a range $2r_c$, where[3]

$$n=\frac{153}{\beta^2}\frac{\rho z}{A}\left[\frac{1}{E_a}-\frac{1}{E_b}\right] \tag{2}$$

one obtains $E_a=400$ electron volts. In view of the somewhat arbitrary criterion used in defining E_b in Eq. (2), the agreement with the calculated value for the lower value limit of sensitivity is quite reasonable. By comparison, Bugg has shown that the measurements of Willis *et al.*[2] in propane give a value $E_a \simeq 680$ electron volts, while the calculated value for propane is $\simeq 530$ electron volts, including kinetic energy terms.

It may be concluded that both the velocity dependence and the actual track density values for charged particles in superheated liquid hydrogen are consistent with the concept of bubble nucleation by delta rays having energies of the order of 400 electron volts. The use of track density measurement as an effective means of distinguishing particles whose velocities are appreciably different is limited only by the statistics associated with the number of bubble gaps to be counted in the tracks observed.

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Theory of Allowed and Forbidden Transitions in Muon Capture Reactions. II*

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The general formalism of the first paper in this series is applied to the calculation of the angular distribution of the recoils in muon capture. Only the unique nth forbidden transitions [spin change $0\rightarrow J$, parity change $(-)^{J+1}$] are considered. As an example the special case of C^{12} is discussed. The angular distribution of the recoils depends strongly on the strength of the induced pseudoscalar interaction, but is rather insensitive to the assumption of conserved vector current.

I. INTRODUCTION

In the first paper of this series,[1] a general formalism was developed for the treatment of muon capture reactions. The application of this formalism to capture by C^{12} to the ground state of B^{12} yielded results which differed by 9–13% in the capture rate from those previously calculated.[2] The difference arose from a detailed consideration of nuclear matrix elements involving the differential operator acting on the nuclear wave function, and of interference terms among the nuclear matrix elements. Both factors had been neglected in earlier calculations by Fujii and Primakoff.[3] The situation in C^{12} was also considered by Wolfenstein.[4]

Recently, Rose and Good[5] have calculated the angular distribution of recoils from muon capture, for the unique nth forbidden transitions [spin change $0\rightarrow J$, parity change $(-)^{J+1}$]. Since their aim was to find parity nonconservation and lepton conservation in muon capture reaction, they considered the sign of the asymmetry and its magnitude, neglecting matrix elements of the type mentioned above.

We apply the formalism of I, taking into account the matrix elements involving the differential operator acting on the nuclear wave functions, to the calculation of the angular distribution of recoils for the same case treated by Rose and Good. The numerical results again bring about 5–20% correction in the asymmetry coefficient for C^{12}. However, the results given both by Rose and Good and by us suffer from a theoretical uncertainty due to the nuclear wave function, which is

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[1] M. Morita and A. Fujii, Phys. Rev. **118**, 606 (1960). We refer to it as I, hereafter.

[2] See Sec. 10 of reference 1.

[3] A. Fujii and H. Primakoff, Nuovo cimento **12**, 237 (1959); H. Primakoff, Revs. Modern Phys. **31**, 802 (1959).

[4] L. Wolfenstein, Nuovo cimento **13**, 319 (1959).

[5] M. E. Rose and R. H. Good, Jr., Ann. Phys. **9**, 211 (1960). The authors would like to express their sincere thanks to them for sending a preprint.

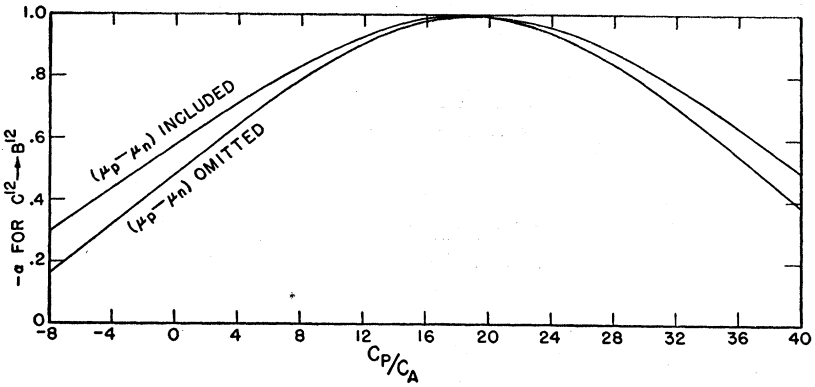

FIG. 1. Calculated asymmetry coefficient for the recoil distribution $1+\alpha P\cos\theta$ in $C^{12}\rightarrow B^{12}$ versus C_P/C_A, relative strength of the induced pseudoscalar and axial vector interactions.

