The Jacobian of the transformation is found from the following identity:

$$\cos\Theta\,\sin\Theta\left(\frac{\partial\Theta}{\partial\theta}\frac{\partial\Phi}{\partial\varphi}-\frac{\partial\Theta}{\partial\varphi}\frac{\partial\Phi}{\partial\theta}\right)$$

$$\equiv\left(\frac{\partial}{\partial\theta}\sin\Theta\,\cos\Phi\right)\left(\frac{\partial}{\partial\varphi}\sin\Theta\,\sin\Phi\right)$$

$$-\left(\frac{\partial}{\partial\varphi}\sin\Theta\,\cos\Phi\right)\left(\frac{\partial}{\partial\theta}\sin\Theta\,\sin\Phi\right). \quad (28)$$

Equation (28) is easily proved by carrying out the differentiations indicated on the right-hand side, collecting terms and simplifying.

Substituting Eqs. (25a) and (25b) into the right-hand side of (28) and dividing through by $V_p\sin\theta$ and $V_p\cos\Theta$ given by Eq. (25c), we get, after much simplification:

$$\frac{\sin\Theta}{\sin\theta}J\begin{pmatrix}\Theta, & \Phi\\ \theta, & \varphi\end{pmatrix}=\frac{u^2}{V_p[V_p^2+v_c^2(A^2-1)]^{\frac{1}{2}}}, \quad (29)$$

which is Eq. (6) of the text.

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Theoretical Study of the Electron Drift Velocity in Binary Gas Mixtures with Applications to A-CO_2 and A-N_2 Mixtures*

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Starting with the Boltzmann equation, a theoretical expression is developed for the electron drift velocity in a binary gas mixture. The theory reduces to that of Morse, Allis, and Lamar in the absence of one of the gases. The theory is applied to pure argon and to mixtures of A–0.1% CO_2, A–0.16% CO_2, A–1.0% CO_2, A–0.1% N_2, A–0.5% N_2, and A–1.0% N_2. The theoretical drift velocity curves for A–CO_2 are in close agreement with experimental data, whereas the A–N_2 curves differ from experimental data. Possible reasons for this discrepancy are discussed.

I. INTRODUCTION

SMALL amounts of nitrogen or of carbon dioxide, when added to argon, alter appreciably the value that the electron drift velocity has in pure argon. Experimental studies of A–CO_2 mixtures have been performed by English and Hanna[1] and by Errett.[2] Experimental studies of A-N_2 mixtures have been performed by Kirshner and Toffollo,[3] Colli and Facchini,[4] Engligh and Hanna,[1] and Errett.[2] "Pure" argon curves, which in reality may be A–N_2–? mixtures, have been published by Allen and Rossi,[5] Kelma and Allen,[6] and Hudson.[7] Pure argon has been studied experimentally by Nielsen,[8] Herreng,[9] Colli and Facchini,[4] Kirshner and Toffollo,[3] and Errett;[2] and theoretically by Allen[10] and Bowe.[11]

In this paper a theoretical expression for the electron drift velocity in a binary gas mixture is derived and applied to A–CO_2 and A–N_2 mixtures. In the limit of one gas the theory accurately predicts the drift velocity curve of pure argon. The agreement between theory and experiment for A–CO_2 mixtures is quite good, lending strong support to the theoretical approach. For A–N_2 mixtures the theory is not in good agreement with any of the published experimental data. There are, however, significant differences between the experimental data reported by different workers. It is postulated that either (1) the A–N_2 mixtures may contain

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different percentages of nitrogen than reported; (2) the A–N_2 mixtures may contain small amounts of additional impurities such as CO_2 or water vapor; (3) a combination of (1) and (2) may exist; or (4) the energy absorption characteristic for nitrogen used in the theoretical calculations is incorrect.

II. THEORY

The derivation to be presented in this paper is an extension of the type of solution first proposed by Morse, Allis, and Lamar.[12] The electron drift velocity will be calculated using the relation

$$\mathbf{v}=\int\int \mathbf{c}f(\mathbf{c},\mathbf{r})d\mathbf{c}d\mathbf{r}\Big/\int\int f(\mathbf{c},\mathbf{r})d\mathbf{c}d\mathbf{r}, \tag{1}$$

where $f(\mathbf{c},\mathbf{r})$ is the electron phase space distribution function. The distribution function for electrons in a binary gas mixture will be obtained from a solution of the Boltzmann equation,

$$\frac{\partial f(\mathbf{c},\mathbf{r})}{\partial t}+\nabla_r\cdot\{\mathbf{c}f(\mathbf{c},\mathbf{r})\} +\nabla_c\cdot\{q/m[\mathbf{E}+\mathbf{c}\times\mathbf{B}]f(\mathbf{c},\mathbf{r})\}=\left[\frac{\partial f(\mathbf{c},\mathbf{r})}{\partial t}\right]_{\text{collisions}}. \tag{2}$$

The following simplifying assumptions are made in solving the Boltzmann equation:

I. The system is in equilibrium under the action of a uniform dc electric field E in the x direction. The electron distribution function is not a function of time or of coordinates, only of velocity.

