to join the two solutions turns out to be just the branching effect we considered in Sec. 3. If we omit the minor stream lines, the result is exactly the one obtained in Sec. 3. The condition under which the minor lines may be omitted is $\sigma/\sigma^m \gg 1$ which is satisfied in our case. This consideration thus justifies the approximate method we used in Sec. 3.

5. ACKNOWLEDGMENT

This problem was pointed out to the author by Professor C. D. Coryell of the Massachusetts Institute of Technology, whose original explanation of the abundance peaks is the basis of this work and to whom the author is greatly indebted for supplying him the problem, references, suggestions, and encouragement.

PHYSICAL REVIEW VOLUME 120, NUMBER 4 NOVEMBER 15, 1960

Mott-Scattering Analysis of Longitudinal Polarization of Electrons from Co^{60}†*

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(Received July 15, 1960)

Mott scattering has been used to analyze the degree of longitudinal polarization of beta particles emitted from radioactive nuclei. The reliability of this method and the influences of the various systematic errors associated with this method on the accuracy of the measurement have been investigated in detail and are discussed. On the basis of a linear extrapolation of the inverse of the Mott asymmetry to zero scatterer thickness, the polarization of 194-kev electrons from Co^{60} was found to be $-(0.994\pm0.057)v/c$ with all known corrections applied. The effects of atomic screening and finite nuclear size have not been included. Using the quoted value for the polarization measured in the pure Gamow-Teller transition in Co^{60} yields $C_A'=(0.7 \text{ to } 1.45)C_A$.

I. INTRODUCTION

THE discovery of the violation of space inversion symmetry in beta decay led to the prediction that electrons were emitted from unoriented nuclei with their spins polarized parallel to their direction of flight.[1–5] The study of the magnitude and direction of the longitudinal polarization of beta particles has been of considerable interest during the past three years as an aid in the investigation of the beta-decay interaction. In addition to providing further unambiguous evidence for parity nonconservation in beta decay, these experiments furnish a measurement of the relative magnitudes of the even and odd beta-coupling constants. The latter quantity is of particular interest in establishing a necessary condition for the validity of the two-component neutrino theory.[6–8] The accuracy attainable with polarization measurements compares very favorably with the other experimental methods that are capable of extracting the same information: (1) the now classical experiments of Wu *et al.*[2] on the beta asymmetry from polarized nuclei, and (2) the measurement of the β-γ (circular polarization) correlation from unpolarized nuclei.[9] Another useful feature of this technique is its applicability to nuclei not suitable for the other two methods.

In this paper the results of a Mott scattering analysis of the longitudinal polarization of beta rays from the allowed Gamow-Teller transition in Co^{60} are presented, with particular emphasis on the systematic errors associated with this technique.

The longitudinal polarization for beta particles in an allowed transition can be written in the following manner[4,10]:

$$P=\frac{Gv/c}{1+bm/E}, \tag{1}$$

where

$$\begin{aligned}\xi G=|M_{\rm F}|^2\Big[&\pm2\,{\rm Re}(C_SC_S'^*-C_VC_V'^*)\\&+\frac{\alpha Z}{p/m}2\,{\rm Im}(C_SC_V'+C_S'C_V^*)\Big]\\+|M_{\rm GT}|^2\Big[&\pm2\,{\rm Re}(C_TC_T'^*-C_AC_A'^*)\\&+\frac{\alpha Z}{p/m}2\,{\rm Im}(C_TC_A'^*+C_T'C_A^*)\Big],\end{aligned} \tag{2}$$

† Sponsored in part by the Office of Naval Research, the Atomic Energy Commission, and the U. S. Air Force Office of Scientific Research.
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$$\xi = |M_{\mathrm{F}}|^2(C_S^2+C_V^2+C_S'^2+C_V'^2) + |M_{\mathrm{GT}}|^2(C_T^2+C_A^2+C_T'^2+C_A'^2), \quad (3)$$

$$b\xi = \pm 2(1-\alpha^2Z^2)\,\mathrm{Re}[|M_{\mathrm{F}}|^2(C_SC_V^*+C_S'C_V'^*) + |M_{\mathrm{GT}}|^2(C_TC_A^*+C_T'C_A'^*)]. \quad (4)$$

The conventional notation of Lee and Yang[1] is being used. C_i and C_i' ($i=S, T, V$, and A) refer to the even and odd beta-coupling constants, respectively; v, m, and E are the velocity, rest mass, and energy of the emitted electron. The upper and lower signs are for electron and positron decays, respectively.

The available experimental evidence from parity experiments,[11,12] in which pseudoscalar quantities are measured, is compatible with complete violation of parity conservation and suggests $C_i = \pm C_i'$. In the two-component neutrino theory, the even and odd coupling constants in Eqs. (2), (3), and (4) are related by $C_i = -C_i'$ ($i=S,T$), and $C_i = C_i'$ ($i=V,A$). Either choice yields a polarization of $-v/c$ for electrons. A similar result for the polarization is obtained from the participation of all four covariants with the maximum parity violation, if one neglects the imaginary part in (2), and sets $b=0$. However, this choice of coupling constants is inconsistent with the two-component neutrino theory with lepton conservation. A measurement of the polarization alone cannot distinguish between these three choices but can only relate the helicity of the neutrino to the types of covariants that are present in the beta interaction: S, $T\rightarrow$ right-handed neutrinos, and left-handed electrons, and V, $A\rightarrow$ left-handed neutrinos and left-handed electrons. In general, this is true for all parity experiments in which either the neutrino helicity or momentum is not measured. If one adopts the choice of coupling constants suggested by neutrino helicity[13] and recoil experiments,[14] experiments on time reversal invariance in the beta interaction,[15–17] and absence of Fierz interference terms,[18–21] i.e.,

$$C_A \gg C_T, \quad C_A' \gg C_T' : C_V \gg C_S, \quad C_V' \gg C_S', \qquad (5)$$
$$C_A/C_V = \text{real number}, \; C_A'/C_V' = \text{real number}, \; b=0,$$

the polarization becomes

$$P = \frac{\mp 2(v/c)|M_{\mathrm{GT}}|^2 C_AC_A' \mp 2(v/c)|M_{\mathrm{F}}|^2 C_VC_V'}{(C_A^2+C_A'^2)|M_{\mathrm{GT}}|^2 + (C_V^2+C_V'^2)|M_{\mathrm{F}}|^2}. \quad (6)$$

For a pure Gamow-Teller or pure Fermi transition, a measurement of the polarization yields a value for the ratio C_i'/C_i ($i=A,V$). An experimental determination of this ratio was part of the purpose of this work.

The experimental observation of the longitudinal polarization of beta rays was first reported by Frauenfelder *et al.*[22] Since then numerous measurements have been performed for both allowed and first forbidden transitions. (See review by Grodzins,[23] and Page.[24]) The renewed interest in the measurement of electron polarization led to an accelerated development of new techniques based on the known spin dependent interactions for electrons and photons. They are: (1) scattering of electrons by the nuclear Coulomb field (Mott scattering); (2) scattering of electrons by electrons polarized in magnetized materials (Möller or Bhabha scattering); (3) spin dependence of Compton scattering; (4) several methods based on the annihilation of positrons in flight, or the formation of positronium. The latter three methods have been employed in the energy region near 1 Mev, and the polarization of electrons (positrons) has been measured to be $-(+)v/c$ within an accuracy of $\pm$(10% to 20%). The accuracy of these measurements has been limited by incomplete knowledge of the depolarization effects and the efficiencies of the polarimeters. The Mott scattering method has been most extensively used by this and other laboratories for electron energies below 500 kev.[22,25–36] Although this method has been employed repeatedly in the past,[37] it possesses many systematic errors and difficulties which in general have not been

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fully investigated or understood. Instrumental asymmetries and multiple scattering effects have led to some recent erroneous interpretations of Mott scattering results.[38–40] The initial measurements reported by us on the polarization of electrons from Co^{60}, Cs^{137}, and Pm^{147} indicated[35,36] that a good absolute quantitative measurement of the polarization by this method, within an accuracy of better than 10%, would require a more careful study of the technique itself. The subsequent experiments on Co^{60} are reported in this paper. Similar investigations, carried on concurrently with this one, have been reported by Cavanagh *et al.*,[25,26] and Bienlein *et al.*[31,32]

