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Pseudoscalar Interaction in Nuclear Beta Decay*

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The experiments on the allowed beta transitions, which lead almost uniquely to the $V-1.2A$ interaction, do not have any bearing on a possible contribution from the pseudoscalar interaction. To determine whether or not any contribution from the pseudoscalar interaction is really needed, an examination has been made of the β longitudinal polarization and the β shape factor in the $0\rightarrow 0$ (yes) beta transitions. The theoretical polarization for the mixture of the pseudoscalar and the axial vector interactions has been developed. In this work, the formulation of the pseudoscalar interaction as given by Rose and Osborn has been used. The numerical results on the β longitudinal polarization and the shape factor depend on two parameters, namely, the coupling constant ratio, C_P/MC_A, and λ, the ratio of the two relevant nuclear matrix elements. M is the nucleon mass in units of the electron mass. The electronic functions occurring in the theoretical formulas for these effects are tabulated for Pr^{144} $(0^-\rightarrow 0^+)$ and Ho^{166} $(0^-\rightarrow 0^+)$. All the electronic radial functions were computed considering the nucleus as a sphere of a uniform charge distribution with a nuclear radius as $1.2A^{\frac{1}{3}}\times 10^{-13}$ cm, and taking into account the finite deBroglie wavelength effect. The results of extensive numerical analysis are presented. We conclude that the absence of the pseudoscalar interaction is consistent with the existing experimental data. The value of C_P/MC_A, which also gives a satisfactory fit to the experimental data depends on λ. The upper limit of the value of $|C_P/MC_A|$ is found to be 0.05 for $|\lambda|=200$. In this work, time-reversal invariance is assumed valid for the weak as well as the strong interactions, and the two-component theory of the neutrino has been used.

I. INTRODUCTION

THE experiments on the allowed beta transitions, during the past three years, lead almost uniquely to the $V-1.2A$ interaction.[1–6] The experiments[7] give the β longitudinal polarization in the allowed transitions as $-v/c$ for the electron, and as v/c for the positron, within an experimental error of about 2%. Here v/c is the ratio of the β-particle velocity to the vacuum velocity of light. To explain these polarization data, the vector and the axial vector interactions require the neutrino to be "left-handed"; whereas the scalar and the tensor interactions demand the neutrino to be a "right-handed" particle. The experimental determination of the neutrino helicity was made by Goldhaber, Grodzins, and Sunyar[8] and the neutrino helicity was found to be negative. The relative sign and the strength of the vector and the axial vector interactions are determined by the nuclear beta transitions where these interactions interfere. Burgy *et al.*[9] measured the anisotropy of the electron with respect to the spin direction of the polarized neutron. The result of this experiment is that the relative sign of the coupling constants of the vector and the axial vector interactions is negative. The comparison of the "*ft* values" (comparative half-lives) of a neutron and O^{14} give 1.21 ± 0.03 as the ratio of the absolute magnitudes of the coupling constants of the axial vector and the vector interactions. The $V-1.2A$ interaction is also consistent with electron-neutrino correlation experiments.[10]

Following different approaches, Marshak and Sudarshan,[11] Feynman and Gell-Mann,[12] also Sakurai[13] proposed the $V-A$ theory.

However, these experiments on the allowed beta transitions do not have any bearing on a possible existence of the pseudoscalar interaction. This can be readily understood because the operator for the pseudoscalar interaction is an irreducible tensor of rank zero and its parity is odd. Thus, for any contribution from the pseudoscalar interaction there has to be a change in the parity of the final nuclear state with respect to the

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1 Many recent review articles appear in the literature, e.g., see M. E. Rose, *Handbook of Physics* (McGraw-Hill Book Company, New York, 1959), pp. 9–90.

2 E. J. Konopinski, *Annual Review of Nuclear Science* (Annual Reviews, Inc., Palo Alto, California, 1959), Vol. 9, p. 99.

3 M. Deutsch and O. Kofoed-Hansen, in *Experimental Nuclear Physics*, edited by E. Segrè (John Wiley & Sons, Inc., New York, 1959), Vol. III, pp. 427–638.

4 Y. Smorodinskii, Uspekhi Fiz. Nauk **67**, 43 (1959) [translation: Soviet Phys-Uspekhi **67** (**2**), 1 (1959)].

5 D. L. Pursey, Proc. Roy. Soc. (London) **A246**, 444 (1958).

6 Invited papers at the *Conference on Weak Interactions, Gatlinburg, Tennessee* [Revs. Modern Phys. **31**, 782 (1959)].

7 C. S. Wu, *Proceedings Rehovoth Conference on Nuclear Structure* (North-Holland Publishing Company, Amsterdam, 1958), p. 359. For a recent summary of the β longitudinal measurements, see A. I. Galonsky, A. R. Brosi, B. Ketelle, and H. B. Willard, Nuclear Phys. (to be published).

8 M. Goldhaber, L. Grodzins, and A. W. Sunyar, Phys. Rev. **109**, 1015 (1958). This result has also been confirmed by I. Marklund and L. A. Page, Nuclear Phys. **9**, 88 (1958).

9 M. T. Burgy *et al.*, Phys. Rev. **110**, 1214 (1958), also see Phys. Rev. Letters **1**, 324 (1958).

10 W. B. Hermannsfeldt *et al.*, Phys. Rev. Letters **1**, 61 (1958). Also see J. S. Allen, Revs. Modern Phys. **31**, 791 (1959), and F. Pleasonton *et al.*, Bull. Am. Phys. Soc. **4**, 78 (1959); see J. B. Gerhart, Phys. Rev. **109**, 897 (1958), and W. B. Hermannsfeldt *et al.*, Phys. Rev. **107**, 641 (1957).

11 R. E. Marshak and E. C. G. Sudarshan, Phys. Rev. **109**, 1860 (1958).

12 R. P. Feynman and M. Gell-Mann, Phys. Rev. **109**, 193 (1958).

13 J. J. Sakurai, Nuovo cimento **7**, 649 (1958).

initial nuclear state in contrast to the allowed transitions with which all previous studies were concerned. To determine whether or not any contribution from the pseudoscalar interaction is really needed, we analyze the experimental data on the $0 \rightarrow 0$ (yes) beta transitions. The $0 \rightarrow 0$ (yes) beta transition is best for this purpose, because the vector interaction rigorously does not make any contribution. Therefore, we consider only the mixture of the axial vector interaction and the pseudoscalar interaction in the beta interaction Hamiltonian for the $0 \rightarrow 0$ (yes) transitions.

The relevant experimental data for the purpose of determining a possible contribution from the pseudoscalar interaction are (1) the β longitudinal polarization and (2) the β shape factor. The pseudoscalar interaction and the axial vector interaction, taken separately, give opposite signs of the beta polarization. This is true provided we take the neutrino helicity as negative. The β shape factor for the pure pseudoscalar interaction and the pure axial vector interaction give different energy dependence. However, the β shape factor, considered alone, is not very sensitive to a small contribution from the pseudoscalar interaction.

