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THEORY OF THE THERMIONIC AMPLIFIER

By H. J. van der Bijl.

I. Introduction.

IN this paper will be given the theory of operation of the thermionic amplifier. This device consists essentially of an evacuated vessel containing a hot cathode, an anode placed at a convenient distance from the cathode, and a third electrode adjacent to the cathode. This electrode, which we shall call the auxiliary electrode, may be in the form of a grid placed between the cathode and anode. It may also be in the form of a plate placed on the side of the cathode opposite to that of the anode, or in the form of a wire or a plurality of wires galvanically connected and placed in the plane of the cathode parallel to that of the anode. The theory to be given here applies to any of these structures, although in particular it will be explained with reference to the case in which the auxiliary electrode takes the form of a grid between cathode and anode.

The cathode usually consists of a filament which can be heated by passing an electric current through it. If the anode be made positive with respect to the filament, the electrons emitted from the hot filament travel to the anode, and the current thus established in the circuit connecting the filament and anode (that is, the output circuit) can be varied by potential variations applied to the auxiliary electrode.

During several years past a considerable amount of research and development work has been done in this laboratory on this device and its various applications. What is given in the following pages is merely an outline of the theory of operation, the framework of which was worked out in the winter of 1913–14 and which formed the basis of development work that has since been going on in this laboratory.

When Dr. H. D. Arnold started making experiments on this device he realized that a necessary condition for satisfactory operation as a telephone relay was that the device must be operated under such conditions that ionization by collision of the residual gas is inappreciable. As is well known to workers in this field, it is difficult to keep a discharge steady and reproducible when ionization by collision is appreciable, and steadiness and reproducibility are conditions that must be complied with by a telephone relay.

It is at present impossible to entirely eliminate ionization by collision. The number of positive ions produced by the discharge in devices of this kind could easily be measured even when the pressure is as low as 10^{-7} mm. Hg.[1] But the amplifier can always be operated under such conditions that the number of positive ions formed by collision ionization is always small compared with the number of electrons moving from cathode to anode.

This condition is presumed in the following. The theory given here has been found to hold with sufficient accuracy, for the computation of circuits involving thermionic amplifiers. It is hardly possible to meet the requirements of efficiency and satisfactory operation of the device without an explicit mathematical formulation of its operation. Take, for instance, the case in which the device is used for relaying telephonic currents. A satisfactory telephone relay must not only amplify power (of course, with the help of local power), but must also be capable of handling sufficient power and have a definite impedance to fit the impedance of the telephone line. Unless the amplifier be properly designed, it will not satisfy these conditions, and so much distortion may be produced as to make the device worthless as a telephone relay. On the other hand, it has been found that a properly designed thermionic amplifier meets the above-named requirements very satisfactorily.

II. Current-voltage Characteristics of Simple Thermionic Devices.

In this type of amplifier, use is made of the thermionic currents obtained from incandescent filaments. We shall not here enter into a discussion of the extensive investigations that have been carried out on thermionics, but merely touch upon those phases of the subject which have a direct bearing on the theory of operation of the thermionic amplifier.

Consider a structure consisting of a heated cathode and an anode, and contained in a vessel which is evacuated to such an extent that the

[1] In this connection it may be stated that the most reliable gauge for the measurement of high vacua is the so-called "ionization manometer" recently devised by O. E. Buckley of this laboratory. In this manometer, which can be used for pressures below 10^{-3} mm. Hg, the pressure is measured by the number of positive ions produced by collision of electrons coming from a hot filament with the residual gas molecules.

residual gas does not play any part in the current convection from cathode to anode. The number of electrons emitted from the cathode is a function of its temperature. If all the electrons emitted from the cathode pass to the anode, the relation between the resulting current I and cathode temperature T is given by a curve of the nature shown in Fig. 1. This curve is obtained provided the voltage between anode and cathode is always high enough to drag all the electrons to the anode as fast as they are emitted from the cathode; that is, I in Fig. 1 represents the saturation current. The saturation current is obtained in the following way: Suppose the cathode be maintained at a constant temperature T_1 and the voltage V between anode and cathode be varied. As this voltage V is raised from zero, the current I to the anode at first increases,

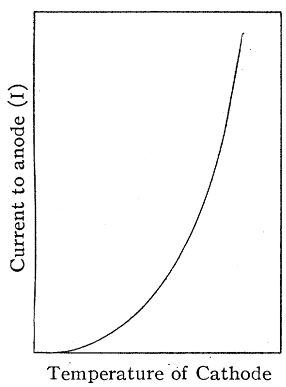

Fig. 1.

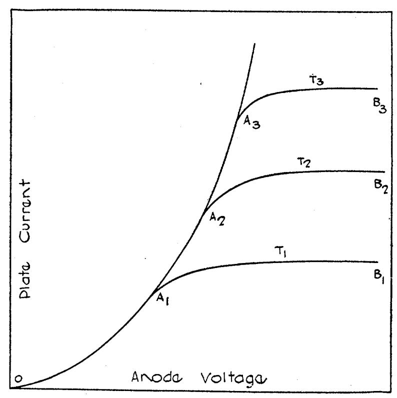

Fig. 2.

the relation between V and I being represented by the curve OA_1 of Fig. 2. Any increase in V beyond the value corresponding to A_1 causes no further increase in I, and we get the part A_1B_1 of the curve. Clearly this part of the curve corresponds to the condition when all the emitted electrons are drawn to the anode as fast as they are emitted from the cathode. If the cathode temperature be increased to T_2, the number of emitted electrons is increased, and we get the curve OA_2B_2. When these values of the saturation current are plotted as a function of the temperature, we obtain the curve of Fig. 1. This curve is represented very approximately by the equation

$$I = aT^{1/2}e^{b/T}, \tag{1}$$

where a and b are constants. This equation was derived by O. W. Richardson in 1901[1] on the basis of the theory that the electrons are

[1] Proc. Camb. Phil. Soc., Vol. II., 285, 1901; Phil. Trans. Roy. Soc., A, 201, 1903.

emitted from the hot cathode without the help of any gas, but solely in virtue of their kinetic energy. The formulation of this theory was the first definite expression of what may be termed a pure electron emission.

In the state in which Richardson's equation holds, the current is independent of the voltage. Under these conditions the device to be treated in the following does not function as an amplifier, since it depends for its operation on the variation of current produced by variation of voltage. The condition under which the current is a function of the voltage is represented by the part OA of Fig. 2. Here the voltage is not high enough to draw all the electrons to the anode as fast as they are emitted from the cathode; in other words, there are more electrons in the neighborhood of the cathode than can be drawn away by the applied voltage. It was first pointed out explicitly by C. D. Child in 1911 that this limitation to the current is due to the space charge effect of the electrons in the space between anode and cathode. Working on the basis that ions of one sign only are present, Child deduced the equation:[1]

$$I = \frac{1}{9\pi}\sqrt{\frac{2e}{m}} \cdot \frac{V^2}{x^2}, \tag{2}$$

where I is the thermionic current per square centimeter of cathode surface, V the voltage between anode and cathode, x the distance between anode and cathode, and e and m are the charge and mass of the ion respectively.

This equation was deduced on the assumption that both cathode and anode are equipotential surfaces of infinite extent.

