Unfortunately however we have not yet been able to calculate $S_{mn}{}^I$ for the general case (one can do all the integrals analytically except for the time integration). We have, however, been able to carry out the calculation in the impulse limit and get the correct answer, thus providing a welcome check on the formal manipulations of Sec. III. We now briefly sketch this calculation. From (5) with (9), or directly from (7) it follows that in the impulse limit[6]

$$S_{mn}{}^I=\int_{-\infty}^{\infty} dx\, \phi_m{}^*(x)e^{iFx}\phi_n(x),$$

where ϕ_m and ϕ_n are harmonic oscillator wave functions. One can carry out the integration by use of the known result[7] for Hermite polynomials that

$$H_m(y)H_n(y)=\sum_{k=0}^{m} 2^k k!\binom{m}{k}\binom{n}{k}H_{m+n-2k}(y), \quad m\leqslant n, \quad (10)$$

and the fact that $\int_{-\infty}^{\infty} dy\, H_L(y) \exp(iyz-y^2)$ is easily evaluated by performing L integrations by parts. The sum introduced by (10) is then recognized as being proportional to an associated Laguerre polynomial and in this way one derives exactly Ludwig's result.

[6] This same integral, for obvious reasons, also occurs in the theory of the infrared catastrophe. See W. Pauli and M. Fierz, Nuovo cimento **15**, 167 (1938), Eq. (18).

[7] A. Erdelyi, *et al.*, *Higher Transcendental Functions* (McGraw-Hill Book Company, Inc., New York, 1953), Vol. 2, p. 195.

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Boson Furry Theorem

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A Furry theorem for heavy mesons and photons is given for a class of highly symmetric interactions, neglecting the Ξ-N mass difference. Because of this neglect most rules are only approximately valid, but a few depend on charge conjugation alone and are absolute.

EXTENSION of the Furry theorem[1] to heavy bosons has proceeded gradually from special to more general cases,[2–6] with considerable duplication and rediscovery along the way. We here base similar remarks on a separately described[7] scheme of seven-dimensional charge space: the conclusions are not all new, but there is some generalization of previous results, and the whole exercise shows how simple and compact is the seven-dimensional scheme for such purposes.

The "antiparticulation" operator A defined in reference 7 has the property that $A^2=1$, and

$$\begin{gathered} Ap=-(\Xi^-)^c, \quad A\Sigma^+=-(\Sigma^-)^c, \\ An=(\Xi^0)^c, \quad A\Sigma^{0+}=(\Sigma^{0-})^c, \\ A\varphi=-\varphi, \\ A\gamma=\gamma, \end{gathered} \quad (1)$$

where φ is any meson field and γ the photon. Invariance rules under A are valid only to the extent that the Ξ-N mass difference Δ can be neglected; according to the scheme of reference 7 this mass difference has an "intrinsic" basis, while there is no asymmetry in the (unrenormalized) strong boson-fermion interactions. Thus A forbiddenness may mean reducing the matrix element of a process by only about $\Delta/M\approx 20\%$; but this is sufficient to be of some practical importance, and in special cases the reduction in the matrix element could be of order $(\Delta/M)^2$. Invariance rules based on the charge conjugation operator C are of course exact

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[1] W. H. Furry, Phys. Rev. **51**, 125 (1937).

[2] Particular cases were considered by H. Fukuda and Y. Miyamoto, Progr. Theoret. Phys. (Kyoto) **4**, 389 (1949); C. B. van Wyck, Phys. Rev. **80**, 487 (1950); K. Nishijima, Progr. Theoret. Phys. (Kyoto) **6**, 614 (1951); L. Michel, Progress in Cosmic-Ray Physics (Interscience Publishers, New York, 1952).

[3] General forms for pion and nucleon systems were given by A. Pais and R. Jost, Phys. Rev. **87**, 871 (1952); L. Michel, Nuovo cimento **10**, 319 (1953); T. D. Lee and C. N. Yang, Nuovo cimento **3**, 749 (1956); with applications to nucleon-antinucleon systems by D. Amati and B. Vitale, Nuovo cimento **2**, 719 (1955); C. Goebel, Phys. Rev. **103**, 258 (1956); S. Barshay, Phys. Rev. **109**, 554 (1958).

[4] D. C. Peaslee, Nuovo cimento **6**, 1 (1957) defines an analogous operator, essentially the A of reference 7, applicable to K mesons and baryons as well as pions and nucleons.

[5] R. E. Pugh, Phys. Rev. **109**, 989 (1958), gives a Furry theorem involving pions, photons, and baryons.

[6] G. Feinberg and R. E. Behrends, Brookhaven National Laboratory Report BNL-4090, 1959 (unpublished), give analogous considerations involving K mesons.

[7] D. C. Peaslee, Phys. Rev. **117**, 873 (1960).

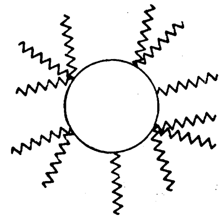

FIG. 1. Virtual baryon loop, with mesons.