We wish to point out that in any investigation of the pseudoscalar interaction a formulation different than the so-called "conventional" one must be used. In 1954, Rose and Osborn[14] suggested that the proper operator for the pseudoscalar interaction is $-\boldsymbol{\sigma}\cdot\mathbf{p}L(\beta\gamma_5)/2M$ in the nucleon space. Here $L(\beta\gamma_5)$ is the pseudoscalar lepton covariant and is equal to $(\psi_e^*\beta\gamma_5[C_P+C_P'\gamma_5]\psi_\nu)$ for e^- emission. Also $\mathbf{p}$ is the momentum *operator*. This pseudoscalar operator was obtained by the application of the Foldy-Wouthuysen transformation to the total Hamiltonian of the system comprised of the decaying nucleon, the lepton $(e-\nu)$ field, and the leptons. In this formulation of the pseudoscalar interaction, the gradient $(\mathbf{p}=-i\nabla)$ appears acting only on the lepton covariant. If we assume the lepton covariant to be a constant (independent of the nucleon coordinates), as is done in the conventional theory, then there is *no* contribution from the pseudoscalar interaction. The Foldy-Wouthuysen transformation, though, also gives additional recoil terms for the axial vector and the vector interactions, but these terms are much smaller than the leading terms and we can neglect them. Then, apart from renaming the nuclear matrix elements, explicit calculations show that we get the same formulas as given by the conventional theory. Thus, the conventional formulation of the A and the V interactions is essentially correct. But the conventional treatment of the pseudoscalar interaction is wrong.[15] Hence, the proper operator for the pseudoscalar interaction, $-\boldsymbol{\sigma}\cdot\mathbf{p}L(\beta\gamma_5)/2M$, must be employed.

The β shape factor for the $0 \rightarrow 0$ (yes) transition with a mixture of the axial vector and the pseudoscalar interaction has been given by Rose and Osborn.[14] But the longitudinal polarization of the β particles in the $0 \rightarrow 0$ (yes) transition, with a mixture of the axial vector interaction and the proper formulation of the pseudoscalar interaction, does not exist in the literature. To derive such an expression, using the relativistic electronic functions for a finite nucleus, is part of the motivation of this work.

Several attempts to investigate the existence of the pseudoscalar interaction in nuclear β decay appear recently in the literature. Tadic[16] analyzed the less accurate ($\sim 22\%$) measurement of the β longitudinal polarization in Pr^{144} $(0^- \rightarrow 0^+)$ due to Geiger *et al.*[17] Cohen and Wiener[18] analyzed their measurement of the β longitudinal polarization in Pr^{144}. Also Mehlhop *et al.*[19] estimated the upper limit on the pseudoscalar contribution by comparing his measurements with the formulas derived by Lee-Whiting.[20] Again using these formulas of Lee-Whiting, Bühring[21] set an upper limit on the pseudoscalar contribution with his β polarization measurement in Ho^{166}. In all these attempts, the conventional pseudoscalar interaction was used. Moreover, the effects due to the finite nuclear size[22] were completely ignored. It is well known that these effects are important for the $0 \rightarrow 0$ (yes) transitions.

In addition, several attempts[23] have been reported in the literature wherein the possible existence of the pseudoscalar interaction was examined by comparing the theoretical shape factor as given by Rose and Osborn[14] with the experimental shape factor of the $0^- \rightarrow 0^+$ transition of Pr^{144}. The general conclusion is that the β shape factor is not very sensitive to the contribution from the pseudoscalar interaction.

However, for a consistent investigation for the pseudoscalar contribution, one must consider *all* the experimental data, namely, the β longitudinal polarization as well as the shape factor. Thus, until now such a consistent treatment for the search of the pseudoscalar interaction did not exist.

The problem considered in this paper, then, is to investigate the existence of the pseudoscalar interaction in the interaction Hamiltonian density for the processes of nuclear beta decay by (i) formulation of the theoretical expressions for the beta longitudinal polarization and

[14] M. E. Rose and R. K. Osborn, Phys. Rev. **93**, 1315 (1954).

[15] For example, see M. Deutsch and O. Kofoed-Hansen, reference 3, p. 516. Also see M. E. Rose and R. K. Osborn, reference 14, for a discussion of this point.

[16] D. Tadic (private communication).

[17] J. S. Geiger *et al.*, Phys. Rev. **112**, 1684 (1958).

[18] S. G. Cohen and R. Wiener, Nuclear Phys. **15**, 79 (1960). In this paper the contribution of γ_5 in the A interaction is neglected.

[19] W. A. W. Mehlhop *et al.*, Bull. Am. Phys. Soc. **5**, 9 (1950). And also see W. A. W. Mehlhop, dissertation, Washington University, Saint Louis, 1959 (unpublished).

[20] G. E. Lee-Whiting, Can. J. Phys. **36**, 1199 (1958).

[21] W. Bühring, Z. Physik **155**, 566 (1959).

[22] M. E. Rose and D. K. Holmes, Phys. Rev. **82**, 389 (1951). Also see M. E. Rose and D. K. Holmes, Oak Ridge National Laboratory Report ORNL-1022 (unpublished).

[23] Graham *et al.*, Can. J. Phys. **36**, 1084 (1958). For a summary of the previous work, see C. P. Bhalla, Oak Ridge National Laboratory Report ORNL-2950 (unpublished).

the β shape factor[24] in the $0\rightarrow 0$ (yes) transitions with the correct form of the pseudoscalar interaction and the axial vector interaction; (ii) making an extensive numerical analysis of the presently available experimental data, using the derived formulas, with the calculated electronic functions which include accurately the nuclear finite size[22] and the finite deBroglie wavelength[25] effects.

In Sec. II, we give the details of the calculation of the β longitudinal polarization in the $0\rightarrow 0$ (yes) beta transitions. The results are specialized by assuming the validity of time-reversal invariance in strong as well as weak interactions, and the two-component theory of the neutrino is used. In Sec. III, the electronic functions occurring in the theoretical expressions for the β longitudinal polarization and the β shape factor are tabulated for Pr^{144} $(0^-\rightarrow 0^+)$ and Ho^{166} $(0^-\rightarrow 0^+)$. These electronic functions were computed considering the nucleus as a sphere of a uniform charge distribution with a nuclear radius of $1.2A^{\frac{1}{3}}\times 10^{-13}$ cm. Also we give graphically the results of large-scale computations for the analysis of the experimental data on Pr^{144} and Ho^{166}. Finally, the discussion and conclusions appear in Sec. IV.

II. FORMULATION OF THE PROBLEM

Throughout, we use the relativistic units: $\hbar=m=c=1$. We use the representation[26] of the Dirac equation corresponding to the free-particle Hamiltonian

$$H_0=-\boldsymbol{\alpha}\cdot\mathbf{p}-\beta.$$

We represent by $\psi_\kappa{}^\mu$ the solution of the Dirac equation for an electron with a central potential $V(r)$, where

$$\begin{aligned} V(r)&=-\alpha Z/r, &&\text{for } r>R,\\ &=-(\alpha Z/2r)(3-r^2/R^2), &&\text{for } r<R. \end{aligned}\tag{1}$$

R is the nuclear radius and it is equal to $0.428\alpha A^{\frac{1}{3}}$ in our units.

$$\psi_\kappa{}^\mu=\begin{pmatrix} -if_\kappa\chi_{-\kappa}{}^\mu \\ g_\kappa\chi_\kappa{}^\mu \end{pmatrix}.\tag{2}$$

In Eq. (2), f_κ and g_κ are the real radial functions. Throughout, the normalization corresponds to one particle in a sphere of unit radius. Here, κ gives both the angular momentum j according to

$$j=|\kappa|-\tfrac{1}{2},$$

and the parity $(-)^{l_\kappa+1}$ according to

$$l_\kappa=|\kappa|+\tfrac{1}{2}(S_\kappa-1),$$

where S_κ is the sign of κ.