The equations that hold in the case in which the cathode is a filament, and therefore not an equipotential surface, were obtained theoretically by W. Wilson, of this laboratory. For the case that the negative end of the filament is connected through the battery E to the anode, and this end of the filament taken as the zero of potential, the potential of the positive end of the filament being V_0 and that of the plate V, Wilson finds:

$$I = \frac{2}{5}\frac{keV^{5/2}}{V_0}\left[1 - \left(1 - \frac{V_0}{V}\right)^{5/2}\right] \text{if } V > V_0, \tag{3}$$

$$I = \frac{2}{5}\frac{keV^{5/2}}{V_0} \text{if } V < V_0, \tag{4}$$

where k is a constant and e the elementary charge.

[1] C. D. Child, PHYS. REV., 32, 498, 1911. The space charge effect has been fully studied by J. Lilienfeld (Ann. d. Phys., 32, 673, 1910); I. Langmuir (PHYS. REV. (2), 2, 450, 1913), who also independently derived the space charge equation (2) and published a clear explanation of the limitation of current by the space charge; and Schottky (Jahrb. d. Rad. u. Elektronik, Vol. 12, p. 147, 1915.)

When the full space charge effect exists, the current is independent of the temperature of the cathode. This can be understood more easily with reference to Fig. 3, which gives the current as a function of the temperature of the cathode for various values of the voltage between anode and cathode. Suppose a constant voltage V_1 be applied between anode and cathode, and the temperature of the cathode be gradually increased. At first when the temperature is still low, the voltage V_1 is large enough to draw all the emitted electrons to the anode, and an increase in the temperature results in an increase in the current. This gives the part OC_1 of the curve of Fig. 3. When the temperature corresponding to C_1 is reached, so many electrons are emitted that the resulting volume density of their charge causes all other emitted electrons to be repelled, and these return to the filament. Obviously any further increase in the temperature of the cathode beyond that given by C_1 causes no further increase in the current, and we obtain the horizontal part C_1D_1. If, however, the voltage be raised to V_2, the current increases, since more electrons are now drawn away from the supply at the filament, the full space charge effect being maintained by less emitted electrons being compelled to return to the filament. It is now clear that the part OC of Fig. 3 corresponds to the part AB of Fig. 2 and CD of Fig. 3 to OA of Fig. 2. The latter represents the condition under which the thermionic amplifier operates.

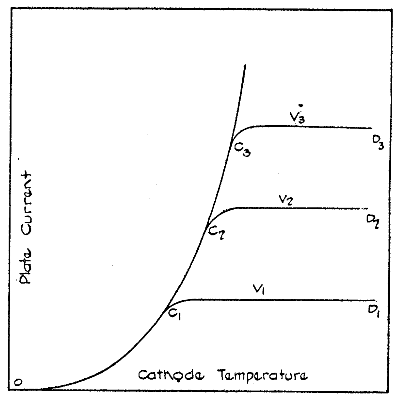

Fig. 3.

It is important to note that the thermionic amplifier operates under the condition characterized by the circumstances that the applied voltage is not sufficiently high to give the saturation current.

III. Effect of Residual Gas on the Characteristic.

In the theory to be set forth in the following, it is assumed, as stated above, that the amplifier operates under the conditions that no appreciable ionization by collision takes place.

In order to see under what circumstances this condition can be obtained, we have to consider the possible means by which gas can influence the discharge and the means whereby gases occluded in the electrodes and walls of the containing vessel are liberated and so brought into a state in which they can influence the discharge.

In the first place the presence of gas in the space between cathode and anode will, when the applied voltage exceeds a small value, produce the well-known phenomenon of ionization by collision. This effect, which we shall call the *volume effect*, alone causes an increase in the number of electrons arriving at the anode, and thus increase the current.

Secondly, the occlusion of gas in the surface of the cathode can cause either an increase or a decrease in the number of electrons emitted from the cathode. This we can refer to as the *surface effect*. Thus hydrogen occluded in a platinum cathode causes an increase in the number of emitted electrons, while oxygen occluded in a lime-covered cathode decreases the number of emitted electrons. Since the number of electrons emitted from the cathode is measured by the saturation current (AB, Fig. 2), the surface effect influences the saturation current.

The third effect to be considered is the liberation of gas occluded in the anode and walls of the vessel, due to the rise in temperature occasioned by the dissipation of energy in the device. If the electrodes and walls of the vessel are not well denuded of gases by heating during evacuation but the device be otherwise evacuated to such a degree that the amount of gas in the space is not sufficient to cause an appreciable volume effect, the heating caused by the operation of the device can, if the applied voltage is high enough, cause the liberation of so much gas that ionization by collision becomes appreciable and thus unduly increase the current. The voltage and current at which such an undue increase in current occurs depends, of course, on the extent to which the electrodes and walls have been denuded of gases during evacuation.

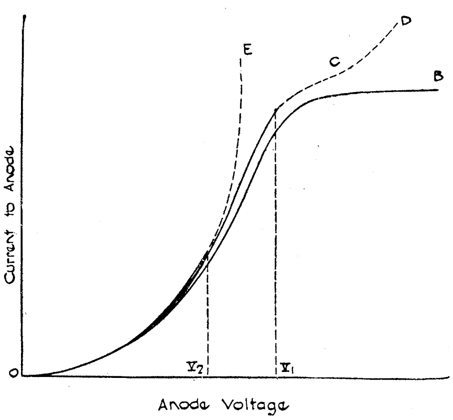

Fig. 4.

Referring to Fig. 4, let the curve OB represent the relation between current and applied voltage for a definite temperature of the cathode when the vacuum is supposed to be perfect and the electrodes and walls are completely denuded of gas. Suppose, now, that the tube contains a small amount of gas and that the electrodes are not completely denuded. If we suppose that the occluded gas and the gas in the space are of such nature as not to affect the emission of electrons from the cathode, the curve OD will be obtained. When the applied voltage becomes equal to the ionization voltage of the gas (of the order of 10 volts), collision ionization sets in, thus increasing the current over that obtained in a perfect vacuum. In view of the small amount of gas that we have supposed to be in the space, this increase in current is inappreciable. As is well known, collision ionization will be inappreciable if the pressure of the gas is such that the distance between the electrodes is small compared with the mean free path of the electrons in the gas. As the voltage is further increased the rise in temperature of the anode due to the bombardment by the electrons liberates more and more gas, thus causing a further increase in ionization by collision. At C so much gas may be liberated that a very appreciable increase in current occurs. For a higher initial pressure and more occluded gas a curve such as OE may be obtained.

When ionization by collision becomes appreciable the device is erratic in its behavior, and a reproduction of the curves is well-nigh impossible, once the voltage has been increased beyond the dotted portions indicated in Fig. 4. The curves in Fig. 4 are therefore merely intended to illustrate the points under consideration. Taking the perfect vacuum curve OB, we see that the device must always be operated (when used as an amplifier) within the voltage limits O to V_1. Obviously the device with a small amount of gas, viz., that to which the curve OCD applies, can also be operated between the voltage limits O to V_1. In the third case the applied voltage should not exceed the limits O to V_2.

This brings out an important point; namely, that a device with an appreciable amount of gas can be operated under such conditions that its characteristic curve over the operating range of voltage is practically the same as that of a hypothetical device with perfect vacuum. But it is seen that, if a steady and reproducible discharge is to be obtained the presence of gas limits the operating range of applied voltage and thus also limits the power that can be obtained from the device.[1] The power limitation of the thermionic amplifier is also due to other important factors involving the structure of the device. These will be discussed later.

IV. Action of the Auxiliary Electrode.

So far we have considered the case of a simple thermionic device consisting of a cathode and anode. When a third electrode is added to the system, the matter becomes more complicated.

