THE RELATION BETWEEN CERTAIN GALVANOMAGNETIC PHENOMENA.

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THE various galvanomagnetic effects have for the most part been studied separately by different experimenters, or where extensive researches along these lines have been conducted it has frequently happened that conditions were such as to prevent the establishing of exact relationships among the effects studied. Nevertheless, a definite relation between some of these effects is indicated qualitatively by experiment. For example, the Hall effect is unusually large in bismuth, and correspondingly, the other related effects are found to be abnormal in this metal also. The relationship is not always simple, however. In tellurium the Hall effect is very large while the other related phenomena are not of corresponding magnitude.

The three chief galvanomagnetic phenomena are ordinarily known as the Corbino effect, the Hall effect, and the change of resistance in a magnetic field. The last two of these are comparatively well known, the first has been less extensively studied. As investigated by Corbino this phenomenon is produced when a flat circular disk in which a radial current is flowing is placed in a magnetic field. Under these conditions a circular current is set up in the plate. At first sight it appears as if this Corbino effect were nothing more than a somewhat unusual aspect of the Hall effect. In the latter phenomenon a potential difference is measured while in the Corbino effect the current resulting from the Hall potential difference is observed. In spite of this apparently simple relation experiment has up to the present time failed to give an exact quantitative relation between these two phenomena. Smith concludes[1] that "while the exact quantitative relation . . . is not obvious it is seen that the sign and order of magnitude of one effect can be predicted if the other is known." Chapman's experiments[2] seem to indicate that "there remains an outstanding difference between the two effects when measured in the same specimen." Chapman, however, did not make his experiments on the two effects in the same specimen, but in different

[1] PHYS. REV., 8, p. 402, 1916.
[2] Phil. Mag., 32, p. 303, 1916.

specimens made up from the same sample of metal, and the electrical conditions were not exactly similar in the two cases. The writer has devised a somewhat simplified method of measuring all three of the above mentioned effects in a single specimen, and the quantitative relations between them have been worked out.

Theory.

If we assume electric fields, X and Y, acting along the x and y axes, respectively, and a magnetic field, H, acting along the z axis, the equations of motion of an electron are:

$$m\frac{d^2x}{dt^2} = Xe - He\frac{dy}{dt},$$

$$m\frac{d^2y}{dt^2} = Ye + He\frac{dx}{dt}. \qquad (1)$$

Assuming Drude's simple theory we may apply these equations to the case of metallic conduction and solve them by Townsend's method.[1] They are, in fact, the same equations which Townsend solves, except for the additional term involving Y. The result obtained for the respective drift velocities, U and V, of the electrons along the x and y axes is

$$U = \frac{eT}{m(1 + \omega^2T^2)}(X - Y\omega T),$$

$$V = \frac{eT}{m(1 + \omega^2T^2)}(Y + X\omega T), \qquad (2)$$

where T is the mean free time of an electron and $\omega = H(e/m)$. These equations may be used to deduce expressions for the various galvanomagnetic effects.

Consider a conducting plate of dimensions a, b and c along the x, y and z axes, respectively. When $V = 0$ we have $Y = -X\omega T = -UH$. Let the current density along the x axis be I_x. Then[2] $I_x = \frac{2}{3}neU\cdot b\cdot c$, where n is the number of free electrons per unit volume, and

$$Y\cdot b = -\frac{3HI_x}{2nec} = \frac{RHI_x}{c}. \qquad (3)$$

The Hall coefficient, R, is thus equal to $-3/2ne$.

For the Corbino effect we consider a circular metal disk of radius r_2 in the center of which is a circular hole of radius r_1. A radial current I_r flows in the disk and a magnetic field H is normal to the plane of the disk. The potential difference, δV, between the edges of an elemental ring of

[1] Electricity in Gases, p. 100.