We first use the 4-component Dirac wave function for the neutrino. We denote by $F_{\kappa\nu}$ and $G_{\kappa\nu}$ the radial functions for the neutrino in a similar representation as for the electron in Eq. (2). Then,

$$\begin{aligned} F_\kappa&=S_\kappa q j_{l(-\kappa)}(qr),\\ G_\kappa&=q j_{l(\kappa)}(qr), \end{aligned}\tag{3}$$

where j_l is the spherical Bessel function and the neutrino energy is $q=W_0-W$. W_0 is the end-point energy and W represents the total energy of the beta particle. After obtaining the formulas using the 4-component theory of the neutrino, we specialize these results for the two-component theory of the neutrino[27] by substituting $C_A=C_A'$, and $C_P=C_P'$.

We also use

$$\begin{aligned} \boldsymbol{\gamma}&=-i\beta\boldsymbol{\alpha},\quad \gamma_4=-\beta,\\ \gamma_5&=\gamma_1\gamma_2\gamma_3\gamma_4, \end{aligned}\tag{4}$$

with

$$\beta=\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix},\quad \gamma_5=\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix},$$

in our representation. For the axial vector and the pseudoscalar interactions, the beta interaction Hamiltonian density in the nucleon space is given by

$$\begin{aligned} H_{\beta^-}=\boldsymbol{\sigma}\cdot(\psi_e{}^*\boldsymbol{\sigma}[C_A+C_A'\gamma_5]\psi_\nu)&\\ -\gamma_5(\psi_e{}^*\gamma_5[C_A+C_A'\gamma_5]\psi_\nu)&\\ +\frac{i}{2M}\boldsymbol{\sigma}\cdot\nabla(\psi_e{}^*\beta\gamma_5[C_P+C_P'\gamma_5]\psi_\nu)&. \end{aligned}\tag{5}$$

In Eq. (5), the first two terms correspond to the (conventional) axial vector interaction and the last term represents the appropriate operator for the pseudoscalar interaction.[28] C_A and C_P are the so-called "parity-conserving" coupling constants for the axial vector and the pseudoscalar interactions, respectively. The primed coupling constants are the so-called "parity-nonconserving" ones. Here, M is the nucleon mass in units of the electron mass.

For the calculation of the β longitudinal polarization in the $0\rightarrow 0$ (yes) beta transitions, we use the first-order perturbation development as given by Rose *et al.*[29] The operator for the longitudinal polarization[30] is $\boldsymbol{\sigma}\cdot\hat{\boldsymbol{p}}$,

[24] This was originally derived by M. E. Rose and R. K. Osborn, reference 14. For a correction of a typographical error, see M. E. Rose and R. K. Osborn, Phys. Rev. **110**, 1484 (1958).

[25] M. E. Rose and C. L. Perry, Phys. Rev. **90**, 479 (1953).

[26] We follow the notation as used by M. E. Rose and R. K. Osborn, reference 14.

[27] T. D. Lee and C. N. Yang, Phys. Rev. **105**, 1671 (1957). Also see A. Salam, Nuovo cimento **5**, 299 (1957), and L. Landau, Nuclear Phys. **3**, 127 (1957).

[28] It is in this respect that the present treatment of the P interaction differs from those appearing in the literature for the calculation of the β longitudinal polarization in the $0\rightarrow 0$ (yes) beta transition. This was originally suggested by Rose and Osborn, reference 14, where the β shape factor was derived for the $0\rightarrow 0$ (yes) transition.

[29] M. E. Rose, L. C. Biedenharn, and G. B. Arfken, Phys. Rev. **85**, 5 (1952).

[30] This operator $\boldsymbol{\sigma}\cdot\hat{\boldsymbol{p}}$ commutes with the free-particle Dirac Hamiltonian. In Eq. (7), the spinor is an eigenfunction of $-\boldsymbol{\alpha}\cdot\mathbf{p}-\beta$, with beta energy W. For a covariant description of the spin, see L. Michel and A. S. Wightman, Phys. Rev. **98**, 1190 (1955); C. Bouchiat and L. Michel, Nuclear Phys. **5**, 416 (1956); also see H. A. Tolhoek, Revs. Modern Phys. **28**, 277 (1956), and R. H. Good, Jr., and M. E. Rose, Nuovo cimento **14**, 879 (1959).

where $\hat{p}$ is a unit vector in the direction of the beta momentum. In the expression of ψ_∞, as given in Eq. (7), $\hat{r}$ may be identified with $\hat{p}$. The β longitudinal polarization, denoted by $P_{\parallel}$, is given by the following:

$$P_{\parallel}=\frac{\langle(\psi_\infty,\boldsymbol{\sigma}\cdot\hat{r}\psi_\infty)\rangle}{\langle(\psi_\infty,\psi_\infty)\rangle}, \tag{6}$$

where

$$\psi_\infty=-i\pi^{\frac{1}{2}}\frac{e^{ipr}}{r}\sum_{\kappa,\mu} e^{i\delta_\kappa}\langle\psi_f|H_{\beta^-}|\psi_i\rangle \times\begin{pmatrix}[(W-1)/p]^{\frac{1}{2}}\chi_{-\kappa}{}^{\mu}(\hat{r})\\ [(W+1)/p]^{\frac{1}{2}}\chi_{\kappa}{}^{\mu}(\hat{r})\end{pmatrix}. \tag{7}$$

In Eq. (7), we have

$$W=(p^2+1)^{\frac{1}{2}},$$

$$\delta_\kappa=\frac{\alpha ZW}{p}\ln(2pr)-\arg\Gamma(\gamma+i\alpha ZW/p)+\eta_\kappa-\tfrac{1}{2}\pi\gamma,$$

and the spin angular function is

$$\chi_\kappa{}^{\mu}(\hat{r})=\textstyle\sum_\tau C(l_\kappa\tfrac{1}{2}j;\mu-\tau,\tau)\chi_{\frac{1}{2}}{}^{\tau}Y_{l_\kappa}{}^{\mu-\tau}(\hat{r}).$$

ψ_∞ represents the probability amplitude for the β particle due to the beta interaction, when a beta transition occurs between ψ_i, the initial nuclear state specified by (J_i,π_i) and ψ_f, the final nuclear state represented by (J_f,π_f). Also ψ_∞ is an outgoing spherical wave and it is the asymptotic form of the solution of the Dirac equation for the central field on the β particle. In Eq. (6), the round brackets denote the scalar product with respect to the spinor indices only. The angular brackets refer (1) to the summation over κ_ν and μ_ν of the neutrino, (2) to the average over the magnetic substates of the initial nuclear state, and (3) to the summation over the magnetic substates of the final nuclear state. In the $0\rightarrow 0$ (yes) transition, (2) and (3) are trivial operations and they give unity. From Eq. (5), for the $0\rightarrow 0$ (yes) transition, we obtain for the β-matrix element

$$\begin{aligned}\langle\psi_f|H_{\beta^-}|\psi_i\rangle &=\frac{1}{4\pi}(-)^{\mu+l+j}\delta_{\mu,-\mu_\nu}\Big\{(iC_A\delta_{\kappa,\kappa_\nu}-S_\kappa C_A{}'\delta_{\kappa,-\kappa_\nu})\\ &\times\Big[(6(2\bar{l}+1))^{\frac{1}{2}}C(\bar{l}1l;00)W(\bar{l}1j\tfrac{1}{2};l\tfrac{1}{2})(f_\kappa G_\kappa+g_\kappa F_\kappa)\\ &\times\int\boldsymbol{\sigma}\cdot\hat{r}+(f_\kappa F_\kappa-g_\kappa G_\kappa)i\int\gamma_5\Big]\\ &+\frac{1}{2M}(iC_P\delta_{\kappa,\kappa_\nu}-S_\kappa C_P{}'\delta_{\kappa,-\kappa_\nu})\\ &\times\frac{d}{dr}(f_\kappa F_\kappa+g_\kappa G_\kappa)\int\boldsymbol{\sigma}\cdot\hat{r}\Big\}.\end{aligned} \tag{8}$$

In Eq. (8) we have also introduced the following notation:

$$l=l_\kappa,\quad \bar{l}=l_{-\kappa}.$$

$\int\boldsymbol{\sigma}\cdot\hat{r}$ and $\int\gamma_5$ are the reduced nuclear matrix elements (independent of the magnetic quantum numbers). S_κ is the sign of κ. $C(\bar{l}1l;00)$ is a Clebsch-Gordon coefficient and $W(\bar{l}1j\frac{1}{2};l\frac{1}{2})$ is a Racah coefficient.[31] $\delta_{\kappa\cdot\kappa_\nu}$ is the Kronecker delta.