[1] The operating range of voltages given above are those defined by Dr. Arnold in 1912 as the necessary range for satisfactory operation of the tube as a telephone relay. Since, in such case, the current is carried almost entirely by electrons, he independently derived and applied the 3/2-power equation (2).

The insertion of a third electrode to control the current between cathode and anode is due to DeForest.[1] DeForest later gave the auxiliary electrode the form of a grid placed between cathode and anode.[2] About the same time von Baeyer[3] used an auxiliary electrode in the form of a wire gauze to control thermionic discharge. The gauze was placed between the thermionic cathode and the anode.

The quantitative effect of the auxiliary electrode was first given by the present writer.[4]

To get an idea of the effect of the auxiliary electrode consider the circuit shown in Fig. 5. F denotes the cathode, P the anode, and G the auxiliary electrode which is in the form of a grid between F and P. Let the potential of F be zero, and that of P be maintained positive by the battery E, and let E_c for the present be zero. Now, although there is no potential difference between F and G, the electric field between F and G is not zero, but has a finite value which depends upon the potential of P. This is due to the fact that the potential of P causes a stray field to act through the openings of the grid. If the potential of P be E_B the field at a point near F is equal to the field which would be sustained at that point if a potential difference equal to γE_B were applied directly between F and an imaginary plane coincident with the plane of G, where γ is a constant which depends on the mesh and position of the grid. If the grid is of very fine mesh γ is nearly zero, and if the grid be removed—that is, if we have the case of a simple valve—γ is equal to unity.

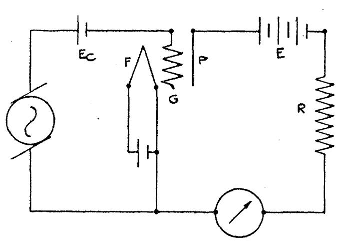

Fig. 5.

These results can be expressed by the following equation:

$$E_s = \gamma E_B + \epsilon. \tag{5}$$

Here ϵ is a small quantity which depends upon a number of factors, such as the contact potential difference between cathode and grid and the power developed in the filament, which is the usual form of cathode used. It is generally of the order of a volt and can be neglected when it is small compared with γE_B. Obviously the current between anode and cathode depends on the value of E_s.

[1] DeForest. U. S. Patent No. 841,387, 1907.

[2] DeForest, U. S. Patent No. 879,532, 1908.

[3] von Baeyer, Verh. d. D. Phys. Ges., 7, 109, 1908.

[4] H. J. van der Bijl, Verh. d. D. Phys. Ges., May, 1913, p. 338. In these experiments which were also performed under such conditions that the current was carried almost entirely by electrons, the source of electrons was a zinc plate subjected to the action of ultra-violet rays. It is obvious that the action of the auxiliary electrode is independent of the nature of the electron source. Hence the results then found apply also to the present case.

Now, suppose a potential E_c be applied directly to the grid G, the cathode F remaining at zero potential. The current is now a function of both E_s and E_c:

$$I = \Phi(E_s, E_c). \tag{6}$$

Before determining the form of this function, let us consider in a general way how the current is affected by E_s and E_c. We have seen that E_s is due to the voltage E_B between anode and cathode, and is less than E_B if the grid is between anode and cathode, since in this case γ is always less than unity. Under the influence of E_s the electrons are drawn through the openings of the grid and are thrown on to the anode by the strong field existing between grid and anode. The effect of E_c on the motion of the electrons between F and G is similar to that of E_s. Whether or not electrons will be drawn away from the cathode depends on the resultant value of E_s and E_c. If $E_s + E_c$ is positive, electrons will flow away from the cathode, and if $E_s + E_c$ is zero or negative, all the emitted electrons will be returned to the cathode, and the current through the tube will be reduced to zero. Now, $E_s + E_c$ will be positive (1) when E_c is positive (E_s is always positive), and (2) if E_c is negative and less than E_s.

1. When E_c is positive some of the electrons moving toward the grid are drawn to the grid, while the rest are drawn through the openings of the grid to the anode under the influence of E_s. The relative number of electrons going through and to the grid depends upon the mesh of the grid, diameter of the grid wire, and the relative values of E_s and E_c. When, for example, E_s is large compared with E_c, the number of electrons going to the grid is comparatively small, but for any fixed value of E_s the current to the grid increases rapidly with increase in E_c. Hence for positive values of E_c current will be established in the circuit FGE_c, Fig. 5.

2. If, however, E_c is negative and less than E_s, as was the case in the above named experiments of the writer, nearly all the electrons drawn away from the filament pass to the plate, practically none going to the grid. In this case the resistance of the circuit FGE_c is infinite.

If, now, an alternating E.M.F. be impressed upon the grid so that the grid becomes alternately positive and negative with respect to the cathode, the resistance of the circuit FGE_c, which may be referred to as the input circuit, will be infinite for the negative half cycle and finite and variable for the positive half cycle. If, on the other hand, the alternating E.M.F. be superimposed upon the negative value, E_c, the values of these voltages being so chosen that the resultant potential of the grid is always negative with respect to the cathode, the impedance of the input circuit is always infinite.

Broadly speaking, the operation of the thermionic amplifier is as follows: The current to the anode we have seen is a function of E_s and E_c, or keeping the potential E_B of the anode constant, the current for any particular structure of the device is a function only of the potential on the grid. Hence, if the oscillations to be repeated are impressed upon the input circuit, variations in potential difference are set up between cathode and grid, and these cause variations in the current in the circuit *FPR*, the power developed in the load *R* being greater than that fed into the input circuit. It is seen then that the device functions broadly as a relay in that variations in one circuit set up amplified variations in another circuit unilaterally coupled with the former.

V. The Current-voltage Characteristic of the Thermionic Amplifier.

It is of importance in deriving the equations for the power amplification, etc., to obtain the correct relation between the current in the output circuit of the amplifier and the voltages applied to the grid and anode. The current is, as we have seen, a function of both E_s and E_c, where E_s depends upon the voltage between cathode and anode, the structure of the grid, and the distance between the grid and anode.

In the amplifier under consideration, the cathode is not an equipotential surface, but consists of a filament which is heated by passing an electric current through it. A theoretical deduction of the relation between the current to the anode and the applied voltages between filament and grid and filament and anode is difficult and leads to very complicated expressions. I have therefore found it more practical to determine the characteristic empirically, and found as the result of a large number of careful experiments that the characteristic can be represented with sufficient accuracy by the following equation:

$$I = \alpha(E_s + E_c)^2, \tag{7}$$

where α is a constant depending on the structure of the device.[1]

With the help of equation (5) this becomes:

$$I = \alpha(\gamma E_B + E_c + \epsilon)^2. \tag{8}$$

This gives the current to the anode as a function of the potentials on the anode and grid, the potential of the filament being zero. If a num-

[1] This equation is sufficiently accurate when the device is used as an amplifier. When the device is operated as a detector a more accurate equation, which I hope to publish in the future, must be used. This is necessary since the detecting action of the device is a function of the second derivative of the characteristic.

ber of voltages be impressed upon the grid and anode, we have generally

$$I = \alpha(\gamma\Sigma E_B + \Sigma E_c + \epsilon)^2. \tag{9}$$

If, for example, an alternating E.M.F., $e \sin pt$ be superimposed upon the grid-voltage, E_c, the equation becomes:

$$I = \alpha(\gamma E_B + E_c + e \sin pt + \epsilon)^2. \tag{10}$$

It must be understood that equation (8) gives the direct current characteristic of the device itself; that is, E_B in equation (8) is the voltage directly between the filament and the anode P (Fig. 5). If the resistance R be zero, E_B is always equal to E, the voltage of the battery in the circuit $EPRE$, which is constant. If R be not zero, the potential difference established between the ends of R by the current flowing in it makes E_B a function of the current. The effect of the resistance R on the characteristic will be explained later. For the present we shall confine ourselves to a discussion of the characteristic of the amplifier itself. This characteristic can always be obtained experimentally by making R equal to zero and using an ammeter in the circuit $FPER$ (Fig. 5), the resistance of which is small compared with the internal output resistance of the amplifier itself.