II. EXPERIMENTAL METHOD

The spin dependent scattering of electrons by the Coulomb field of a nucleus was first calculated by Mott in 1929.[41] He showed that a transverse spin polarization of the electron gave rise to a left-right asymmetry in the scattering intensity due to a spin orbit force acting on the magnetic moment of the electron moving in the rapidly varying Coulomb field of the nucleus. Under the proper conditions this asymmetry can be as high as 50%, and therefore Mott scattering is capable of providing a sensitive measure of the transverse polarization. For electrons the asymmetry is largest for high Z scatterers, and in general large scattering angles. For positrons, both the magnitudes of the asymmetry and of the scattering cross section are not as favorable as for electrons, due to the Coulomb repulsion of the positron by the scattering nucleus. In general, the Mott method is not suited for measuring positron polarization. Calculations on the asymmetry have also been made by Bartlett and Watson,[42,43] Sauter,[44] and more recently by Sherman.[45,46]

The Mott cross section can be written in the form

$$\frac{d\sigma\,(\theta,\phi)}{d\Omega}=\frac{d\sigma_0\,(\theta)}{d\Omega}\left[1-PS(\theta)\sin(\phi-\phi_0)\right], \qquad (7)$$

where $d\sigma_0(\theta)/d\Omega$ represents the polarization independent cross section, P is the degree of transverse polarization of the incident electrons along the azimuth ϕ_0, $S(\theta)$ is the asymmetry function defined by Sherman,[45] θ is the polar angle of scattering, and ϕ is the azimuthal angle of scattering. The polarization of a partially polarized beam of electrons along a specified axis is defined here as

$$\frac{N(\boldsymbol{\sigma})-N(-\boldsymbol{\sigma})}{N(\boldsymbol{\sigma})+N(-\boldsymbol{\sigma})}=P, \qquad (8)$$

where $N(\boldsymbol{\sigma})$ and $N(-\boldsymbol{\sigma})$ refer to the number of electrons which possess a spin along the direction of this axis and opposite to this direction, respectively. If $I(\phi)$ and $I(\phi+\pi)$ are the measured scattered intensities at azimuth ϕ and $(\phi+\pi)$, then the measured asymmetry, as defined below, is proportional to the polarization.

$$A\equiv\frac{I(\phi)-I(\phi+\pi)}{I(\phi)+I(\phi+\pi)}=-PS(\theta)\sin(\phi-\phi_0). \qquad (9)$$

In principle then, a measurement of these scattered intensities yields a value for the polarization. Appropriate values for $S(\theta)$ used in this work were supplied by Rawitscher.[46] Screening corrections for Mott scattering are not included in this calculation.

The above simple approach is, of course, modified by the many experimental difficulties associated with this technique. Some of these problems have previously been encountered and recognized in the double scattering experiments in which one scattering is used as a polarizer, and the second as an analyzer.[37] The earliest double scattering experiments did not agree with Mott's predictions and failed to show any polarization effects. Later experiments reported only qualitative agreement with predicted cross sections; even in the most recent experiments,[47] quantitative agreement at angles greater than 90° has been poor. The results show consistently smaller asymmetries than predicted by theory, and the source of the discrepancies is never clear. Plural and multiple scattering effects are usually quoted as possible sources of this disagreement. The investigation of these and other sources of error is discussed in the sections that follow.

A. Apparatus

Since the Mott method analyzes transverse polarization, the longitudinal polarization of the beta rays has to be transformed into a transverse one. We chose to use a radial electric field set up between quarter sections of two concentric cylinders to precess the momentum vector relative to the spin, a method first suggested by Tolhoek and DeGroot.[48] This device also fulfilled the function of an energy analyzer. Rotation of the momentum vector through 90° by the electrostatic field however does not completely transform a longitudinal polarization into a transverse one because the electron experiences a magnetic field in its own

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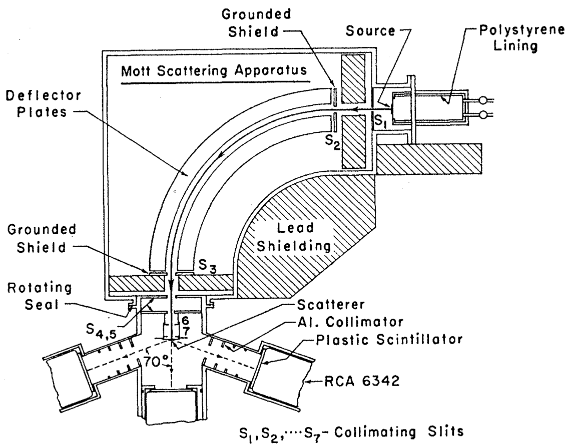

FIG. 1. Mott scattering apparatus. Diagram not to scale.

coordinate frame due to its motion through the electric field of the analyzer. Its spin undergoes a rotation about an axis perpendicular to the plane formed by the electric field vector and its momentum. For the 90° electrostatic analyzer used in this experiment the angle ψ between the electron spin and the original momentum vector has been shown by Tolhoek[37] to be,

$$\psi=\left(\frac{mc^2-m_0c^2}{mc^2}\right)\frac{\pi}{2}.$$

This angle is small for the energies of interest here: $\simeq 18°$ for $v/c=0.6$, and $\simeq 25°$ for $v/c=0.7$. Since the cosine of this angle enters on the right-hand side of Eq. (9), the loss in magnitude to the measured asymmetry is slight. Other forms of spin precessors have been used by other investigators.

The general layout of the apparatus is shown schematically in Fig. 1. The whole apparatus was evacuated to a pressure of approximately 10^{-5} mm of Hg. The mean radius of the analyzer was 20 cm with a gap between the plates of 1.8 cm. Its dimensions were chosen so as to allow a large amount of lead shielding to be placed between source and detector, and to suit the available high-voltage supplies of ± 25 kv applied symmetrically to the plates. The high voltage was adjusted manually to within $\frac{1}{2}\%$. The effect of the fringing fields at the entrance and exit of the analyzer was reduced by placing appropriate grounded shields at these positions. For a cylindrical analyzer the focusing action is only in the plane containing the electric field vector. This limits its transmission, which in this case was approximately 5×10^{-5}. However, this lack of focusing in the plane perpendicular to the plane containing the spin and momentum of the electron, enabled one to minimize one of the principal geometric asymmetries by a not too arduous leveling of the source and scattering foil positions. (One recalls that the maximum Mott asymmetry occurs in this plane. This effect is discussed further in the section on instrumental asymmetries.) For relativistic electrons, the cylindrical electrostatic analyzer is chromatic even for first order focusing. With appropriate collimation, this did not turn out to be very serious. Placing a detector at the position of the scatterer, an energy calibration was performed by making use of the K and L conversion lines from Ag^{110m} (112.5, 90.5 kev), In^{114m} (188.1, 164.1 kev), and Tl^{203m} (263.7, 192.8 kev). Using a thin In^{114m} source the energy resolution of the analyzer was measured to be approximately 5%, and the magnification approximately one.

The beam entered the scattering chamber through a series of circular collimators, designed to define the beam and minimize scattering from the last collimator. The beam diameter was limited to about 1 cm at the position of the scatterer. The scatterer's plane was oriented at 90° to the beam direction. The scattering foils were mounted on aluminum rings which were set into a foil changer so that five different scatterers could be used without breaking the vacuum. Utilizing the scatterers as a mirror in an optical lever arrangement, the orientation of their planes was adjusted perpendicular to the axis of rotation of the scattering chamber. The scattering chamber was constructed with a rotary seal, so that the azimuthal angle of scattering was varied by rotation of the whole scattering head. The relative position of counters and foil remained constant as a function of the azimuthal angle setting. This arrangement assured constant solid angle for the counters relative to the scatterer as they were rotated, and minimized asymmetries due to any nonuniformities in the scattering foil. The whole apparatus was aligned using a cathetometer, and the alignment of the scattering chamber axis (axis of rotation), with the central axis of the analyzer (beam axis), was determined in the same way.

