experiments can be interpreted as indicating the system H^3+p to have two broad levels $J=2^-$, isotopic spin $T=0$ at 22 Mev and $J=1^-$, $T=0$ about 1–2-Mev higher. These experiments have been performed with proton beams from 1 to 12 Mev, which correspond to excitation energies in He^4 from about 21 to about 29 Mev.

(c) $He^4(p,p)$

Experiments performed with 32 Mev[13] and 40 Mev[14] protons give no evidence of excited states in He^4. The experiments performed by Selove *et al.*[15] with 95-Mev protons could indicate the existence of a broad virtual level, or group of levels, with $\Gamma\sim 10$ Mev about 25 Mev above the ground state. The proton energy resolution was of the order of 1 Mev.

It is seen that most of the experiments quoted above give evidence of broad resonances with a Γ of several Mev. None give evidence of the narrow resonances found in the present work. It should be noted however that of all the quoted experiments only the $T(p,n)$ measurements reported by Bogdanov *et al.*[11] had sufficiently good proton energy resolution (about 0.1 Mev) and an appropriate He^4 excitation energy interval to have given evidence of the resonances found here. The fact that narrow resonances are distinguished only in the present experiment may be due to the relatively good energy resolution employed and to the selective nature of the (γ,p) process.

At photon energies in the neighborhood of the giant resonance, electric dipole absorption is expected to be predominant. Thus in the reaction $He(\gamma,p)$ the most important contribution should to come from $J=1^-$ He^{4*} states of $T=1$, since the selection rules do not allow $\Delta T=0$ dipole absorption by nuclei with $A=2Z$.

These states may be expected in He^4 at excitation energies as high as found in the present work (>20 Mev) because in the nuclei O^{16}, C^{12}, and Be^8 ($A=4n$ $=2Z$) the lower excited states with $J=1$ and $T=1$ are located many Mev above the ground state and the corresponding excitation energies are higher for lighter nuclei [O^{16}: $E_x=13.09$ Mev, $J=1^-$ ($T=1$). C^{12}: $E_x=15.11$ Mev, $1^+(1)$; $E_x=17.23$ Mev, $1^-(1)$. Be^8: $E_x=17.64$ Mev, $1^+(1)$.][16,17] In the reaction $T(p,n)$, He^4 states having $T=0$ and $T=1$ may be excited (T transitions $0\rightarrow 0$ and $1\rightarrow 1$). The fact that peaks have been found only in the $He(\gamma,p)$ reaction might indicate that transitions involving the states with $J=1^-$, $T=1$ are weak in the $T(p,n)$ reaction in comparison with other transitions.

We expect that the $T=1$ levels distinguished in the photoproton spectrum would be also revealed by the $T(p,\gamma)$ reaction at proton energies higher than have been used in experiments already reported. Also experiments on the photoneutron spectra from He should give the same indication as the photoproton spectra; this experiment is in process.

It is relevant to note that a level whose width is only 100 kev has been found recently at 16.7 Mev in He^5 by means of the $He^4(n,n)$ reaction.[18]

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[12] A. J. Baz and J. A. Smorodinskii, J. Exptl. Theoret. Phys. (U.S.S.R) **27**, 382 (1954).
[13] J. Benveniste and B. Cork, Phys. Rev. **83**, 894A (1951).
[14] R. M. Eisberg, Phys. Rev. **102**, 1104 (1956).
[15] W. Selove and J. M. Teem, Phys. Rev. **112**, 1658 (1958).
[16] F. Ajzemberg-Selove and T. Lauritsen, Nuclear Phys. **1**, 11 (1959).
[17] W. E. Burcham, *Encyclopedia of Physics* (Springer-Verlag, Berlin, 1957), p. 182.
[18] T. W. Bonner, F. B. Prosser, and J. Slattery, Phys. Rev. **115**, 398 (1959).

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Interpretation of Isomeric Cross-Section Ratios for (n,γ) and (γ,n) Reactions*

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The relative probability of forming each member of a pair of nuclear isomeric states has been compared with theoretical predictions in order to learn which nuclear parameters can be determined from these data. For thermal neutron capture reactions, the observed ratios do not give much information about the dependence of the nuclear level density on spin, but they are consistent with a spin cutoff factor, $\exp[-(J+\frac{1}{2})^2/2\sigma^2]$, where $\sigma\leq 5$. The calculations are sufficiently consistent with experiment to make their predictions usable as a guide for assigning spins to the compound states formed in thermal or resonant energy neutron capture. For (γ,n) reactions, the calculations reproduce the energy dependence of the experimentally observed isomeric cross-section ratios. In order to obtain quantitative information about the spin dependence of the nuclear level density, it is necessary to consider reactions where particles are emitted which can carry off enough angular momentum to reach many spin states of the residual nucleus.