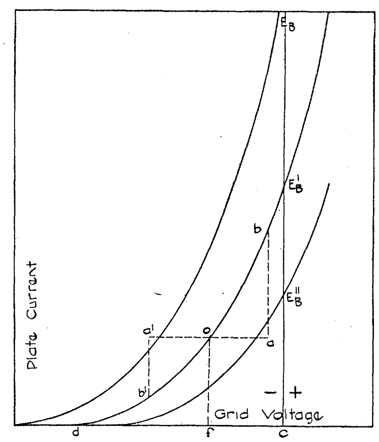

Fig. 6.

A graphical representation of equation (8) is given in Fig. 6. The curves give the current to the anode as a function of the grid voltage E_c for different values of the perameter, E_B.

Referring to equation (8) and Fig. 6, we see that the current is finite for negative values of the grid voltage E_c, and is only reduced to zero when

$$E_c = -(\gamma E_B + \epsilon).$$

Differentiating I (equation 8), first with respect to E_B, keeping E_c constant, and then with respect to E_c, keeping E_B constant, we get:

$$\frac{\partial I}{\partial E_B} = 2\alpha\gamma(\gamma E_B + E_c + \epsilon) = Q, \tag{11}$$

$$\frac{\partial I}{\partial E_c} = 2\alpha(\gamma E_B + E_c + \epsilon) = S. \tag{12}$$

Hence

$$\frac{Q}{S} = \gamma = \text{constant}, \tag{13}$$

from which it follows that for equivalent values of E_B and E_c, a change in the anode voltage E_B produces γ times as great a change in the current to the anode as an equal change in the grid voltage E_c.

The output impedance of the tube is obtained from the admittance K which is given by

$$K = \frac{1}{2\pi}\int_0^{2\pi} \frac{\partial I}{\partial E_B}\, dt,$$

or putting in the value of $\partial I/\partial E_B$ from (11):

$$K = \frac{1}{2\pi}\int_0^{2\pi} 2\alpha\gamma(\gamma E_B + E_c + \epsilon + e \sin pt)dt.$$

It is seen that $\partial I/\partial E_B$ is not constant but depends upon the instantaneous value of the input voltage $e \sin pt$. This is also obvious since the characteristic is curved. The admittance and impedance, however, are independent of the input voltage, as is seen readily by integrating the expression for K:

$$R_0 = \frac{1}{K} = \frac{1}{2\alpha\gamma(\gamma E_B + E_c + \epsilon)}. \tag{14}$$

Comparing this with equation (11), it is seen that the impedance can readily be obtained by taking the slope of the characteristic at a point corresponding to the D.C. values E_B and E_c at which it is desired to operate the tube.

Equation (14) can be expressed in a more convenient form by multiplying numerator and denominator of (14) by $(\gamma E_B + E_c + \epsilon)$:

$$R_0 = \frac{\gamma E_B + E_c + \epsilon}{2\alpha\gamma(\gamma E_B + E_c + \epsilon)^2},$$

which, with the help of equation (8) becomes

$$R_0 = \frac{E_B + \mu_0(E_c + \epsilon)}{2I}, \tag{15}$$

where

$$\mu_0 = \frac{1}{\gamma}. \tag{16}$$

We shall see that μ_0 is the maximum voltage amplification obtainable from the device.

Comparing (14) with (11) it is seen that $R_0 = 1/Q$ and therefore from (13) and (16) the slope of the I, E_c-curve is given by

$$S = \frac{\mu_0}{R_0}. \tag{13a}$$

This constant is very important. It will be shown later that the quality of the device is determined by the value of S, that is, the slope of the curve giving the current to the plate as a function of the grid voltage.

VI. Experimental Verification of the Characteristic Equation.

In order to experimentally verify equation (8) it is necessary to know the values of the constants γ and ϵ. Both these constants can be determined by methods which do not depend on the exponent of the equation. The linear stray field relation

$$E_s = \gamma E_B + \epsilon, \tag{4}$$

which is involved in equation (8) is also independent of the exponent. The constants γ and ϵ can be determined and the relation (4) tested as follows:

Let us assume an arbitrary exponent β for equation (8)

$$I = \alpha(\gamma E_B + E_c + \epsilon)^{\beta}, \tag{8a}$$

Taking the general case in which both E_B and E_c are variable, we have:

$$\frac{dI}{dE_c} = \frac{\partial I}{\partial E_B}\frac{dE_B}{dE_c} + \frac{\partial I}{\partial E_c}.$$

Now

$$\frac{\partial I}{\partial E_B} = \alpha\beta\gamma(\gamma E_B + E_c + \epsilon)^{\beta-1},$$

$$\frac{\partial I}{\partial E_c} = \alpha\beta(\gamma E_B + E_c + \epsilon)^{\beta-1}.$$

Hence

$$\frac{dI}{dE_c} = \alpha\beta(\gamma E_B + E_c + \epsilon)^{\beta-1}\left(\gamma\frac{dE_B}{dE_c} + 1\right).$$

Now, let I be constant, then either

$$\gamma E_B + E_c + \epsilon = 0,$$

i. e.,

$$-E_c = \gamma E_B + \epsilon = E_s, \tag{18}$$

or

$$\frac{dE_B}{dE_c} = -\frac{1}{\gamma}.$$

Integrating and putting $1/\gamma$ equal to μ_0, we get

$$E_B' = E_B + \mu_0 E_c. \tag{19}$$

Equations (18) and (19) are therefore independent of the exponent of (8). Equation (18) gives the case in which the current has the constant value zero. It shows that E_s in equation (4) is simply the absolute value of the grid potential E_c, which suffices to reduce the current to the anode to zero when the anode has a potential E_B. (The potentials are referred to that of the filament which is always grounded.) Referring to Fig. (6), we see that equation (18) gives the relation between the intercepts of the curves on the axis of the grid potential E_c and the corresponding values of the anode potentials E_B.[1]

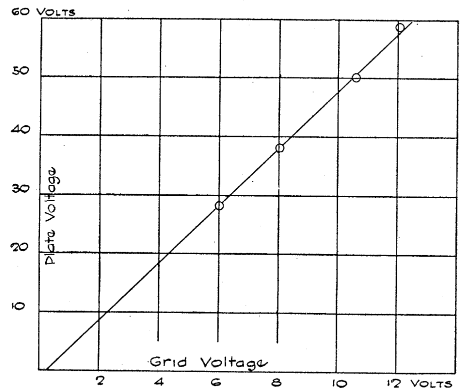

Fig. 7.

In Fig. 7 are given some results obtained with a thermionic amplifier. The corresponding observations are given in Section A of Table I.