In our notation, the energy spectrum is given by

$$N(W)=\frac{4}{\pi}r^2\langle(\psi_\infty,\psi_\infty)\rangle. \tag{9}$$

Substituting ψ_∞, as given in Eq. (7), in Eq. (6), we obtain,[32] after some simplification,[33]

$$P_{\parallel}=\frac{\sum_{\kappa,\kappa_\nu}\exp[i\delta_\kappa-i\delta_{-\kappa}](2j+1)\mathfrak{F}^*(-\kappa,\kappa_\nu)\mathfrak{F}(\kappa,\kappa_\nu)}{\sum_{\kappa,\kappa_\nu}(2j+1)\mathfrak{F}^*(\kappa,\kappa_\nu)\mathfrak{F}(\kappa,\kappa_\nu)}, \tag{10}$$

where

$$\begin{aligned}\mathfrak{F}(\kappa,\kappa_\nu)=(iC_A\delta_{\kappa,\kappa_\nu}-S_\kappa C_A{}'\delta_{\kappa,-\kappa_\nu})&\Big\{[6(2\bar{l}+1)]^{\frac{1}{2}}\\ &\times C(\bar{l}1l;00)W(\bar{l}1j\tfrac{1}{2};l\tfrac{1}{2})(f_\kappa G_\kappa+g_\kappa F_\kappa)\int\boldsymbol{\sigma}\cdot\hat{r}\\ &+(f_\kappa F_\kappa-g_\kappa G_\kappa)i\int\gamma_5\Big\}\\ &+\frac{1}{2M}(iC_P\delta_{\kappa,\kappa_\nu}-S_\kappa C_P{}'\delta_{\kappa,-\kappa_\nu})\\ &\times\frac{d}{dr}(f_\kappa F_\kappa+g_\kappa G_\kappa)\int\boldsymbol{\sigma}\cdot\hat{r},\end{aligned} \tag{10'}$$

and the radial functions are, of course, evaluated at $r=R$.

Now we assume[34] the validity of time-reversal invariance in the weak as well as in the strong interactions. This implies that all the coupling constants are real and the combination of nuclear matrix elements $i\int\gamma_5\cdot(\int\boldsymbol{\sigma}\cdot\hat{r})^*$ is real.

Carrying out the calculations[32] in Eq. (10), we find that the main contribution comes from terms[35] for $\kappa=1$ and $\kappa=-1$. We neglect terms of relative order p^2R^2 (or higher orders). Then we obtain, for the β longitudinal

[31] We follow the notation and the conventions as given by M. E. Rose, *Elementary Theory of Angular Momentum* (John Wiley & Sons, Inc., New York, 1957).

[32] The details of the calculations in this paper are given by C. P. Bhalla, reference 23.

[33] For the application of this formalism to the calculation of the polarization "vector" of the conversion electrons following β decay, see R. L. Becker and M. E. Rose, Nuovo cimento **13**, 1182 (1959).

[34] For the weak interactions, see M. A. Clark *et al.*, Phys. Rev. Letters **1**, 100 (1958), and also see T. D. Lee and C. N. Yang, Brookhaven National Laboratory Report BNL-443 (T-91), 1957 (unpublished). For the reality condition on the combination of the nuclear matrix elements, see, for example, L. Longmire and A. M. L. Messiah, Phys. Rev. **83**, 464 (1951), and also see L. C. Biedenharn and M. E. Rose, Revs. Modern Phys. **25**, 729 (1953).

[35] To check these formulas for $Z=0$, terms which vanish for $\kappa=\pm1$ have to be considered for the pseudoscalar interaction.

polarization in the $0 \to 0$ (yes) transition,

$$P_{||} = -\frac{a_0 + a_1\lambda^2 + a_2\lambda - a_3\xi^2 + (a_4 + a_5\lambda)\xi}{b_0 + b_1\lambda^2 + b_2\lambda + b_3\xi^2 + (b_4 + b_5\lambda)\xi}. \quad (11)$$

The β shape factor is given by

$$C_{\beta^-} = b_0 + b_1\lambda^2 + b_2\lambda + b_3\xi^2 + (b_4 + b_5\lambda)\xi. \quad (11')$$

We have introduced the following definitions in Eqs. (11):

$$\lambda = i\int\gamma_5 \Big/ \int \boldsymbol{\sigma}\cdot\mathbf{r}, \quad \xi = C_P/MC_A.$$

$$a_0 = B_0 + \tfrac{1}{3}qD_0 - \tfrac{1}{9}q^2A_0, \quad a_1 = -A_0, \quad a_2 = D_0 - \tfrac{2}{3}qA_0, \quad (12)$$

$$a_3 = \tfrac{1}{4}\{(U^2-1)B_0 + \tfrac{1}{3}q[8UB_0 - (U^2+1)D_0 - 2UC_0] + \tfrac{1}{9}q^2[16B_0 - 4(UD_0 + C_0) - (U^2-1)A_0]\}, \quad (12')$$

$$a_4 = B_0 + \tfrac{1}{3}q(UC_0 + D_0) + \tfrac{1}{9}q^2(2C_0 - A_0),$$
$$a_5 = \tfrac{1}{2}\{UC_0 + D_0 + \tfrac{2}{3}q(2C_0 - A_0)\}, \quad (12'')$$

and

$$b_0 = M_0 - \tfrac{2}{3}qN_0 + \tfrac{1}{9}q^2L_0, \quad b_1 = L_0, \quad b_2 = -2(N_0 - \tfrac{1}{3}qL_0). \quad (13)$$

$$b_3 = \tfrac{1}{4}\{(U^2+1)M_0 - 2UQ_0 + \tfrac{2}{3}q[4(UM_0 - Q_0) + (U^2-1)N_0] + \tfrac{1}{9}q^2 \times [(U^2+1)L_0 - 2UP_0 + 16M_0 + 8(UN_0 - R_0)]\}, \quad (13')$$

$$b_4 = M_0 - UQ_0 - \tfrac{2}{3}q(N_0 + 2Q_0) + \tfrac{1}{9}q^2(-UP_0 + L_0 - 4R_0),$$

$$b_5 = -\{UR_0 + N_0 - \tfrac{1}{3}q[-UP_0 + L_0 - 4R_0 + R^2(UQ_0 - M_0)] + \tfrac{1}{9}q^2R^2(UR_0 - N_0 - 4Q_0)\}. \quad (13'')$$