1. INTRODUCTION

THE relative probability of forming each state of an isomeric pair seems to be governed mainly by the spin differences between the states which decay to the isomers and the isomer spins themselves. In the many cases (encountered in radioactivity) in which a third low-lying state can decay to either of the isomers, the well-known preference of the photon transition of

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low multipolarity is a valuable guide to the spin of this third excited state. In as much as the preference for low multipolarity is assumed to persist even for the higher energy gamma rays emitted in a gamma-ray cascade (e.g., following neutron capture), the relative formation of the isomeric states is an indication of the spins both of the states participating in the cascade and of the initial state. The existence of a multistep cascade tends to broaden the distribution of spins of the states participating but a qualitative connection between the relative formation probability of the isomers and the spin of the initial state persists. Inferences from the isomeric ratio for neutron capture have in some cases been used to decide which isomeric level has a spin close to that of the capturing state.[1–3]

The purpose of the present work is to show the degree to which the isomeric ratio can give quantitative or semiquantitative information about the dependence of the energy level density on spin and the spins of the initial compound states formed in a nuclear reaction. In view of the lack of information about the detailed description of nuclear states at high excitation energy, and the complexity of the de-excitation processes, any calculation must be rather crude. Various workers[4–6] have performed calculations relating isomeric ratios to the initial angular momentum deposited in the nuclear reaction and the final isomer spins. In this and a companion paper[7] a more detailed treatment of the de-excitation process has been attempted in order to relate isomeric cross-section ratios more quantitatively to the spin dependence of the nuclear level density and the multiplicity and multipole character of the gamma-ray cascade.

Among the important factors which determine the isomeric ratio are: (1) the spins of the compound nuclear states, (2) the number and types of steps in the de-excitation of the compound state; this depends on the excitation energy, (3) the angular momentum carried away at each step, (4) the probability of forming states of different spins during each step of the cascade, and finally (5) the spins of the isomeric states.

It has been suggested[1] that if the energy of the neutrons which are captured is increased so that capture occurs into many levels of all possible angular momenta, the isomeric cross-section ratio σ_m/σ_g might approach as a limit the ratio of the "statistical weights" $(2I_m+1)/(2I_g+1)$. In view of the above discussion the reason why this should be expected is not clear.

2. NEUTRON CAPTURE

In order to make any detailed calculations more specific assumptions have to be made in terms of the general considerations outlined above.

(1) It is assumed that for thermal or resonant energy neutrons only s-wave neutrons are captured, so that the spin of the capturing state in the compound nucleus is given by $I\pm\frac{1}{2}$, where I is the spin of the target nucleus.

(2) The average number of steps in the gamma-ray cascade (the gamma-ray multiplicity) has been measured[8–10] for quite a few nuclei and is approximately 3 to 4. The multiplicity has been kept as a variable in the calculations, although the calculations are in general consistent with the measured multiplicity.

(3) The gamma-ray cascade is believed to consist mostly of dipole radiation.[11–13] In the bulk of the calculations pure dipole radiations have been assumed, although a few calculations for quadrupole radiations have been performed and will be mentioned later. It has been assumed that levels of both parity are present in equal number, so that the parity changes have not been followed in the cascade process. The parity of the initial compound state and of the final isomeric states might become important if one makes the more restrictive assumption of electric dipole radiation. However, this effect would be largely washed out if the distribution about the average number of gamma rays per cascade is broad enough so that there are approximately equal numbers of cascades with even and odd numbers of transitions.

(4) Of the different factors that have to be taken into account, the relative probability of forming states of different spins is most model dependent.

The total radiation width for emission of dipole radiation from a state of spin J_c and initial excitation energy B can be written as[14]

$$\Gamma_\gamma=C\int_0^B f(E,B,J_c,J_f)E^3\frac{\rho'(B-E)}{\rho(J_c,B)}dE, \qquad (1)$$

where C is constant, $f(E,B,J_c,J_f)$ is a model-dependent

[1] E. Segrè and A. C. Helmholz, Revs. Modern Phys. **21**, 271 (1949).
[2] M. Goldhaber and R. D. Hill, Revs. Modern Phys. **24**, 179 (1952).
[3] E. der Mateosian and M. Goldhaber, Phys. Rev. **108**, 766 (1957).
[4] L. Katz, L. Pease, and H. Moody, Can. J. Phys. **30**, 476 (1952).
[5] J. W. Meadows, R. M. Diamond, and R. A. Sharp, Phys. Rev. **102**, 190 (1956).
[6] S. M. Bailey, University of California Radiation Laboratory Report UCRL-8710, April 1959 (unpublished).
[7] R. Vandenbosch and J. R. Huizenga, following paper [Phys. Rev. **120**, 1313 (1960)].
[8] C. O. Muehlhause, Phys. Rev. **79**, 277 (1950).
[9] T. E. Springer and J. E. Draper, Bull. Am. Phys. Soc. **4**, 35 (1959).
[10] L. V. Groshev, A. M. Demidov, V. N. Lutsenko, and V. I. Pelekhov, *Proceedings of the Second United Nations International Conference on the Peaceful Use of Atomic Energy, Geneva, September, 1958* (United Nations, Geneva, 1958), Vol. 15, Paper P/2029.
[11] A. G. W. Cameron, Can. J. Phys. **35**, 666 (1957).
[12] V. M. Strutinski, L. V. Groshev, and M. K. Akimova, Nuclear Phys. **16**, 657 (1960).
[13] B. B. Kinsey and G. A. Bartholomew, Phys. Rev. **101**, 1328 (1956).
[14] B. B. Kinsey, *Handbuch der Physik*, edited by S. Flügge (Springer-Verlag, Berlin, Göttingen, Heidelberg, 1957), Vol. 40, Part I, pp. 314–319.

factor, E is the transition energy of the radiation,

$$\rho'(B-E)=\sum_{|J_c-1|}^{J_c+1}\rho(J,B-E)$$

is the total density of levels at excitation energy $B-E$ which are accessible to the initial state (spin J_c) by emission of dipole radiation, and $\rho(J_c,B)$ is the density of levels of spin J_c at an excitation energy B. Since we are concerned only with the relative probability for decaying to different spin states, the constant C and the level density factor $\rho(J_c,B)$ can be ignored.