estimated by Wolfenstein.[4] Since the asymmetry coefficient depends strongly on the strength of the induced pseudoscalar interaction, but is rather insensitive to the assumption of the conserved vector current, the experimental data (if obtainable) give us some information concerning the induced pseudoscalar interaction.

As in I, we use the nonrelativistic form of the Hamiltonian density. We assume the muon wave function for a point nucleus, neglecting its small component.[6] A formula for the angular distribution of the recoils is given in Sec. II. Finally, in Sec. III, we take the C^{12} and B^{12} nuclear wave functions of the j-j coupling shell model for a harmonic oscillator potential.[7] Although it is expected that both approximations for muon and nuclear wave functions bring some uncertainty, and though this uncertainty will be considerably diminished by cancellation in taking the ratio of matrix elements for the calculation of the angular distributions of recoils, they must nevertheless be kept in mind when considering the numerical results.

II. ANGULAR DISTRIBUTION OF RECOILS

The most general Hamiltonian density is given by I(1).

$$H=\bar{\psi}_n\mathcal{H}\psi_p,$$

with

$$\begin{aligned}\sqrt{2}\mathcal{H}=\gamma_\lambda&[C_V(\bar{\psi}_\nu\gamma_\lambda\psi_\mu)+C_V'(\bar{\psi}_\nu\gamma_\lambda\gamma_5\psi_\mu)]\\&+i\gamma_\lambda\gamma_5[C_A(\bar{\psi}_\nu i\gamma_\lambda\gamma_5\psi_\mu)+C_A'(\bar{\psi}_\nu i\gamma_\lambda\psi_\mu)]\\&+\gamma_5[C_P(\bar{\psi}_\nu\gamma_5\psi_\mu)-C_P'(\bar{\psi}_\nu\psi_\mu)]\\&+\sigma_{\lambda\rho}[C_Mp_\rho(\bar{\psi}_\nu i\gamma_\lambda\psi_\mu)+C_M'p_\rho(\bar{\psi}_\nu i\gamma_\lambda\gamma_5\psi_\mu)].\qquad(1)\end{aligned}$$

This is reduced to the nonrelativistic form given in I(11). We also take $C_i=C_i'$. In the case of the unique nth forbidden transition, this becomes

$$H_{fi}=\langle u_f|\mathbf{H}|u_i\rangle=\mathbf{V}\cdot\mathbf{S}+\mathbf{V}_1\cdot\mathbf{S}_1+V_2S_2,\qquad(2)$$

where

$$\begin{aligned}\sqrt{2}\mathbf{V}&=C_A\langle u_f|e^{-i\mathbf{q}\cdot\mathbf{r}}e^{-\alpha Zm_\mu' r}\boldsymbol{\sigma}|u_i\rangle,\\ \sqrt{2}\mathbf{V}_1&=(C_V/M)\langle u_f|e^{-i\mathbf{q}\cdot\mathbf{r}}e^{-\alpha Zm_\mu' r}\mathbf{p}|u_i\rangle,\\ \sqrt{2}V_2&=(C_A/M)\langle u_f|e^{-i\mathbf{q}\cdot\mathbf{r}}e^{-\alpha Zm_\mu' r}\boldsymbol{\sigma}\cdot\mathbf{p}|u_i\rangle,\\ \mathbf{S}&=2(\alpha Zm_\mu')^{\frac{3}{2}}(4\pi)^{-\frac{1}{2}}\chi_\nu^\dagger(1+\gamma_5)\\&\quad\times\bigg[\boldsymbol{\sigma}-\frac{q}{2M}\bigg(1-\frac{C_P}{C_A}\bigg)\gamma_4\hat{q}-i\frac{q}{2M}\frac{C_V}{C_A}\\&\qquad\times(1+\mu_p-\mu_n)(\hat{q}\times\boldsymbol{\sigma})\bigg]\chi_\mu,\qquad(3)\\ \mathbf{S}_1&=2(\alpha Zm_\mu')^{\frac{3}{2}}(4\pi)^{-\frac{1}{2}}\chi_\nu^\dagger(1+\gamma_5)\boldsymbol{\sigma}\chi_\mu,\\ S_2&=2(\alpha Zm_\mu')^{\frac{3}{2}}(4\pi)^{-\frac{1}{2}}\chi_\nu^\dagger(1+\gamma_5)\chi_\mu.\end{aligned}$$

