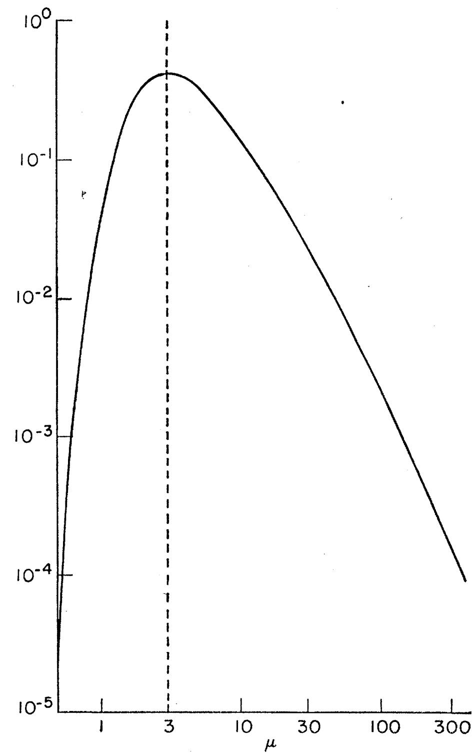

Fig. 4. The classical energy fluctuation per unit spectral frequency, per detector area within the diffraction pattern and per system time constant, drawn for a source at 3000°K. The units on the ordinate are arbitrary.

TABLE I. Results of correlation measurements.[a]

Run	Integration time[b] (min)	Optical system	Filter	Comparison run	Expected correlation (v^2 sec)	Measured correlation (v^2 sec)	Measured S/N
1	20	1	Ge	unsuperposed detectors	100	44 ± 107	0.41
2	20	1	Ge	unsuperposed detectors	100	$137\pm\ 81$	1.68
3	20	1	Ge	unsuperposed detectors	100	$235\pm\ 99$	2.37
4	10	1	Ge	$1\ \mu$ bandpass filter	35	$-4\pm\ 67$	-0.05
5	10	1	Ge	$1\ \mu$ bandpass filter	90	54 ± 140	0.38
6	10	1	Ge	diffuser	90	$68\pm\ 67$	0.63
7	240	2	$2.65\ \mu$	$90°$ rotation	160	220 ± 373	1.01
8	160	1	Ge	diffuser	250	201 ± 183	1.10
9	160	1	Ge	diffuser	250	508 ± 183	2.78
10	160	1	Ge	diffuser	250	190 ± 216	0.88
					Total 1425	1657	
11[c]	160	1	Ge	diffuser	0	-249 ± 204	-1.22

[a] The total measured correlator output signal-to-noise ratio is 3.9. There is a probability less than 10^{-4} that this signal-to-noise ratio or a higher one, could have occurred by chance.
[b] The short runs represent early measurements in which cooling was slow and the system could be run only on an intermittent basis. In later runs continuous operation was possible. The integration time may be somewhat misleading. The total time required to obtain all of the data was usually about ten hours for each of the reported runs.
[c] Run No. 11 offers an extra comparison with Runs 8 to 10. One of the detectors was moved 0.2 mm out of alignment so that no correlation was expected. Runs with the Ge filter again were compared with runs with the diffuser. (The somewhat high negative value obtained in Run 11 probably is not significant.)

2. No half-silvered mirror was used; the filtered radiation was directly incident on two very small detectors mounted side by side. (See Fig. 3.) The filament thickness, X, was about $80\ \mu$; its exposed length, Y, was about $500\ \mu$; the height of the detectors, y, was $12\ \mu$; their width, x, was $50\ \mu$; and the separation, d, between detector centers was $60\ \mu$; λ was about $2.5\ \mu$; and R was about 1 cm. Under these conditions the products Xx, Yy, and Xd were much less than λR, and expression (17) approached unity. Hence, with the detectors arranged as in Fig. 3(a) the factor $(\lambda^2/\omega a)$ in Eq. (16) is replaced by a quantity (17), whose value is close to unity. However, if the detectors are arranged as in Fig. 3(b), a greatly reduced correlation is expected, because although the product Xy is extremely small, Yx and Yd are larger than λR. At the same time, the roles of X and Y and of Φ and Ψ in expression (17) are interchanged in this detector configuration, so that the argument of the cosine function in the first integral can become large. The value of this integral drops off very rapidly as a function of Yx and Yd. Hence the entire expression (17) approaches zero, and the expected correlation in Fig. 3(b) is much less than in Fig. 3(a).