In Eqs. (12), we have used the following combinations[36] of the electron radial functions:

$$\begin{aligned}
A_{k-1} &= (p^2F_0)^{-1}R^{2-2k}f_kg_{-k}\sin(\delta_k - \delta_{-k}),\\
B_{k-1} &= (p^2F_0)^{-1}R^{-2k}f_{-k}g_k\sin(\delta_k - \delta_{-k}),\\
C_{k-1} &= (p^2F_0)^{-1}R^{1-2k}(f_kf_{-k} + g_kg_{-k})\sin(\delta_k - \delta_{-k}),\\
D_{k-1} &= (p^2F_0)^{-1}R^{1-2k}(f_kf_{-k} - g_kg_{-k})\sin(\delta_k - \delta_{-k}),
\end{aligned} \quad (14)$$

and the following combinations, which appear in the literature[37]:

$$\begin{aligned}
L_{k-1} &= (2p^2F_0)^{-1}R^{2-2k}(g_{-k}^2 + f_k^2),\\
M_{k-1} &= (2p^2F_0)^{-1}R^{-2k}(g_k^2 + f_{-k}^2),\\
N_{k-1} &= (2p^2F_0)^{-1}R^{1-2k}(f_{-k}g_{-k} - f_kg_k),\\
P_{k-1} &= (2p^2F_0)^{-1}R^{2-2k}(g_{-k}^2 - f_k^2),\\
Q_{k-1} &= (2p^2F_0)^{-1}R^{-2k}(g_k^2 - f_{-k}^2),\\
R_{k-1} &= (2p^2F_0)^{-1}R^{1-2k}(f_{-k}g_{-k} + f_kg_k).
\end{aligned} \quad (15)$$

In Eqs. (12) and Eqs. (13), we have

$$U = W - V_c - q.$$

For e^- and e^+, $V_c = -\alpha Z/R$ and $V_c = \alpha Z/R$, respectively. Here, F_0 is the Fermi function.

This completes the first part of the problem considered in this paper.

III. NUMERICAL RESULTS

Out of the five $0 \to 0$ (yes) beta transitions reported in the literature,[38] namely, Pr^{144}, Ho^{166}, Ce^{144}, Eu^{152}, and Tl^{206}, only Pr^{144} $(0^- \to 0^+)$ has been studied in detail. Several measurements of the shape factor of the $0^- \to 0^+$ branch appear in the literature.[39] We analyze the β^- shape factor as given by Porter and Day. This shape factor can be fitted by the following cubic in p:

$$C_{\beta^-} = 9459.32 - 375.752p + 89.84p^2 - 8.4994p^3. \quad (16)$$

The mean sum of the squared residuals[40] of this fit from the experimental data is 1.217. The most accurate measurement of the β^- longitudinal polarization in Pr^{144} is due to Mehlhop *et al.*[19] and they give

$$\langle P_{||}/(v/c)\rangle = -0.986 \pm 0.03$$

averaged over an interval of β kinetic energy from 1 Mev to an energy near end point ($\sim$3 Mev).

An accurate measurement of the β^- longitudinal polarization in Ho^{166} has been reported by Bühring[41] and in this measurement,

$$\langle P_{||}/(v/c)\rangle = -0.99 \pm 0.02,$$

for β kinetic energy from 0.18 Mev to near the beta end-point energy ($\sim$1.8 Mev). There are no accurate measurements[42] on the β^- shape factor in Ho^{166} $(0^- \to 0^+)$.

We give the tabulated functions for the β longitudinal polarization and the shape factor, as given in Eqs. (11), in Table I and Table II for Pr^{144} $(0^- \to 0^+)$, and Ho^{166} $(0^- \to 0^+)$. The details of the actual computation of the electronic radial functions are given elsewhere.[36]

In the theoretical expressions for the β longitudinal polarization and the β shape factor, as given in Eqs. (11), we have two parameters, namely, ξ and λ. It is not possible, as yet, to calculate λ with much confidence. Several attempts have been made to evaluate λ by using

[36] See, for example, C. P. Bhalla and M. E. Rose, Oak Ridge National Laboratory Report ORNL-2954 (unpublished). These tables give f_κ and g_κ for $\kappa = \pm 1$ (the nuclear finite size effects and the finite deBroglie wavelength effects have been taken into account). In addition, F_0 and $\sin(\delta_1 - \delta_{-1})$ are also calculated.

[37] See, for example, Rose and Osborn, reference 14.

[38] See, for example, D. Strominger *et al.*, Revs. Modern Phys. **30**, 585 (1958). Tl^{206} $(0^- \to 0^+)$ has been reported by L. N. Zyrianova, Izvest. Akad. Nauk S.S.S.R. Ser. Fiz. **20**, 1399 (1956) [translation: Bull. Acad. Sciences U.S.S.R. **20**, 1280 (1956)]. This assignment in Tl^{206} needs confirmation.

[39] See F. T. Porter and P. P. Day, Phys. Rev. **114**, 1286 (1959), and N. F. Freeman, Proc. Phys. Soc. **73**, 600 (1959). Graham *et al.*, footnote 23, give references to the previous works.

[40] The mean sum of the squared residuals is defined to be equal to $\sum_{i=1}^{46}[(n_i)_c - n_i]^2/(46-4)$. Here $(n_i)_c$ and n_i are the computed values and the experimental values of the shape factor, respectively. There were 46 experimental points in the shape factor of Porter and Day.

[41] W. Bühring, Z. Physik **155**, 566 (1959).

[42] Dr. R. L. Graham has advised us that more thorough experimental work needs to be done, as hitherto reported on Ho^{166}.

TABLE I. Pr^{144} ($0^- \rightarrow 0^+$). Numerical coefficients for beta longitudinal polarization and shape factor formulas.[a]

p	a_0	a_1	a_2	a_3	a_4	a_5	b_0	b_1	b_2	b_3	b_4	b_5
1.0	112.3	0.6400	16.97	14 290	91.54	1.922	153.4	0.9026	23.53	20 780	2260	175.7
1.5	131.5	0.7487	19.85	17 740	108.5	2.182	153.9	0.8976	23.50	21 830	1768	141.1
2.0	140.7	0.7992	21.21	20 250	117.7	2.261	153.8	0.8917	23.43	23 050	1445	116.7
2.5	145.1	0.8234	21.86	22 390	123.3	2.248	153.6	0.8854	23.32	24 480	1217	99.48
2.783	146.6	0.8310	22.08	23 520	125.7	2.229	153.3	0.8816	23.25	25 320	1116	91.95
3.0	147.3	0.8348	22.19	24 370	127.3	2.203	153.1	0.8787	23.18	26 010	1050	87.00
3.5	148.5	0.8395	22.33	26 380	130.5	2.128	152.5	0.8720	23.05	27 680	923.5	77.60
4.0	148.9	0.8402	22.37	28 410	133.2	2.047	151.8	0.8651	22.92	29 490	825.4	70.33
4.5	148.9	0.8385	22.35	30 510	135.8	1.964	151.1	0.8582	22.76	31 400	746.6	64.52
5.0	148.7	0.8354	22.29	32 730	138.2	1.870	150.3	0.8512	22.62	33 480	681.8	59.83
5.5	148.2	0.8312	22.20	35 020	140.6	1.770	149.5	0.8442	22.45	35 670	627.9	55.94
6.0	147.6	0.8264	22.09	37 430	142.9	1.690	148.7	0.8372	22.31	37 990	582.0	52.67
6.5	147.0	0.8211	21.97	39 960	145.3	1.595	147.8	0.8302	22.15	40 460	542.6	49.89

[a] Equations (11) and (11′). These coefficients have been calculated considering (1) the nuclear radius to be $0.428\alpha A^{\frac{1}{3}}(\hbar/mc)$, (2) the corrections due to the finite nuclear size, and (3) the finite deBroglie wavelength effects.

simple nuclear models. Rose and Osborn,[43] Ahrens and Feenberg,[44] and Pursey[45] give

$$\lambda = -30 \text{ to } -37, \tag{17}$$

for Pr^{144} and Ho^{166}. Pearson[46] estimates

$$\lambda = 2.5 \text{ to } 8,$$

by using two different types of assumptions. The Coulomb contribution[43–45] provides the dominant term for the value of λ and this circumstance favors a value of λ as given in Eq. (17). However, in our analysis we consider a wide range of the values of λ.