[2] Swann, Phil. Mag., 27, p. 441, 1914.

width δr, and distant r from the center, is given by

$$\delta V = \frac{\rho I_r \delta r}{2\pi r \cdot d}, \tag{4}$$

where d is the thickness of the disk and ρ is its specific resistance. The radial electric field is thus

$$X_r = \frac{\rho I_r}{2\pi r \cdot d}, \tag{5}$$

and since there is no applied tangential electric field equations (2) above give the tangential drift velocity of the electrons:

$$V_t = X_r \frac{e}{m} T \frac{\omega T}{1 + \omega^2 T^2}. \tag{6}$$

The tangential current in the elemental ring is

$$\delta I_t = \tfrac{2}{3} ne V_t \cdot d \cdot dr. \tag{7}$$

The radial current is given by

$$I_r = \tfrac{2}{3} ne U \cdot 2\pi r \cdot d, \tag{8}$$

from which it follows that

$$\rho = \frac{3}{2} \frac{m}{ne^2} \frac{(1 + \omega^2 T^2)}{T}. \tag{9}$$

This equation, combined with (5) and (6) above, enables us to integrate (7), giving

$$I_t = \frac{H I_r T e}{2\pi m} \log \frac{r_2}{r_1}. \tag{10}$$

This expression is the same as Corbino's when $(e/m)T$ is replaced[1] by Corbino's constant, E.

It is evident that the Hall effect and the Corbino effect yield us information about two different concepts in the theory of metallic conduction, namely the number of electrons per unit volume and the mean free time of these electrons. Hence we should not expect, in general, that there would always be correspondence between the Hall coefficient and the Corbino coefficient. To get a relation on the basis of the theory we may proceed as follows:

Consider a conducting plate of dimensions a, b and c along the x, y and z axes, respectively. Eliminating Y from equations (2) we get

$$V = \frac{X(e/m)T - U}{\omega T}. \tag{11}$$

[1] Adams finds (Phil. Mag., 27, p. 244, 1914) the constant E to be $\frac{1}{2}(e/m)T$, but his deduction assumes the average drift velocity of electrons in the direction of an electric force, X, to be $\frac{1}{2}X(e/m)T$ instead of $X(e/m)T$.

When $V = 0$ let $U = U_1$ and $X = X_1$. Then we have

$$\frac{e}{m}T = \frac{U_1}{X_1},$$

so that

$$V = \frac{1}{\omega T}\left(\frac{XU_1}{X_1} - U\right). \tag{12}$$

Since the density of the electric current is proportional to the drift velocity of the electrons we may write

$$i_y = \frac{1}{\omega T}\left(\frac{X}{X_1}i_x' - i_x\right), \tag{13}$$

where i_x and i_y are the current densities along the x and y axes, respectively, and i_x' is the value of i_x when $i_y = 0$. If two mutually perpendicular currents are sent through a metal plate by separate batteries, and if this plate is then placed in a magnetic field, i_x is equal to i_x' within the limits of ordinary observation. Equation (13) may, therefore, be put into the simplified form

$$\omega T = \frac{i_x}{i_y}\left(\frac{X - X_1}{X_1}\right). \tag{14}$$

The specific resistance of the metal plate along the x direction is given by $\rho = X_1/i_x$. It is obvious that $X - X_1$ gives the potential difference of the ordinary Hall effect, though usually this latter is measured when $X_1 = 0$. The Hall coefficient, R, is given by the relation $X - X_1 = RHi_y$, hence equation (14) becomes

$$E = \frac{e}{m}T = \frac{R}{\rho}. \tag{15}$$

Both Smith and Chapman have compared values of $E\rho$ with R for different metals and different magnetic fields. Their conclusions are as stated above. It appears, however, that there may be good reason for this failure of Smith's and Chapman's results to fit equation (15) exactly. In the first place they assume the value of ρ to be independent of H, and while this assumption may be legitimate in the case of many metals, it introduces an error where such metals as bismuth are being studied. Furthermore, by taking values of ρ from tables instead of determining them experimentally another error is likely to be introduced. It is not surprising, therefore, that they should get somewhat inexact agreement between $E\rho$ and R.

EXPERIMENT.

If a specimen is arranged in the proper way and all the quantities on the right-hand side of equation (14) are obtained experimentally then sufficient data is at hand for determining all three of the galvanomagnetic effects. Experiments were made on bismuth, copper, zinc, and natural graphite crystals. In all cases the specimen under examination was cut to the form of a fairly thin and almost square plate. This plate was then sawed into the form of a symmetrical, flat cross and each arm of the cross was divided by a number of fine saw-cuts, each cut being parallel to the cross arm in which it was made. The result was a square or rectangular plate from each edge of which extended a number of parallel arms of the same material as the plate. To the extremities of these arms were then soldered wires, the ends of which were joined together and connected in an electric circuit as shown in Fig. 1. In this diagram A and A' indicate ammeters and G is a galvanometer. In adjusting the apparatus it was found necessary to make all the wires from the ends of the arms of about the same resistance and to secure a fairly symmetrical specimen, otherwise the lines of current flow in the plate were not parallel. In order to prevent an appreciable current from flowing out of one arm through the leading-in wires and back to the plate by another arm these leading-in wires were made to have a resistance large compared with that of the plate itself.