II. The gas molecules are at rest.

III. Only electron-molecule collisions are considered; electron-electron collisions are ignored.

IV. The distribution function is expanded in powers of (c_x/c). Only the first term in the expansion is retained:

$$f(c,c_x)=f_0(c)+[c_x/c][f_1(c)].$$

When assumption I is used, Eq. (2) reduces to

$$\frac{qE}{m}\frac{\partial f}{\partial c_x}=\left[\frac{\partial f}{\partial t}\right]_{\text{collisions}}. \tag{3}$$

Substitution of assumption IV in (3) yields

$$\frac{qE}{m}\left[\frac{c_x}{c}\frac{\partial f_0}{\partial c}+\frac{c_x^2}{c}\frac{\partial}{\partial c}\left(\frac{f_1}{c}\right)+\frac{f_1}{c}\right]=\left[\frac{\partial f}{\partial t}\right]_{\text{coll}}. \tag{4}$$

The term c_x^2 in Eq. (4) can be replaced by its spherical average $c^2/3$ since harmonics in (c_x/c) greater than first order are not being considered.

The right-hand side of Eq. (4) when multiplied by a small volume in phase space, $d\mathbf{c}d\mathbf{r}$, yields the increase per unit time of the number of electrons in the phase space volume $d\mathbf{c}d\mathbf{r}$ caused by scattering encounters with the gas molecules. The increase per unit time in the number of electrons in $d\mathbf{c}d\mathbf{r}$ is equal to the electrons scattered into minus the electrons scattered out of $d\mathbf{c}d\mathbf{r}$ per unit time:

$$[\partial f/\partial t]_{\text{coll}}d\mathbf{c}d\mathbf{r}=[\alpha-\beta]d\mathbf{c}d\mathbf{r}. \tag{5}$$

The number of electrons scattered out of $d\mathbf{c}d\mathbf{r}$ per unit time is the usual function of the densities and cross sections of the two types of gas molecules:

$$\beta d\mathbf{c}d\mathbf{r}=cf(c,c_x)[N_1\sigma_1+N_2\sigma_2]d\mathbf{c}d\mathbf{r}, \tag{6}$$

where

$$\sigma_1(c)=\int\int I_1(c,\theta)\sin\theta d\theta d\phi,$$

$$\sigma_2(c)=\int\int I_2(c,\theta)\sin\theta d\theta d\phi,$$

and I is the differential cross section.

The calculation of α is more complex. To be scattered into a volume element in velocity space represented by speed c, an electron must have made a collision while having a speed c', where $c'>c$, since an electron loses energy and speed in colliding with a stationary gas molecule. For an elastic collision with, say, gas 1 the relation between electron speed before and after collision is

$$c_1'=c_1[1+(1-\cos\theta)m/M_1], \tag{7}$$

where $(m/M_1)^2$ is ignored and M_1 is the mass of the gas molecule of type 1. The loss in speed Δc_1 is

$$\Delta c_1=c_1'-c_1=c_1(m/M_1)(1-\cos\theta). \tag{8}$$

The loss of speed in an electron collision with a gas molecule of type 2 can be written as

$$\Delta c_2=c_2(mb/M_1)(1-\cos\theta), \tag{9}$$

where the term b will be designated the energy absorption characteristic. If the collisions with gas 2 are elastic, b is equal to M_1/M_2. If the collisions with gas 2 are inelastic, b is to be determined empirically from energy absorption data using Eq. (9) as a definition. Since $d\mathbf{c}$ and $d\mathbf{c}'$ are small volumes in velocity space, one finds, using Eq. (7), that

$$(d\mathbf{c}/d\mathbf{c}_1')=(c/c_1')^3;\quad (d\mathbf{c}/d\mathbf{c}_2')=(c/c_2')^3. \tag{10}$$

The number of electrons scattered per unit time into $d\mathbf{c}d\mathbf{r}$ is

$$\begin{aligned}\alpha d\mathbf{c}d\mathbf{r} &=N_1c_1'\int_\theta\int_\phi I_1(c_1',\theta)f(c_1',c_{1x}')\sin\theta d\theta d\phi d\mathbf{c}_1'd\mathbf{r}\\ &\quad+N_2c_2'\int_\theta\int_\phi I_2(c_2',\theta)f(c_2',c_{2x}')\sin\theta d\theta d\varphi d\mathbf{c}_2'd\mathbf{r},\end{aligned} \tag{11}$$

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[12] P. M. Morse, W. P. Allis, and E. S. Lamar, Phys. Rev. **48**, 412 (1935).

or, using Eq. (10),

$$\alpha=N_1\left(\frac{c_1'^4}{c^3}\right)\int_\theta\int_\phi I_1(c_1',\theta)f(c_1',c_{1x}')\sin\theta d\theta d\phi$$

$$+N_2\left(\frac{c_2'^4}{c^3}\right)\int_\theta\int_\phi I_2(c_2',\theta)f(c_2',c_{2x}')\sin\theta d\theta d\phi. \quad (12)$$

After substitution of Eqs. (6) and (12) into Eq. (5), and division by $d\mathbf{c}d\mathbf{r}$, the collision term becomes

$$\left[\frac{\partial f}{\partial t}\right]_{\text{coll}}$$

$$=N_1\left(\frac{c_1'^4}{c^3}\right)\int\int I_1(c_1',\theta)f(c_1',c_{1x}')\sin\theta d\theta d\phi$$

$$+N_2\left(\frac{c_2'^4}{c^3}\right)\int\int I_2(c_2',\theta)f(c_2',c_{1x}')\sin\theta d\theta d\phi$$

$$-N_1\left(\frac{c^4}{c^3}\right)\int\int I_1(c,\theta)f(c,c_x)\sin\theta d\theta d\phi$$

$$-N_2\left(\frac{c^4}{c^3}\right)\int\int I_2(c,\theta)f(c,c_x)\sin\theta d\theta d\phi. \quad (13)$$