The electron beam uniformity at the position of the scatterer, and its inclination and divergence with respect to the scattering chamber axis were studied with the aid of x-ray film. Optical densitometer traces showed that the beam was uniform within a few

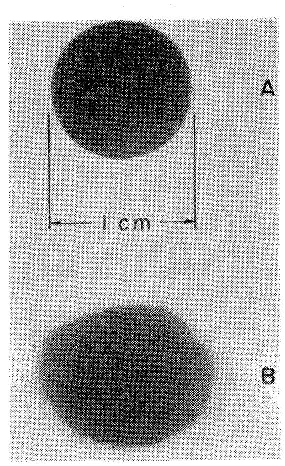

FIG. 2. (A) Beam cross section at the position of the scattering foil. (B) Beam cross section at a position 5 centimeters beyond the scattering foil in the direction of the beam.

percent over its cross section. Its inclination to the scattering chamber axis was found to be less than 1° by this method. Figure 2 is a representative photograph of the beam cross section taken at the position of the scatterer and 5 centimeters beyond this point in the direction of the beam.

Figure 3 shows that from the point of view of obtaining a maximum asymmetry, there exists an optimum choice of polar scattering angle for a given energy. This angle is greater than 90° for the energy of interest. On the other hand, the scattering cross section also decreases rather rapidly with increasing angle. Other considerations in choosing the scattering angle were the higher rate at which the differential cross section changes with angle at forward scattering angles (which is one of the principle sources of instrumental asymmetries), the larger multiple and plural scattering effects in the backward scattering direction suggested by the double scattering experiments, and the greater difficulty at backward angles of preventing electrons scattered from the walls of the scattering chamber from entering the counters. Mostly because of the intensity considerations, and the desire to stay in the transmission direction while utilizing two counters separated 180° in azimuth, the average scattering angle was chosen to be 70°. This angle is not so close to 90° as to emphasize scattering in the plane of the scattering foil. In addition to the obvious advantage of a factor of two in the rate of data accumulation, the use of two counters enabled one to deal with the ratio of counting rates instead of absolute counting rates in handling the data. In this way, spurious asymmetries that were caused by instabilities in the apparatus that similarly affected both counters (e.g., a change in analyzer beam intensity) were eliminated to first order. A third counter located on the beam axis was used as a beam monitor to detect rapid drifts.

Plastic scintillators were used as the electron

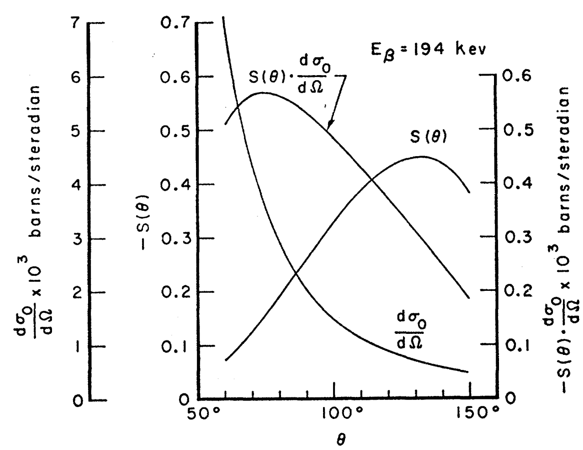

FIG. 3. The asymmetry function $S(\theta)$, and the differential cross section $d\sigma_0(\theta)/d\Omega$ plotted as a function of polar scattering angle θ, for a Au scatterer ($Z=79$) and electron energy $E_\beta=194$ kev.

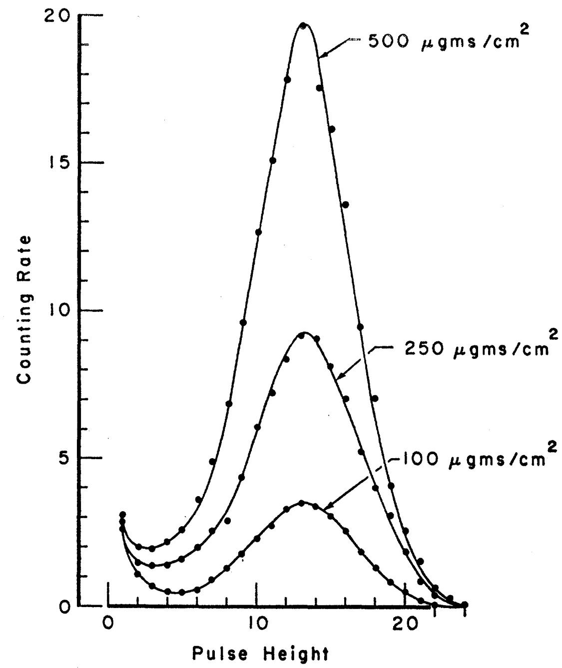

FIG. 4. Pulse-height spectra from scintillation counters for beta particles scattered by various thicknesses of Au scatterer. $E_\beta=188$ kev.

detectors. A series of aluminum collimators in front of the scintillators limited the acceptance apertures of the counters to ±10°. These collimators were also designed to minimize the scattering of electrons from the scattering chamber walls into the counters. Figure 4 shows pulse-height spectra of electrons scattered from gold scatterers of various thicknesses. Low-energy tails exist which are probably due to spurious electrons that enter the counters after they have undergone scattering in the scattering chamber, and counter collimators. These low-energy tails were excluded from the pulses that were counted. The electronics used was conventional, consisting of photomultipliers, linear amplifiers, differential discriminators and scalers. In the latter stages of the experiment, data were taken continuously with a multichannel analyzer.

Because of the importance of multiple and plural scattering in the interpretation of the results, a range of foil thicknesses from 50 to 715 micrograms/cm² was used. Except for the 509 microgram/cm², and 715 microgram/cm² foils, the scattering foils were prepared by vacuum evaporation of gold on Formvar backings ranging in thickness from 15 to 20 micrograms/cm². The latter foils were constructed from multiple layers of 160 microgram/cm² beaten gold foil. The surface density of the foils was determined by weighing to ±5 micrograms/cm². The uniformity of the evaporated foils was better than 1% and they were wrinkle free.

The results of our initial experiments pointed out the necessity of thin and uniform sources. To meet this requirement, Co^{60} sources of thicknesses ranging up to 2 mg/cm^2 were carefully prepared by electroplating them onto 0.8 mg/cm^2 Cu backing foils. The foils were clamped onto aluminum rings which were mounted onto brass source mounts whose dimensions were large compared to the source. An arrangement was provided to change the thickness of the backing so that the depolarization in the source could be studied as a function of both source and backing thickness. The inside of the source mount was lined with polystyrene or teflon to reduce back-scattering from the holder. Pump holes were provided so that the space behind the source could be evacuated.

B. Instrumental Asymmetries

The importance of a correct evaluation of spurious instrumental asymmetries not associated with spin-dependent scattering has already been emphasized. The origins of these asymmetries, and their evaluation are described below.

Regardless of the care taken with the physical alignment of the apparatus, the electron beam axis may not coincide with the axis of rotation of the scattering head. In general this misalignment may be in both position and angle. Either gives rise to asymmetries which vary sinusoidally with the azimuthal angle ϕ.

The asymmetry due to positional misalignment is produced by a variation in the solid angle and the polar scattering angle as the azimuth is varied. In addition, there is an energy spread across the beam profile at the exit of the analyzer. Hence, any lateral movement of the scattering chamber axis relative to the beam axis as the azimuth is varied results in a variation in the average energy of the scattered beam with the azimuthal position of the counters. The Mott scattering cross section is strongly energy dependent, so that this dependence of the average energy of the scattered beam on the azimuthal position of the counters results in a spin independent scattering probability which also depends on the azimuthal angle of scattering. The latter asymmetry is eliminated by using the two counters described earlier, since a change in the scattering probability is mirrored in both counters. This can be seen if the asymmetry is written as

$$A=\frac{[I_1(\phi)I_2(\phi+\pi)/I_2(\phi)I_1(\phi+\pi)]^{\frac{1}{2}}-1}{[I_1(\phi)I_2(\phi+\pi)/I_2(\phi)I_1(\phi+\pi)]^{\frac{1}{2}}+1}\equiv\frac{(L/R)-1}{(L/R)+1}, \quad (10)$$

where I_1 and I_2 refer to the counting rates in counters No. 1 and No. 2, and ϕ and $(\phi+\pi)$ refer to the azimuthal positions of counter No. 1 only.