χ_ν and χ_μ are the neutrino and muon spin wave functions. $\hat{q}$ is the unit vector in the direction of the neutrino momentum. The other symbols are the same as those in I. The transition rate to a state with neutrino momentum in solid angle $d\hat{q}$ is given by

$$\mathcal{W}(\theta)d\hat{q}=2\pi\sum\nolimits_M\sum\nolimits_m p_m|H_{fi}|^2q^2d\hat{q}/(2\pi)^3.\qquad(4)$$

Here M denotes the spin state of the final nucleus, m, the spin state of the muon, and p_m the probability for the muon to be in state m. If we quantize along the direction of polarization $\mathbf{P}$, we have

$$p_m\chi_\mu^m=\tfrac{1}{2}(1+\boldsymbol{\sigma}\cdot\mathbf{P})\chi_\mu^m.\qquad(5)$$

The transition rate can then be expressed as a trace[5]

$$\begin{aligned}\mathcal{W}(\theta)d\hat{q}=(32\pi^2)^{-1}q^2d\hat{q}\,\mathrm{Tr}[&(\mathcal{S}\cdot\mathbf{V}+\mathcal{S}_1\cdot\mathbf{V}_1+\mathcal{S}_2V_2)\\&\times(1+\boldsymbol{\sigma}\cdot\mathbf{P})(1+\gamma_4)(\mathcal{S}^\dagger\cdot\mathbf{V}^*+\mathcal{S}_1^\dagger\cdot\mathbf{V}_1^*+\mathcal{S}_2^\dagger V_2^*)\\&\times(1-\boldsymbol{\sigma}\cdot\hat{q})],\qquad(6)\end{aligned}$$

where $\mathcal{S}$, $\mathcal{S}_1$, etc., denote the operator part of the S's defined in (3), such that $\mathbf{S}_1=\chi_\nu^\dagger\mathcal{S}_1\chi_\mu$. The result is expressed in terms of the reduced nuclear matrix

[6] G. Flamand and K. W. Ford [Phys. Rev. **116**, 1591 (1959)] have indicated that the inclusion of the small component of the muon wave function and the finite size correction decreases the muon capture rate in C^{12} by a few percent.

[7] They are also adopted by Rose and Good. These wave functions yield a result for the beta-decay rate of $B^{12}\rightarrow C^{12}$ which differs from the experimental value by a factor of five.

elements of Table II in I, and is

$$\mathcal{W}(\theta)d\hat{q}=(2/3\pi)C_A{}^2(\alpha Zm_\mu{}')^3(2J+1)\times[A+B\mathbf{P}\cdot\hat{q}]q^2d\hat{q}, \quad (7)$$

in units of $\hbar=m_e=c=1$. The asymmetry coefficient α is defined by writing in the form

$$\mathcal{W}(\theta)d\hat{q}=\text{const}(1+\alpha P\cos\theta)d\hat{q}, \quad (8)$$

with $\alpha=B/A$. P is the degree of the muon polarization in the K orbit. θ is the angle of $\mathbf{P}\cdot\mathbf{q}$. A and B are given below.[8]

$$\left.\begin{matrix}A\\B\end{matrix}\right\}=g_2{}^2([1\ J-1\ J]^2+[1\ J+1\ J]^2)+(\pm g_1{}^2-g_2{}^2)\times\{J^{\frac{1}{2}}[1\ J-1\ J]+(J+1)^{\frac{1}{2}}[1\ J+1\ J]\}^2\times(2J+1)^{-1}-(2/M)(C_V/C_A)g_2[1\ J\ J\ p]\times\{(J+1)^{\frac{1}{2}}[1\ J-1\ J]-J^{\frac{1}{2}}[1\ J+1\ J]\}\times(2J+1)^{-\frac{1}{2}}\pm(2/M)3^{\frac{1}{2}}g_1[0\ J\ J\ p]\times\{J^{\frac{1}{2}}[1\ J-1\ J]+(J+1)^{\frac{1}{2}}[1\ J+1\ J]\}\times(2J+1)^{-\frac{1}{2}}, \quad (9)$$