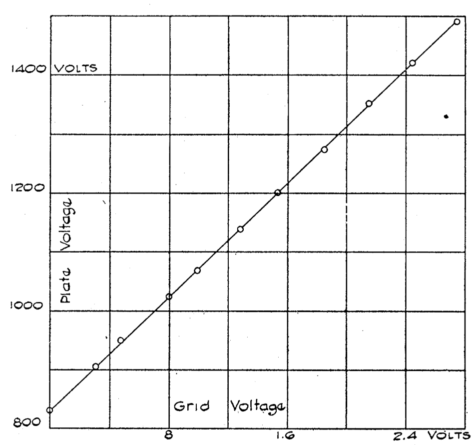

Fig. 8.

The factor μ_0, which plays a very important part in the theory of operation of the thermionic amplifier, can be more easily and accurately

[1] This is the method which I used several years ago to test the linear stray field relation (4). The accuracy with which this relation is obeyed is seen from Figs. 3 and 5 of my above mentioned publication.

determined with the help of equation (19), which gives the relation between the anode and grid potentials necessary to maintain the current at some convenient constant value. In Sections B and C of Table I. are given results obtained by applying various potentials, E_B, to the anode, and each time adjusting the grid potential, E_c, so as to keep the current to the anode constant. The results give a perfectly linear relation between E_B and E_c, as is shown by Fig. 8. The values of μ_0 given in the first column of Table I. were obtained from the slope of the E_B, E_c-curve, in accordance with equation (19).

Table I.

	E_B (Volts).	E_c (Volts).
A. Oct. 20, 1913	28.2	− 6.0
$I = 0$,	38.2	− 8.0
$\mu_0 = 5$.	50.2	−10.6
	59.2	−12.6
B. May 9, 1914	59.5	+ 3
$I = 4.10^{-3}$ amp.,	69.5	+ 1
$\mu_0 = 5.1$.	81.0	− 1.2
	90.0	− 3.0
	100.0	− 5.0
C. Nov. 21, 1914.	830	0.0
$I = 60.10^{-6}$ amp.,	905	0.31
$\mu_0 = 241$.	950	− 0.48
	1,025	− 0.80
	1,070	− 0.98
	1,140	− 1.28
	1,200	− 1.54
	1,275	− 1.85
	1,350	− 2.15
	1,420	− 2.45
	1,491	− 2.75

Another method of determining μ_0 is with the help of equation (13):

$$\frac{Q}{S} = \gamma = \frac{1}{\mu_0}. \tag{13}$$

S is the slope of the curve which gives the current to the anode as a function of the grid potential, and Q is the slope of the curve which gives the current as a function of the potential of the anode. Since both of these slopes depend upon the anode and grid potentials E_B and E_c, they must be measured for the same values of E_B and E_c. This method gives quite reliable values of μ_0, but is not as convenient as the one explained above.

The fundamental equation (8) was now tested as follows:

A convenient negative potential was applied to the grid and kept constant, while the current to the anode was observed for different anode potentials, the potential of the filament remaining zero. The grid being negative with respect to the filament, no current could be established in the filament-grid circuit. The current to the anode was observed by means of an ammeter placed in the circuit *FPE*, Fig. 5. There was no resistance in the circuit except that of the ammeter, and this was small compared with the internal output resistance of the amplifier, so that the voltage of the battery E was always equal to E_B, the voltage between the filament and anode. The observed relative values of current and voltage therefore gave the true characteristic of the amplifier itself. From the curve giving the current as a function of the anode potential, the value of the anode potential could be determined, for which the current was reduced to zero. This potential, of course, depends upon that of the grid. Hence, putting I equal to zero in equation (8), we get

$$\gamma E_B + E_c + \epsilon = 0$$

or since

$$\gamma = \frac{1}{\mu_0},$$

$$\epsilon = -\left(\frac{E_B}{\mu_0} + E_c\right).$$

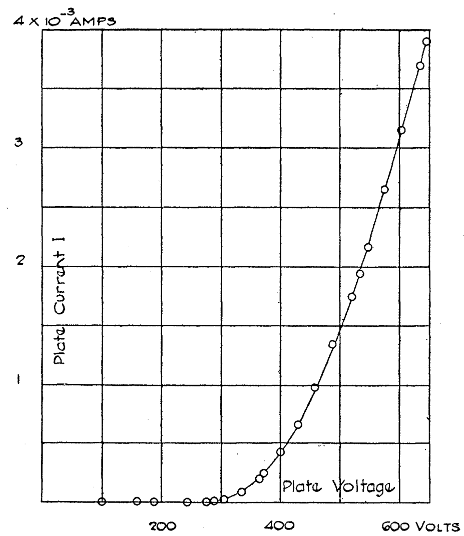

Fig. 9.

Now, μ_0 could be measured independently by the method explained above; hence ϵ could be determined.

Fig. 9 shows a typical characteristic of the thermionic amplifier. In this case, the grid was kept 5 volts negative with respect to the filament. With the known values of μ_0 and ϵ, the current I could be plotted against the expression

$$\left(\frac{E_B}{\mu_0} + E_c + \epsilon\right)^2 \tag{20}$$

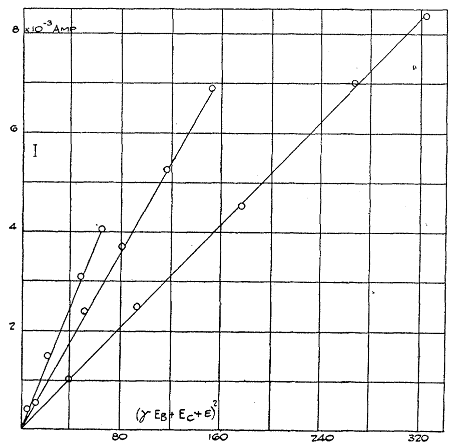

Fig. 10.

for arbitrary values of E_B when E_c is kept constant, or for arbitrary values of E_c when E_B is kept constant.

In Table II. are given the results of a few differently constructed amplifiers.

Table II.

	E_B (Volts).	I (Amp.).	$\left(\frac{E_B}{\mu_0} + E_c + \epsilon\right)^2$.
$E_c = -5$ volts,	400	$0.44 \cdot 10^{-3}$	5.3
$\mu_0 = 43$,	500	1.5 "	21.2
$\epsilon = -2$.	600	3.1 "	47.6
	650	4.05 "	65.6
$E_c = -20$ volts,	140	$1.05 \cdot 10^{-3}$	39.7
$\mu_0 = 6$,	160	2.5 "	94.0
$\epsilon = 3$.	180	4.65 "	177
	200	7.05 "	266
	210	8.4 "	324
$E_c = -12$ volts,	160	$0.56 \cdot 10^{-3}$	12.2
$\mu_0 = 11$,	200	2.4 "	51.8
$\epsilon = 1$.	220	3.7 "	81
	240	5.25 "	117
	260	6.9 "	151

Fig. 10 which gives the current I as a function of the expression (20) shows the accuracy with which equation (8) is obeyed. The agreement was as good in the case of a tube which was specially constructed to test the parabolic relation between current and voltage up to higher voltages. This tube had a short filament, about 2.5 cm. long, and a circular plate as anode 2.5 cm. in diameter, and placed at a distance of 2 cm. from the filament. The auxiliary electrode consisted of a fine wire gauze placed between filament and anode at a distance of 6 mm. from the filament. The grid had a constant negative potential equal to ϵ. Hence, in Fig. 11

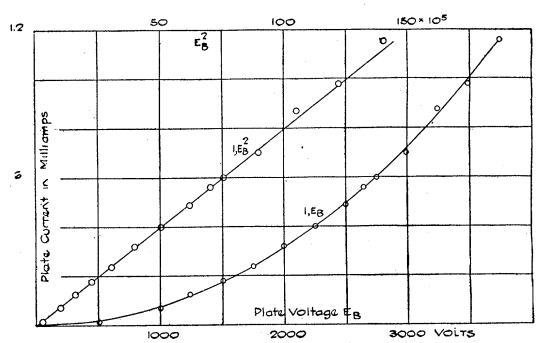

Fig. 11.

which shows the results obtained with this tube, the current was plotted simply as a function of the anode potential E_B. The straight line gives the current as a function of $E_B{}^2$. It is seen that the parabolic relation is obeyed.