It can readily be seen how an angular misalignment can lead to a large instrumental asymmetry. The spin-independent differential cross section varies approximately as the inverse fourth power of $\sin(\theta/2)$. If θ' is the angle between the beam axis and the axis of rotation for the scattering head, then

$$\frac{d\sigma_0(\theta-\theta')}{d\Omega}\bigg/\frac{d\sigma_0(\theta+\theta')}{d\Omega}$$

can be quite large. For example, for $\theta'=1°$, $\theta=70°$, and an electron energy of 194 kev, from Fig. 3 this ratio for a gold scatterer is approximately 1.1. Consider the geometric situation illustrated in Fig. 5. Writing the differential cross section,

$$\frac{d\sigma(\theta,\phi)}{d\Omega}=\eta-P\zeta\sin(\phi-\phi_0), \quad (11)$$

where

$$\eta=d\sigma_0/d\Omega \quad \text{and} \quad \zeta=S\eta,$$

then for

$$\left|\frac{\partial\eta}{\partial\theta}\right|\bigg/\left|\frac{\partial\zeta}{\partial\theta}\right|\gg 1 \quad \text{and} \quad \left|\frac{\partial^2\eta}{\partial\theta^2}\right|\theta'\ll\left|\frac{\partial\eta}{\partial\theta}\right|,$$

$$A\simeq\left[\theta'\frac{|\partial\eta/\partial\theta|}{\eta}\sin\phi'-PS\cos\phi_0\right]\sin\phi+\left[\theta'\frac{|\partial\eta/\partial\theta|}{\eta}\cos\phi'+PS\sin\phi_0\right]\cos\phi. \quad (12)$$

Adding to this a similar analysis of the positional misalignment previously mentioned only changes the amplitude and phase of the instrumental terms in Eq. (12). Therefore in general one can write

$$A\simeq(X-PS\cos\phi_0)\sin\phi+(Y+PS\sin\phi_0)\cos\phi. \quad (13)$$

The effect of the variation of polar angle with azimuth on the spin dependent part of the cross section has not been included in the treatment given above. These terms amount to a few tenths percent correction to Eq.

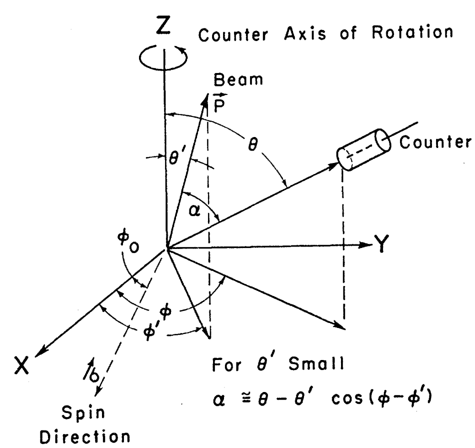

FIG. 5. Geometric considerations for angular misalignment of beam axis with scattering chamber axis.

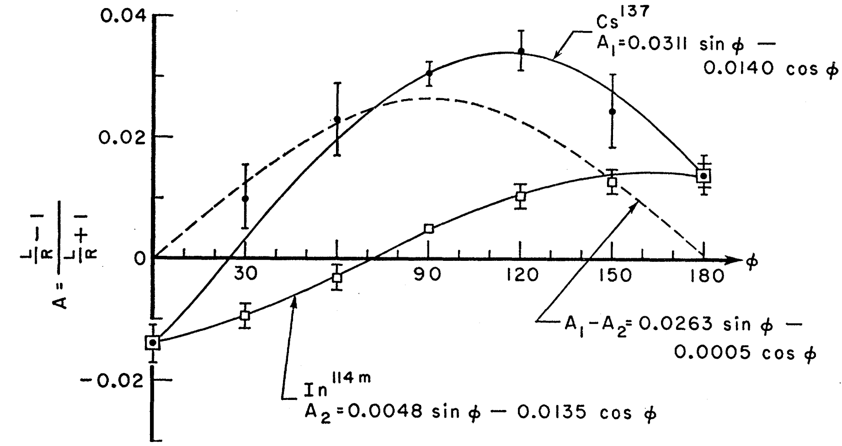

FIG. 6. Measured asymmetry for beta particles from Cs^{137} and K-shell internal conversion electrons from the 192-kev isomeric state in In^{114}. Solid lines are least-squares fits to the data, and the broken line is the difference between the two solid lines.

(13) for our experimental conditions. The error incurred in setting $\phi_0=0$ is of the same magnitude. For the considerations that follow, we assume ϕ_0 to be zero.

Measurements of the beam characteristics with x-ray film, and positional alignments with the cathetometer indicated that the in-phase term X would be small compared to $-PS$. This was confirmed by measurements described below. $|\partial\eta/\partial\theta|/\eta$ varies very slowly with energy in the energy range of interest. Therefore, within the accuracy of the present measurements, one expected that the out-of-phase term Y would remain substantially independent of energy if the geometric parameters did not vary with energy. In effect, a measurement of Y as a function of energy served to monitor the variation with energy of the above geometric asymmetries. In addition, the dependence on scatterer thickness also had to be investigated, due to a possible variation of the geometric asymmetry associated with multiple and plural scattering effects. The geometric term ($X \sin\phi + Y \cos\phi$) was determined experimentally by using the unpolarized K, and L conversion electrons from the decay of the 192-kev isomeric state in In^{114}. The contribution to the asymmetry by the polarized electrons, from the decay of the ground state of In^{114}, was estimated to be $0.0007 \sin\phi$ within a factor of two. This latter uncertainty introduces an almost negligible error in the final result.

Figure 6 illustrates some typical data obtained for the asymmetry A as a function of azimuthal angle ϕ for 128-kev electrons from Cs^{137}, and 188-kev conversion electrons from In^{114}. Gold was used as a scatterer. The solid lines are least square fits to the data. For the Cs^{137} electrons, a phase shift of approximately 25° is introduced by the spin independent asymmetries. A least-squares analysis of the In^{114} internal conversion electron data yielded the following values for X and Y: $X=0.0048\pm0.0001$; $Y=-0.0135\pm0.0002$. On the assumption that the largest contribution to the geometric assymetry came from angular misalignment (which was suggested by the beam profile studies), the above values for X and Y were used to compute that $\theta' \lesssim 0.32°$, and $\phi' \simeq 160°$. One can see that even a very small θ' can lead to a large geometric asymmetry. The broken curve in Fig. 6 represents a subtraction of the other two, and according to this analysis should represent the spin dependent asymmetry and be symmetric about $\phi=90°$.

The possible dependence of the geometric asymmetry on the gold scatterer thickness was investigated by measuring A at 90° and 180° with the conversion electrons from In^{114m}. This measurement was also done for Co^{60} at $\phi=180°$ alone. The scatterer thickness was varied from 100 microgram/cm^2 to 715 microgram/cm^2. No effect was detected within the statistical accuracy of the measurement indicated in Fig. 6. Similarly, measuring the quantity Y for electrons from Co^{60}, Cs^{137}, and In^{114m}, showed no detectable dependence of the asymmetry on energy within the energy limits of measurement. They were the following: 128 and 194 kev for Co^{60}; 128 kev for Cs^{137}; 164 and 188 kev for In^{114m}.

Other possible sources of polarization independent asymmetries were studied. One important one is the asymmetry that could be introduced if both the incident beam and scatterer were nonuniform. This effect was reduced by insuring that the detector was fixed with respect to the scatterer, so that the detector always received electrons scattered from the same position of the scatterer. It has already been stated that the scatterers were uniform to better than 1%, and the beam was uniform over its circular cross section at the position of the scatterer to a few percent. A measurement of the magnitude of this asymmetry was performed by rotating the scatterers through 180° and repeating the asymmetry measurements. No difference was detected for the two positions of each scatterer. A similar procedure carried out with the source orientation showed that no measurable asymmetry was being introduced by a possible off-axis source location.