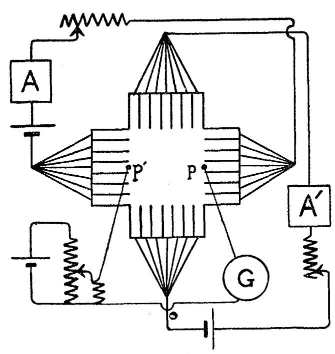

Fig. 1.

To the points p and p' other wires were soldered and led through a potentiometer circuit to the galvanometer G. The points p and p' were located so as to be on an equipotential line when the current flowed through the circuit containing the ammeter A', and the circuit containing A was open. When connections were properly made the plate was placed between the poles of an electromagnet so that the magnetic lines of force were as nearly as possible perpendicular to the face of the plate. The pole-pieces of the magnet had rectangular faces of dimensions 2.4×1.5 cm., and were in most experiments from 0.5 to 0.8 cm. apart. The strength of the magnetic field was determined by the use of a bismuth spiral.

The method of taking observations was as follows: With the electromagnet excited the ammeters A and A' were read, thus allowing the respective current densities, i_x and i_y to be calculated. The current through the electromagnet was then broken and i_x also made zero. Under these conditions it sometimes happened that the galvanometer G showed a small deflection because of thermal electromotive forces at the junctions p and p'. By adjusting the potentiometer arrangement in series with the galvanometer this thermal E.M.F. was counterbalanced and the reading of the galvanometer again made zero. In most cases, however, there was very little effect of this kind, so that except for an initial setting to make a small compensation for inaccurate placing of the terminals, p and p', the potentiometer seldom required adjustment. The magnetizing current was next thrown on and the reading of G observed. The current i_y was next broken, the current i_x made, and the reading of G again noted. Since the galvanometer deflections are proportional to the potential drop in the plate these deflections enable the ratio $(X - X_1)/X_1$ of equation (14) to be calculated. Several sets of observations were taken by allowing i_x to flow constantly, observing the galvanometer, and noting the change in the galvanometer deflection when i_y was made or broken. These observations agreed with those obtained in the previously described manner.

In all cases the final data recorded is a mean of results obtained with i_x and i_y flowing first in one direction then in the other, though it was found that the directions of the currents had little effect on the magnitude of the results. In some cases, however, a magnetic field in one direction gave a much larger effect than when in the opposite direction. This difference was probably due to the lines of equipotential in the plate not being parallel, for it was observed that this unsymmetrical effect of the magnetic field could be largely obviated by making all connections at the ends of the arms of about equal resistance, and by making the plate itself of uniform thickness and structure.

Measurements made as described enable the quantity $(e/m)T$ of equation (14) to be calculated. To get values of ρ or R of equation (15) it was necessary to make a further observation. By means of the potentiometer it was determined what E.M.F. in the galvanometer circuit produced unit deflection of the galvanometer. Knowing this constant and the distance between p and p' the value of $X - X_1$ and of X_1 could be determined from the corresponding galvanometer deflections. Hence R and ρ could be easily calculated.

In making measurements on the Hall effect it is ordinarily considered desirable to connect the Hall electrodes to the plate with bars of the same material as the plate itself in order to minimize the effects of thermal E.M.F. For the sake of comparison the Hall effect was determined in a specimen of bismuth with the terminals of the galvanometer connected to the respective ends of a pair of arms instead of at the points p and p'. In this case the values obtained for R were slightly larger than those previously secured. One would expect a result of this kind if some of the primary current flowing through A' passed outside the points p and p' instead of between them. However, errors arising from causes of this kind and from thermal effects were considered as small enough to be neglected in this experiment.