These curves were of course obtained after taking every precaution to evacuate the tubes to such an extent that under the conditions of operation there was no appreciable ionization by collision of the residual gas. For this purpose it is necessary to make use of the well-known methods of denuding the metallic parts in the tube and the glass walls of the tube itself by heating during evacuation.

VII. Characteristic of Circuit Containing Thermionic Amplifier and Ohmic Resistance in Series.

In discussing the behavior of the thermionic amplifier in an alternating current circuit, we shall make two assumptions:

First. The alternating current established in the circuit $FPER$ (Fig. 5) is a linear function of the voltage impressed upon the input circuit FGE_c. This implies that the power amplification is independent of the input. This is the condition for an ideal amplifier.

Second. The thermionic amplifier shows no reactance effect. This implies that if the amplifier be inserted in a non-inductive circuit, the power amplification produced is independent of the frequency.

No proof is needed to establish the validity of the second assumption. The first is, however, not true except under certain conditions, and it remains to determine these conditions and operate the amplifier so that they are satisfied. Let the external resistance of the output circuit *FPE* (Fig. 5) be zero, and the resistance of the current-measuring device negligibly small compared with the internal output resistance of the amplifier itself. Under these circumstances E_B, which denotes the potential difference between filament and anode, is independent of the current in the circuit *FPE*, and always equal to the voltage E of the battery in the circuit. Hence, if the current be plotted as a function of E_c, the potential difference between filament and grid, the parabola given by equation (8) and Fig. 6 is obtained. If, now, the potential of the grid be varied about the value E_c equal to *cf* (Fig. 6), it is obvious from the curve that the increase, *ab*, in current to the anode due to a decrease, *oa*, in the negative potential of the grid is greater than the decrease $a'b'$ in current caused by an equal increase, oa', in the negative grid potential. In this case, the output current consists of the following parts: Let the alternating input voltage superimposed upon E_c be $e \sin pt$; then

$$I_0 = \alpha(\gamma E_B + E_c + \epsilon + e \sin pt)^2. \tag{10}$$

Expanding this we get

$$I_0 = \alpha(\gamma E_B + E_c + \epsilon)^2 + 2\alpha(\gamma E_B + E_c + \epsilon)\, e \sin pt + \frac{\alpha e^2}{2} \cos(2\,pt + \pi) + \frac{\alpha e^2}{2}. \tag{21}$$

The first term represents the steady direct current maintained by the constant voltages E_B and E_c when the input voltage e is zero (equation 8). The second term gives the alternating output current oscillating about the value of direct current given by (8). It is in phase with and has the same frequency as the input voltage. When using the device as an amplifier, this is the only useful current we need to consider. The first harmonic represented by the third term is present, as was to be expected in virtue of the parabolic characteristic. The last term, which is proportional to the square of the input voltage, represents the change in the D.C. component due to the alternating input voltage, and is the only effective current when using the device as a wireless wave detector. If a direct current meter were inserted in the output circuit, it would show a current which is greater than that given by equation (8) by an amount equal to $\alpha e^2/2$, the last term of equation (21). This is the state of matters when the device works into a negligibly small resistance.

If, on the other hand, the output circuit contains an appreciable resistance,[1] R, the voltage E_B between filament and plate is not constant, but is a function of the current, and an increase in the current due to an increase in the grid potential sets up a potential drop in the resistance R, with the result that E_B decreases, since the battery voltage E is constant. E_B is now given by

$$E_B = E - RI. \tag{22}$$

In order to obtain the characteristic of the circuit containing the tube and a resistance R, let us substitute (22) in (10):

$$I_0 = \alpha[\gamma(E - RI) + E_c + \epsilon + e \sin pt]^2$$

and put $\gamma E + E_c + \epsilon = V$. This gives

$$I_0 = \frac{1 + 2\alpha\gamma R(V + e \sin pt) - \sqrt{1 + 4\alpha\gamma R(V + e \sin pt)}}{2\alpha\gamma^2 R^2}. \tag{23}$$

This expression can be expanded into a Fourier series:

$$I_0 = \left\{ \begin{aligned} & 1 + 2\alpha\gamma RV - \sqrt{1 + 4\alpha\gamma RV} \\ & \quad + \left(1 - \frac{1}{\sqrt{1 + 4\alpha\gamma RV}}\right)\frac{e}{\gamma R}\sin pt \\ & \quad + \frac{(-1)^{n+1}2^n \underline{|(2n-1)}\alpha^n\gamma^{n-1}R^{n-1}e^{n+1}\sin^{n+1} pt}{\underline{|n+1}(1 + 4\alpha\gamma RV)^{2n+1/2}}. \end{aligned} \right. \tag{24}$$

From this it is seen that the rate of convergence of the series increases as R is increased. Actual computations show that when the tube is made to work into an impedance equal to or greater than that of the tube the harmonic terms become negligibly small compared with the second term of (24), which is the only useful term when using the tube as an amplifier, so that we can assume that the amplification is independent of the input voltage.[2]

[1] The insertion of a suitable resistance in the output circuit to straighten out the characteristic and so reduce distortion was, I believe, first suggested by Dr. Arnold, who also showed experimentally that distortion is almost negligible when the external resistance is equal to the impedance of the tube.

[2] In this connection I want to point out that although the parabolic relation used here represents the characteristic of the tube with sufficient accuracy when using the tube as an amplifier, and indeed with quite a good degree of accuracy, as shown by the experimental curves, yet the approximation is not close enough to accurately represent the second and higher derivatives of the characteristic, and therefore too much reliance should not be placed on the actual values of the several harmonic terms represented by the last term of equation (24). This equation is merely intended to show, as it does, in a general way how the insertion of a resistance in the output circuit of the tube tends to straighten out the characteristic.

When the tube works into a large external resistance it can show a blocking or choking effect on the current. This is seen from the following: The voltage E_B which is effective in drawing electrons through the grid to the anode is given by equation (22). If now the current be increased, not by increasing the electromotive force in the circuit *FPR* (Fig. 5), but by increasing the potential difference between filament and grid, the current I increases, while E, the electromotive force in the plate circuit, remains constant, from which it follows that E_B must decrease while E_C, the grid voltage, increases. The result of this is that more electrons that otherwise would have come through the grid to the anode are now drawn to the grid. If the input voltage becomes large enough the anode circuit *FPR* may be robbed of so many of its electrons that no further increase in current in the anode circuit results no matter how much the grid voltage is increased. Under these conditions equation (24) does not apply. Its application is limited to the conditions stated by equations (25) and (26).

Even if the series represented by the last term of equation (24) were zero, distortionless transmission can only be obtained if the input voltage is kept within certain limits.