C. Depolarization Effects

1. Multiple and plural scattering in foils of practical thickness modifies the asymmetry predicted by a Mott

scattering calculation based on single scattering. The difficulties of estimating the contributions of these two effects has already been pointed out in the case of the double scattering experiments.[37] Even foils as thin as 50 mg./cm^2 did not seem to be thin enough in our measurements to ensure single scattering. We have investigated the effect of foil thickness on the measured asymmetry in an effort to determine the above corrections to the single scattering theory. Similar considerations were also given to the depolarization effects in the source material and source backing. To determine the latter, the polarization was measured for various source and backing thicknesses.

2. The measured polarization can be influenced by electrons which are scattered into the transmitted beam from the source holder or analyzer entrance flange. These electrons would possess varied spin orientations which would depend on the scattering processes by which they arrived in the beam. It is, of course, difficult to make a quantitative estimate of their effect on the polarization. Their contribution to the beam intensity at the scatterer was measured by suitably baffling a carefully prepared small source (0.1 cm in diameter) of Co^{60} so that only electrons scattered from the source holder, and in turn, the entrance flange could enter the analyzer. This measurement showed that the scattered electrons only contributed approximately 0.2% to the total beam intensity, and therefore their effect on the polarization was neglected.

3. Scattering of electrons by the spin rotator plates can be another source of error, since electrons with energies greater than the analyzer's transmitting energy are able to reach the position of the scatterer in this manner. The magnitude of this effect was measured using the internal conversion electrons from the 392-kev isomeric state in In^{113}. The isomer was extracted from a sample of Sn^{113}. Except for line broadening due to finite source thickness, this transition provided a very suitable line source of electrons. With the analyzer adjusted for various energies less than the energy of the conversion line, it was found that the electron intensity at the position of the scatterer was less than 1% of those that would be transmitted by the instrument if set to accept electrons from the conversion line. Since the Co^{60} spectrum has an end point energy of only 313 kev, and the intensity is decreasing rapidly above 200 kev, these spurious electrons do not appreciably dilute or influence the polarization of the electron beam. It is estimated that they introduce an uncertainty of less than 1% in the polarization measurement. This uncertainty has been included in the quoted error. The influence of the presence of gamma rays was also found to be negligible.

III. EXPERIMENTAL PROCEDURE

A program of measurement was arranged to minimize the effects due to electronic drifts and other systematic errors already discussed. To measure the asymmetry at an azimuthal angle ϕ, the scattering chamber was set at the desired angle, and both the scattered electron and background events were each accumulated for a period of 0.5 hour. The chamber was then rotated by 180° and the process was repeated. The background measurement involved replacing the scattering foil by a Formvar foil backing, maintaining all other structural characteristics in the scattering head unchanged. The asymmetry was calculated using Eq. (10) for reasons previously discussed. The above measurements were repeated (in varying sequence to eliminate systematic correlations in time) until sufficient statistical accuracy was obtained. A gain calibration of the counting system (excluding the photomultipliers) was performed every two hours. It was found that the gain remained stable to better than a tenth of a percent over this period. This is to be compared with a measurement which indicated that shifts in gain by 1% would produce counting rate changes of approximately 0.2%. Drifts in the photomultiplier gain were found to have a long term character compared to 2 hours, and were monitored periodically by checking the spectrum of the scattered electrons with a multichannel analyzer.

Measurements were performed with scattering foils of thickness 95, 150, 206, 300, 509, and 715 microgram/cm^2. Using the 509 microgram/cm^2 scattering foil, the dependence of asymmetry upon source backing and source thickness was measured by backing the source with nickel foils of thickness 0.8, 2.4, 3.2, and 4.0 mg/cm^2, and using sources with surface densities of 1 mg/cm^2 and 2 mg/cm^2.

IV. RESULTS AND DISCUSSION

The results obtained for the variation of the asymmetry as a function of scatterer thickness for 194-kev electrons from Co^{60} are summarized in Fig. 7. Here the inverse of the asymmetry, $(L+R)/(L-R)$, for $\phi=90°$ is plotted. The geometric asymmetry measured with the In^{114m} source has been suitably subtracted. The

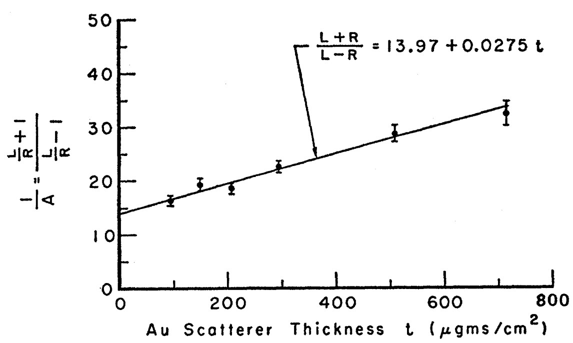

FIG. 7. Inverse of the measured asymmetry versus Au scatterer thickness for 194-kev beta particles from the decay of Co^{60}. The solid line is a least-squares fit to the data, $1/A=13.97+0.0275t$. Source thickness = 1 mg/cm^2, electroplated on a 0.8-mg/cm^2 copper backing.

source had a surface density of 1 mg/cm², and was electroplated on a 0.8 mg/cm² Cu backing foil.

The solid line in Fig. 7 was obtained by fitting the data points to a linear equation of the form $(L+R)/(L-R)=a+bt$. A weighted least square analysis was used. On the basis of a χ^2 test, such a linear relationship is in fact supported by the data for the range of scatterer thicknesses shown in Fig. 7, supplying some empirical justification for a linear extrapolation to zero thickness scatterer. A discussion of the validity of this procedure is given in Appendix A. There it is indicated that if one considers terms in the scattering probability no higher than second order in the thickness of the scatterer, the dependence of the inverse of the asymmetry on the scatterer thickness should be closely linear. The accuracy with which the intercept a can be predicted using a linear extrapolation is not yet clear, however, and awaits a numerical computation. It is also not yet clear to what extent higher order terms than the second in the scattering probability will influence the linearity for the range of scatterers used in this experiment.

The experiment indicates substantial linearity out to 715 micrograms/cm². With the view towards minimizing any effect on the slope due to nonlinearity for thick scatterers, a separate least square analysis was performed for scatterer thicknesses up to 300 micrograms/cm² only. This altered the intercept by 1.3%. Therefore, within the assumption that a linear extrapolation is valid for scatterer thicknesses up to 715 micrograms/cm², the polarization for 194-kev electrons was found to be $P=-(0.954\pm0.053)v/c$. This value for P has been corrected for the geometric asymmetry and finite scatterer thickness in the manner discussed above, but it has not been corrected for depolarization in the source, and source backing materials. The error quoted includes the statistical error in counting, the uncertainties in determining the geometric asymmetry, and an upper limit on the errors that are introduced by the effects discussed in Sec. II-C 2, 3 of this paper. The asymmetry parameter S that was used was an appropriate average over the finite solid angle of the counter

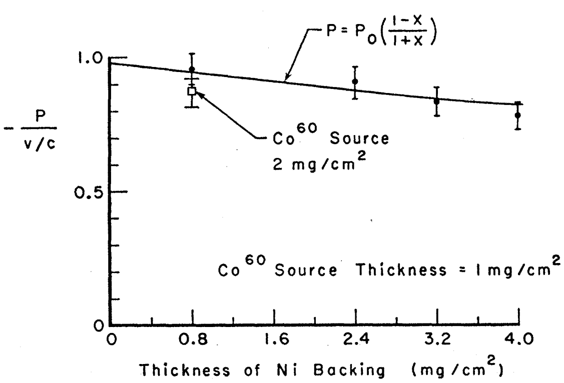

FIG. 8. Measured polarization versus Ni backing thickness.

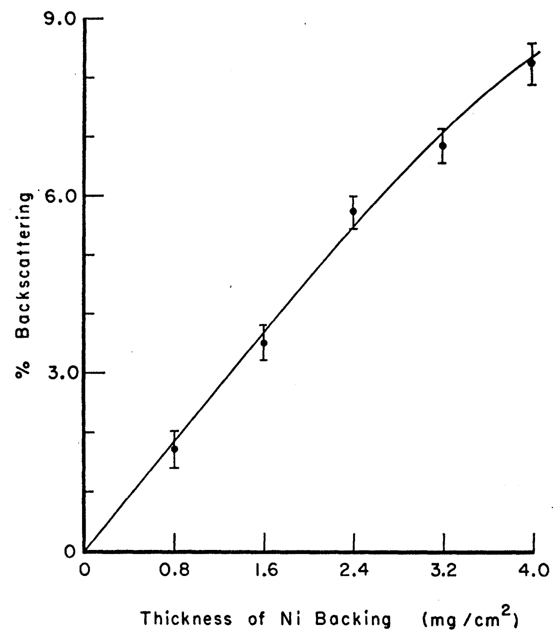

FIG. 9. The dependence of the increase in the beam intensity transmitted by the electrostatic analyzer as a function of the source backing thickness. Mean energy of the analyzer was set at 194 kev.