One other source of error must be considered in measuring the Hall effect. Properly speaking no current should be allowed to flow from the Hall terminals as a result of the potential difference set up by the magnetic field. When a current is allowed to flow through a galvanometer the assumption made in deducing equation (3), that $V = 0$, is no longer absolutely correct. Solving the second of equations (2) for Y gives

$$Y = \frac{mV}{eT}(1 + \omega^2T^2) - X\omega T. \tag{16}$$

Let the current through the galvanometer be I_g. Then

$$I_g = \tfrac{2}{3}neV\cdot a\cdot c.$$

Hence

$$Y = \frac{3mI_g(1 + \omega^2T^2)}{2a\cdot c\cdot ne^2T} - X\omega T. \tag{17}$$

Applying equation (9) to this expression we get

$$Y = \frac{\rho}{a\cdot c}I_g - X\omega T. \tag{18}$$

In the experiment as performed a sensitive galvanometer of comparatively high resistance was used so that I_g was very small—in all cases the value of $\rho I_g/ac$ was small compared with the potential difference observed. Hence this term may be neglected and we get approximately the same results for Y as would have been obtained by a balancing potentiometer method. Experiment confirmed this conclusion.

Experimental Results.

The results obtained are tabulated below. The bismuth specimen was cast from the metallic crystals as supplied by Merck—about 98 per cent. pure—and the central square plate of the specimen had dimensions 1.05 × 1.05 × 0.20 cm. The zinc and copper specimens were cut from sheets of the commercial metal and were of dimensions 1.6 × 0.8 × 0.046 cm. and 1.35 × 1.35 × 0.059 cm. respectively. The natural graphite was obtained from Orange County, N. Y., through the kindness of Mr.

A. C. Hawkins, formerly of Brown. This graphite consisted of numerous thin plates superposed upon each other, and was very perfect and free from impurities. A specimen was cemented to a glass plate with sealing wax, cut to the desired form and copper-plated at all points where wires were to be affixed by soldering. After one or two trials no difficulty was experienced in making contacts by this method, though when pressure contacts were used satisfactory results could not be obtained. The specimen used had dimensions 1.26 × 1.20 × 0.04 cm.

Copper. $\rho = 1.56 \times 10^3$ (from Baedeker's table).

H, Gausses.	E.	$R = E\rho$.	T, Seconds.	n.
5000	-2.81×10^{-7}	-4.38×10^{-4}		
9100	-2.68 "	-4.18 "		
13100	-2.65 "	-4.13 "		
14700	-2.77 "	-4.32 "		
Mean.....	-2.73×10^{-7}	4.25×10^{-4}	1.54×10^{-14}	2.15×10^{23}

Zinc. $\rho = 5.71 \times 10^3$ (from Baedeker's table).

10300	$+12.8 \times 10^{-8}$	$+7.30 \times 10^{-4}$		
13400	$+12.4$ "	$+7.08$ "		
17200	$+9.8$ "	$+5.60$ "		
Mean.....	$+11.7 \times 10^{-8}$	$+6.66 \times 10^{-4}$		

Bismuth.

H, Gausses.	E.	R.	$\rho = R/E$, C. G. S. E. M. U.	T, Seconds.	n.
800	-10.9×10^{-5}	-15.4	14.1×10^4	6.16×10^{-12}	6.09×10^{18}
2450	-6.5 "	-10.0	15.4 "	3.67 "	9.38 "
6150	-4.42 "	-7.85	17.7 "	2.50 "	11.9 "
9750	-3.12 "	-6.40	20.5 "	1.76 "	14.6 "
11100	-2.92 "	-6.15	21.1 "	1.65 "	15.2 "
12850	-2.63 "	-5.92	22.5 "	1.49 "	15.8 "

Graphite.

2600	-6.15×10^{-6}	-0.612	9.95×10^4	3.47×10^{-13}	1.53×10^{20}
4200	-6.05 "	-0.662	10.95 "	3.42 "	1.42 "
5500	-5.52 "	-0.690	12.50 "	3.12 "	1.36 "
6550	-5.52 "	-0.760	13.8 "	3.12 "	1.23 "
7600	-5.07 "	-0.780	15.4 "	2.86 "	1.20 "
8400	-4.90 "	-0.810	16.5 "	2.77 "	1.15 "
9100	-4.85 "	-0.858	17.7 "	2.74 "	1.09 "
9900	-4.60 "	-0.890	19.4 "	2.60 "	1.05 "
10550	-4.43 "	-0.910	20.5 "	2.50 "	1.03 "
11300	-4.32 "	-0.946	21.8 "	2.44 "	0.99 "

The effect to be measured was so small in zinc and copper that accurate measurements could not be made. The arrangement of apparatus is not the best for determining small changes of resistance such as occur when copper or zinc are placed in a magnetic field. Both the Hall effect and the resistance effect have been carefully measured for these metals, however, and it has been found that the Hall coefficient is independent of the magnetic field. Since the effect of H on ρ is very small in these metals we should expect the Corbino constant to be practically independent of H—a conclusion which the above data appear to corroborate in the case of copper. The variations of E in zinc are no larger than the experimental errors. It should be noted that both the Corbino effect and the Hall effect in zinc have a sign opposite to that predicted by the theory. In the other metals studied the sign is such as to make ωT in equation (14) have a positive value.