The additional assumption is now made that arbitrary functions of c_1' and of c_2' can be expanded as follows:

$$\begin{aligned}F(c_1')&=F(c)+[\partial F(c)/\partial c]\Delta c_1,\\ F(c_2')&=F(c)+(\partial F(c)/\partial c]\Delta c_2,\end{aligned} \quad (14)$$

where Δc_1 and Δc_2 are defined by Eqs. (8) and (9), respectively. After substitution of Eqs. (8), (9), and (14) into Eq. (13) with

$$F(c')=c'^4I(c',\theta)f(c',c_x'),$$

the collision term becomes

$$\left[\frac{\partial f}{\partial t}\right]_{\text{coll}}$$

$$=\frac{N_1}{c^3}\int\int\Big[c^4\{f(c,c_{1x}')-f(c,c_x)\}I_1(c,\theta)$$

$$+\frac{m}{M}(1-\cos\theta)c\frac{\partial}{\partial c}\{c^4f(c,c_{1x}')I_1(c,\theta)\}\Big]\sin\theta d\theta d\phi$$

$$+\frac{N_2}{c^3}\int\int\Big[c^4\{f(c,c_{2x}')-f(c,c_x)\}I_2(c,\theta)$$

$$+\frac{mb}{M}(1-\cos\theta)c\frac{\partial}{\partial c}\{c^4f(c,c_{2x}')I_2(c,\theta)\}\Big]\sin\theta d\theta d\phi, \quad (15)$$

where M_1 is now designated as M. If the function f in Eq. (15) is replaced by its expansion as given in assumption IV and the terms in

$$\frac{m}{M}\frac{\partial}{\partial c}\left(c^4\frac{c_x}{c}f_1I\right)$$

are neglected in comparison with the terms in

$$\frac{m}{M}\frac{\partial}{\partial c}(c^4f_0I),$$

Eq. (15) becomes

$$\left[\frac{\partial f}{\partial t}\right]_{\text{coll}}$$

$$=\frac{N_1}{c^3}\int\int\Big[\frac{c^4}{c}\{c_{1x}'-c_x\}f_1(c)I_1(c,\theta)$$

$$+\frac{m}{M}(1-\cos\theta)c\frac{\partial}{\partial c}\{c^4f_0(c)I_1(c,\theta)\}\Big]\sin\theta d\theta d\phi$$

$$+\frac{N_2}{c^3}\int\int\Big[\frac{c^4}{c}\{c_{2x}'-c_x\}f_1(c)I_2(c,\theta)$$

$$+\frac{mb}{M}(1-\cos\theta)c\frac{\partial}{\partial c}\{c^4f_0(c)I_2(c,\theta)\}\Big]\sin\theta d\theta d\phi. \quad (16)$$

Define

$$\cos\omega=c_x/c. \quad (17)$$

From the geometry of the collision, as shown in Fig. 1, one obtains

$$\cos\omega'=\cos\omega\cos\theta+\sin\omega\sin\theta\cos(\phi-\chi),$$

or

$$(\cos\omega'-\cos\omega)=\cos\omega(\cos\theta-1)+\sin\omega\sin\theta\cos(\phi-\chi). \quad (18)$$

After substitution of (18) into the first and third terms of Eq. (16) and interchange of the order of differentiation and integration in the second and fourth terms of Eq. (16) (noting that the integral over $\cos\phi$ vanishes), Eq. (16) becomes

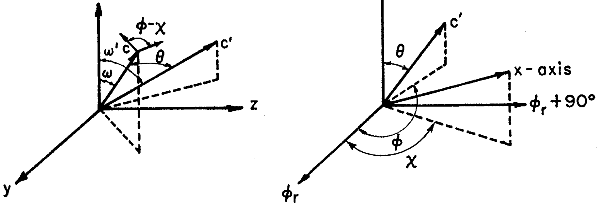

FIG. 1. The geometry of the electron-molecule collision. ϕ_r is the reference axis for the measurement of ϕ.

$$\left[\frac{\partial f}{\partial t}\right]_{\text{coll}} = -N_1\sigma_{1D}cf_1\cos\omega - N_2\sigma_{2D}cf_1\cos\omega + \frac{m}{M}\frac{N_1}{c^2}\frac{\partial}{\partial c}(c^4\sigma_{1D}f_0) + \frac{mb}{M}\frac{N_2}{c^2}\frac{\partial}{\partial c}(c^4\sigma_{2D}f_0), \tag{19}$$

where "diffusion" cross section σ_D is defined as

$$\sigma_D = \int\int I(1-\cos\theta)\sin\theta d\theta d\phi.$$

After substitution of Eq. (19) into Eq. (4), with $c_x^2 = c^2/3$, the Boltzmann equation becomes

$$\frac{qE}{m}\left[\frac{c_x}{c}\frac{\partial f_0}{\partial c} + \frac{c}{3}\frac{\partial}{\partial c}\left(\frac{f_1}{c}\right) + \frac{f_1}{c}\right] = -N_1\sigma_{1D}cf_1\frac{c_x}{c} - N_2\sigma_{2D}cf_1\frac{c_x}{c} + \frac{m}{M}\frac{N_1}{c^2}\frac{\partial}{\partial c}(c^4\sigma_{1D}f_0) + \frac{mb}{M}\frac{N_2}{c^2}\frac{\partial}{\partial c}(c^4\sigma_{2D}f_0). \tag{20}$$