A. Analysis of Pr^{144} ($0^- \rightarrow 0^+$) Data

First we investigate whether or not the pure axial vector interaction can explain the data on the β longitudinal polarization of Mehlhop *et al.*,[19] and the β shape factor of Porter and Day.[39] In Fig. 1, we plot the calculated β^- longitudinal polarization divided by $-v/c$ versus the beta momentum for $\lambda = 10$, 30, 110, -30, -50, and -150. In this figure, the region of the beta momentum which corresponds to the data of Mehlhop *et al.* is indicated. Clearly, the upper limit of the polarization datum of Mehlhop *et al.*, namely, 1.016, can be easily explained by the pure axial vector interaction.

We define a reasonable fit to the beta shape factor as follows. We normalize the shape factor as given by the cubic fit, Eq. (16), and the calculated shape factor to unity at $p = 5.0$. For $p = 1.0$ to $p = 6.5$, in steps of 0.5, we compute

$$\bar{\Delta} = \frac{1}{11} \sum_{p=1.0}^{p=6.5} \left(\frac{\Delta X_i}{X_i} \right)^2,$$

where ΔX_i is the difference of the calculated shape factor from the corresponding value X_i given by the cubic fit. We take the calculated shape factor as a satisfactory fit, if

$$\bar{\Delta} \leqslant 0.005.$$

This, generally, corresponds to the value of $|\Delta X_i / X_i|$ as being less than 4%. We find that the pure axial vector interaction gives a satisfactory fit to the experimental shape factor for

$$\lambda > 0,$$

and for

$$-\lambda > 50.$$

However, there is no satisfactory fit for

$$-50 < \lambda < -10. \tag{18}$$

TABLE II. Ho^{166} ($0^- \rightarrow 0^+$). Numerical coefficients for beta longitudinal polarization and shape factor formulas.[a]

p	a_0	a_1	a_2	a_3	a_4	a_5	b_0	b_1	b_2	b_3	b_4	b_5
0.76	95.95	0.5323	14.30	16 800	84.46	1.441	152.6	0.8758	23.12	28 130	2809	217.9
1.0	111.8	0.6200	16.66	20 050	99.02	1.661	152.8	0.8734	23.10	28 670	2505	195.1
1.5	130.8	0.7239	19.47	24 820	117.5	1.882	153.1	0.8673	23.05	30 080	1983	156.2
2.0	139.6	0.7713	20.76	28 220	127.4	1.936	152.9	0.8602	22.93	31 750	1615	128.7
2.5	143.9	0.7932	21.37	31 080	133.7	1.919	152.4	0.8525	22.80	33 620	1354	109.3
3.0	145.9	0.8026	21.65	33 700	138.1	1.869	151.6	0.8446	22.62	35 620	1163	95.15
3.5	146.8	0.8055	21.76	36 290	141.6	1.801	150.9	0.8364	22.47	37 820	1018	84.52
4.0	147.0	0.8046	21.76	38 910	144.7	1.724	149.9	0.8282	22.28	40 120	905.6	76.26

[a] Equations (11) and (11′). These coefficients have been calculated considering (1) the nuclear radius to be $0.428\alpha A^{\frac{1}{3}}(\hbar/mc)$, (2) the corrections due to the finite nuclear size, and (3) the finite deBroglie wavelength effects.

[43] M. E. Rose and R. K. Osborn, Phys. Rev. **93**, 1326 (1954).
[44] T. Ahrens and E. Feenberg, Phys. Rev. **86**, 64 (1952).
[45] D. L. Pursey, Phil. Mag. **42**, 1193 (1951).
[46] J. M. Pearson, Can. J. Phys. **38**, 148 (1960).

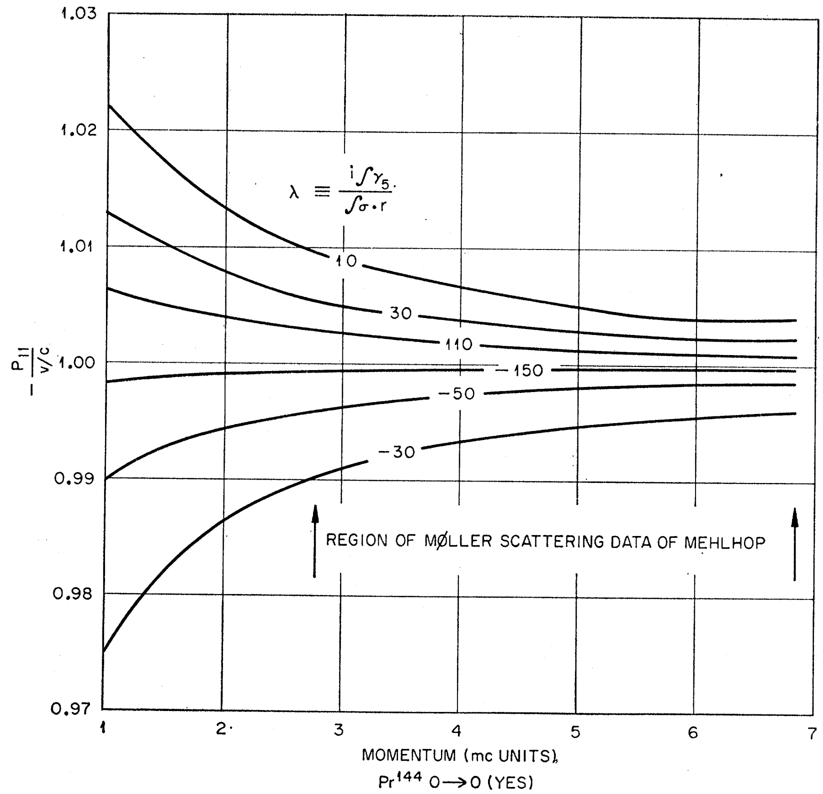

FIG. 1. Calculated longitudinal polarization in units of $-v/c$ versus β momentum for the axial-vector interaction. The numbers attached to the curves give λ, the ratio of the nuclear matrix elements.

Therefore, we conclude that the pure axial vector interaction can explain the experimental data on Pr^{144} $(0^- \rightarrow 0^+)$.

We may determine the value of ξ and λ, which also gives a satisfactory fit to these data. The results of extensive computation are summarized in Fig. 2. In the (ξ,λ) plane, the overlapping regions of satisfactory fits to the shape factor and to the polarization datum are shown as crosshatched. The values of ξ in this crosshatched region depend on λ, the ratio of the two nuclear matrix elements. In Fig. 2, the lines denoted by L and U represent the loci for the lower and the upper limits of the polarization datum of Mehlhop *et al.* It is interesting to observe that we can find values of ξ for $\lambda=-35$ which are also consistent with the experimental data. In the previous work, no such fit was reported.