Let the input voltage, $e \sin pt$, be superimposed upon the negative grid voltage, E_c (Fig. 6). Theoretically speaking, one condition of operation is that the grid should never become so much positive with respect to the filament that it takes appreciable current, for if this happens the current established in the grid circuit would lower the input voltage, and therefore the amplification. In actual practice the extent to which the grid can become positive before taking appreciable current depends upon the value of the plate voltage and the structure of the tube. We can therefore state that a condition for distortionless transmission is $e \leqq |E_c| + |g|$, where g is the positive voltage which the grid can acquire without taking enough current to cause distortion. Another condition is that the input voltage must not exceed the value given by df (Fig. 6); otherwise the negative peaks of the output current wave will be chopped off. Now cd is given by $\gamma E_B + \epsilon$. This is obtained by equating the current I to zero in equation (8). We therefore have the conditions

$$\left| \begin{aligned} &e \leqq |E_c| + |g|, \\ &e \leqq |\gamma E_B + \epsilon| - |E_c|, \end{aligned} \right. \tag{25}$$

or when the tube is working at full capacity—that is, when operating over the whole curve,

$$e = |E_c| + |g| = |\gamma E_B + \epsilon| - |E_c|. \tag{26}$$

VIII. Amplification Equations of the Thermionic Amplifier.

On the strength of the two assumptions discussed in the previous paragraph, namely, that the amplification is independent of the input and the frequency, it is possible to derive the equations of amplification in a very simple way. Referring to Fig. 5, let the current in the external resistance R be varied by variations produced in the grid potential, E_c. Then, as was shown in the last paragraph, E_B is also a variable depending on the current I, as shown by

(22) $$E_B = E - RI,$$

where E is the constant voltage of the battery in the output circuit $FPER$. Hence

$$I = \Phi(E_B, E_c),$$

from which

$$\frac{dI}{dE_c} = \frac{\partial I}{\partial E_B} \cdot \frac{dE_B}{dE_c} + \frac{\partial I}{\partial E_c}.$$

This gives the variation of current in R as a function of the variation in the grid voltage.

Substituting from (11) and (12),

$$\frac{dI}{dE_c} = 2\alpha(\gamma E_B + E_c + \epsilon)\left(\gamma\frac{d(E - RI)}{dE_c} + 1\right),$$

that is,

$$\frac{dI}{dE_c} = \frac{2\alpha(\gamma E_B + E_c + \epsilon)}{1 + 2\alpha\gamma R(\gamma E- + E_c + \epsilon)}.$$

Multiplying throughout by R, and putting $\gamma = 1/\mu_0$ we obtain by a simple transformation

(27) $$R\frac{dI}{dE_c} = \frac{\mu_0 R}{R + \dfrac{E_B + \mu_0(E_c + \epsilon)}{2I}}.$$

Now, $R \cdot dI$ is the voltage change set up in the resistance R, and dE_c is the change in the input voltage. Hence equation (27) gives the voltage amplification produced by the device, which we shall call μ. Furthermore, it follows from paragraph V. that the output impedance of the amplifier is given by

(15) $$R_0 = \frac{E_B + \mu_0(E_c + \epsilon)}{2I}.$$

Hence the voltage amplification μ is given by

(28) $$\mu = \frac{\mu_0 R}{R + R_0}.$$

From this equation it is seen that the voltage amplification asymptotically approaches a finite value μ_0, which is attained when the external resistance R becomes infinitely large compared with the output impedance of the amplifier.

In order to find the power amplification it is necessary to know the input impedance of the amplifier; that is, the impedance of the circuit FGE_c (Fig. 5). Now, the amplifier is operated, as was stated above, under such conditions that no current is established in the circuit FGE_c. The impedance of this circuit is therefore infinite, and the power developed in it is indeterminate.

In order to give the input circuit a definite constant resistance, Mr. Arnold suggested shunting the filament and grid with a high resistance. This can be considered as the input resistance R_i of the amplifier. The input voltage is that developed between the ends of this shunt resistance.

If, now, e and e_i represent the voltages established between the ends of the output and input resistances R and R_i respectively, the power developed in R and R_i is e^2/R and e_i^2/R_i. Hence the power amplification is

$$\eta = \frac{e^2 R_i}{e_i^2 R} = \mu^2 \frac{R_i}{R},$$

which, with the help of (28), becomes

$$\eta = \frac{\mu_0^2 R_i R}{(R + R_0)^2}. \tag{29}$$

The amplification is therefore a maximum when R is equal to R_0.

The power developed in R is

$$P = \frac{\mu_0^2 e_i^2 R}{(R + R_0)^2}, \tag{30}$$

from which it follows, as was to be expected, that the power in R is a maximum when the external output resistance R is equal to the output impedance R_0 of the tube.

It is readily seen that the current amplification is given by

$$\xi = \frac{\mu_0 R_i}{R + R_0}, \tag{31}$$

from which it follows that the current amplification asymptotically approaches zero as R is increased, the maximum current amplification being obtained when R becomes infinitely small compared with R_0.

Putting $R = R_0$ in (29) and $R = 0$ in (31) and remembering that the slope of the curve giving the relation between plate current and grid voltage is given by

$$S = \frac{\mu_0}{R_0}, \tag{13a}$$

we get for the maximum energy amplification

$$\eta' = \frac{\mu_0 R_i}{4} \cdot S; \tag{29a}$$

and for the maximum current amplification

$$\xi' = R_i \cdot S. \tag{31a}$$

These equations show the important part played by the slope S of the curve giving plate current as a function of the grid voltage. The factor S is equally important in the operation of the tube as oscillation generator and detector.

IX. Experimental Verification of Amplification Equations.

We shall now describe some experiments that were performed to test the equations derived in the previous paragraph.

In order to satisfactorily make these measurements, a special circuit had to be used. The reasons for this can be gathered from the following. In accordance with the views expressed in the previous paragraphs, the grid was kept E_c volts negative with respect to the filament by a battery C inserted in the filament-grid circuit (Fig. 12), and an alternating voltage $e \sin pt$ was superimposed upon E_c, while the anode was kept positive with respect to the filament by the battery B_1, the filament being grounded. Care was taken to keep the maximum value e of the input voltage within the limits given by equation (25).

The current in the output circuit is given by equation (24). In deriving the amplification equations in the previous paragraph, we have assumed that the last term of equation (24) is zero. If $e \sin pt$ is also zero, the current in the output circuit is not zero, but has a finite constant value, given by the first term of equation (24). For finite values of $e \sin pt$, the resulting alternating current established in the output circuit which is to be measured can not be separated in the usual way from this direct current, with the help of appropriate inductances and capacities, since then the amplification would be largely determined by the constants of the circuit. On the other hand, it is not possible to simply make the amplifier work into a straight non-inductive resistance alone, since the direct current that would flow through the galvanometer (which must necessarily be in series with such resistance) is in most cases large compared with the output alternating current, due to the input, so that a galvanometer which would be capable of carrying the direct current would not be sensitive enough to measure the output alternating current with any degree of accuracy.

