$|\Delta I|=\frac{1}{2}$ Rule and the Weak Four-Fermion Interaction*

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Although the usually considered diagram for the $\Lambda \rightarrow N+\pi$ decay arising from the interaction $(\bar{p}\Lambda)(\bar{n}p)$ can explain the decay rate, branching ratio, and asymmetry parameter of Λ decay, it fails to explain (a) the approximate validity of the $|\Delta I|=\frac{1}{2}$ rule, and (b) that the leptonic decay rates of the strange particles are slower than the universal rate, while the nonleptonic modes have nearly the universal rate. Introducing the effect of renormalization at the vertices of the strongly interacting particles phenomenologically, we have estimated the contributions to Λ decay from a set of diagrams satisfying the strict $|\Delta I|=\frac{1}{2}$ rule for both local and nonlocal Fermi interactions. It is found that they are considerably more important than the usual diagram. This makes it easier to explain the approximate $|\Delta I|=\frac{1}{2}$ rule. Moreover, since these diagrams do not contribute to leptonic modes, one can understand (b) by associating the strangeness-nonconserving current with a weaker coupling constant. These important classes of diagrams lead to different restrictions on the chiralities of the currents involved in Λ decay for local and nonlocal interactions.

THE hyperon decays into leptons seem to be about ten times slower than the universal rate[1]. The rather slow rates of K-meson decays into leptons are also consistent with the above experimental results.[2] This situation may indicate that the values of the weak Fermi coupling constants do depend upon the change of strangeness. However, accepting this fact, if one assumes that *all* the strangeness-nonconserving interactions are characterized by a weaker coupling constant, then the faster rates of strangeness-nonconserving nonleptonic processes appear hard to understand. In a previous work,[3] it has been stressed that this difficulty may be solved in connection with the $|\Delta I|=\frac{1}{2}$ rule.[4] In this paper we shall demonstrate this possibility in more detail considering the Λ decay and shall also study the implication for this problem of the charged vector meson[5] which may mediate the four-fermion interactions.

For simplicity, more or less in the spirit of the Sakata model,[6] we shall introduce the primary weak interaction only among the leptons, the nucleon, and the Λ particle. The same approach may be extended to other models. If the four-fermion interaction is mediated by a charged vector meson B_α with mass m_B, then assuming negative chiral currents only, the interaction is given by

$$H_{\text{weak}}=J_\alpha B_\alpha+\text{H.c.}, \quad (1)$$

where the total current J_α in the present model has the *unique* form

$$J_\alpha=F[\bar{e}\gamma_\alpha(1+\gamma_5)\nu+\bar{\mu}\gamma_\alpha(1+\gamma_5)\omega +\bar{n}\gamma_\alpha(1+\gamma_5)p+(F'/F)\bar{\Lambda}\gamma_\alpha(1+\gamma_5)p]. \quad (2)$$

We could obtain the local Fermi interaction in the limit $m_B \rightarrow \infty$ and $F^2/m_B^2 \rightarrow G/\sqrt{2}$ where G denotes the usual Fermi-coupling constant. F' denotes the strength of the strangeness-nonconserving currents. Now if we assume the two-component theory of the neutrino with the usual lepton number conservation, it is known that the observed rate of $\mu \rightarrow e+\gamma$ decay[7] requires a very large value of m_B. However, as proposed before,[8] we could use another assignment of lepton numbers[9] consistent with experiments, where e^- is a lepton, while

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[1] F. S. Crawford *et al.*, Phys. Rev. Letters **1**, 377 (1958). P. Nordin *et al.*, Phys. Rev. Letters **1**, 380 (1958). J. Leitner *et al.*, Phys. Rev. Letters **3**, 186 (1959).

[2] S. Oneda, Nuclear Phys. **9**, 476 (1958). C. H. Albright, Phys. Rev. **114**, 1648 (1959). B. Sakita, Phys. Rev. **114**, 1650 (1959). Z. Maki, Soryushiron Kenkyu (in Japanese) **19**, 369 (1959).

[3] B. Sakita and S. Oneda, Nuclear Phys. (to be published).

[4] This possibility was also speculated by R. P. Feynman and M. Gell-Mann. See, for instance, a brief remark in Gell-Mann's article in Revs. Modern Phys. **31**, 834 (1959). See also M. Gell-Mann, *1958 Annual International Conference on High-Energy Physics at CERN*, edited by B. Ferretti (CERN Scientific Information Service, Geneva, 1958); T. Obayashi, Progr. Theoret. Phys. (Kyoto) (to be published); R. E. Marshak, Ninth Annual International Conference on High-Energy Physics, Kiev, 1959 (unpublished); and S. Oneda and Y. Tanikawa, Phys. Rev. **113**, 1354 (1959). In the last reference the nonleptonic processes have been discussed solely on the basis of an effective $(\bar{n}\Lambda)$ interaction (derived from the intermediate spin-zero boson). However, the same order of magnitude obtained for the matrix element of Fig. 1(b) in the present model as in the last reference seems to indicate the plausibility of the estimation.