When the terms with and without c_x/c are equated separately, Eq. (20) reduces to two equations:

$$\frac{qE}{m}\frac{\partial f_0}{\partial c} = -N_1\sigma_{1D}cf_1 - N_2\sigma_{2D}cf_1, \tag{21}$$

$$\frac{qE}{3m}\frac{\partial}{\partial c}(c^2f_1) = \frac{mN_1}{M}\frac{\partial}{\partial c}(c^4\sigma_{10}f_0) + \frac{mbN_2}{M}\frac{\partial}{\partial c}(c^4\sigma_{2D}f_0). \tag{22}$$

Equations (21) and (22) are essentially momentum and energy balance equations, respectively. That Eq. (22) represents an energy balance can be seen more easily after it is integrated with respect to speed:

$$\frac{qE}{3}f_1 = \frac{2m}{M}N_1\sigma_{1D}f_0\epsilon + \frac{2mb}{M}N_2\sigma_{2D}f_0\epsilon + \frac{mK}{c^2}, \tag{23}$$

where $\epsilon = \frac{1}{2}mc^2$ and K is the constant of integration. The left-hand side of (23) is related to the energy gained from the field. The function f_1 is a measure of the number of electrons whose motion is influenced by the electric field and, hence, of the number of electrons which gain energy from the electric field. The first two terms on the right-hand side of (23) are related to the energy lost in collision. The constant $2m/M$ is the average fractional electron energy loss per collision, $N\sigma$ is essentially the collision probability, and f_0 is a measure, to first approximation, of the number of electrons available for collisions. The third term on the right-hand side of (23) is related to the energy extracted from the electrons by means other than collisions. In equilibrium, therefore, $K=0$. If Eqs. (23) and (21) are written in energy coordinates with $K=0$, one obtains

$$\frac{qE}{3}f_1 = \frac{2m}{M}\epsilon f_0[N_1\sigma_{1D} + bN_2\sigma_{2D}], \tag{24}$$

$$\frac{\partial f_0}{\partial\epsilon} = -\frac{f_1}{qE}[N_1\sigma_{1D} + N_2\sigma_{2D}]. \tag{25}$$

Equations (24) and (25) reduce to those of Morse, Allis, and Lamar[12] if $N_2=0$.

To solve Eqs. (24) and (25) for f_0 and f_1, the form of σ_{1D}, σ_{2D}, and b must be inserted into the equations. The cross sections σ_{1D}, σ_{2D}, and the energy absorption characteristic b as functions of energy are each represented by a series of seven straight-line approximations:

$$\begin{aligned}\sigma_{1D} &= a\epsilon + s,\\ \sigma_{2D} &= d\epsilon + z,\\ b &= g\epsilon + h,\end{aligned} \tag{26}$$

where a, s, d, z, g, and h each have seven values. After putting (26) into Eqs. (24) and (25) and solving for f_0 and f_1, one obtains

$$f_0 = G\exp\left[-\frac{6m}{M}\left(\frac{N_1}{E}\right)^2(A\epsilon^5 + B\epsilon^4 + C\epsilon^3 + D\epsilon^2)\right], \tag{27}$$

$$f_1 = \frac{6m}{M}\left(\frac{N_1}{E}\right)[H\epsilon^3 + L\epsilon^2 + J\epsilon]f_0, \tag{28}$$

where

$$A = \frac{1}{5}\left(\frac{N_2}{N_1}gd\right)\left(a + \frac{N_2}{N_1}d\right),$$

$$B = \frac{1}{4}\left[\left(a + \frac{N_2}{N_1}d\right)\left(a + \frac{N_2}{N_1}dh + \frac{N_2}{N_1}gz\right) + \left(\frac{N_2}{N_1}gd\right)\left(s + \frac{N_2}{N_1}z\right)\right],$$

$$C = \frac{1}{3}\left[\left(a + \frac{N_2}{N_1}d\right)\left(s + \frac{N_2}{N_1}hz\right) + \left(s + \frac{N_2}{N_1}z\right)\left(a + \frac{N_2}{N_1}dh + \frac{N_2}{N_1}gz\right)\right],$$

$$D = \frac{1}{2}\left(s + \frac{N_2}{N_1}z\right)\left(s + \frac{N_2}{N_1}hz\right),$$

$$H = \frac{N_2}{N_1}gd,$$

$$L = \left(a + \frac{N_2}{N_1}dh + \frac{N_2}{N_1}gz\right),$$

$$J = \left(s + \frac{N_2}{N_1}hz\right).$$

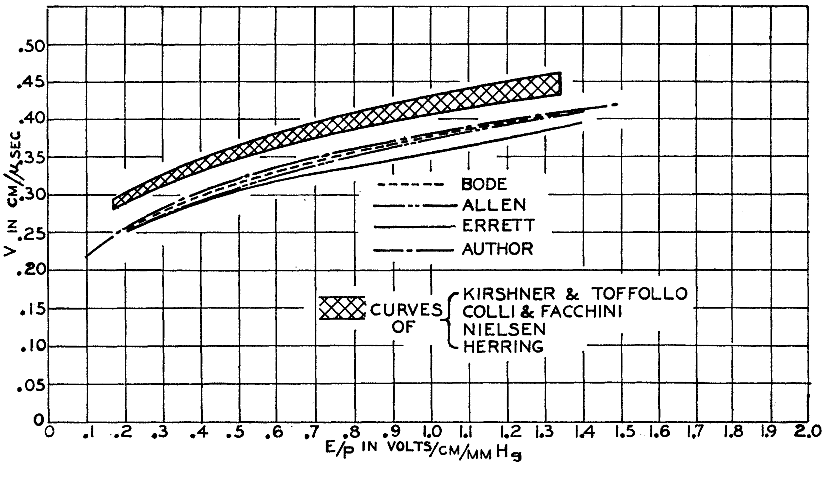

FIG. 2. Drift velocity vs E/p in argon.