The data in the case of bismuth show that both the Hall coefficient and the Corbino constant diminish in magnitude as the strength of the magnetic field is increased. The value of E diminishes more rapidly than R—a result to be expected when the resistance of a metal is increased by a magnetic field. In the case of graphite we have a substance which increases its resistance even more than bismuth, and for which the variation of E with H is not so rapid as the field changes. The result of these conclusions is that the coefficient, R, instead of decreasing, is found to increase as H increases.

The method used in this experiment for getting E is based upon equation (14). If this equation is put into the form

$$E = \frac{i_x}{Hi_y}\left(\frac{X - X_1}{X_1}\right) \qquad (19)$$

it is evident that the sign of E changes with the sign of $X - X_1$, and since this latter quantity determines the Hall coefficient we see that the Corbino constant and the Hall coefficient must change sign together. Chapman finds in the case of an alloy of bismuth and tin that as H increases one of these effects changes sign before the other. His measurements of the two effects were made separately, however, so we might expect some variations because of experimental errors. If for some reason a constant error in the value of R were introduced (as was obtained in the present experiment by shifting the position of the Hall terminals) then a result as obtained by Chapman might be expected—the curve for R would not cross the H axis at the same point as the curve for E. By the conditions of Chapman's experiment it is also possible for thermal effects in the measurement of R and E to be different, thus introducing an error in the comparison of the two effects.

As regards the disagreement of the experimental with the theoretical sign of the Hall and Corbino effects in zinc (and other metals) no perfectly satisfactory explanation has yet been given. According to J. J. Thomson[1] we might expect the value of H in equation (14) to have a sign different from that of the applied external magnetic field, especially where intramolecular fields are called into play by the magnetizing force.

With the observed values of R and E it is of interest to use equations (3) and (14) and calculate values of n and T on this theory of electron conduction, though in making these calculations one makes the assumption—possibly incorrect—that the magnetic field in which the electrons move is the same as that produced by the electromagnet of the experiment. As seen in the last columns of the above tables both the number of free electrons per unit volume and the mean free period of these electrons may be dependent upon the strength of the magnetic field. As has been previously pointed out,[2] one would expect T to be dependent upon H, but the reasons for variations in n are not so clear. In the case of graphite crystals it appears that n decreases with an increasing magnetic field, while for bismuth the increasing field produces an increasing n. Evidently the molecular structure of the metal is important in connection with this question. One may imagine a magnetic field influencing the stability of the orbits of electrons revolving in a molecule and thus, by varying the absorbing or emitting power of molecules for electrons, causing variations in n. Also, by causing molecular systems to orient themselves differently a magnetic field might change intra-molecular forces so as to affect the electronic radiation of molecules, and thus n would be changed. Our knowledge of these processes, however, is at present too vague to permit of any great specialization of the theory as described above.

Summary.

1. The electron theory of metallic conduction has been used in developing a relationship between the three galvanomagnetic phenomena,—the Hall effect, the Corbino effect, and the change of resistance in a magnetic field.

2. It is found that the Hall coefficient divided by the specific resistance of the metal is equal to the Corbino constant.

3. A simple experiment has been devised so that all three of these galvanomagnetic effects may be determined in the same specimen with relatively simple operations.

4. In zinc and copper the Corbino constant is probably independent

[1] Corpuscular Theory of Matter.

[2] Phys. Rev., X., 4, p. 366, 1917.

of the magnetic field. In bismuth the three galvanomagnetic effects are found to behave in the usual manner but in crystalline graphite the resistance increases rapidly with the magnetic field, the Corbino constant decreases, and the Hall constant increases.

5. According to the usual interpretation of these experiments we may conclude that the number of free electrons per unit volume in a metal may be affected by placing the metal in a magnetic field. The free period is also affected. It appears, therefore, that both of these factors must be taken into account when one is considering the effect of a magnetic field on resistance.

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