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[8] S. Oneda and J. C. Pati, Phys. Rev. Letters **2**, 125 (1959).

[9] J. Schwinger, Ann. Phys. **2**, 407 (1957). K. Nishijima, Phys. Rev. **108**, 907 (1958). M. Konuma, Nuclear Phys. **5**, 504 (1958).

μ^- is an antilepton. This scheme, of course, demands a four-component theory of the massless neutrino, where the neutrinos associated with e^- and μ^+ have opposite helicities [i.e., in Eq. (2), $\omega=\nu^c$]. With this assignment the process $\mu\rightarrow e+\gamma$ is absolutely forbidden without any restriction on m_B. We shall, however, show that the analysis of Λ decay may impose a restriction on m_B.

Figure 1(a) represents the usual type of Feynman diagram for Λ decay discussed thoroughly so far. The branching ratio and the asymmetry parameter computed from this diagram are in surprisingly good agreement with experiments.[10] However, this may not be regarded as a success due to the following reasons. (a) The type of diagram Fig. 1(A) contains appreciable amounts of $|\Delta I|=\frac{3}{2}$ in addition to $|\Delta I|=\frac{1}{2}$ transitions and hence to explain especially the $K\rightarrow 2\pi$ decays, we have to expect an unreasonably large suppression of the $|\Delta I|=\frac{3}{2}$ part compared to the $|\Delta I|=\frac{1}{2}$ part by the higher order corrections. (b) Furthermore, if one estimates the black box of Fig. 1(A) from the pion decay rate, then it yields a rate for the $\Lambda\rightarrow p+\pi^-$ decay, nearly half the observed value[11] provided $F=F'$. However, since leptonic decay rates require $F<F'$, this diagram alone cannot provide a consistent explanation of both the leptonic and the nonleptonic decays. We wish, however, to demonstrate that a class of diagrams of type 1(B), satisfying the strict $|\Delta I|=\frac{1}{2}$ rule, are more important than those of 1(A) and hence may provide a natural explanation to avoid the difficulties (a) and (b).

It is also interesting to note that from the point of view of dispersion theory the lowest mass state that can connect the final $\pi-N$ system with the Λ particle is a neutron state as in Fig. 1(b). Thus, the importance of the class of diagrams 1(b) is rather obvious.

Now, for the interaction current given by (2), the contribution of Fig. 1(B) without pionic or kaonic renormalization vanishes in the limit $m_B\rightarrow\infty$. However, the inclusion of renormalization effects leads to a rather large contribution from Fig. 1(B) even in the limit $m_B\rightarrow\infty$. We shall estimate this effect approximately as follows: We write phenomenologically the renormalized vertices 1 and 2 of Fig. 1(B) as $F''\gamma_\alpha(1+A\gamma_5)$ and $F\gamma_\alpha(1+B\gamma_5)$, respectively.[12] For the β decay the renormalization effect changes[13] the bare nucleonic current into $F\bar{n}\gamma_\alpha(1+1.25\gamma_5)p$. We here assume[14] the conservation of the vector part of the nucleon current, which guarantees the absence of any renormalization for the vector part. For the axial-vector part we may replace the renormalization effect for the vertex 2 in Fig. 1(B) by putting $B=1.25$, as in β decay.

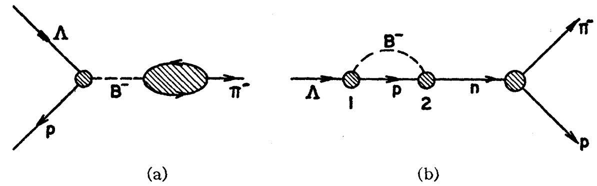

FIG. 1. Typical diagrams for the $\Lambda^0\rightarrow p+\pi^-$ decay.

The renormalization effect on the vertex 1 is not yet clear. Tentatively we assume here $F''\approx F'$ and $A\approx 1$.[15] The above procedure is similar *in spirit* to the Feynman and Speisman argument of the electromagnetic mass shift of the nucleon,[16] and likewise we introduce a Feynman cutoff $[-\lambda^2/(k^2-\lambda^2)]^2$ for the evaluation of the matrix element of Fig. 1(B), which is proportional to

$$\bar{p}(p_\pi\gamma)(\alpha+\beta\gamma_5)\Lambda, \quad (3)$$

where

$$\begin{aligned}\alpha&=(A-B)O_1-(A+B)(m_\Lambda/m_p)O_2,\\ \beta&=(1-AB)O_1+(1+AB)(m_\Lambda/m_p)O_2.\end{aligned} \quad (4)$$