The constants G are determined by the total number of electrons in the system and by the continuity of f, or, what is approximately equivalent, by the continuity of f_0. The constants A, B, C, D, H, L, J, and G each have seven values. In the above expressions all quantities are in the mks system except ϵ which is expressed in electron volts.

The drift velocity expression, Eq. (1), can be written in terms of the electron energy. Since f_0 and f_1 are functions only of speed and therefore are even functions of velocity, Approximation (IV) can be substituted into Eq. (1) yielding

$$\mathbf{v}=\int c_x \frac{c_x}{c} f_1 d\mathbf{c} \Big/ \int f_0 d\mathbf{c}.$$

Since $c_x^2=c^2/3$ and $d\mathbf{c}\rightarrow 4\pi c^2 dc$ (for an isotropic integrand), the drift velocity becomes

$$\mathrm{v}=\int_0^\infty \frac{c^3}{3} f_1 dc \Big/ \int_0^\infty c^2 f_0 dc. \tag{29}$$

When speed coordinates are transformed to energy coordinates, Eq. (29) becomes

$$\mathrm{v}=\frac{1}{3}\left(\frac{2}{m}\right)^{\frac{1}{2}} \int_0^\infty \epsilon f_1 d\epsilon \Big/ \int_0^\infty \epsilon^{\frac{1}{2}} f_0 d\epsilon. \tag{30}$$

Equation (30) is the expression from which the drift velocity is calculated. The actual computation of the integrals in Eq. (30) was done on an IBM 650 digital computer.

III. DISCUSSION OF APPROXIMATIONS

Three mathematical approximations of questionable validity have been made:

A. The termination of the expansion for f (approximation IV) at two terms implies that

$$(c_x/c)f_1 \ll f_0.$$

B. The termination of the expansions of Eq. (14) at two terms implies that

$$\frac{1}{2}\frac{\partial^2 F}{\partial c^2}(\Delta c_1)^2 \ll \begin{cases} \dfrac{\partial F}{\partial c}\Delta c_1 & \text{(31a)} \\ \dfrac{\partial F}{\partial c}\Delta c_2 & \text{(31b)} \end{cases},$$

$$\frac{1}{2}\frac{\partial^2 F}{\partial c^2}(\Delta c_2)^2 \ll \begin{cases} \dfrac{\partial F}{\partial c}\Delta c_1 & \text{(31c)} \\ \dfrac{\partial F}{\partial c}\Delta c_2. & \text{(31d)} \end{cases}$$

C. The approximation used in deriving Eq. (16)

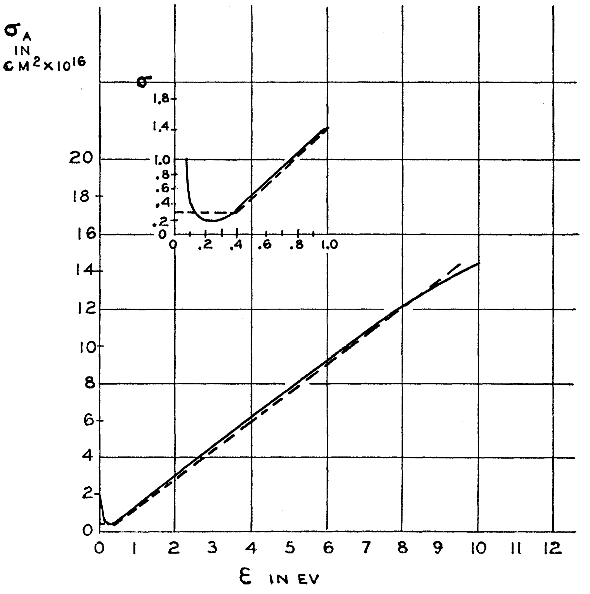

FIG. 3. Argon cross section. Dashed lines represent the approximation used.

TABLE I. Approximations to the argon and to the carbon dioxide cross sections and to the carbon dioxide energy absorption characteristic, where $\sigma_A = a\epsilon + s$, $\sigma_{CD} = d\epsilon + z$, $b_{CD} = g\epsilon + h$, and the values for a, s, d, z, g, and h are expressed in square angstrom units.

	0.0–0.25 ev	0.25–0.4 ev	0.4–1.0 ev	1.0–2.4 ev	2.4–4.0 ev	4.0–5.75 ev	5.75– ev
a	0.00	0.00	1.61	1.61	1.61	1.61	1.61
s	0.30	0.30	−0.344	−0.344	−0.344	−0.344	−0.344
d	−14.70	−14.70	−14.70	0.00	5.44	−3.71	0.827
z	21.00	21.00	21.00	6.30	−6.76	29.83	3.75
g	8000.00	0.00	−450.00	−153.30	−153.30	−153.30	−153.30
h	400.00	2400.00	2580.00	2267.99	2267.99	2267.99	2267.99

from Eq. (15) necessitates that

$$\frac{\partial}{\partial c}[c^4 f_0 I] \gg \frac{\partial}{\partial c}\left[c^4 \frac{c_x}{c} f_1 I\right]. \tag{32}$$

The restrictions which the above approximations may impose will now be examined:

A. Equation (28) yields

$$f_1/f_0 = \frac{6m}{M}\left(\frac{N_1}{E}\right)[H\epsilon^3 + L\epsilon^2 + J\epsilon]. \tag{33}$$

For all data used in this paper, f_1/f_0 is less than 0.2. For the cases of 0.1% N_2 or CO_2, f_1/f_0 is always less than 0.1. If c_x/c is assumed to be of order unity, f_0 is always significantly greater than $(c_x/c)f_1$ and this approximation is substantially valid.