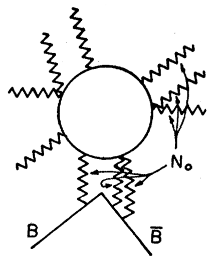

FIG. 2. Baryon-antibaryon annihilation.

(C valid); those based on the combined operator AC are only A valid, i.e., as approximate as those based on A.[8]

Consider first a virtual baryon loop attached to any number N of real meson lines, as in Fig. 1. A contribution of equal magnitude comes from the same loop with $\Psi \rightarrow A\Psi$ for the baryons. The relative sign of the two loops follows from $A\varphi = -\varphi$ and the fact that the 1-, 2-, $\cdots$ N-meson vertices[7] are invariant under A: this relative sign is thus $(-1)^N$, implying the rule

$$N = \text{even}. \tag{2}$$

Any internal boson line would have two vertices on the baryon loop and leave this result unchanged.

The restriction on Fig. 1 under C is less clear-cut and depends on the number of K_4 mesons ($CK_4 = -K_4$) involved; for $N(K_4) = \text{even}$, the mesons in Fig. 1 must be symmetric[9] in $+$ and $-$, while $N(K_3) = \text{even}$ to conserve strangeness. Then in combination with Eq. (2) we have the A rule

$$\begin{aligned} &N(K_4) = \text{even},\\ &N(K_3) = \text{even},\\ &N(K^+) = N(K^-),\\ &N(\pi^+) = N(\pi^-),\\ &N(\pi^0) = \text{even}. \end{aligned} \tag{3}$$

The other alternative, $N(K_4) = \text{odd}$, can be considered by examining in detail the forms of $\text{Tr}(\rho_1\sigma^\pm\rho_1\sigma_3\cdots)$ associated with $K^\pm K_3\cdots$ etc., according to the scheme of reference 7. One finds[10] that in fact $N(K^+ + K^- + K_3) = \text{odd}$, $N(\pi^+ + \pi^- + \pi^0) = \text{odd}$; this is incompatible with Eq. (2), which makes this case A forbidden.

Equation (3) can be applied indirectly to baryon-antibaryon annihilation as follows: for any baryon-antibaryon state compute the minimum number N_0 of bosons that can be produced; higher allowed states follow by adding even numbers of mesons according to Eq. (3). This is illustrated in Fig. 2 for $N_0 = 3$: the baryon-antibaryon line is connected by N_0 lines to the loop, which must have at least N_0 free meson lines emerging, or a total of $2N_0 = \text{even}$ for the loop. One can then add even numbers of external meson lines.

A special example is the case of $\bar{\Xi}$-N annihilation, where by the exclusion principle the initial state has

$$A = -P_L P_S P_T = (-1)^{L+S+T+1}, \tag{4}$$

where the P are exchange operators. This must equal $(-1)^N$, where N is the number of emitted mesons. Here $T = 0$ or 1 is the conventional isotopic spin of the $(\bar{\Xi}N)$ combination.

To Fig. 1 may be added n external γ-ray lines to yield the following, which are all A-rules with the single exception noted:

(i) Equation (2) is unchanged for any n;
(ii) for $n = \text{even}$, Eq. (3) is valid with symmetry of charged mesons;
(iii) for $n = \text{odd}$, Eq. (3) is valid, but with antisymmetry of charged mesons this implies a C-rule that not all charged mesons can vanish for $n = \text{odd}$, $N(K_4) = \text{even}$. (5)

These are elementary modifications of Eqs. (2) and (3) according to $A = +1$, $C = -1$ for photons. The C forbiddenness of $\pi^0 \rightarrow \text{odd } \gamma$ is contained in (5 iii); the A forbiddenness[4,11] of $\pi^0 \rightarrow \text{even } \gamma$ is in (5 ii). One can see also that (diagonal) neutral mesons have no connection with odd powers of the electromagnetic field, as π^0, $K^0 \rightarrow \pi^0$, $K^0 + \text{odd } \gamma$ is C forbidden by (5 iii) when the K^0 is identical on both sides; on the other hand π^0, $K^0 \rightarrow \pi^0$, $K^0 + \text{even } \gamma$ is not forbidden at all. Since the absence of $N(K_4) = \text{odd}$ terms is merely an A rule in the above, $K_4 \leftrightarrow K_3 + \text{odd } \gamma$ is in principle only inhibited but not forbidden, suggesting a possible interaction form

$$eA_\mu f(\Box^2)[\varphi_4\partial_\mu\varphi_3 - \varphi_3\partial_\mu\varphi_4] \tag{6a}$$

$$= ieA_\mu f(\Box^2)[\varphi^*\partial_\mu\varphi - \partial_\mu\varphi^*\varphi], \tag{6b}$$

[8] The one present experimental datum on A forbiddenness seems to show practically no reduction at all: the isotopic scalar part of the nuclear charge distribution ρ should be A forbidden relative to the isotopic vector part, while approximate equality of the two is suggested by the observation that $\rho(\text{neutron}) \approx 0$.