p_π denotes the four-momentum of the pion and O_1 and O_2 are known functions of m_B, λ, m_Λ, and m_p. The behavior of O_1 and O_2 with respect to m_B for any reasonable choice[17] of the cutoff λ is found roughly to be as follows: O_1 and O_2 are comparable to each other for values of m_B around the nucleon's mass, the former having positive values only, the latter only negative ones. O_1 is an increasing function of m_B and reaches a finite positive limit as $m_B\rightarrow\infty$; O_2, on the other hand, decreases in magnitude with increase in m_B and reaches the zero limit from negative values as $m_B\rightarrow\infty$. This behavior of O_1 and O_2 and their absolute magnitudes are such that, with our choice of A and B, the contribution of Fig. 1(B) *always dominates* over that of Fig. 1(A) either in the case of nonlocal interaction with reasonably finite value of m_B (m_B less than $3m_p$ for example) or local interactions ($m_B\rightarrow\infty$). At present the experimental[18] sign of the asymmetry parameter of $\Lambda\rightarrow p+\pi^-$ decay demands that α and β should have the same sign. With this in mind, it is found that with our choice of the renormalization effects, $A\approx 1$, $B\approx +1.25$, which corresponds to a choice of negative bare chiral currents, there exists no way to explain the observed sign of the asymmetry parameter of Λ decay for reasonably finite values of m_B.

[10] J. J. Sakurai, Nuovo cimento **7**, 649 (1958). S. Okubo, R. E. Marshak, and E. C. G. Sudarshan, Phys. Rev. **113**, 944 (1959).

[11] S. Oneda and A. Wakasa, Nuclear Phys. **1**, 445 (1956).

[12] This, of course, leaves the importance of other induced terms at the vertices 1 and 2 in Fig. 1(B) as an open question.

[13] A. N. Sosnovskii, P. E. Spivak, Yu. A. Prokof'ev, I. E. Kutikov, and Yu. P. Dobrynin, *1958 Annual International Conference on High-Energy Physics at CERN*, edited by B. Ferretti (CERN Scientific Information Service, Geneva, 1958).

[14] For Sakata model this is automatically satisfied. See L. B. Okun, *1958 Annual International Conference on High-Energy Physics at CERN*, edited by B. Ferretti (CERN Scientific Information Service, Geneva, 1958). C. Iso, Progr. Theoret. Phys. (Kyoto) **22**, 62 (1959).

[15] Our knowledge of hyperon beta decay is still poor. In the present model, the $(\bar{\Lambda}p)$ vertex can only be renormalized either through exchange of at least two pions or one kaon.

[16] R. P. Feynman and G. Speisman, Phys. Rev. **94**, 500 (1954).

[17] The choice of cutoff $\lambda\approx 1.5m_p$ for the present problem is consistent with R. P. Feynman and G. Speisman's choice to explain the proton-neutron mass difference.

[18] E. Boldt, H. S. Bridge, D. O. Caldwell, and Y. Pal, Phys. Rev. Letters **1**, 256 (1958). R. W. Birge and B. Fowler, Bull. Am. Phys. Soc. **4**, 355 (1959).

TABLE I. The ratio of the absolute squares of the matrix elements of Figs. 1(B) and 1(A).[a]

	$\lambda=m_\Lambda$		$\lambda=1.5m_p$		$\lambda=2m_p$
	$m_B=m_p$	$m_B\to\infty$	$m_B=m_p$	$m_B\to\infty$	$m_B\to\infty$
$\left\|\frac{M[\text{Fig. 1(B)}]}{M[\text{Fig. 1(A)}]}\right\|^2$	11	3.0	89	9.1	36

[a] The numbers in this table correspond to choosing either $A=+1$, $B=+1.25$ or $A=-1$, $B=-1.25$. M[Fig. 1(A)] has been estimated by comparison with the $\pi\to\mu+\nu$ decay rate.

However, for sufficiently large values of m_B, which yield the local Fermi interactions in the limit $m_B\to\infty$, it is found that only with the above choice of A and B, one can obtain the right sign and magnitude of the asymmetry parameter for Fig. 1(B) with negative chiral currents.[19] The ratio of the absolute squares of the matrix elements for Figs. 1(B) and 1(A) with $m_B=m_p$ and ∞ are given in Table I for a few values of λ.

The table shows that the contribution of Fig. 1(B) satisfying the strict $|\Delta I|=\frac{1}{2}$ rule dominates over Fig. 1(A), which contains appreciable amounts of $|\Delta I|=\frac{1}{2}$ and $\frac{3}{2}$ transitions. This would certainly make it easier[20] to explain the approximate validity of the $|\Delta I|=\frac{1}{2}$ rule[21] for other nonleptonic processes. Moreover, since Fig. 1(A) yields a decay rate about half the observed value for $F=F'$; we could explain the observed faster rates of nonleptonic as well as the rather slow rates of leptonic decay modes of the strange particles by assuming, for example, $F'^2\approx F^2/10$.