($\bar{S}=-0.120$). It does not include the effect of atomic electron screening, since a calculation is presently not available at this energy.

The results obtained from measuring the polarization as a function of the thickness of the backing, for a source with a surface density of 1 mg/cm², are shown in Fig. 8. Also plotted is one point for a source with a surface density of 2 mg/cm². Any attempt to calculate the absolute magnitude of a theoretical correction for the depolarization in the source material and its backing, is frustrated by the same difficulties encountered with the calculations on the effect of a scatterer of finite thickness on the asymmetry. In addition, the calculation is further complicated by the fact that one has to consider the effect on the polarization of electrons that are emitted from the nucleus with energies higher than 194 kev, but are degraded and scattered into the energy region acceptable by the analyzer by inelastic processes in the source and backing materials.

An attempt was made to estimate the depolarizing effect of the source backing material by correlating the magnitude of back-scattering with the measured polarization. The increase in intensity due to back-scattering is shown in Fig. 9. The data were obtained by monitoring the electron intensity through the electrostatic analyzer as the backing thickness was varied.

An obvious overestimate of the depolarization due to the backing would be obtained, if one considered the back-scattered electrons to have the relative direction of their spin and momentum reversed in the back-scattering process. This would lead to a depolarization equal to approximately twice the percentage increase in intensity due to back-scattering, i.e., the polarization $P=P_0(1-x)/(1+x)$, where x is the percentage increase in back-scattering, and P_0 is the polarization of the electrons with the absence of any back-scattering. This is represented by the solid line in Fig. 8. That the depolarization is as large as this is very surprising, though the statistical accuracy of the experimental points in Figs. 8 and 9 does not exclude a depolarization of half this limit. Using this as a guide, and taking the data obtained with the 1 and 2 mg/cm^2 sources as an indication of the depolarization in the source material, it is estimated that the depolarization in the 1 mg/cm^2 source backed by a 0.8 mg/cm^2 copper foil is approximately 4% with a possible 50% error. Adding this correction to the value of the polarization obtained from the linear extrapolation of the inverse of the asymmetry to zero thickness scatterer yields $P=-(0.994\pm0.057)v/c$.

The largest uncertainties in the present measurement, excluding those due to counting statistics, arise from the depolarization effects in the source and scatterer. One must remember that the linear extrapolation, on which the result is based, is only an approximate procedure, and its range of validity is unknown. In this spirit, the above error should be expanded to include this uncertainty. With some changes, the present apparatus would enable one to improve on the statistical accuracy of the experiment, but, as has been shown, this is not the limiting factor on the over-all accuracy that can be achieved. To perform an absolute measurement of the polarization with an accuracy of 1% would require a detailed understanding of the depolarization processes, and in addition, a more meticulous evaluation of some of the other systematic errors. The effect of screening on S would also have to be evaluated. All the above corrections increase rapidly in magnitude with decreasing electron energy. Therefore, their evaluation becomes increasingly important in a measurement of the v/c dependence of the polarization in the energy region where v/c is varying rapidly with energy. In view of the considerations presented in this work, the sometimes quoted accuracies of 1 or 2% in the literature we consider presently optimistic for the absolute measurement of the polarization by the Mott method for electron energies less than a few hundred kev.

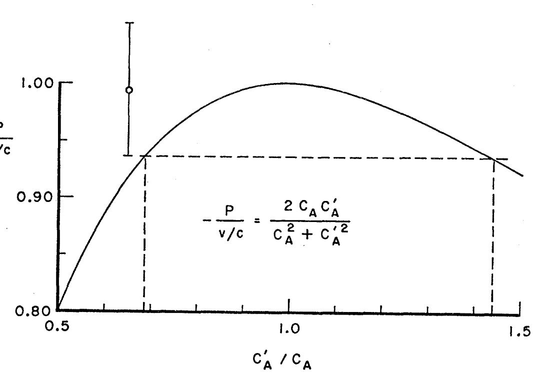

FIG. 10. Polarization predicted by Eq. (6) for beta particles from a pure Gamow-Teller transition as a function of C_A'/C_A. Single datum point shows the measured polarization of 194-kev beta particles from Co^{60}.

Equation (6) can be used to determine the ratio of C_A'/C_A from the experimental value for the longitudinal polarization. Co^{60} is a pure GT transition so that $C_V=C_V'=0$. Thus simplified, Eq. (6) is shown in graphic form in Fig. 10. In the neighborhood of C_A'/C_A close to unity, the polarization is quite insensitive to this ratio. This is equally true for the other pseudoscalars measured in parity experiments, since they all exhibit the same functional behavior with respect to C_A'/C_A. Consequently, the accuracy with which this ratio has been determined has not been very good. This experiment yields $C_A=(0.7 \text{ to } 1.45)C_A'$, and represents one of the more accurate measurements of C_A'/C_A. Reduction of this error by an order of magnitude would be very difficult with the present techniques.

ACKNOWLEDGMENTS

The authors thank Dr. G. Rawitscher for the loan of his IBM-650 computer program, which was used to calculate the Mott scattering asymmetries and cross sections. We would also like to thank Dr. A. R. Quinton for his participation in the earlier phases of this work. We wish to express our thanks to Dr. G. Pieper for the use of a multichannel analyzer, and R. Herman, J. Henkel, and G. Chamberlain for assistance in taking the data. One of the authors (RLG) would also like to thank Dr. H. Wegener and Professor G. Breit for several interesting discussions.

APPENDIX A

In investigating the dependence of the cross section and asymmetry on scatterer thickness it is natural to expect that the first order correction will come from two scattering events in the scattering foil, and will be approximately linear with foil thickness. This would imply, for example, for the number of electrons scattered into the solid angle $d\Omega_s$ at an angle θ_s

$$dn=Nt\sigma(\theta_s)+\frac{N^2t^2}{2}d\Omega_s\int d\Omega_1\frac{\sigma(\theta_1)\sigma(\theta_{1s})}{|\cos\theta_1|}\times[1+S(\theta_1)S(\theta_{1s})\cos\phi_{1s}]-\frac{N^2t^2}{2}\left(1+\frac{1}{|\cos\theta_s|}\right)\times\sigma(\theta_s)d\Omega_s\int\sigma(\theta)d\Omega, \quad \text{(A1)}$$

where Nt is the foil thickness in atoms/cm^2, $\mathbf{\Omega}_1$ is the direction of the scattering, and ϕ_{1s} is the azimuthal angle of $\mathbf{\Omega}_s$ with respect to the plane containing $\mathbf{\Omega}_1$ and the incident direction. The differential scattering cross section $d\sigma/d\Omega$ is denoted by $\sigma(\theta)$ and the asymmetry function $S(\theta)$ is the same as that in Eq. (7).

The first term on the right side of Eq. (A1) represents the contribution of single scattering, the second represents those electrons scattered into the direction $\mathbf{\Omega}_s$ after first being scattered into the direction $\mathbf{\Omega}_1$, and the third term represents those electrons which are scattered out of $\mathbf{\Omega}_s$ after (or before) a first scattering into $\mathbf{\Omega}_s$. The bracketed quantity in the second term contains the effect of the polarizing action of the first scattering, as derived by Mott.[41]

Were it not for the fact that the integrals extend over regions in $\mathbf{\Omega}_1$ for which Eq. (A1) is clearly not valid, this equation (and a similar one involving the product of asymmetry and cross section) would give an expected linear dependence of the correction to the cross section (and asymmetry) due to finite scatterer thickness. However, the following difficulties with Eq. (A1) exist:

(1) The first integral in Eq. (A1) diverges for $\theta_1=\pi/2$. This corresponds to an electron first being scattered into the plane of the foil, thereby giving an infinite probability for a second scattering.