The factor $f(E,B,J_c,J_f)$ should take into account the giant resonance and other specific properties of the matrix elements connecting the states involved in the transition. Because of the complexity of the nuclear states at these excitation energies and our lack of a detailed description of these states, the factor $f(E,B,J_c,J_f)$ has been taken as unity in most of our calculations. The variations in the nuclear matrix elements are assumed to have been averaged out by considering a sufficiently large number of initial and final states of the nucleus. The factor $\rho'(B-E)$ contains the spin dependence of the nuclear level density. The spin distribution is predicted theoretically to be of the form[15,16]

$$\rho(J)\propto\rho(0)(2J+1)\exp[-(J+\tfrac{1}{2})^2/2\sigma^2], \tag{2}$$

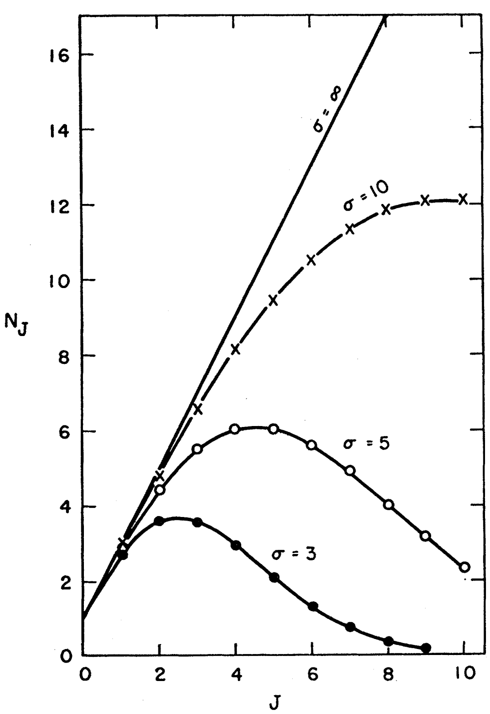

FIG. 1. Illustration of the theoretically predicted spin dependence of the nuclear level density for selected values of the parameter σ. N_J is the relative level density for levels with spin J.

[15] H. A. Bethe, Revs. Modern Phys. **9**, 84 (1937).

[16] C. Bloch, Phys. Rev. **93**, 1094 (1954).

TABLE I. Examples of spin distributions after emission of N_γ dipole radiations from compound states of $J=1$, 3, and 5 for selected values of the parameter σ.

$J\rightarrow$	0	1	2	3	4	5	6	7	8
Initial state $J=1$									
$N_\gamma=1$									
$\sigma=3$	0.14	0.37	0.49						
$\sigma=5$	0.13	0.34	0.53						
$\sigma=\infty$	0.11	0.33	0.56						
$N_\gamma=2$									
$\sigma=3$	0.09	0.37	0.36	0.18					
$\sigma=5$	0.07	0.33	0.37	0.23					
$\sigma=\infty$	0.06	0.31	0.37	0.26					
$N_\gamma=3$									
$\sigma=3$	0.08	0.30	0.38	0.19	0.05				
$\sigma=5$	0.06	0.26	0.37	0.23	0.08				
$\sigma=\infty$	0.05	0.23	0.35	0.26	0.11				
Initial state $J=3$									
$N_\gamma=1$									
$\sigma=3$			0.35	0.36	0.29				
$\sigma=5$			0.28	0.34	0.38				
$\sigma=\infty$			0.24	0.33	0.43				
$N_\gamma=2$									
$\sigma=3$		0.10	0.25	0.38	0.20	0.07			
$\sigma=5$		0.06	0.19	0.36	0.26	0.13			
$\sigma=\infty$		0.05	0.16	0.33	0.29	0.17			
$N_\gamma=3$									
$\sigma=3$	0.02	0.10	0.27	0.31	0.21	0.07	0.02		
$\sigma=5$	0.01	0.06	0.20	0.29	0.27	0.13	0.04		
$\sigma=\infty$	0.01	0.05	0.16	0.24	0.28	0.17	0.09		
Initial state $J=5$									
$N_\gamma=1$									
$\sigma=3$					0.47	0.33	0.20		
$\sigma=5$					0.34	0.34	0.32		
$\sigma=\infty$					0.27	0.33	0.40		
$N_\gamma=2$									
$\sigma=3$				0.19	0.32	0.33	0.13	0.03	
$\sigma=5$				0.11	0.23	0.35	0.22	0.09	
$\sigma=\infty$				0.07	0.18	0.34	0.26	0.15	
$N_\gamma=3$									
$\sigma=3$			0.07	0.20	0.32	0.25	0.13	0.03	0.00
$\sigma=5$			0.03	0.11	0.24	0.28	0.22	0.09	0.03
$\sigma=\infty$			0.02	0.07	0.18	0.26	0.26	0.15	0.06

where $\rho(J)$ is the density of levels with spin J, $\rho(0)$ is the density of levels with spin zero [$\rho(0)$ contains most of the dependence of the nuclear level density on excitation energy], and σ is the parameter which characterizes the distribution in spin. Theoretically, σ^2 is proportional to the product of the moment of inertia and the nuclear temperature. Figure 1 shows the form of this distribution for various values of σ. The most probable value of the spin J is $\sigma-\frac{1}{2}$. Although the limiting form of $\rho(J)=(2J+1)\rho(0)$ for $\sigma=\infty$ has often been assumed for cases where high spins are not expected to be encountered, the "$2J+1$ law" must break down for large values of J. In all the calculations described here, the parameter σ has been assumed to be constant, independent of excitation energy.