with $g_1=1+[(C_A-C_P)/C_A](q/2M)$,

$$g_2=1-(1+\mu_p-\mu_n)(C_V/C_A)(q/2M).$$

Here the upper (lower) sign refers to A (B). The matrix elements, $[1\ J\pm1\ J\ \pm]$, are approximately replaced by $[1\ J\pm1\ J]$. $[0\ J\ J]^2M^{-2}$ and $[1\ J\pm1\ J]^2M^{-2}$ are kept in (9), because the formula is much simpler keeping these terms. All other terms of order $(p/2M)^2$ are neglected. (9) is equivalent to (2) of reference 5, if we set $[O\ J\ J\ p]$ and $[1\ J\ J\ p]$ to be zero.[8]

Integrating over $d\hat{q}$, we have transition rate, $\mathcal{W}$, of the muon capture for $0\rightarrow J$,

$$\mathcal{W}=\int\mathcal{W}(\theta)d\hat{q}. \quad (10)$$

[8] In reference 5, the term of order p/M arising from the vector interaction is neglected, although it is the same order of magnitude as that arising from the conserved vector current interaction, and remains when one sets $C_M=0$. Note that the symbol $(\mu_p-\mu_n)$ in reference 5 involves both the anomalous as well as Dirac magnetic moments and is equal to 4.70, while we use the same symbol for the anomalous magnetic moment, which alone appears in C_M and is 3.70. The remaining 1 corresponding to the Dirac moment arises from the vector interaction.

TABLE I. Theoretical asymmetry coefficient in $1+\alpha P\cos\theta$ for $C^{12}\rightarrow B^{12}$.

Assumptions		$-\alpha$		
C_P/C_A	$(\mu_p-\mu_n)$ terms	Present work	Rose and Good[a]	Wolfenstein[b]
0	omitted	0.48	0.40[c]	0.42±0.21
0	included	0.58	0.54	
8	omitted	0.79	0.73	0.75±0.13
8	included	0.84	0.80	0.80±0.10

[a] M. E. Rose and R. H. Good, Jr., Ann. Phys. 9, 211 (1960).
[b] L. Wolfenstein, Nuovo cimento 13, 319 (1959). There, $C_V=-0.83C_A$ is adopted.
[c] 0.43 for $\mu_p-\mu_n=3.7$ (see reference 8).

This is consistent with I(55), and

$$C_A{}^2A=3P_0, \quad (11)$$

where P_0 is given in I(58).

III. NUMERICAL RESULTS FOR $C^{12}\rightarrow B^{12}$

The reduced matrix elements of interest are [1 0 1], [1 2 1], [0 1 1 p], and [1 1 1 p]. These are evaluated in Sec. 10 of I, for j-j coupling harmonic oscillator nuclear wave functions, and are

$$[1\ 0\ 1]=-0.138, \qquad [1\ 2\ 1]=0.0048,$$
$$[0\ 1\ 1\ p]=0.0058M, \quad [1\ 1\ 1\ p]=0.0030M.$$

Here the M is the nucleon mass. We also use $q=91.4$ Mev/c and $C_A=-1.24C_V$. The calculated asymmetry coefficient is given in Fig. 1. It is also given in Table I for particular values of the induced pseudoscalar interaction, in comparison with previous calculations.[4,5] Although the relativistic correction to the results in reference 5 is about 5–20%, this would not be important, because an uncertainty of the same order of magnitude would come from the inaccuracy of the matrix elements (see Wolfenstein's value). The small component of the muon wave function, which we neglected, may also bring about a correction of order $\alpha Z\sim4\%$. As is seen, the asymmetry coefficient depends strongly on the strength of the induced pseudoscalar interaction, but is rather insensitive to the assumption of the conserved vector current.

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