[9] This statement takes all meson lines in Fig. 1 as outgoing; conversion to real situations is by $\pi^\pm$ (out) $\equiv \pi^\mp$ (in), and likewise for $K^\pm$. There is no such distinction between incoming and outgoing π^0, K_3, or K_4, although there would be for K^0 and $\bar{K}^0$. By "symmetric in $+$ and $-$" is meant invariance on reversal of charge sign for all charged mesons, without any other change in the (real or charge space) meson state function. This restriction can always be restated in terms of angular momentum and isotopic spin, by virtue of the mesons' Bose statistics; such restatement is not germane to the present argument, however.

[10] For the one- and two-meson interactions of reference 7; presumably the N-meson interactions have the same property.

[11] J. Tiomno, Nuovo cimento 6, 255 (1957).

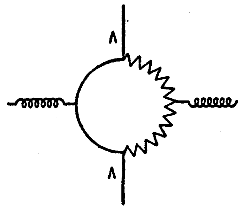

FIG. 3. Electromagnetic interaction of Λ (or Σ^0).

where $\varphi=(\varphi_3+i\varphi_4)/\sqrt{2}$ represents a K^0 meson. Electrical neutrality of the K^0 assures by Eq. (6b) that[12] $f(0)=0$ but tells nothing about higher moments.[13] If $f(\Box^2)$ does not vanish identically, the possibility exists of $\theta^2\rightarrow\theta^1$ conversion by atomic Coulomb fields; it is unlikely to be observable, however, according to the discussion in the Appendix.

The combined operator (AC) is of interest in application to Λ and Σ^0, which are eigenfunctions of (AC). Consider Fig. 3, in which the "closed loop" consists of a baryon and boson, with two external baryon lines. The relative signature of the loop under the operation AC follows by applying AC to external lines, since all vertices are invariant: $(AC)^2=+1$ for the two baryon lines, so that the signature is $(-1)^n$, where n is the number of photons. Thus an A rule is

$$n=\text{even},\tag{7}$$

which implies vanishing magnetic moments[4,5,14] (first order electromagnetic effect) for the Λ and Σ^0. The same procedure also leads[5,14] to

$$\mu(\Sigma^+)+\mu(\Sigma^-)=\mu(\Xi^-)+\mu(p)=\mu(\Xi^0)+\mu(n)=0.$$

Of course all these conclusions are only A valid.

APPENDIX

If interaction (6a) is present, $\theta^2\rightarrow\theta^1$ conversion can be induced by atomic Coulomb fields.[15] Write

$$f(\Box^2)=f_0+f_2(\Box^2/M^2)+f_4(\Box^4/M^4)+\cdots,\tag{A1}$$

where $f_0=0$ as remarked in the text, and $M\approx1.1$ Bev is the baryon mass. The matrix element of the second term in (A1) for conversion in the Coulomb field of a nucleus of charge Z is

$$\mathfrak{M}=\frac{4\pi f_2Ze^2(k+k')_\mu(K+K')_\mu}{(16\omega\omega'\Omega\Omega')^{\frac{1}{2}}M^2},\tag{A2}$$

where k_μ, k_μ' and K_μ, K_μ' represent the initial and final states of θ and recoil nucleus, respectively, and ω, ω', Ω, Ω' are their time-like components. Taking $(K+K')_4\approx2\Omega\approx2\Omega'\gg(K+K')$; we have

$$\mathfrak{M}\approx4\pi f_2Ze^2/M^2.\tag{A3}$$

The corresponding conversion cross section is

$$\begin{aligned}\sigma&=\pi a^2\\ |a|&=4f_2(Ze^2/Mc^2)(E_\theta/Mc^2)\\ &\approx5\times10^{-3}f_2ZE_\theta\text{ (Bev)},\end{aligned}\tag{A4}$$

where a is in fermis (10^{-13} cm). For a Pb target and $E_\theta\approx1$ Bev, this is $|a|\approx0.4\ f_2$; but the amplitude for nuclear conversion is presumably on the order of the nuclear radius, $|a(\text{nuclear})|\sim R\approx8$ fermi. Thus Coulomb conversion seems scarcely feasible to observe if

$$|f_2|\lesssim10.\tag{A5}$$

Although nothing is known about the value of f_2, one could hardly call it anomalous offhand if $|f_2|\sim0.1$ to 1.

[12] The authors thank Professor K. M. Watson for a helpful discussion.

[13] If $\mathfrak{L}=\partial_\mu\varphi^*\partial_\mu\varphi+\kappa^2\varphi^*\varphi$ were an adequate Lagrangian for the K^0 meson, then $f(\Box^2)=$const would be required for gauge invariance; in particular, $f(\Box^2)\equiv f(0)=0$. Because of strong interactions with baryons, however, $\mathfrak{L}$ is inadequate to give the correct K^0 propagator, and considerations of gauge invariance are less immediate: see for example N. N. Bogoliubov and D. V. Shirkov, *Introduction to the Theory of Quantized Fields* (Interscience Publishers, New York, 1959), Chap. 7. The perturbation expansion of the K^0 change-current vector on the basis of a gauge invariant baryon-photon interaction yields a form like Eq. (6).

[14] H. Katsumori, Progr. Theoret. Phys. (Kyoto) **18**, 375 (1937).

[15] One-photon conversion can involve only scalar photons because the θ has spin zero.