It can be shown[14] that the total radiation width [Eq.

TABLE II. Comparison of experimental and calculated isomeric cross-section ratios for thermal neutron capture. The results of the calculations are only shown for selected values of the parameters N_γ and σ. The experimental errors are often lower limits as they do not include uncertainties in the decay schemes. Unless otherwise indicated the experimental data is taken from Hughes and Schwartz.[a]

			$\sigma_{\text{high spin}}/\sigma_{\text{total}}$	Calculated					
				$J_c=I-\frac{1}{2}$			$J_c=I+\frac{1}{2}$		
Target	Target spin I	Competing levels	Experimental	$N_\gamma=3$ $\sigma=3$	$N_\gamma=4$ $\sigma=3$	$N_\gamma=5$ $\sigma=5$	$N_\gamma=3$ $\sigma=3$	$N_\gamma=4$ $\sigma=3$	$N_\gamma=5$ $\sigma=5$
Ge^{74}	0	1/2−7/2	0.18±0.05[b]				0.26	0.37	0.54
Ge^{76}	0	1/2−7/2	0.47±0.14[b]						
Se^{76}	0	1/2−7/2	0.06±0.03						
Se^{80}	0	1/2−7/2	0.09±0.04						
Zn^{68}	0	1/2−9/2	0.09±0.02				0.13	0.23	0.38
Kr^{84}	0	1/2−9/2	0.37±0.17						
Se^{82}	0	1/2−9/2	0.07±0.05						
Cd^{114}	0	1/2−11/2	0.11±0.04				0	0.09	0.21
Sn^{120}	0	3/2−11/2	0.007±0.007				0	0.04	0.13
Sn^{122}	0	3/2−11/2	0.006±0.004						
Sn^{124}	0	3/2−11/2	0.02±0.01						
Te^{126}	0	3/2−11/2	0.10±0.05						
Te^{128}	0	3/2−11/2	0.10±0.06						
Ce^{136}	0	3/2−11/2	0.087±0.036						
Ce^{138}	0	3/2−11/2	0.012±0.010						
Hg^{196}	0	5/2−13/2	0.044±0.008[c] 0.32±0.06[d]				0	0	0.022
Pd^{108}	0	5/2−11/2	0.019±0.004[d]				0	0	0.04
Rh^{103}	1/2	2−5	0.08±0.03	0	0	0.08	0	0.05	0.15
Ag^{109}	1/2	1−6	0.03±0.01	0	0	0.08	0	0.05	0.15
Br^{79}	3/2	2−5	0.25±0.06	0	0.05	0.15	0.11	0.14	0.28
Ir^{191}	3/2	1−4	0.54[e]	0.18	0.25	0.41	0.37	0.42	0.56
Ta^{181}	7/2	5−8	0.0004[f]	0	0	0.012	0	0.01	0.05
Cs^{133}	7/2	5−8	0.10 0.09±0.02[g]						
Co^{59}	7/2	2−5	0.56±0.09	0.28	0.30	0.47	0.66	0.50	0.66
Sc^{45}	7/2	4−7	0.45±0.20	0	0.01	0.07	0.05	0.06	0.18
In^{115}	9/2	1−5	0.75±0.14	0.71	0.65	0.76	0.91	0.83	0.89
In^{113}	9/2	1−5	$0.97^{+0.03}_{-0.37}$						
Eu^{151}	5/2	0−3	0.81[h]	0.76	0.76	0.83	0.90	0.88	0.92

[a] *Neutron Cross Sections*, compiled by D. J. Hughes and R. B. Schwartz, Brookhaven National Laboratory Report BNL-325 (Superintendent of Documents, U. S. Government Printing Office, Washington, D. C., 1958), 2nd. ed. and Suppl. No. 1.
[b] See reference 3.
[c] See reference 7.
[d] M. L. Sehgal, H. S. Hans, and P. S. Gill, Nuclear Phys. **12**, 261 (1959).
[e] G. Sharf-Goldhaber and M. McKeown, Phys. Rev. Letters **3**, 47 (1959).
[f] See reference 23. The decay scheme information does not exclude the possibility that the competing levels have spins 6 and 9 rather than 5 and 8.
[g] C. T. Bishop and J. R. Huizenga (unpublished results).
[h] R. Hayden, J. H. Reynolds, and M. C. Inghram, Phys. Rev. **75**, 1500 (1949).

(1)] is independent of the J value of the capturing state if $f(E,B,J_c,J_f)$ is set equal to unity and σ in Eq. (2) is ∞. In fact, one has to have a very low σ value (less than 3) and a high capturing spin state before the exponential cutoff of the spin dependence becomes important. Thus, total radiation widths cannot give much information about the parameter σ, but certain cases of low capturing spin do show the existence of the pre-exponential $2J+1$ factor in Eq. (2).[14,17]

A. Sample Calculation

The calculations are performed in the following way for dipole emission. The relative probabilities for a compound nucleus of spin J to decay to states with spin $J+1$, J, and $J-1$ are determined by the level density factor $(2J_f+1)\exp[-(J_f+\frac{1}{2})^2/2\sigma^2]$ for the particular spin states. An example of the spin distribution after the emission of the first gamma ray can be seen in the first line of Table I. Each spin population then can emit another radiation and feed three spin groups again. Examples of the spin distribution after various numbers of gamma-ray de-excitations are shown in Table I for selected values of the initial compound spin J_c and the parameter σ. It is assumed that the excited nucleus just prior to the last gamma de-excitation chooses to feed the metastable or ground state depending on which transition has the smaller spin change. For example, let us assume $\sigma=3$ and consider a compound nucleus with spin $J_c=3$ which emits three gamma rays before the final (fourth) gamma ray decides to populate either a spin 2 metastable state or a spin 5 ground state. States which after the third gamma-ray de-excitation have spins 0, 1, 2, or 3 will populate the spin 2 state and states with spin 4 or greater will populate the spin 5 state. From Table I we see that the population of the spin 2 state will be 0.70 and the population of the spin 5 state will be 0.30.