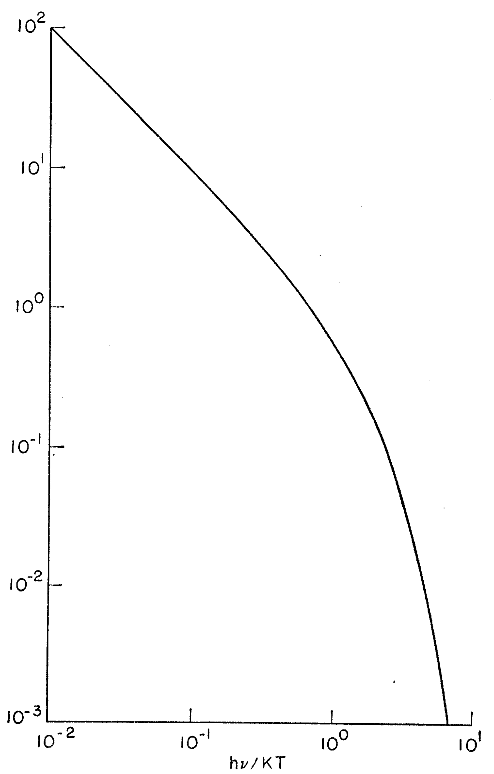

FIG. 5. Ratio of classical to shot noise fluctuation plotted as a function of $h\nu/kT$.

The entire optical system was cooled to minimize radiation from the optical components and the optical housing.

Low-noise Tektronix 122 preamplifiers formed the first stage of amplification in each of the two amplifier channels. Further gain was obtained with a modified Fisher PR-66 two-channel amplifier designed for stereophonic reproduction. These two stages were completely battery-powered, since ripple, even from a regulated power supply, would contribute prohibitively to the correlator signal. The combined gain of the two stages was about 10^5, and the output signal was of the order of one or two volts. The signal from each channel was further amplified in two stages of Philbrick K2-W operational amplifiers,[13] which gave distortion-free gain of 20 to 30. Finally, the signal was multiplied in a Philbrick model HM multiplier before being integrated in a modified K2-W operational amplifier stage.

Elaborate shielding of the detectors and electronic

[13] K. Eklund, Rev. Sci. Instr. 30, 328, 331 (1959).

filtering were necessary to minimize 60-cps line frequency pickup and other unwanted signals that might contribute to the measured correlation. To eliminate the effects of systematic correlator drifts or other spurious correlation signals, a series of comparison conditions was established. For example, when the detector bias on one of the detectors is reversed, the correlator output signal should be negative rather than positive; when the detector images are superposed (Fig. 1), the observed correlation should be much greater than when they are slightly apart; when the detector assembly in Fig. 3(a) is rotated by 90° [Fig. 3(b)] the measured correlation should vanish; with a long-wavelength filter, between source and detector, more correlation is expected than with a short-wavelength filter; if a diffuser is inserted between source and detector, coherence is lost and the observed correlation is expected to vanish. Differences in measurements obtained under these comparable conditions are summarized in Table I.

6. RESULTS OF THE EXPERIMENT

A number of correlation runs in which the integration time t varied from 10 min to 240 min were made, with a variety of different optical systems. In a typical set of runs the expected correlation was determined by using Eqs. (16) and (17):

$$\mathcal{S}=\beta M[V_1V_2\gamma_1\gamma_2\delta_1\delta_2F_1F_2]\Delta f\Delta\nu^{-1}\Gamma t, \tag{18}$$

where the quantity $V\gamma$ replaces the product $\epsilon\alpha n$ and represents the detector response in volts per incident photon, multiplied by the average number of photons incident on a detector during response time τ; $\beta=1.04$ is a measure of the source polarization, $M=0.4$ is a correlator parameter, $V_1V_2=0.422$ v^2 is the product of the detector bias voltages, $\gamma_1\gamma_2=0.015$ is the product of the detector response terms, $\delta_1\delta_2=0.18$ is the product of impedance matching terms at the amplifier inputs, $F_1F_2\Delta f=2\times10^{14}$ sec^{-1} is the integrated product of amplifier gain, $\Delta\nu=9\times10^{12}$ sec^{-1} is a measure of the spectral radiation bandwidth, $\Gamma=0.89$ is a coherence factor which lies between zero and unity, and $t=14\,400$ sec is the total integration time.

Substituting these values in Eq. (18) we obtain $\mathcal{S}=160\pm80$ v^2 sec. The measured value was 220 ± 373 v^2 sec. The possible error in the predicted value is largely due to the uncertainty in the detector time constant, and the uncertainty in the measured value is given by the scatter of the correlation integral sampled at 480 points during the integration.

Ten experimental runs of this type were performed. (See Table I.) Within the experimental error, they are in agreement with the predicted values. However the predicted values themselves are uncertain by approximately 50% because the lead sulfide detector characteristics could not be determined more accurately under the experimental conditions.

The signal-to-noise ratio of all the integrations taken together was 3.9. The probability of obtaining such a high signal-to-noise ratio accidentally is less than 10^{-4}. Furthermore, the large number of internal checks and comparison measurements described in Sec. 4 were designed to rule out spurious correlations caused by some peculiarity of any one particular optical arrangement.

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