(2) If one uses a Rutherford or Mott form for $\sigma(\theta)$, the integrals also diverge for $\theta_1=0$ and $\theta_{1s}=0$ ($\mathbf{\Omega}_1=\mathbf{\Omega}_s$). However, one might imagine this divergence to be removed by considering the effect of electron shielding. Nevertheless, even with a modification of $\sigma(\theta)$ to include effects of electron shielding, the probability of electron scattering at small angles is high, and Eq. (A1) must be modified to include multiple scattering effects. These events of higher multiplicity of electron scattering can give corrections of low order in t.

In the treatment of this problem by Wegener[49] the first difficulty listed above is resolved by evaluating the integral in the second term on the right side of Eq. (A1) near $\theta_1=\pi/2$. The integral is kept from diverging by taking into account the fact that an electron scattered into the plane of the foil has only a finite "life expectancy" in the foil because of multiple scattering. Using reasonable approximations to the form $\sigma(\theta)$ and assuming $S(\theta)\rightarrow 0$ as $\theta\rightarrow 0$, Wegener obtains

$$dn_1=Nt\sigma(\theta_s)d\Omega_s[1+\eta_2 f\rho t], \qquad \text{(A2)}$$

where ρt is the foil thickness in g/cm², η_2 is a function of θ_s only, and f is related to L, the mean path length of an electron scattered into the plane of the scattering foil at the center of the foil. Specifically, f is given by

$$f=\int_{\theta_{\min}}^{1}\frac{d\cos\theta_1}{\cos\theta}=\ln\frac{1}{\cos\theta_{\min}}=\ln(2L/t), \qquad \text{(A3)}$$

with $L=\frac{1}{2}t/\cos\theta_{\min}$. Wegener evaluates L by using the usual multiple scattering description of the angular and spatial spreading of a beam in traversing a medium, and obtains

$$f\simeq 1.5-\tfrac{1}{3}\ln(2\lambda t), \qquad \text{(A4)}$$

where $2\lambda t$ is the mean square scattering angle for multiple scattering in a depth t.

In the treatment of the difficulty (2) listed above, Wegener expresses Eq. (A1) in the more general form[50]

$$dn=d\Omega_s N\int_0^t dz\int d\Omega_1\int d\Omega_2 \times\frac{f_1(0,z;\,\mathbf{\Omega}_0,\mathbf{\Omega}_1)\sigma(\theta_{12})f_2(z,t;\,\mathbf{\Omega}_2,\mathbf{\Omega}_s)}{|\cos\theta_{12}|}. \qquad \text{(A5)}$$

Here $f_1(0,z;\,\mathbf{\Omega}_0,\mathbf{\Omega}_1)d\Omega_1$ is the probability of an electron changing direction from $\mathbf{\Omega}_0$ to $\mathbf{\Omega}_1$ in a depth z with normal incidence on the foil. Similarly $f_2(z,t;\,\mathbf{\Omega}_2,\mathbf{\Omega}_s)d\Omega_s$ is the probability of an electron changing direction from $\mathbf{\Omega}_2$ to $\mathbf{\Omega}_s$ and emerging from a depth z in the foil. (The present application is to forward scattering, with $\cos\theta_s>0$. Analogous expressions can be used for $\cos\theta_s<0$ which is the focus of Wegener's attention.) The $\cos\theta_{12}$ terms in Eq. (A5) are now expanded both around $\theta_1=0$ and $\mathbf{\Omega}_2=\mathbf{\Omega}_s$ to give up to quadratic terms in t, using the multiple scattering distributions for f_1, f_2 along the lines of Bothe.[51] This leads to a generalization of Eq. (A2) of the form

$$dn=Nt\sigma(\theta_s)d\Omega_s[1+(\eta_1 h+\eta_2 f)\rho t], \qquad \text{(A6)}$$

where η_1 is a function only of θ_s and[52]

$$C\frac{1-\beta^2}{\beta^4}h\rho t \qquad \text{(A7)}$$

is the mean square scattering angle for multiple scattering in a gold foil of thickness t.

Similar treatment for the spin dependent term in the scattering leads to a generalization of Eq. (A6),

$$dn=Nt\sigma(\theta_s)d\Omega_s\{[1+(\eta_1 h+\eta_2 f)\rho t][1+PS(\theta_s)\sin\phi] +(\nu_1 h+\nu_2 f)\rho tPS(\theta_s)\sin\phi\}, \qquad \text{(A8)}$$

where ν_1 and ν_2 are θ_s dependent terms, P is the transverse polarization of the electron beam, ϕ is the azimuthal angle referred to the direction of the transverse polarization, and $S(\theta_s)$ is the asymmetry function tabulated by Sherman.[45] The result for a right left asymmetry measurement is

$$\frac{L+R}{L-R}=\frac{1+(\eta_1 h+\eta_2 f)\rho t}{PS[1+(\eta_1+\nu_1)h\rho t+(\eta_2+\nu_2)f\rho t]} =\frac{1-(\nu_1 h+\nu_2 f)\rho t}{PS(\theta_s)} \qquad \text{(A9)}$$

[49] H. Wegener, Z. Physik **151**, 252 (1958).

[50] Since θ_1 and θ_{2s} are considered small in Eq. (A5), the term involving the product of asymmetries in Eq. (1) is neglected. Here θ_{2s} is the angle between the directions $\mathbf{\Omega}_2$ and $\mathbf{\Omega}_s$.

[51] W. Bothe, Sitzber. heidelberg. Akad. Wiss. Math-naturw. Kl. Abhandl. **7**, 305 (1951).

[52] The constant C (18.9 cm²/g for gold) is $4\pi N_0(e^2/mc^2)^2(Z^2/A)$ where N_0 is Avogadro's number and Z, A are the atomic number and weight of the scatterer.

valid in the spirit of $(\nu_1 h+\nu_2 f)\rho t$ and $(\eta_1 h+\eta_2 f)\rho t$ being small compared to 1, a limitation which should be no more serious than the neglect of higher terms in the expansion of Eq. (A5).

The following features of Wegener's results should be mentioned:

(1) Since h and f are determined from multiple scattering calculations, both are logarithmically dependent on scatterer thickness and caution must be used in any linear extrapolation to zero thickness to obtain P.

(2) There appear to be several errors in sign in the expressions for η_1 and ν_1 for $\cos\theta_s>0$ only. These have entered the numerical tables.

(3) In the calculation of f, multiple scattering theory has been used and the mean square width of the spatial distribution appears to have been taken too large by a factor of 2.

(4) The contributions of the regions outside the multiple scattering range have been neglected.

Since several of the assumptions and approximations made in the above treatment are questionable, the calculation has been repeated along the following lines:

In Eq. (A5) the expansion of θ_{12}, for small θ_1 and θ_{2s} was carried out, but the final result is expressed in terms of the average of θ_x^2, $\theta_x x$, x^2, etc., over the distribution

$$f(\theta_x,x,\theta_y,y),$$

where θ_x, θ_y, and x, y are the angular deflection and lateral displacement of the particles in the distribution f. One can evaluate these averages without assuming that only multiple scattering contributes to f. For example, one can find the average value of θ^2 as follows (the integrals over x and y have been done, the explicit dependence on z has been suppressed, and θ_1 stands for both θ_{1x} and θ_{1y}).