B. The most stringent condition in this set of four conditions is (31c),

$$\frac{1}{2}\frac{\partial^2 F}{\partial c^2}(\Delta c_2)^2 \ll \frac{\partial F}{\partial c}\Delta c_1.$$

After substitution of $F(c)$, Δc_1, Δc_2 into (31c), inequality (31c) becomes

$$\left\{\frac{1}{2}\frac{\partial^2}{\partial c^2}[c^4 f I_2]\right\}\left\{\left(\frac{mb}{M}\right)^2(1-\cos\theta)^2\right\} \ll \left\{\frac{\partial}{\partial c}[c^4 f I_1]\right\}\left\{\frac{m}{M}(1-\cos\theta)\right\}. \tag{34}$$

To assure that (34) is satisfied, it is necessary that

$$m/M \gg (mb/M)^2,$$

or

$$b \ll (M/m)^{\frac{1}{2}}. \tag{35}$$

Condition (35) is violated for both cases examined in this paper. The agreement between theory and experimental data indicates that either (34) can be satisfied without satisfying (35) or that the restriction of Eq. (34) is not important to the theoretical derivation.

C. If Eq. (28) is substituted into the inequality (32), and speed terms are transformed into energy terms, inequality (32) becomes approximately

$$\frac{\partial}{\partial \epsilon}[\epsilon^2 f_0 I] \gg \left\{\frac{\partial}{\partial \epsilon}([\epsilon^2 f_0 I][H\epsilon^3 + L\epsilon^2 + J\epsilon])\right\}\left\{\frac{6m}{M}\frac{N_1}{E}\right\}. \tag{36}$$

For all data used in this paper, the inequality of (36) appears to be valid. As the N_2 or CO_2 content of argon increases past 1.0%, the inequality approaches an equality.

IV. RESULTS AND DISCUSSION

A. Pure Argon

For pure argon, Eqs. (27) and (28) reduce to the equations of Morse, Allis, and Lamar[12] for the electron distribution function in a single gas. The drift velocity curve for pure argon shown in Fig. 2 was computed from the value for the argon diffusion cross section listed in Table I or Table II and shown in Fig. 3. The straight-line approximations in Table I and Table II to the argon diffusion cross section are based on the data of

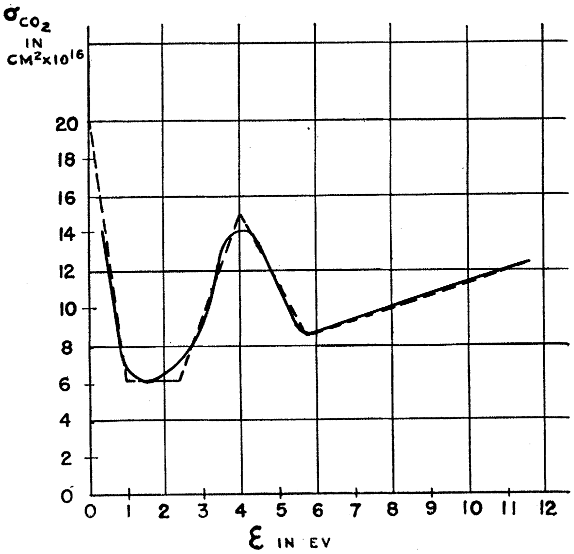

FIG. 4. Carbon dioxide cross section. Dashed lines represent the approximation used.

TABLE II. Approximations to the argon and to the nitrogen cross sections and to the nitrogen energy absorption characteristic, where $\sigma_A = a\epsilon + s$, $\sigma_N = d\epsilon + z$, $b_N = g\epsilon + h$, and the values for a, s, d, z, g, and h are expressed in square angstrom units.

	0.0–0.4 ev	0.4–0.8 ev	0.8–1.3 ev	1.3–1.7 ev	1.7–2.4 ev	2.4–3.4 ev	3.4– ev
a	00.00	1.61	1.61	1.61	1.61	1.61	1.61
s	0.30	−0.344	−0.344	−0.344	−0.344	−0.344	−0.344
d	−12.00	−12.00	0.00	15.18	15.18	−15.30	0.00
z	17.50	17.50	7.90	−11.83	−11.83	61.32	9.30
g	0.00	0.00	1292.30	1292.30	0.00	0.00	0.00
h	36.90	36.90	−997.90	−997.90	1200.00	1200.00	1200.00

Barbiere[13] and of Kivel.[14] Barbiere has computed the diffusion cross section of argon using the angular scattering data of Ramsauer and Kollath.[15] Kivel has indicated a value for the Ramsauer-effect minimum. It should be noted that the cross-section approximations are inaccurate for energies above 12 electron volts and below 0.1 or 0.2 electron volt (actual values are not accurately known in this range). No appreciable errors are introduced in the computations since a negligible number of electrons occupies these energies for the range of E/p studied.