B. Analysis of Ho^{166} $(0^- \rightarrow 0^+)$ Datum

We do not attempt to analyze the β shape factor as no accurate measurement exists. In Fig. 3, we plot the calculated beta longitudinal polarization in units of $-v/c$ versus β^- momentum for $\lambda=10$, 30, 130, -30, -50 and -130 for the pure axial vector interaction. Again, we find that a large number of the values of λ can be found for which the calculated values lie well within the measurement of Bühring.

In Fig. 4, the shaded region represents the permissible values of ξ and λ for a satisfactory fit to the datum of Bühring. In this figure, L and U denote the loci in the (ξ,λ) plane for which the calculated values give the lower and the upper limits of the polarization datum.

We summarize, below, the upper limits on C_P/MC_A, which is also consistent with the experimental data. We get for Pr^{144} $(0^- \rightarrow 0^+)$

(i) $\xi=-0.05$, for $\lambda=200$,

and

(ii) $\xi=0.045$, for $\lambda=-200$.

For Ho^{166}, we obtain

(i) $\xi=0.048$, for $\lambda=200$,

and

(ii) $\xi=-0.04$, for $\lambda=-200$.

For any other value of λ, the ranges of ξ can be immediately obtained from Fig. 2 and Fig. 4.

IV. DISCUSSION AND CONCLUSIONS

1. We have developed the theoretical formulas for the β longitudinal polarization and the β shape factor[47] in the $0 \rightarrow 0$ (yes) transitions, without any significant ap-

[47] This was derived by M. E. Rose and R. K. Osborn, see reference 24.

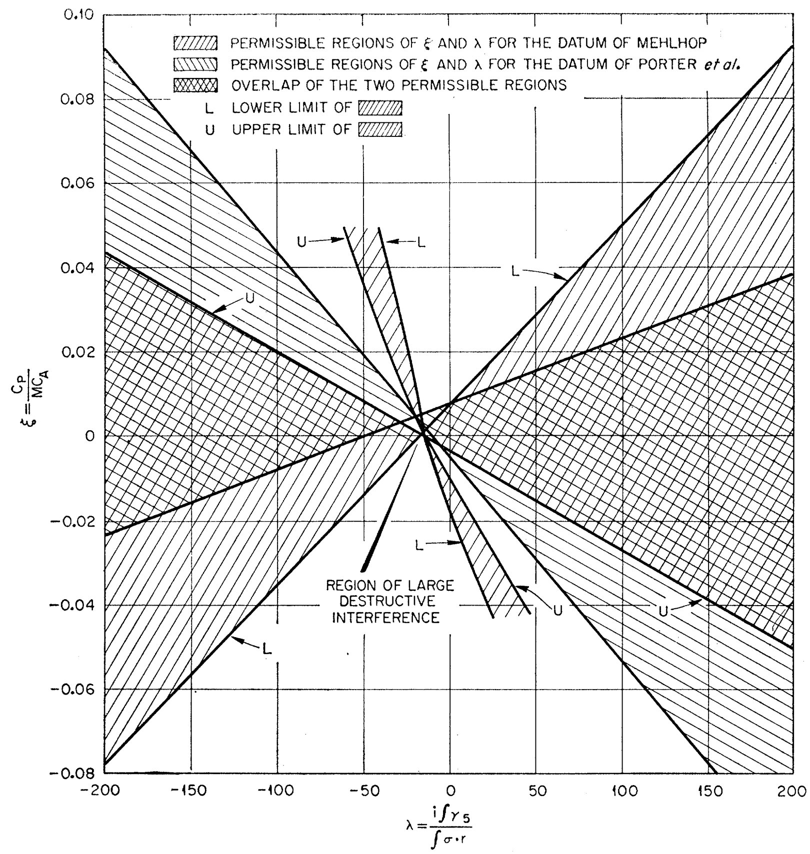

FIG. 2. The permissible values of the parameters $\xi = C_P/MC_A$ and λ the ratio of the nuclear matrix elements, for the polarization and the shape factor data of Mehlhop, and Porter *et al.*, for Pr^{144} $(0^- \rightarrow 0^+)$.

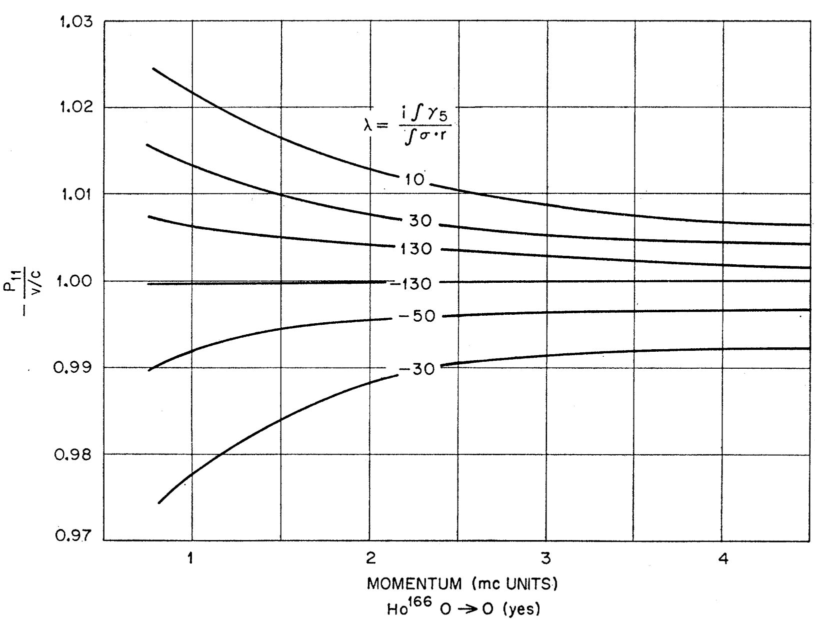

FIG. 3. Calculated longitudinal polarization in units of $-v/c$ versus β momentum for A interaction only. The numbers attached to the curves give λ, the ratio of the nuclear matrix elements.

proximations, using the Rose-Osborn formulation of the pseudoscalar interaction taken together with the axial vector interaction.

2. By the application of these formulas to the most accurate *existing* experimental data on $0 \rightarrow 0$ (yes) beta transitions, we have been able to conclude the following:

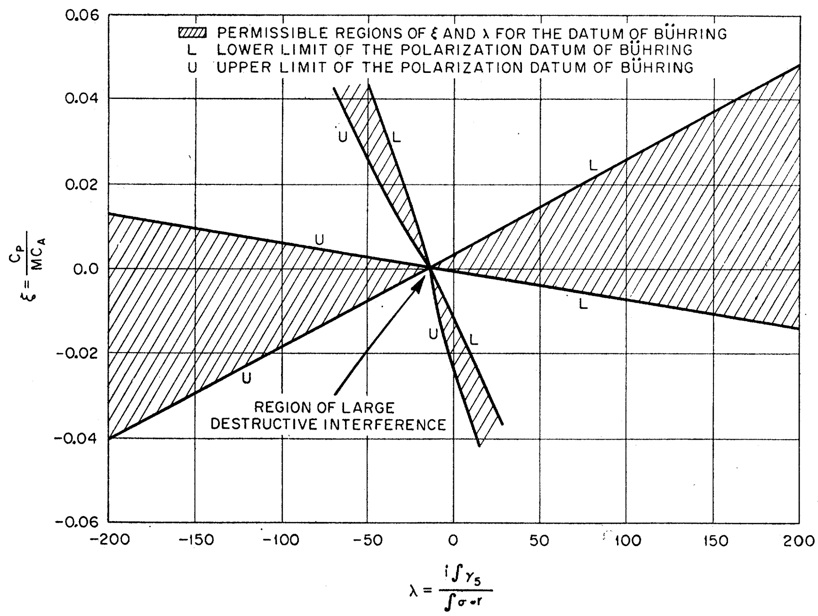

FIG. 4. The permissible values of the parameters ξ and λ for the polarization datum of Bühring for Ho^{166} ($0^- \to 0^+$).