The above discussion also reveals that, if we stick to the negative bare chiral currents only, there is no way to explain the sign and magnitude of the asymmetry parameter of Λ decay unless we take the B meson as extremely heavy. This in a certain sense may be taken as an evidence against the existence of the charged vector meson in the present model and is in parallel with the conclusion drawn from the analysis of $\mu\to e+\gamma$ decay in the framework of the two-component theory of the neutrino.

However, if we insist on the idea of the intermediate charged vector meson, then the following possibility may be added provided we abandon the universal $V-A$ form of weak interactions. In the framework of nonlocal interactions, it is possible to explain the desired asymmetry parameter of $\Lambda\to p+\pi^-$ decay and the importance of Fig. 1(B) compared to Fig. 1(A), if either (i) both $(\bar{p}\Lambda)$ and $(\bar{n}p)$ currents entering into the Λ-decay matrix element have positive chirality of the form $\bar{A}\gamma_\alpha(1-\gamma_5)B$ or (ii) the former has negative chirality, whereas the latter has positive chirality.[22] This might suggest the existence of two charged vector mesons, one mediating the strangeness-conserving processes (B) and the other the strangeness-nonconserving ones (B'). The latter must be associated with a weaker coupling constant and should couple the $(\bar{\Lambda}p)$ vertex to $(\bar{n}p)$, $(e^-\nu)$, and $(\mu^-\omega)$ vertices. It may be noted that although in this scheme $(\bar{n}p)$ and $(\bar{\Lambda}p)$ should couple with B' as given by either (i) or (ii), $(\mu^-\omega)$ must couple with B' in the same way as with B (i.e., in the negative chiral form) due to the observed similarity of asymmetry parameters in $K\to\mu\to e$ and $\pi\to\mu\to e$ chains and helicity measurements in the latter. It may therefore be suggested that one couple $(e^-\nu)$ to B' also in negative chiral form. However, we shall not go into details in this paper.

As regards the decays of the K^+ and $K_{1,2}{}^0$ mesons into their various modes, it is *a priori* expected that the inclusion of Fig. 1(B) for Λ decay will lead to qualitative explanation of at least the various features of the $|\Delta I|=\frac{1}{2}$ rule in K-meson decays. It is, however, rather striking to find that the inclusion of Fig. 1(B) provides also a quantitative explanation of the relative rates of *all* the observed modes of K^+ and $K_{1,2}{}^0$ decays in reasonably good agreement with experiments, provided one assumes consistently that the matrix element is damped by a factor $\sim\sqrt{3}$ whenever a pion is emitted from a closed baryon-antibaryon loop. This work will be discussed in detail in a forthcoming paper.[23]

In conclusion we would like to stress that in either case of local ($m_B\to\infty$) or nonlocal (m_B finite) four-fermion interactions, the inclusion of Fig. 1(B) makes it easier to explain the approximate validity of the $|\Delta I|=\frac{1}{2}$ rule along with the slower rates of strangeness-nonconserving leptonic processes and the faster rates of the strangeness-nonconserving nonleptonic processes.

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[19] The choice of A for $B=+1.25$ is rather restricted in the local limit (i.e., $0.95<A<1.14$).

[20] Although $|M[\text{Fig. 1(B)}]/M[\text{Fig. 1(A)}]|^2\approx 10$, the branching ratio of Λ decay still deviates from the value obtained by the strict $|\Delta I|=\frac{1}{2}$ rule due to the interference effect of the matrix elements of the two diagrams. This arises since the diagram shown in Fig. 1(A) contains a very large amount of $|\Delta I|=\frac{3}{2}$ part. However, if the higher order corrections reduce the $|\Delta I|=\frac{3}{2}$ part to some extent, the approximate $|\Delta I|=\frac{1}{2}$ rule may be valid. This problem remains to be investigated. At any rate, the point is that unless we do not consider the Fig. 1(B) we must expect an unreasonably large suppression of $|\Delta I|=\frac{3}{2}$ part (especially for forbidding the $K^+\to\pi^++\pi^0$ decay).

[21] It may be noted that even if the $|\Delta I|=\frac{1}{2}$ rule owes its origin to a different model of weak interaction from what has been considered here, the consideration of Fig. 1(B) cannot be ignored for good reasons.

[22] Incidentally, this possibility works also for the local limit for which the contribution of Fig. 1(A) is almost negligible compared to that of Fig. 1(B).

[23] J. C. Pati, S. Oneda, and B. Sakita (University of Maryland, Physics Department Technical Report No. 171, submitted to Nuclear Physics).