[17] L. M. Bollinger and R. E. Coté, Bull. Am. Phys. Soc. **5**, 294 (1960).

B. Comparison with Experiment

Most of the experimental data in the literature on isomer cross-section ratios for thermal neutron capture was reported before detailed decay scheme data were available, and is therefore of qualitative rather than quantitative significance. Table II presents the available experimental data and for comparison predictions of the calculations for selected values of the parameters σ and N_γ, the number of gamma rays in the gamma cascade. It can be seen from this table that much of the experimental data can be approximately reproduced assuming an average number of gamma rays of three or four and values of the parameter σ of 3 to 5. The results compiled in Table II serve to illustrate that matching calculated and experimental isomer cross-section ratios for thermal neutron capture does not allow a precise determination of σ. The isomeric cross-section ratios are much more sensitive to the parameter σ for reactions induced by energetic particles which produce compound nuclei of higher angular momentum, or in reactions in which neutrons (which can carry more angular momentum and sample more spin states) are emitted.[7]

The best available check of the internal consistency of these calculations is obtained by a comparison with some isomeric cross-section ratios measured[18] for thermal and resonant energy neutron capture in Eu^{151}. Eu^{151} has a nuclear spin of $\frac{5}{2}$, so that s-wave neutron capture can give compound nuclei with spins $J=2$ or $J=3$. The experimental data are summarized in the first part of Table III. The cross-section ratios seem to fall into two groups, consistent with the hypothesis that one group corresponds to spin $J=2$ for the compound nucleus and the other group corresponds to spin $J=3$. The second part of Table III summarizes the calculations, which not only reproduce the absolute value of the cross-section ratios, but for a particular σ also reproduce the variation with the spin of the compound system.[19] A similar study of the isomeric ratios for resonant energy neutron capture in In^{115} has been performed by Domanic and Sailor.[20] They report that the ratio of the high spin isomer to the low spin isomer is approximately 3.5 times larger for the 1.456-ev resonance than for the 3.86-ev resonance. Calculations similar to those presented above for Eu^{151} predict a ratio of 2.9 for $N_\gamma=4$ and $\sigma=4$ if the spin associated with the 1.456-ev resonance is $J=5$ and with 3.86-ev resonance is $J=4$. These spin assignments are in agreement with those determined recently for these resonances by Stolovy.[21] It can be seen that in favorable cases this type of calculation can distinguish which of the two possible compound nucleus spin states contributes most to the thermal neutron capture cross section.

It is interesting to compare the results of these calculations with some recent experimental measurements of the relative population of different rotational levels in deformed nuclei.[22] Even-even deformed nuclei exhibit

TABLE III. Comparison of experimental[a] and calculated isomeric cross-section ratios for different resonances in Eu^{151}. Assuming the thermal neutron cross section to be due only to the -0.011 and $+0.327$ ev resonances (not completely correct), absolute cross section ratios have been obtained from a normalization factor obtained from the average of Wood's relative ratios[a] for the -0.011 and $+0.327$ ev resonances and the absolute ratio[b] for thermal neutron capture. The errors listed refer only to the relative ratios. The predicted values were calculated assuming $\bar{N}_\gamma=4$.

Experimental	
Resonance energy (ev)	$\sigma_{I=0}/(\sigma_{I=0}+\sigma_{I=3})$
-0.011	0.22 ± 0.03
$+0.327$	0.17 ± 0.02
$+0.461$	0.10 ± 0.01
$+1.055$	0.07 ± 0.03

Calculated				
Compound spin	$\sigma_{I=0}/(\sigma_{I=0}+\sigma_{I=3})$ $\sigma=3$	$\sigma=4$	$\sigma=5$	$\sigma=\infty$
2	0.24	0.20	0.18	0.18
3	0.12	0.08	0.07	0.05

[a] See reference 18.
[b] R. Hayden, J. H. Reynolds, and M. C. Inghram, Phys. Rev. 75, 1500 (1949).

TABLE IV. Comparison of experimentally determined and calculated intensity ratios for the $I=6$ to $I=4$ and $I=4$ to $I=2$ transitions in Er^{168} and Hf^{178}. A small correction for internal conversion has been made to the gamma-ray intensity data.[a] Both the experimental and calculated intensities for the $I=4$ to $I=2$ transition include the population from the de-excitation of the $I=6$ level.

	Experimental Resonance energy (ev)	Int.(6+ → 4+) / Int.(4+ → 2+)
Er^{168}	Thermal	0.13 ± 0.02
	0.47	0.15 ± 0.04
	0.58	0.11 ± 0.04
	6.0	0.08 ± 0.03
Hf^{178}	Thermal	0.33 ± 0.05
	1.10	0.09 ± 0.05
	2.38	0.30 ± 0.08

	Calculated Int.(6+ → 4+)/(Int.(4+ → 2+) $\sigma=3$	$\sigma=5$	$\sigma=\infty$
$J=3$	0.08	0.11	0.15
$J=4$	0.18	0.28	0.35

[a] See reference 22.

