(4) we see that the radial coordinate r_∞ of this particle (the suffix denotes that the meeting takes place in the infinite future) is given by

$$r_\infty = R\tan\chi. \tag{19}$$

The acceleration α corresponding to an η as in (18) is found, from (10), to be

$$\alpha = R^{-1}\tan 2\chi, \tag{20}$$

and this allows us to eliminate χ from (19). Thus we easily find

$$r_\infty = \alpha^{-1}\{(1+\alpha^2R^2)^{\frac{1}{2}}-1\}. \tag{21}$$

The same result was obtained analytically in paper I, which should be consulted for the connection between r_∞ and the "α horizon." If α is infinite, the "accelerating" particle is a photon. This corresponds, by (19) and (20), to $r_\infty = R$. Hence even a light-signal does not overtake all fundamental particles on its line of motion, which is, of course, a well-known fact.

It is also clear from the diagram that there exists a photon which, sent after the accelerating particle, only intercepts it in the infinite future. Its projection on to the (x,y) plane is QQ', parallel to PP' and tangent to the waist-circle. The coordinates of Q are evidently $(0, R\operatorname{cosec}2\chi, 0)$, while the coordinates of the point on $\mathfrak{H}$ whose projection is Q are $(0, R\operatorname{cosec}2\chi, R\cot 2\chi)$. By 3 (ii), this corresponds to a time

$$t_{\text{crit}} = R\ln(\operatorname{cosec}2\chi+\cot 2\chi),$$

which, by (20), is equivalent to

$$t_{\text{crit}} = R\ln[\alpha^{-1}R^{-1}\{(1+\alpha^2R^2)^{\frac{1}{2}}+1\}]. \tag{22}$$

The subscript denotes that this is a critical time for photons sent from the origin-particle ($r=0$) to intercept an accelerating particle that was released there from rest at $t=0$. Photons emitted earlier intercept the receding particle, while photons emitted later do not This result too was obtained analytically in paper I.

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Absence of Bound States in a Gravitational Field*

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The Klein-Gordon and Dirac equations for a particle in the gravitational field of a point mass are investigated. It is shown that the geometrical properties of the Schwarzschild metric prevent the normalization of any bound-state solutions.

CALLAWAY[1] has recently investigated the general relativistic Klein-Gordon and Dirac equations for an attractive center of mass M and charge e, in the case $M<e$.[2]

The opposite case, $M>e$, displays wholly different features: Equation (C2) shows that the metric is singular at $r=M\pm(M^2-e^2)^{\frac{1}{2}}$. We shall take here $e=0$, since this does not essentially differ from $0<e<M$.

The Klein-Gordon equation (C4) is singular at $r=2M$. The indicial equation at this point shows that the solutions behave there as $(r-2M)^{\pm 2iEM}$. Since the current density is

$$J^k = i(-g)^{\frac{1}{2}}g^{kn}(\bar\psi\psi_{,n}-\bar\psi_{,n}\psi)/2m,$$

then it follows from (C1) that $\int J^0 drd\theta d\phi$ diverges at $r=2M$.

In the case of the Dirac equation, one has[1]

$$J^0 = \bar\psi(-g)^{\frac{1}{2}}\gamma^0\psi \sim (-g)^{\frac{1}{2}}(F^2+G^2)/r^2g_{00}.$$

Expansion of (C7) about $r=2M$ shows that the leading terms are

$$(r-2M)F'+2MEG=0 \quad \text{and} \quad (r-2M)G'-2MEF=0.$$

It follows that F and G behave as $(r-2M)^{\pm 2iEM}$, so that here also the wave function is not normalizable.

Thus, there are no bound states in the gravitational field of a point mass.

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[1] J. Callaway, Phys. Rev. **112**, 290 (1958), hereafter referred to as C. There is a misprint in Eq. (C7); see D. Brill and J. A. Wheeler, Revs. Modern Phys. **29**, 465 (1957).

[2] Natural units are used: $c=G=\hbar=1$.