(i) The absence of the pseudoscalar interaction in nuclear beta decay is consistent with the existing data. Therefore, the data on the $0 \to 0$ (yes) transitions do not require any supplementation of the $V-1.2A$ interaction, which is well established by the experiments on the allowed beta transitions.

(ii) A new upper limit on the ratio of the coupling constants of the pseudoscalar interaction and the axial vector interaction, divided by the nucleon mass, can be set and this is

$$|C_P/MC_A| < 0.05,$$

which is about half the previous estimates as reported in the literature. For $C_P/MC_A = 0.05$, the contribution[48] from the pseudoscalar interaction is <0.002.

3. Within the framework of the developed formulas, it is conceivable to improve the estimate of the upper limit of the pseudoscalar contribution to nuclear beta decay provided that

(i) the β longitudinal polarization in the $0 \to 0$ (yes) transition is measured with an accuracy of about 1% at four or five different values of the beta momentum throughout the spectrum, and

(ii) more accurate beta spectrum measurements are performed for the $0 \to 0$ (yes) transitions.

This paper presents a consistent and detailed analysis of the possible pseudoscalar contribution to the nuclear beta interaction. The essential limitations which influence the results of this analysis are the following:

(a) The ratio of the nuclear matrix elements has to be treated as an adjustable parameter.

(b) The presently available measurements of the β longitudinal polarization are not sensitive to a possible contribution from the pseudoscalar interaction. This is mainly so, because these polarization measurements give the average of $P_{11}/(v/c)$ over a large portion of the beta spectrum. The axial vector interaction can, therefore, easily explain these "average" polarization measurements within the stated errors.

However, a plot of longitudinal polarization datum versus beta momentum would be most informative. This can be understood as follows. The values of λ would be restricted[49] for the pure axial vector interaction, so that the calculated values of the polarization give a satisfactory fit to the experimental data. Moreover, additional restrictions[50] on the values which λ can take on arise from the condition that the experimental shape factor be accounted for. It is in this respect that accurate measurements on the shape factor would be extremely useful. Thus, if no value of λ can be found which gives a satisfactory fit to the experimental data for the pure axial vector interaction, then it would imply the existence of a nonvanishing contribution of the pseudoscalar interaction.

[48] We define the contribution of the P interaction as the ratio of the calculated shape factor for the P interaction to the calculated shape factor for the pure A interaction at β kinetic energy of 1 Mev for Pr^{144} ($0^- \to 0^+$). In this case, $\lambda = -200$.

[49] For example, see Fig. 1. Here, for $\lambda > 0$, the values of $-P_{11}/(v/c)$ are >1.00 for low-energy beta particles in contrast to a case when the values of $-P_{11}/(v/c)$ are <1.00, for $\lambda < 0$.

[50] For example, in the case of Pr^{144}, there is no fit to experimental shape factor for the pure axial vector interaction, and λ as given in Eq. (18).

V. ACKNOWLEDGMENTS

One of us (C.P.B.) is grateful to the administration of the Oak Ridge National Laboratory, who made their facilities available for the completion of this work, and to Dr. R. L. Graham for a private communication on Ho^{166}. It is a pleasure to acknowledge our thanks to Dr. T. A. Pond for making available a copy of the dissertation of Dr. W. A. W. Mehlhop.

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Photoneutron Cross Sections of Cobalt and Manganese*

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The total photoneutron yields of Mn^{55} and Co^{59} have been measured from threshold to approximately 30 Mev. Analysis of these data using the Leiss-Penfold matrix indicates that the cross sections for both elements show a splitting in the giant resonance region in accord with the predictions of the classical hydrodynamic model. The Mn^{55} peaks occur at energies of 16.8 ± 0.25 Mev and 19.75 ± 0.25 Mev corresponding to cross sections of 90 mb and 77 mb, respectively. Co^{59} maxima occur at 16.75 ± 0.25 Mev and 18.75 ± 0.25 Mev with cross sections of 109 mb and 92 mb. The cross sections $\sigma(\gamma,n)+\sigma(\gamma,2n)+\sigma(\gamma,np)+\cdots$ integrated to 25 Mev are 627 Mev-mb for Mn^{55} and 709 Mev-mb for Co^{59}. Breit-Wigner resonance lines were fitted to both cross sections and the intrinsic quadrupole moments determined from these fits are $+0.78\pm0.10$ barn for manganese and $+0.76\pm0.11$ barn for cobalt.

INTRODUCTION

As initially pointed out by Okamoto[1] and Danos,[2] if the classical hydrodynamic model of the nucleus affords a reasonable description of the nuclear photoeffect, one might expect, for strongly deformed nuclei, the giant resonance to be split into two separate resolvable resonances. The detailed calculations as performed by Danos[3] show that over the range of nuclear deformations, the splitting of the energy eigenvalues is accurately given by

$$\frac{\omega_b}{\omega_a}=0.911\frac{a}{b}+0.089, \tag{1}$$

where ω_a and ω_b refer to the resonance energies associated with the axes a and b of the spheroid chosen to represent the nuclear shape, a being the axis of rotational symmetry. If an eccentricity ϵ is defined as $\epsilon R^2=a^2-b^2$, where R is the radius of a sphere of equal volume $R^3=R_0^3A$, the intrinsic quadrupole moment of a spheroid with uniform charge distribution can be written as

$$Q_0=\tfrac{2}{5}R_0^2\epsilon ZA^{\frac{2}{3}}. \tag{2}$$

In an effort to substantiate the predictions by Okamoto and Danos, the initial experiments[4–6] were conducted on rare earth elements having large intrinsic quadrupole moments. Recently, Spicer[7] has pointed out that the splitting of the giant resonance of deformed nuclei into two components should be readily observable in the region $9\leq Z\leq 30$. Deformations in this region are comparable to those of rare earth nuclei. In addition, Spicer has re-examined the published cross sections for a number of nuclei of $9\leq Z\leq 30$ and interpreted the results as showing a splitting of the resonance consistent with the hydrodynamic model.

Using the published values of Q_0, the intrinsic quadrupole moment, obtained from microwave spectroscopy or Coulomb excitation, Spicer suggests five other nuclei in the chosen atomic number range in which a splitting of the giant resonance should be clearly observable.

Two of these suggested elements, cobalt and manganese, have been selected and closely examined as to the detailed shape of the total neutron production cross section in the giant resonance region.

EXPERIMENTAL PROCEDURE

Figure 1 is a schematic diagram of the synchrotron area. The x-ray beam is collimated to $\frac{7}{8}$ inch at the sample position by an eight-inch lead collimator located 80 cm from the tungsten target. The center of the neutron house was approximately two and one-half meters from x-ray source.

Photoneutrons are detected by BF_3 counters embedded in a paraffin cube. A thorough description of this method has been published by Halpern.[8] Eight counters were placed symmetrically on a cylinder of

* Supported by the Air Force Office of Scientific Research.
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