[18] R. E. Wood, Phys. Rev. 95, 453 (1954).

[19] Recently there appeared a preliminary report [A. Stolovy, Bull. Am. Phys. Soc. 5, 294 (1960)] assigning spin values to the capturing states for resonant capture in Eu^{151}. Dr. Stolovy has informed us that recent work [R. M. Bozworth and J. H. Van Vleck (to be published)] indicates that the magnetic behavior of Eu at low temperatures is not well enough understood to determine the direction and magnitude of the field at the nucleus. Thus Dr. Stolovy feels that his measurements indicate only that the 0.461-ev and 1.055-ev levels have the same spin while the negative and 0.327-ev levels have the opposite spin.

[20] F. Domanic and V. L. Sailor, Phys. Rev. 119, 208 (1960).

[21] A. Stolovy, Phys. Rev. 118, 211 (1960).

[22] C. A. Fenstermacher, J. E. Draper, and C. K. Bockelman, Nuclear Physics 10, 386 (1959).

a simple pattern of rotational energy levels built upon the ground state with a spin sequence, $I=0$, 2, 4, 6, etc. These states de-excite by emission of quadrupole radiation to the next lower state rather than by crossover transitions. The possibility that there may be transitions between rotational levels built upon vibrational states have been neglected in the calculations. Table IV lists the intensity ratios for the $I=6$ to $I=4$ and $I=4$ to $I=2$ transitions for capture in Er^{167} and Hf^{177}, both of which have $I=\frac{7}{2}$. In doing the calculations it was assumed that levels with spin greater than 5 populated the 6+ or higher rotational levels, spin 5 levels divided between the 6+ and 4+ levels, spin 4 levels decay to the 4+ rotational level, and spin 3 levels divide between the 3+ and 4+ rotational levels. It was also assumed that there were three gamma rays per capture before transitions within the rotational band. The calculations are summarized in Table IV. The calculations fit the data nicely if all the resonances in Er^{167} and the 1.10-ev resonance in Hf^{177} produce $J=3$ compound states while thermal and 2.38-ev neutrons capture in $J=4$ compound states of Hf^{178}. It might be remarked that one does not expect all resonances capturing into states of the same spin to have exactly the same intensity ratios, as nuclear levels are not completely characterized by spin and excitation energy.

C. Discussion of Certain Specific Aspects of the Analysis

(a) Dependence on the Average Number of Gamma Rays $\bar{N}_\gamma$, and the Dispersion in the Number of Gamma Rays per Capture

If one of the isomeric states has a spin value which is quite different from that of the capturing state, the calculations predict rather different results depending on the average number of gamma rays per cascade. The entries in Table II for nuclei with competing levels of $I=3/2$ and $I=11/2$ are an example of this effect. The observation that Sn^{121}, Sn^{123}, and Sn^{125}, which have a closed proton shell, and Ce^{139}, which has one less neutron than the 82 neutron closed shell, appear to have abnormally low isomeric cross-section ratios suggests a lower level density and a smaller number of gamma rays per capture for these nuclei. However, with the uncertainties in the experimental data and the absence of measurements for $\bar{N}_\gamma$ for these nuclei, such observations are rather tentative.

The fact that, in a given nucleus, all captures may not give the same number of γ rays does not in general appreciably change the calculated results. There are a few cases, particularly Ta^{182}, for which the isomer having a high spin, far removed from the spin of the capturing nucleus, has a very low yield.[23] Such exceptionally low yields ($\sigma_{I=8}/\sigma_{\mathrm{Total}}\sim 0.0004$) enables one to estimate a limit for the distribution about the average in the number of γ rays emitted (or alternatively the contribution of quadrupole radiation, [see (*b*) below]). If $\sigma\geq 3$, which seems quite likely for this mass region (see reference 7), then less than 20% of the captures have 5 or more gamma rays in their cascade.

[23] A. W. Sunyar and P. Axel (private communication).

(b) Contributions due to Radiation of Higher Multipolarity than Dipole

It has been assumed for most of the calculations that only dipole emission occurs, with the exception of the final transition which has to decide whether to go to the ground or metastable state. The very low yield of the high spin isomer for capture in Ta^{181}, mentioned above, enables an estimate of the contribution of quadrupole radiation, as quadrupole emission can change the spin by two units for each gamma-ray transition as compared with only one spin unit change for dipole radiations. If we assume that $\sigma\geq 3$ and that $\bar{N}_\gamma=4$, the model predicts that less than 1% of the cascades are pure quadrupole, or considering another of the many possibilities, that if the first two transitions are dipoles then less than 10% of the third transitions are quadrupole. It seems reasonable therefore to consider the cascade to consist only of dipole radiations.

(c) Competing Levels

It is assumed that the last gamma ray transition in the cascade chooses between populating the metastable or ground state. Occasionally there is an energy level between the metastable and ground state, usually having a spin value intermediate between the spins of the two isomers. In these cases it is assumed that the competing spins are that of the intermediate level and that of the metastable state. (See for example the discussion in the following paper[7] on the Hg^{197} decay scheme.) It is conceivable that there may be an unknown level of intermediate spin just above the metastable state which decays by a crossover transition to the ground state. If such a state is far away from other more excited states and might therefore be strongly populated in the cascade process, a perturbation may arise which is not taken into account in the calculations. One can only hope that in most nuclei such states are not populated in a high percentage of the cascades.