The argon drift velocity curve of Fig. 2 is in excellent agreement with the theoretical curves of Allen[10] and Bowe.[11] The authors' theoretical curve also approximates the experimental drift velocity curve of Errett.[2] As is shown in Fig. 2 the experimental plots of Colli and Facchini,[4] Kirshner and Toffollo,[3] Herreng,[9] and Nielsen[8] are quite close to each other and are slightly higher than the curves of Allen,[10] Bowe,[11] and the authors. Possible reasons for this discrepancy are discussed in the A-N_2 mixtures section.

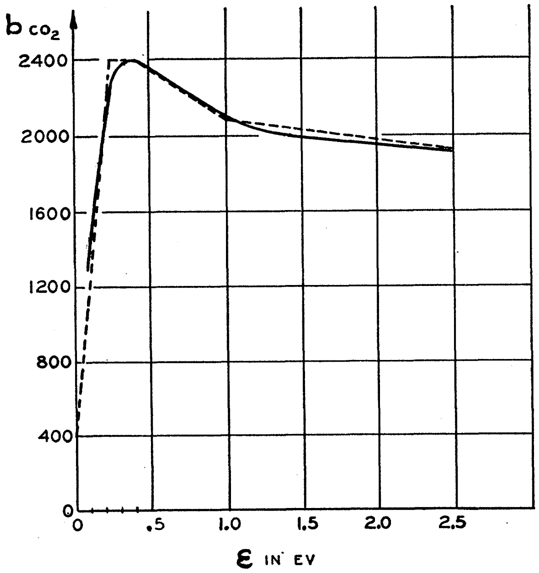

FIG. 5. Carbon dioxide energy absorption characteristic. Dashed lines represent the approximation used.

B. Argon-Carbon Dioxide Mixtures

Mixtures of A–0.1% CO_2, A–0.16% CO_2, and A–1.0% CO_2 have been investigated theoretically. The straight-line approximations to the CO_2 cross section and to the energy absorption characteristic listed in Table I and shown in Figs. 4 and 5 are based on the data of Brode[16] and of Healy and Reed,[17] respectively. The theoretical A–CO_2 drift velocity curves of Fig. 6 are in close agreement with the experimental data of English and Hanna[1] and of Errett.[2]

It should be noted that condition (35) has been violated in the case of CO_2. The correspondence between theoretical and experimental data lends strong confirmation to the theory.

C. Argon-Nitrogen Mixtures

Mixtures of argon plus 0.1%, 0.5%, 1.0% N_2 have been investigated theoretically. The straight-line approximations to the nitrogen cross section and to the energy absorption characteristic listed in Table II and plotted in Figs. 7 and 8 are based on the data of Normand[18] and of Errett,[2] respectively. The authors' A–N_2 drift velocity curves and the experimental curves of Kirshner and Toffollo,[3] Colli and Facchini,[4] and Errett[2] are shown in Fig. 9.

The disparity between the various experimental curves probably indicates an inadequate knowledge of the amount of nitrogen and of other impurities present in the argon. While the experimentalist may be careful to purify the argon of any impurity nitrogen before adding controlled amounts of nitgogen to the argon, he has possibly not paid enough attention to the minute quantities of residual CO_2 and water vapor which may be present in the argon and in the nitrogen. This possibility is also suggested in comparing the theoretical and experimental curves. The authors' theoretical data shows no pronounced drift velocity peak as does the experimental work. In addition, the theoretical work of Errett,[2] using an "average electron" model, shows no pronounced peak. Minute quantities of CO_2 or of water vapor[2] when added to argon will cause a pronounced

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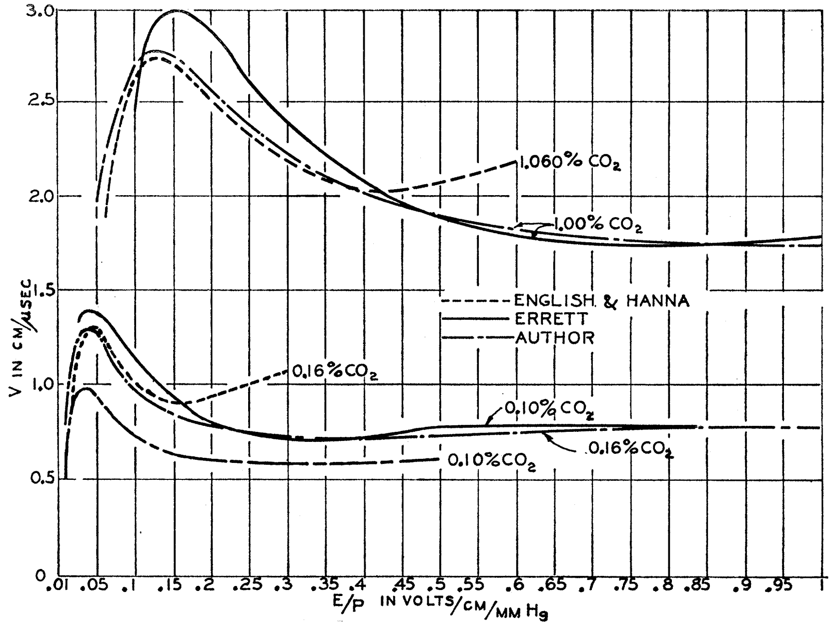

FIG. 6. Drift velocity vs E/p in A-CO_2 mixtures.