The distribution $f(\theta_1)$ satisfies the equation

$$\frac{\partial f(\theta_1)}{\partial z}=-N\frac{f(\theta_1)}{|\cos\theta_1|}\int\sigma(\theta_2)d\Omega_2+N\int\frac{f(\theta_2)}{\cos\theta_2}\sigma(\theta_{12})d\Omega_2, \quad \text{(A10)}$$

where the terms on the right represent those electrons which are scattered out of and into the direction $\mathbf{\Omega}_1$, respectively. If one assumes that $f(\theta)$ is restricted to forward angles only, multiplies by $\cos\theta_1$ and integrates over $d\Omega_1$, one finds

$$\frac{\partial}{\partial z}\langle\cos\theta_1\rangle=-N\int\sigma(\theta_2)d\Omega_2+N\int\int d\Omega_1 d\Omega_2\frac{\cos\theta_1}{\cos\theta_2}\sigma(\theta_{12})f(\theta_2), \quad \text{(A11)}$$

where $\langle\ \rangle$ signifies an average over the distribution. Defining $\theta_3=\theta_{12}$ and measuring the direction $\mathbf{\Omega}_3$ relative to $\mathbf{\Omega}_2$, one can rewrite the second integral on the right as

$$\int\int d\Omega_2 d\Omega_3\frac{\cos\theta_2\cos\theta_3+\sin\theta_2\sin\theta_3\cos\phi_{23}}{\cos\theta_2}\sigma(\theta_3)f(\theta_2).$$

Using the azimuthal symmetry of $\sigma(\theta_3)$, the term in the azimuthal angle ϕ_{23} vanishes and one gets, since $\int f(\theta_2)d\Omega_2=1$,

$$\int d\Omega_3\cos\theta_3\sigma(\theta_3).$$

One then can write

$$\frac{\partial}{\partial z}\langle 1-\cos\theta\rangle=N\int d\Omega\sigma(\theta)(1-\cos\theta) \quad \text{(A12)}$$

a result exactly independent of z since $f(\theta)$ no longer appears on the right side. For small angles Eq. (A12) can be written as

$$\langle\theta^2\rangle=Nt\int\theta^2\sigma(\theta)d\Omega=C\frac{1-\beta^2}{\beta^4}h'\rho t. \quad \text{(A13)}$$

In the spirit of the exact result of Eq. (A12) the upper limit for the integral in Eq. (A15) is to be taken as being of order unity, rather than the upper limit for the multiple scattering region. This change in the upper limit implies that contributions from the plural scattering region add to those from the multiple scattering region. The two effects together are correctly described by an h' which is independent of t, since the integral in Eq. (A13) does not depend on the angle which divides plural from multiple scatterings.

Consideration of the terms $\langle\theta_x x\rangle$ and $\langle x^2\rangle$ leads to results analogous to Eq. (A13). Eventually one finds that the correct inclusion of plural scattering leads to results identical with Eqs. (A6), (A8), and (A9) except that h defined in Eq. (A7) is to be replaced by h' as defined by Eq. (A13). If θ_0 is the minimum angle dictated, as in multiple scattering theory, by screening considerations and $\theta_{\max}$ is the maximum angle (of order unity), h' may be written as

$$h'\cong 2\ln(\theta_{\max}/\theta_0). \quad \text{(A14)}$$

It therefore appears that the h in Wegener's expression should be replaced by a thickness independent h' corresponding to the mean square scattering angle for the plural as well as multiple scattering regions. Since the $\ln t$ term in f was much less prominent than that previously in h, it appears that a linear extrapolation to zero thickness is a much safer procedure than previously imagined.

One should now consider the fact that the separation of the contributions of the integrals in Eq. (A1) into three regions around $\theta_1=\frac{1}{2}\pi(\theta_{1s}=\frac{1}{2}\pi-\theta_s)$, $\theta_1=0$, and $\theta_{1s}=0$ ($\mathbf{\Omega}_1=\mathbf{\Omega}_s$) is not well defined. In fact for $\theta_s=70°$, it might seem reasonable to consider the regions around $\theta_1=\frac{1}{2}\pi(\theta_{1s}=\frac{1}{2}\pi-\theta_s)$ and $\theta_{1s}=0$ to correspond to cones of half-angle $\frac{1}{2}(90°-70°)=10°$, in order to prevent any extensive overlap between them. Similarly it may be reasonable to consider the region around $\theta_1=0$ to correspond to a cone of half angle about 35°. To be sure, this procedure is quite crude, but it may suggest the direction in which the previous numerical results should be modified.

In the above spirit one is led to a modification of Eq. (A6) [and of Eq. (A8)] of the form

$$dn=Nt\sigma(\theta_s)d\Omega_s[1+(\eta_{1a}h_a+\eta_{1b}h_b+\eta_2\bar{f})\rho t], \quad \text{(A15)}$$

where

$$\bar{f}\cong f+\ln[\cos\tfrac{1}{2}(\tfrac{1}{2}\pi-\theta_s)], \quad \text{(A16)}$$

$$h_a\cong 2\ln[\theta_s/\theta_0], \quad \text{(A17)}$$

$$h_b\cong 2\ln[\tfrac{1}{2}(\tfrac{1}{2}\pi-\theta_s)/\theta_0]. \quad \text{(A18)}$$

Equation (A16) follows from the dependence of f on the upper limit of the integral in Eq. (A3), and Eqs. (A17) and (A18) follow from Eq. (A15). The θ_s-dependent quantities η_{1a} and η_{1b} are

$$\eta_{1a}=4.7\frac{\text{cm}^2}{\text{g}}\frac{1-\beta^2}{\beta^4}\left[1-\frac{\sigma'}{\sigma}\cos\theta_s+\frac{\sigma''}{\sigma}\frac{\sin^2\theta_s}{2}\right], \quad \text{(A19)}$$

$$\eta_{1b}=4.7\frac{\text{cm}^2}{\text{g}}\frac{1-\beta^2}{\beta^4}\times\left[\sec^2\theta_s-\frac{\sigma'}{\sigma}\sec\theta_s+\frac{\sigma''}{\sigma}\frac{\sin^2\theta_s}{2}\right]\Big/|\cos\theta_s|, \quad \text{(A20)}$$

where the prime indicates differentiation with respect to $\cos\theta_s$ and the cross sections are evaluated at θ_s. Similarly

$$\nu_{1a}+\eta_{1a}=4.7\frac{\text{cm}^2}{\text{g}}\frac{1-\beta^2}{\beta^4}\left[1-\tfrac{1}{2}\cot^2\theta_s-\frac{(S\sigma)'}{S\sigma}\cos\theta_s+\frac{(S\sigma)''}{S\sigma}\frac{\sin^2\theta_s}{2}\right], \quad \text{(A21)}$$

$$\nu_{1b}+\eta_{1b}=4.7\frac{\text{cm}^2}{\text{g}}\frac{1-\beta^2}{\beta^4}\left[\sec^2\theta_s-\tfrac{1}{2}\csc^2\theta_s-\frac{(S\sigma)'}{S\sigma}\sec\theta_s+\frac{(S\sigma)''}{S\sigma}\frac{\sin^2\theta_s}{2}\right]\Big/|\cos\theta_s|. \quad \text{(A22)}$$

A term in β^2 arising from relativistic spin rotation in the small angle scattering has been neglected in ν_{1a} and ν_{1b}. It is numerically unimportant.

Because of the uncertainty in the limits needed for the calculation of h_a, h_b, and $\bar{f}$, a numerical estimate of the correction to be applied for finite foil thickness is dangerous. The primary value of the preceding discussion is that it makes plausible an approximately linear extrapolation to zero thickness.

Wegener has recently[53] extended his previous work and obtains agreement with the Oak Ridge[54] and Erlangen[55] experiments, both at backward angles. A more detailed analysis of the in-plane scattering has been developed, including some consideration of the overlap between the in-plane and small-angle regions. It is likely that these considerations are more accurate for backward scattering since the effects on the asymmetry are in the same direction for the in-plane and small-angle scattering. For forward scattering it may prove necessary to use Eq. (A1) directly for reliable numerical corrections in order to avoid uncertainties in the division between the various regions. However, Wegener's recent work should lead to a more reliable cutoff for the region near $\theta_1=\pi/2$, to which he has applied detailed multiple-scattering theory.

[53] H. Wegener, Bull. Am. Phys. Soc. **5**, 238 (1960).
[54] H. B. Willard (private communication).
[55] H. Wegener (private communication).

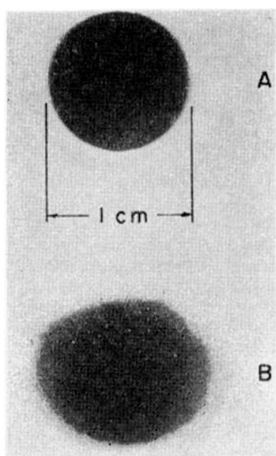

FIG. 2. (A) Beam cross section at the position of the scattering foil. (B) Beam cross section at a position 5 centimeters beyond the scattering foil in the direction of the beam.