(d) Applicability of the Statistical Model

For the model described here to be appropriate one must assume that a very large number of levels are present so that many different cascade paths are available. If such a situation exists the gamma-ray energy spectrum should show a broad, featureless distribution as exhibited by the gamma-ray spectrum of neutron capture in europium.[24] Certain nuclei, for example some

[24] L. V. Groshev, A. M. Demidov, V. N. Lutsenko, and V. I. Pelekhov, *Atlas of γ-ray Spectra from Radiative Capture of Thermal Neutrons*, translated from the Russian by J. B. Sykes (Pergamon Press, New York, 1959).

of the isotopes of iron and lead, exhibit gamma-ray spectra with a prominent line structure with most of the de-excitation occurring through a few high-energy transitions.[24] The calculations described here are not appropriate for such cases where a statistical description is not valid.

(e) *Improbable Processes (Small Branches)*

As was brought out under the discussion under (*a*) and (*b*) above, the calculations are most sensitive to some of the less well understood details when one of the products is produced in very low yield. Thus, one does not expect as quantitative agreement when the observed isomeric ratios are either very high or very low.

(f) *Model Dependence of the Transition Probability*

In the form in which we have written the radiation width above, the model dependence of the transition probability, apart from the level density factor, is contained in the factor $f(E,B,J_c,J_f)$. This factor was set equal to unity in most of our calculations. However, more specific models, for example the single particle model, predict a J dependence of this factor.[25–27] Sample calculations were made where $f(E,B,J_c,J_f)$ was arbitrarily assigned a J dependence of $2J_f+1$ (a somewhat stronger dependence than predicted by the single particle model[25,27]). One finds that the experimental isomeric ratios are now reproduced by values of the parameter σ which are one or two units smaller than those deduced if $f(E,B,J_c,J_f)$ is set equal to unity. It is shown in the following paper, that in reactions where particles are emitted any J dependence of the gamma-ray transition probability is relatively unimportant.

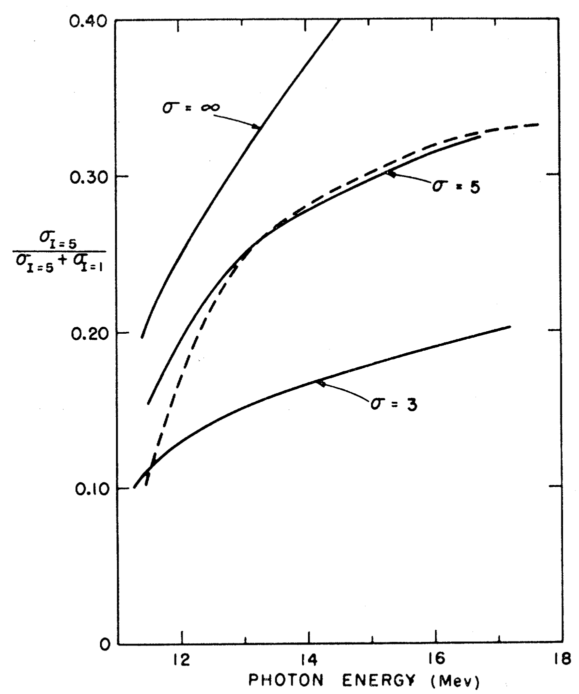

FIG. 2. Comparison of the experimental (dashed line) and calculated (solid line) isomeric cross-section ratios $\sigma_{I=5}/(\sigma_{I=1}+\sigma_{I=5})$ for the $Br^{81}(\gamma,n)$ reaction. Experimental data taken from reference 4. A nuclear temperature of 0.6 Mev was used in the calculations.

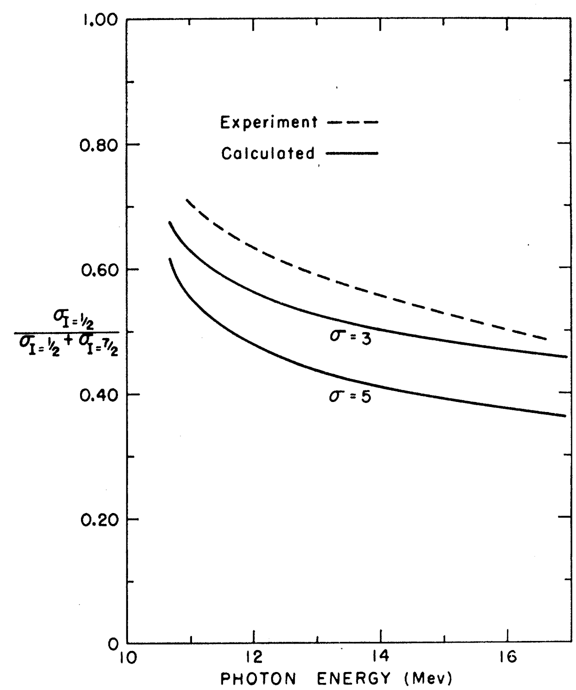

FIG. 3. Comparison of the experimental and calculated isomeric cross-section ratios $\sigma_{I=\frac{1}{2}}/(\sigma_{I=\frac{1}{2}}+\sigma_{I=7/2})$ for the $Se^{82}(\gamma,n)$ reaction. Experimental data taken from reference 32. A nuclear temperature of 0.6 Mev was used in the calculations.