peak in the drift velocity curve. An "average electron" analysis of the type used by Errett[2] and by English and Hanna[1] indicates that amounts of CO_2 as small as 0.05% and of water vapor as small as 0.02% added to an A–N_2 mixture will cause pronounced peaking as well as raising of the drift velocity curve from its theoretically predicted value. An amount of CO_2 impurity considerably less than 0.05% or of water vapor impurity considerably less than 0.02% in otherwise pure argon could have caused the rise in the drift velocity curves for pure argon of Colli and Facchini,[4] Herreng,[9] Nielsen,[8] and Kirshner and Toffollo[3] above those of Allen,[10] Errett,[2] Bowe,[11] and the authors. It should be noted that condition (35) is more nearly fulfilled by N_2 than by CO_2 so that there is no reason to believe the theory incorrect.

Another explanation of the disparity between the theoretical and the experimental drift velocity data for A–N_2 mixture could lie in the inaccuracy of the cross section and energy absorption data for nitrogen appearing in the literature. In particular, the energy absorption characteristic b for nitrogen appears only

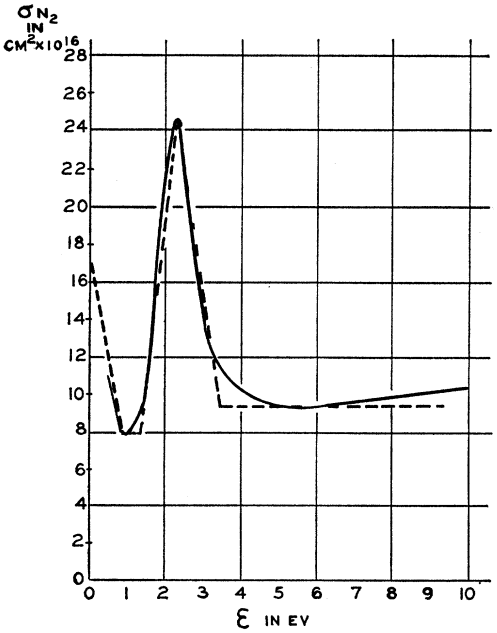

FIG. 7. Molecular nitrogen cross section. Dashed lines represent the approximation used.

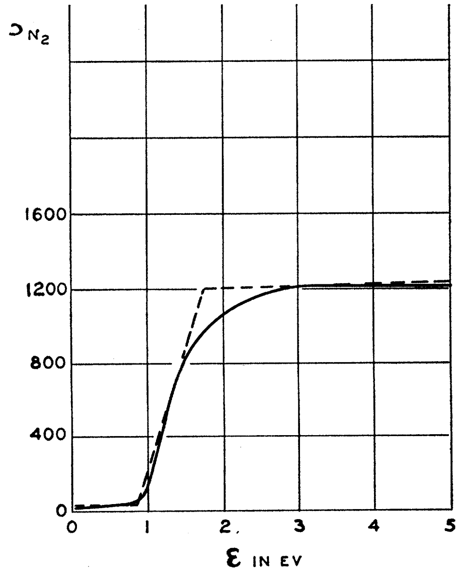

FIG. 8. Molecular nitrogen energy absorption characteristic. Dashed lines represent the approximation used.

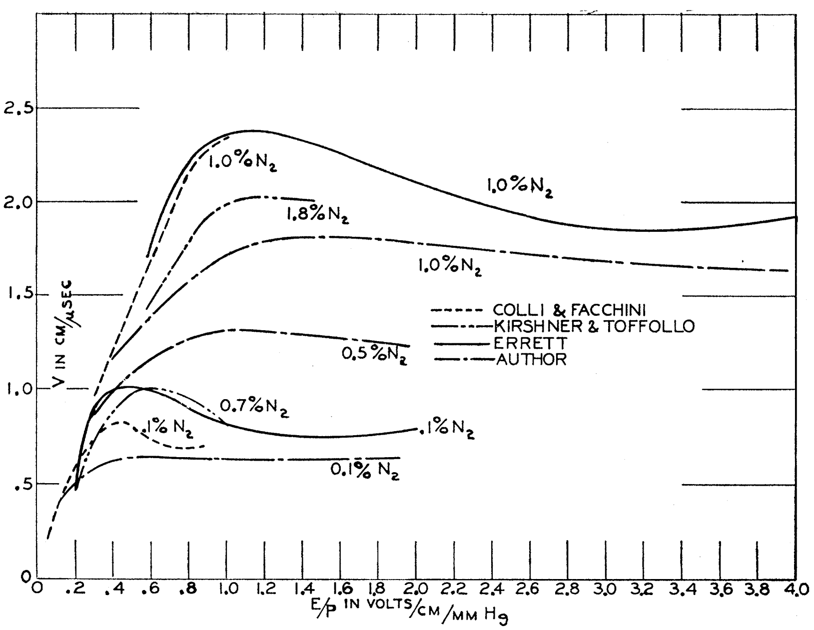

FIG. 9. Drift velocity vs E/p in A-N_2 mixture.

twice in the literature, in Healy and Reed[17] and in Errett.[2] These two sets of values are not in agreement. Those of Errett yield a more reasonable theoretical drift velocity and have been utilized for this paper. By altering the energy absorption characteristic, the shape of the experimental curves can be reproduced. If further experimentation proves the qualitative shape of the experimental drift velocity curves to be correct, the theory presented in this paper could provide a means to determine the correct energy absorption characteristic.

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