To overcome this, the circuit shown in Fig. 12 was used. The output circuit was closed through the non-inductive resistances R and R'. These were straight wires stretched upon a board. Parallel to R' was shunted a sensitive A.C. galvanometer, G_2, and a balancing battery B_2. This battery was so adjusted that when the input was zero, that is, when

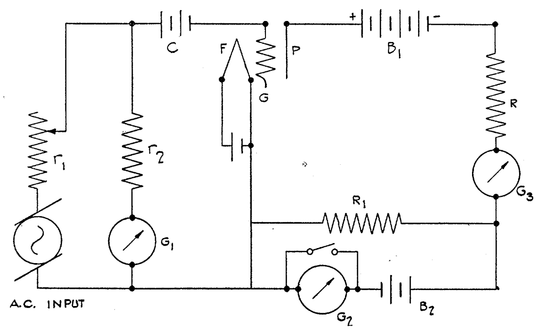

Fig. 12.

only the direct current was established in the circuit $FPRR'$, the current in the galvanometer G_2 was zero. The resistance R' was large compared with that of the galvanometer, G_2; hence when the input was on practically all the output alternating current went through the galvanometer G_2 and could then be easily measured. It is obvious that the effective external output resistance in this case was given by R. The input was varied with the help of the resistances r_1 and r_2, and measured with the galvanometer G_1. The whole system was carefully shielded, and care was taken to avoid any effects due to mutual and shunt capacity of the leads and resistances.

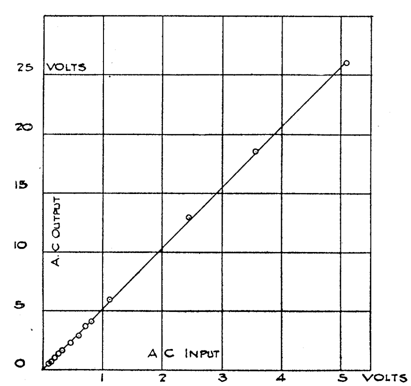

Fig. 13.

With these precautions, the amplification was found to be independent of the input and the frequency, which justifies the assumptions made in deriving the amplification equations. The voltage was varied from a few hundredths of a volt to several volts, while the frequency ranged from 200 to 350,000 cycles per second.

Fig. 13 gives the output voltage (that is, the voltage between the ends of the external resistance R) as a function of the input voltage, the frequency being 1,000 cycles per second. The linear relation between the two shows that the voltage amplification is independent of the input voltage; hence also the power amplification must be independent of the input power.

In the following table is given the voltage amplification μ obtained for various frequencies.

μ.	Frequency.
4.98	2,500
4.95	2,000
5.00	1,500
5.03	1,000
4.94	500
5.15	200

The mean of these values for μ is 5.0. The value of μ_0 for this amplifier was found by equation (19) to be 10.2. Under the conditions of the experiment, the internal output impedance R_0 of the amplifier as determined by the direct current methods explained in paragraph V. was 14,800 ohms. The external resistance R was 15,000 ohms. Hence with these values of R_0 and R, the voltage amplification μ as calculated from equation (28) is 5.1, which is in good agreement with the directly measured value, 5.0.

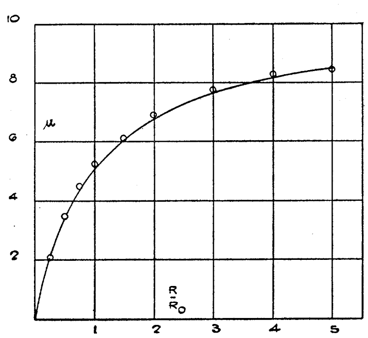

Fig. 14.
$\mu_0 = 10.2$, input = 3.55 volts.

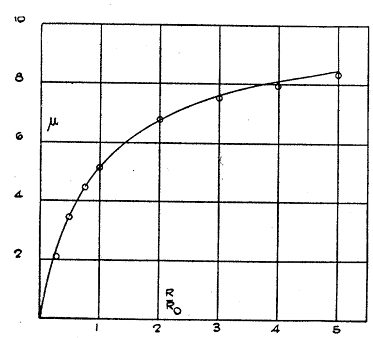

Fig. 15.
$\mu_0 = 10.2$, input = 0.45 volt.

As a further test of equation (28), the voltage amplification μ was determined for various values of the external output resistance R (Fig. 12), and the accuracy with which equation (28) holds can be seen from Figs. 14 and 15, which give the results for two different values of the input voltage (0.45 and 3.55 volts). The circles show the observed values, while the curves were calculated from equation (28). The external resistance R is plotted in terms of the internal output impedance R_0 of the amplifier. Now R_0 depends on the direct current I, that is, upon the voltage E_B between filament and anode (equation 15), and this voltage depends for constant values of E, the voltage of the battery B_1 (Fig. 12), upon the external resistance R. Hence for the different values of R in the experiment, the voltage E of the battery B_1 was always so adjusted as to keep the internal output impedance R_0 constant.

In Fig. 16 is given the A.C. power i_0^2R developed in the external resistance R as a function of R. Here, too, the resistance of the amplifier was kept constant throughout the experiment. The values of the power given here were determined by measuring the output alternating current, that is, i_0 given by the galvanometer G_2. The power developed in the external resistance R should be a maximum when R is equal to the impedance R_0 of the amplifier. From Fig. 16 it is seen that the power

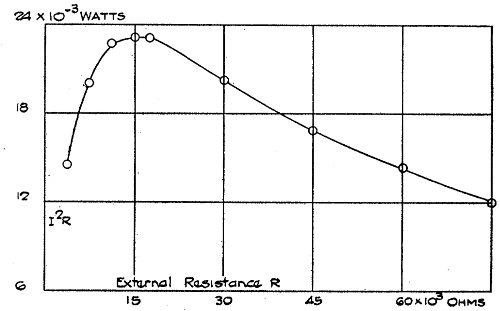

Fig. 16.

is a maximum for $R = 15{,}000$ ohms. The impedance R_0 of the amplifier given by the direct current method was found to be 14,800. Furthermore, from Fig. 16 the maximum power is 23.10^{-3} watt. Now in this experiment we had

e_i equal to 3.55 volts,

R equal to 15,000 ohms,

R_0 equal to 14,800 ohms,

μ_0 equal to 10.2.

Hence the power calculated from equation (30) is $22.2 \cdot 10^{-3}$ watt, which is in good agreement with the observed value.

Equation (30) does not give the maximum power that can be handled by the amplifier, but merely the power developed in R for a given value e_i of the input voltage. The maximum power is obtained when the input voltage has the value given by equation (26). In the case of high power amplifiers from which it is desirable to obtain an output power of 100 watts and more, the construction of the device is such that the grid can be made appreciably positive with respect to the filament without producing distortion, thus increasing the range of the input voltage with a consequent increase in the output power.

By proper choice of the structural parameters, tubes have been designed to give voltage or power amplification covering a wide range. A voltage amplification of several hundred fold is not difficult to obtain, while a power amplification of 3,000 fold was found possible using a plate voltage of only about 100 volts. Tubes used as wireless detectors can be designed to operate on very low voltages, a very efficient type of detector having been made to give satisfactory operation with two volts on the filament and a plate voltage of 12 volts and less.

It is evident from the foregoing that the structure of the device performs a very important function. On it depend the constants μ_0 and R_0 which appear in the amplification equations and which are involved explicitly and implicitly in the fundamental equation of the characteristic (equation 9). Proper structural design manifests many latent possibilities of this type of device, and enables us to meet the many conditions that must be complied with in order to obtain satisfactory operation in its ever increasing number of applications.

The writer wishes to express his indebtedness to Mr. E. H. Colpitts and Mr. H. D. Arnold for valuable advice and kind interest which greatly facilitated the work; and to Mr. H. W. Everitt for able assistance in carrying out the experiments.

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