3. PHOTONEUTRON REACTIONS

A few determinations of isomeric cross-section ratios have been reported for (γ,n) reactions.[4,28–32] An attempt has been made to treat these reactions using the considerations developed in this and the following paper. It has been assumed that all of the gamma-ray absorption proceeds by electric dipole absorption. If the target is even-even, this always results in formation of a compound nucleus with $J_c=1$. If the target has nonzero spin, the distribution in J_c is assumed to be divided between $J_c=I-1$, $J_c=I$, and $J_c=I+1$ in the proportions $2J_c+1$, by reasoning similar to that for the

[25] J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley & Sons, Inc., New York, 1952), p. 626.
[26] M. Deutsch, *Contribution to Experimental Nuclear Physics*, edited by E. Segrè (John Wiley & Sons, Inc., New York, 1959), Vol. III.
[27] S. A. Moszkowski, Phys. Rev. **89**, 474 (1953).
[28] L. Katz, R. G. Baker, and R. Montalbetti, Can. J. Phys. **31**, 250 (1953).
[29] J. Goldemberg and L. Katz, Phys. Rev. **90**, 308 (1953).
[30] P. Axel and J. D. Fox, Phys. Rev. **102**, 400 (1956).
[31] J. D. Fox, Ph.D. thesis, University of Illinois, 1960 (unpublished).
[32] E. Silva and E. J. Goldemberg, Anais acad. brasil. cienc. **28**, 275 (1956).

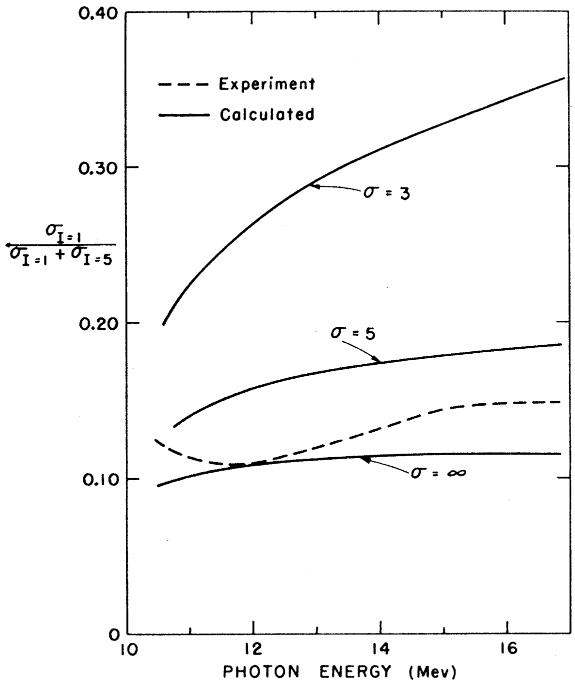

FIG. 4. Comparison of the experimental and calculated isomeric cross-section ratios $\sigma_{I=1}/(\sigma_{I=1}+\sigma_{I=5})$ for the $In^{115}(\gamma,n)$ reaction. Experimental data taken from reference 29. A nuclear temperature of 0.6 Mev was used in the calculations.

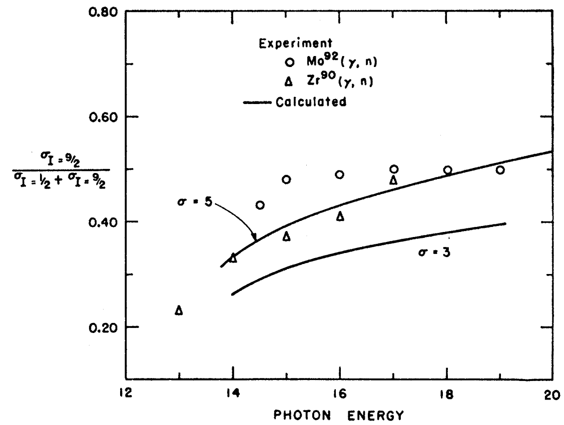

FIG. 5. Comparison of the experimental and calculated isomeric cross-section ratios $\sigma_{I=9/2}/(\sigma_{I=\frac{1}{2}}+\sigma_{I=9/2})$ for the $Zr^{90}(\gamma,n)$ and $Mo^{92}(\gamma,n)$ reactions. Experimental data taken from reference 31. A nuclear temperature of 1.0 Mev was used in the calculations.

particle-induced case discussed in reference 7. The neutron emission is also treated as described in reference 7. The results of the calculations are compared with the experimental results in Figs. 2, 3, 4, and 5. The calculated curves for the (γ,n) reactions induced in Br^{81}, Se^{81}, and In^{115}, were all computed using a constant nuclear temperature of 0.6 Mev, suggested by temperatures determined from inelastic scattering of neutrons. If the temperature is taken to be 1.0 Mev, the calculated values change by less than 10%. For the closed shell nuclei Zr^{90} and Mo^{92} a constant temperature of 1 Mev was chosen. The calculations do not have much meaning at the threshold or for the first few Mev above the threshold as there are not enough levels present for a statistical treatment. The region just above the threshold has been treated using a somewhat different model by Axel and Fox.[30,31]

As can be seen in Figs. 2–5, the dependence of the isomeric cross-section ratios with photon energy is very well reproduced in all of the cases. This may be taken as an indication that σ does not vary strongly with excitation energy. The variation in the values of σ indicated to fit the data is much larger than would be expected, but as no estimates of the experimental error in the Br, Se, or In cross-section ratios have been reported, it is probably premature to attach much significance to the variations in σ. Since isomeric ratios for (γ,n) reactions are not as sensitive to details of the gamma-ray cascade such as the contribution of quadrupole radiations, reliable isomeric cross-section ratios for (γ,n) reactions may reveal valuable information about the parameter σ.

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